A MULTINEPHRON MODEL OF THE RENAL CONCENTRATING MECHANISM

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Abstract. The renal inner medulla has a very particular shape, which has never been adequately appreciated in mathematical models (Kriz, 1983). It is characteristic for the inner medulla to taper from a broad basis to the papilla. These features were incorporated in a new multinephron model of the renal countercurrent system. The model is complex enough to allow the investigation of an interesting hypothesis presented by Kriz (1983), too. The simulations demonstrate, that this hypothesis is capable to explain the sharp rise of the NaCl concentration in the inner medulla which is observed in measurements.

Keywords. Renal countercurrent system; loops of Henle; boundary value problem; differential equations.

INTRODUCTION

The present mathematical model of the renal concentrating mechanism is a successor of the kidney models of Stephenson et al. (1974), Mejia and Stephenson (1979), Moore and Marsh (1980), Lory et al. (1983), and Horster et al. (1984). It is presented in detail in Lory (1985). So, only its main features will be described here.

THE MATHEMATICAL MODEL

The present model differs from its above mentioned predecessors in a consequent consideration of the medullary architecture. Experimental morphological results of Becker (1978) and Kriz (1967,1981,1983) show: Two thirds of the loops of Henle turn at or just before the outer-inner medullary junction ("short" loops). The other third of the loops reaches the inner medulla ("long" loops). Their number decreases exponentially in this zone.

In order to model the medullary architecture of the tubular system consistently with these results, the inner medulla is subdivided into 8 equal parts: b_1=4.5 (outer-inner medullary junction), b_2=5.25,...,b_8=9.75, b_9=10.5 (papillary tip). The number of loops of length b_j is given by n_j: n_1=1024, n_2=256, n_3=128,...,n_8=4, n_9=4. So, altogether 1536 loops are present in the model. This number is chosen according to the fact that 1536 loops eventually merge into one collecting duct.

The model is designed for application in current physiological research. It has been used for testing a recent hypothesis of Kriz (1983). Based on morphological observations and assuming salt secretion into the descending limbs of Henle's loops, Kriz has hypothesized that this active transmural NaCl transport might enhance the NaCl concentration at the papillary tip. In order to model this hypothesis consistently with Kriz's assumptions, active transport of NaCl into certain portions of the long descending limbs of Henle's loops is included in the model. These portions are $[1.5,4.5]$ for j=2,3,4 and $[1.5,b_{j-3}]$ for j=5,...,9.

The mathematical formulation of the kidney model is based on the laws of non-equilibrium thermodynamics and results in a two point boundary value problem for a system of 67 simultaneous ordinary differential equations. Its solution on a computer makes great demands on the numerical method. A new especially adapted algorithm has been developed by Lory (1985). It combines features of finite element and shooting methods.

RESULTS

The results of the simulations are summarized in the following table. It gives the computed NaCl concentrations [mmole/liter] in the central core for differing rates of active NaCl transport (Vm) into the descending limbs of Henle's loops. In all the cases the active NaCl transport out of the thick ascending limbs is Vm=17.6.

TABLE. Computed NaCl concentrations

	Vm	0.0	3.3	6.6	9.9
0.0 (cortico-med. junction)		140	140	140	140
4.5 (outer-inner med. jct.)		661	653	632	599
8.25		575	598	601	584
10.5 (papillary tip)		664	758	805	813

Vm $[10^{-6}$ mmole cm^{-2} $sec^{-1}]$

CONCLUSION

The hypothesis as advanced on a morphological basis indeed leads to an increase

of the NaCl concentration in the inner medullary central core, especially at the papillary tip. This is consistent with measurements made by Koepsell et al. (1974) using the electron microprobe.

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