MASS AND INERTIA PARAMETERS FOR NUCLEAR FISSION

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Abstract

MASS AND INERTIA PARAMETERS FOR NUCLEAR FISSION. The effective mass parameter and the moments of inertia for a deformed nucleus are evaluated using the cranking-model formalism. Special attention is paid to the dependence of these quantities on the intrinsic structure, which may arise due to shells in deformed nuclei. It is found that these inertial parameters are very much influenced by the shells present. The effective-mass parameter, which appears in an important way in the theory of spontaneous fission, fluctuates in the same manner as the shell-energy corrections. Its values at the fission barrier are up to two or three times larger than those at the equilibrium minima. This correlation comes about because for the effective mass the change in the local density of single-particle states is very important, much more so than the change in the pairing correlation. The moments of inertia which enter in the theory of angular anisotropy of fission fragments, also fluctuate as a function of the deformation. At low temperatures, the fluctuation is large and shows a distinct but more complicated correlation with the shells. At high temperatures, the moments of inertia fluctuate with a smaller amplitude about the rigid-body value in correlation with the energy-shell corrections. For the first and second barriers, the rigid-body values are essentially reached at a nuclear temperature of 0.8 to 1.0 MeV.

1. INTRODUCTION

Large-scale collective motion associated with the fission process puts it a special position among other nuclear phenomena. The shape of the nucleus changes very appreciably and to describe the related flow of the nuclear matter, one should know the dynamics of this non-stationary process.

To study the dynamics of such a process, its multi-dimensionality is very important. Consequently, the trajectory can be found only if all $\frac{1}{2}n(n+1)$ mass parameters are known for the n degrees of freedom introduced in the definition of the shape of the nucleus. It is not simple to solve the relevant dynamic equations even for the very unrealistic case of the incompressible classical liquid-drop nucleus with an irrotational flow. In real nuclei, it is even more complicated because the effective mass parameters will be greatly influenced by the intrinsic structure, particularly by the shells present in the deformed nucleus.

At lowest excitations, especially in the case of spontaneous fission, an adiabatic motion is assumed. This allows one to use a simple cranking model theory [1] for the mass parameters related to generalized deformation co-ordinates. Some results of the numerical calculations of these quantities are described here. Special emphasis is laid on providing simple physical interpretations in order to pave a way for the complete theory of

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the process in the future and to give a more solid basis for extrapolations to large and fancy distortions of the nucleus in fission or to unknown regions of nuclei. To illustrate these points, some calculations were performed for a simple case of the large ellipsoidal distortion of the Nilsson potential, a case considered recently also by Sobiczewski et al. [2]. Calculations with the Saxon-Woods type potential have been performed also but will not be presented here.

There is another closely related problem which is also relevant to other aspects of the fission process. This is the problem of the effect of intrinsic structure on the moments of inertia of a nucleus, which appear, for example, in the theory of angular distribution of the fission fragments. To study the dependence of the moment of inertia on deformation and excitation of a nucleus, we have performed calculations, also with the cranking-model formula, by using a Nilsson potential with ellipsoidal deformation. Calculations performed with the Saxon-Woods type potential, for a more general deformation of the nuclear surface, are in progress.

More detailed results will be published elsewhere. Qualitative discussion of the subject matter can also be found in Ref. [3].

2. EFFECTIVE MASS PARAMETER

In the adiabatic description of the collective behaviour of a nucleus, the nucleons are assumed to move in a uniform average non-spherical field. Vibrations and rotational motion are then described in terms of changes in this average field. We start with a Hamiltonian which may include effects of residual interaction, such as the pairing interaction. Next, the potential in which the particles move is set in motion. One obtains the increase in the energy of the system in the second-order terms in the time derivatives of the collective co-ordinates q_i:

$$K = \frac{1}{2} \sum_{i,j=1}^{n} B_{ij} \dot{q}_i \dot{q}_j \tag{1}$$

which is identified with the kinetic energy of the collective motion. The formula for the effective mass parameters is [1]

$$B_{ij} = 2\hbar^2 \sum_{m} \frac{\langle 0 | \partial/\partial q_i | m \rangle \langle m | \partial/\partial q_j | 0 \rangle}{E_m - E_0} \tag{2}$$

where $|0\rangle$ is the ground state and $|m\rangle$ is an excited state of the system.

In Eq. (2), no specific assumptions are made concerning the wave functions $|m\rangle$ of the system, provided that their dependence on the collective variables q_i is known.

The collective degrees of freedom q_i are usually introduced by means of the Lagrange multiplier method [4]. Thus, the nucleons are put into an external field which restricts the intrinsic motion in such a way that the collective variables are kept constant and have given values.

However, in practical applications when the shell model is used, one has already such a field; this is the average field of the model which is assumed to be the same for all single-particle states near the Fermi energy. Therefore, the parameters which appear in the definition of the average field, especially those which describe its shape, can be considered as collective adiabatic variables. The response of the system to slow changes of the shape can be determined directly from the cranking-model formula (2), where the wave functions are adiabatic solutions for the fixed deformed shell-model field.

In practical applications of the cranking-model formula, one usually assumes that the excited states of the even-even system are combinations of two-quasiparticle excitations $|\mu\nu\rangle$ with the energy $E_\mu + E_\nu$, where $E_\mu = \sqrt{(\epsilon_\mu - \lambda)^2 + \Delta^2}$. Here, ϵ_μ is the single-particle energy, λ and Δ are the Fermi energy and the pairing gap of the system. With this assumption, we obtain the matrix elements for the operator $\partial/\partial q_i$ as [5]

$$\left\langle \mu\nu \left| \frac{\partial}{\partial q_i} \right| 0 \right\rangle = - \frac{U_\mu V_\nu + U_\nu V_\mu}{E_\mu + E_\nu} \left(\frac{\partial H}{\partial q_i}\right)_{\mu\nu} \qquad \text{for } \mu \neq \nu \tag{3}$$

where $(\partial H/\partial q_i)_{\mu\nu}$ is the matrix element of $\partial H/\partial q_i$ between single-particle states $|\mu\rangle$ and $|v\rangle$. In the case when $E_\mu = E_\nu$ (or when one quasi-particle is the time reversed state of the other), one has non-vanishing matrix elements of $\partial/\partial q_i$ due to the variation of the occupation amplitudes U and V with respect to deformation. The result is [5]

$$\left\langle \nu\nu \left| \frac{\partial}{\partial q_i} \right| 0 \right\rangle = \frac{1}{2E_\nu^2} \left[- \Delta \left(\frac{\partial H}{\partial q_i}\right)_{\nu\nu} + \Delta \frac{\partial \lambda}{\partial q_i} + (\epsilon_\nu - \lambda) \frac{\partial \Delta}{\partial q_i} \right] \tag{4}$$

where

$$\frac{\partial \lambda}{\partial q_i} = \frac{-(ac_i + bd_i)}{a^2 + b^2}; \qquad \frac{\partial \Delta}{\partial q_i} = \frac{ad_i - bc_i}{a^2 + b^2}$$

$$a = \Delta \sum_\nu \frac{1}{E_\nu^3}, \qquad b = \sum_\nu \frac{\epsilon_\nu - \lambda}{E_\nu^3}$$

$$c_i = - \Delta \sum_\nu \frac{(\partial H/\partial q_i)_{\nu\nu}}{E_\nu^3} \qquad d_i = \sum_\nu \frac{(\partial H/\partial q_i)_{\nu\nu}(\epsilon_\nu - \lambda)}{E_\nu^3}$$

Therefore, the effective mass is given by

$$\begin{aligned} B_{ij} = 2\hbar^2 \Bigg\{ & \sum_{\mu\nu} \frac{(\partial H/\partial q_i)_{\mu\nu} (\partial H/\partial q_j)_{\nu\mu}}{(E_\mu + E_\nu)^3} (U_\mu V_\nu + U_\nu V_\mu)^2 \\ & + \frac{1}{8} \sum_\nu \frac{1}{E_\nu^5} \Bigg[\Delta^2 \frac{\partial \lambda}{\partial q_i} \frac{\partial \lambda}{\partial q_j} + (\epsilon_\nu - \lambda)^2 \frac{\partial \Delta}{\partial q_i} \frac{\partial \Delta}{\partial q_j} \\ & + \Delta(\epsilon_\nu - \lambda) \left(\frac{\partial \lambda}{\partial q_i} \frac{\partial \Delta}{\partial q_j} + \frac{\partial \lambda}{\partial q_j} \frac{\partial \Delta}{\partial q_i} \right) - \Delta^2 \left\{ \left(\frac{\partial H}{\partial q_i}\right)_{\nu\nu} \frac{\partial \lambda}{\partial q_j} + \left(\frac{\partial H}{\partial q_j}\right)_{\nu\nu} \frac{\partial \lambda}{\partial q_i} \right\} \\ & - \Delta(\epsilon_\nu - \lambda) \left\{ \left(\frac{\partial H}{\partial q_i}\right)_{\nu\nu} \frac{\partial \Delta}{\partial q_j} + \left(\frac{\partial H}{\partial q_j}\right)_{\nu\nu} \frac{\partial \Delta}{\partial q_i} \right\} \Bigg] \Bigg\} \end{aligned} \tag{5}$$

which, for the sake of convenience, can be written as

$$B_{ij} = \sum_{\mu\nu} (M_{ij})_{\mu\nu}$$

The effective mass formula (5) can be generalized to the case where the excitation of a nucleus can be described in terms of a nuclear temperature T. The resulting formula is

$$B_{ij}(T) = \frac{1}{2} \sum_{\mu\nu} (M_{ij})_{\mu\nu} \left(\tanh \frac{E_\mu}{2T} + \tanh \frac{E_\nu}{2T} \right)$$

$$+ \sum_{\mu \neq \nu} (U_\mu U_\nu - V_\nu V_\mu)^2 \frac{(\partial H/\partial q_i)_{\mu\nu} (\partial H/\partial q_j)_{\nu\mu}}{(E_\mu - E_\nu)^3} \left(\tanh \frac{E_\mu}{2T} - \tanh \frac{E_\nu}{2T} \right) \quad (6)$$

where the occupation amplitudes U and V, and E depend on the temperature indirectly through the dependence of Δ and λ on T.

It is easy to see that, for the pure independent-particle motion, the values of the mass parameters obtained with the formulas (2) or (5) should be abnormally small. Indeed, the mass parameters are directly related to the derivatives of the wave functions with respect to the deformation parameters. In the pure independent-particle model (IPM), these are known to be very small. (An exceptional case occurs when two proper levels cross.) This is different when there are residual interactions the most important of which is the pair correlation. With the pair correlations, the composition of the nuclear wave functions changes more strongly with the deformation. In this cause, the predominant contribution to the effective mass comes from the diagonal matrix elements within an energy interval of 2Δ near the Fermi energy. This corresponds to a relatively small energy denominator in Eq.(5), of the order of 2Δ, instead of a value of $2\hbar\omega$ for the pure IPM, and leads to increased values of the mass parameters, in comparison with the very low values of the IPM. This is so, however, only because it is the IPM value which is too low, and as soon as some pairing correlations are present, the dependence of the mass parameters on the strength of the residual interaction is much more moderate. In fact, the mass parameters decrease with further increase of the pair correlation strength.

The pairing effect disappears when a certain critical temperature is reached. In this case, it is inappropriate to apply formula (6) because residual interactions other than pairing become important. The treatment of these residual interactions is beyond the scope of the present study. We shall therefore limit our attention to the cases with a significant pairing gap, in the hope that the other residual interactions are less important.

A simple approximate expression is known for the mass parameter, when the pairing gap is sufficiently large ($\Delta \gg G$, where G is the pairing matrix element). The latter condition ensures that the terms with $\partial\Delta/\partial q$ and $\partial\lambda/\partial q$ in Eq.(5) are small so that the main contribution comes from the first sum. There, the most important are the diagonal matrix elements arising from single-particle states in an energy interval of 2Δ at the Fermi sea. Let g_{eff} be some effective local density of single-particle states near the Fermi sea and $\overline{|\partial H/\partial q|^2}$ the average of the square of the matrix elements for these states. Since the factor involving the occupation numbers U and V is of the order of unity and the energy denominator is of the order of 2Δ, we have

$$B \sim \frac{\hbar^2}{2} \overline{\left|\frac{\partial H}{\partial q}\right|^2} \frac{g_{eff}}{\Delta^2} + \delta \tag{7}$$

where the second term, which is approximately constant and very small compared to the first term, denotes all other contributions.

In some cases in the deformed region, there is a significant shell in the single-particle spectrum so that the pairing gap is very small. We have then essentially the case of the IPM. If now proper levels cross each other at the Fermi sea, the terms in Eq.(5) involving $\partial\lambda/\partial q$ and $\partial\Delta/\partial q$ become much larger than the first sum since the wave function changes drastically with deformation. No simple expression such as Eq.(7) is obtained as the mass parameter becomes singular. In that case, it is inappropriate to apply expressions (6) and (7) because residual interactions other than pairing become important. The treatment of these residual interactions is beyond the scope of the present study. We shall therefore limit our attention to the cases when a significant pairing gap is present ($\Delta > 0.3$ MeV, say) in the hope that the other residual interactions are less important.

The pairing effect disappears when a certain critical temperature is reached. The method cannot be applied to temperatures higher than the critical temperature for reasons mentioned above. In this work, we shall consider mass parameters at zero temperature only.

The correspondence between this equation and the numerical calculations is illustrated in Figs 1-3. There, the mass parameters are shown evaluated for the case of ellipsoidal distortion of the Nilsson potential well. As the deformation coordinate ρ, we have taken one half of the distance between the centres of mass of the two halves of the nucleus, divided by the value of the undeformed radius[1]. The specific choice of the deformamation parameter is not very essential[2]. For another deformation parameter x, the relevant mass coefficient is related to B in the following way:

$$B_x = B_\rho \left(\frac{d\rho}{dx}\right)^2 \tag{8}$$

Figure 1 shows the dependence of the calculated mass parameter B_ρ on the parameter $\tilde{\Delta}$ which characterizes the strength of the pairing correlation ($\tilde{\Delta}$ is the energy gap parameter for a uniform distribution

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[1] A convenient unit of reference for the effective mass is the reduced mass for two equal fragments at large distance, which is equal to

$$B_\rho = 0.0240\ r_0^2 A^{5/3} (\hbar^2/\text{MeV})$$

[2] The use of the ϵ-parameter of the Nilsson model is rather inconvenient. With the ϵ-parameter defined in a finite interval $\epsilon \lesssim 1.5$, one obtains [2] a spurious divergence of B_ϵ at larger values of ϵ, which makes it difficult to see any finer structure.

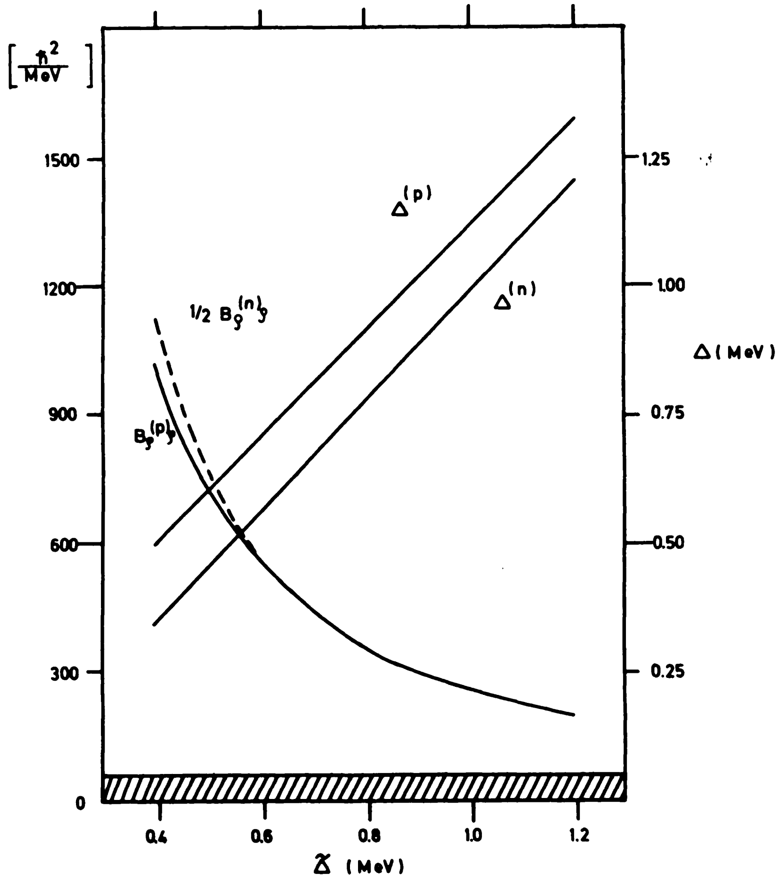

FIG. 1. Mass parameter B_ρ multiplied by ρ shown as a function of Δ. The calculation was performed for N = 146 and Z = 94 at a deformation of $\rho = 0.45$ corresponding to $\epsilon = 0.28$. The gap parameters $\Delta^{(n)}$ and $\Delta^{(p)}$ are also shown here. It can be seen that after subtracting a value represented by the shaded region, the quantity B_ρ^ρ behaves like $\tilde{\Delta}^{-2}$.

of the single-particle states). For larger values of Δ, they are in clear agreement with Eq. (7). This seems, however, to contradict the results shown in Fig. 2 where the same quantities together with the shell corrections to the nuclear binding energy ($\delta U + \delta P$) are presented as functions of the deformation. Very significant fluctuations are clearly seen, the larger values of B_ρ^ρ coinciding approximately with the larger values of the Δ parameter, evaluated for the same deformations. This apparent contradiction is resolved if the fact is taken into account that the effective energy region in Eq. (5) is very small — of the order of 2Δ — and the level density for such an energy interval g_{eff} shows very strong oscillations due to shell structure (see, e.g. Ref. [6]). This is a more important effect than the changes of Δ. The importance of the shell structure is further evidenced by the correlations between the fluctuations of the effective-mass parameters and their corresponding shell-energy corrections which are known to be roughly proportional to the fluctuations of the local level density near the Fermi energy [6].

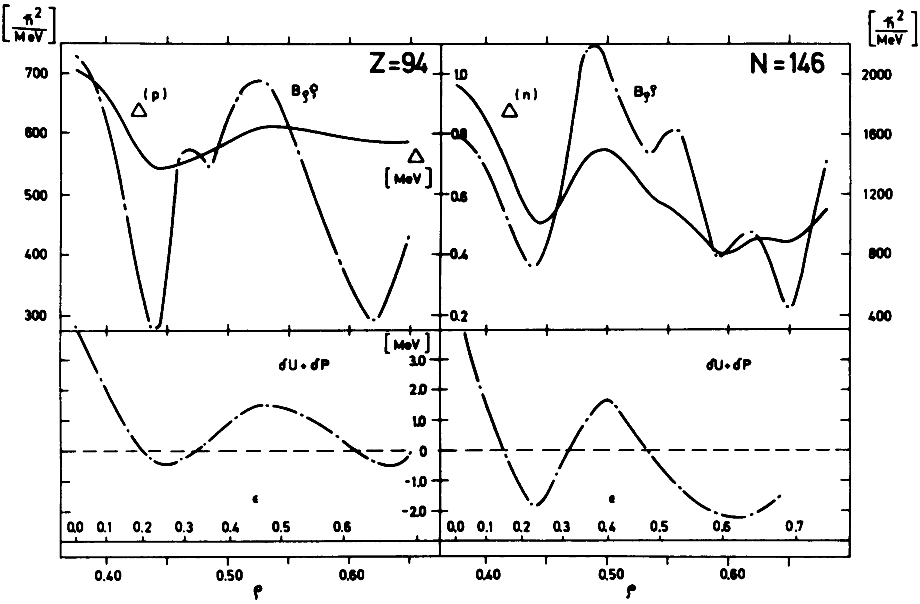

FIG. 2. In the upper diagrams the quantities B_ρ^ρ and Δ for Z = 94 and N = 146 are shown as functions of the parameters ρ and ε. The corresponding shell corrections are shown below.

Even though the energy gap Δ has a strong exponential dependence on the density of single-particle states, the charge in Δ is a less important factor here. This can be understood because, in the BCS pairing theory, and energy interval much larger than 2Δ is essential. Therefore, the effective level density which appears in the BCS equation should be identified with a much more smoothed density function rather than the local density g_{eff} which appears in Eq.(7) and also in the energy shell corrections.

In Fig. 3, some results obtained by Sobiczewski et al. are also shown, These data were re-evaluated by means of Eq.(8) from B_ϵ values presented in Ref.[2]. While the average value, which is equal to about 15 units of the reduced mass, seems to agree with our results for $\tilde{\Delta}$ = 0.6 MeV (which gives the correct value for Δ at the ground-state deformation), some essential discrepancy is evident[3]. This should change appreciably the estimates of same spontaneous fission lifetimes given in Ref.[2].

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[3] The equation used in Ref.[2] for evaluating the effective mass can be written schematically as follows

$$B_\epsilon = (2\hbar^2 \Sigma_3)\left(\frac{dQ/d\epsilon}{2\Sigma_1}\right)^2 \tag{9}$$

It can be shown that up to the first order in ε the second factor in Eq.(9) should be equal to the square of the constant κ which characterizes the strength of the coupled deformed field. (This is true also when the corrections due to pairing are taken into account.) Therefore, Eq.(9) is, up to a smooth function of the deformation, identical to our Eq.(5). In Ref. [2], however, the ratio in the second factor was determined numerically. The result may be erroneous owing to some inaccuracy in evaluating the poorly converging term Σ_1.

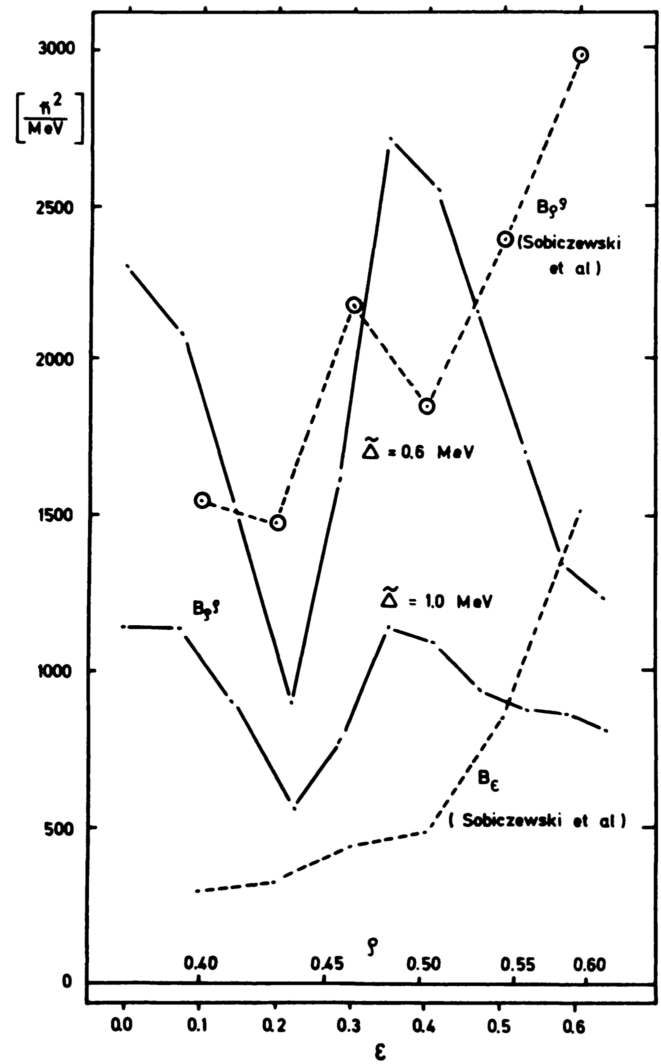

FIG. 3. The solid curves are the calculated values of B_{ρ}^{ρ} for ^{240}Pu for the cases when $\tilde{\Delta}$ is equal to 0.6 and 1.0 MeV. The lower dotted curve is the quantity B_{ϵ} obtained by Sobiczewski et al. It is a rapidly increasing function of ϵ. This quantity B_{ϵ} can be converted to B_{ρ}^{ρ} as shown in the upper dotted curve and should be compared with our results.

The results shown in Figs 2 and 3 lead us to the important conclusion that the effective mass parameters are abnormally large near the top of the shell maxima in the deformation energy where the local level density is large. They decrease when one moves away from this point and the lowest value is obtained near the stationary shape minima of the deformation energy, corresponding to the ground-state deformation or the second minimum. The increased inertia of the nuclear matter in the region of the potential barriers is very important for estimates of the penetrability, especially in the superheavy elements.

For more definite estimates of the penetration factor, the problem of the trajectory must be considered, which requires the knowledge of the mass parameters related to other degrees of freedom. In addition, other single-particle models must also be considered, as the Nilsson model is too ambiguous to be used without reservation for the extrapolations to large deformations and new regions of nuclei.

To approach the solution of these problems, an attempt was made to develop fast numerical methods for solving the IPM with a rather generally defined average field and the shape of the nuclear surface [7], and, at the same time, to evaluate the mass parameters related to the generalized deformation co-ordinates q_i which might appear in the definition of the nuclear shape. In these calculations, the surface of the nucleus was defined by the equation

$$\Pi_{q_1 \ldots q_n}(u, v) = 0 \tag{10}$$

where u and v are the two cylindrical co-ordinates. The operators $\delta H/\delta q_i$ which appear in Eq.(5) are also computed in a rather general manner. The calculations with the Saxon-Woods model are now in progress and will be published elsewhere.

3. MOMENTS OF INERTIA

The anisotropy of the angular distribution of the fission fragments at higher excitations is determined by the value of the so-called effective moment of inertia [8]

$$\mathscr{I}_{\text{eff}} = \left(\frac{1}{\mathscr{I}_{\parallel}} - \frac{1}{\mathscr{I}_{\perp}}\right)^{-1} \tag{11}$$

where $\mathscr{I}_{\parallel}$ and $\mathscr{I}_{\perp}$ are two moments of inertia for rotation about the symmetry axis (or the fission axis) and the axis perpendicular to the symmetry axis, respectively. For $\mathscr{I}_{\parallel}$ and $\mathscr{I}_{\perp}$, the rigid-body values are usually assumed, and $\mathscr{I}_{\text{eff}}$ is then rather strongly dependent on the shape of the nucleus at the top of the fission barrier [8]. It becomes infinite when the saddle shape is spherical. This should be the case of a nucleus which is very unstable against fission. For such a nucleus, isotropic angular distribution is predicted and recently some attempts were made to determine the limits of stability of nuclei by measuring the angular anisotropy of highly excited nuclei produced in nuclear reactions with α-particles and heavy ions [10-12].

Experimental studies of $\mathscr{I}_{\text{eff}}$ are very important in view of the fact that they are probably the most direct way of investigating the shape of the nucleus at the barrier. However, this quantity may be also affected by the shell structure, which may be misinterpreted as due to a different shape of the nucleus.

For the inertia parameters, the following equation holds (which is analogous to Eq.(6))

$$\mathscr{I}_i = \sum_{\mu\nu} \left\{ \frac{(U_\mu V_\nu - U_\nu V_\mu)^2}{2(E_\mu + E_\nu)} \left(\tanh \frac{E_\mu}{2T} + \tanh \frac{E_\nu}{2T} \right) + \frac{(U_\mu V_\nu + V_\mu V_\nu)^2}{2(E_\mu - E_\nu)} \left(\tanh \frac{E_\mu}{2T} - \tanh \frac{E_\nu}{2T} \right) \right\} |\langle \mu | M_i | \nu \rangle|^2 \tag{12}$$

where the operator M_i is

$$M_\perp = j_x$$

for $\mathscr{I}_\perp$ and (13)

$$M_\parallel = j_z$$

for $\mathscr{I}_\parallel$. Here $\hat{j}$ is the single-particle angular momentum operator. From Eqs (12) and (13) one obtains a known expression for $\mathscr{I}_\parallel$

$$\mathscr{I}_\parallel = \frac{1}{4T} \sum_\nu \frac{K_\nu^2}{\cosh^2(E_\nu/T)} \tag{14}$$

where $K_\nu = \langle \nu | j_z | \nu \rangle$.

The moment of inertia $\mathscr{I}_\parallel$ can also be expressed in the following way

$$\mathscr{I}_\parallel = \overline{K^2}\, g_T^{\mathrm{eff}} \tag{15}$$

where

$$g_T^{\mathrm{eff}} = (1/4T) \sum_\nu \{1/\cosh^2(E_\nu/2T)\} \tag{16}$$

and

$$\overline{K^2} = \left(\sum_\nu K_\nu^2/\cosh^2(E_\nu/2T) \right) \Big/ \sum_\nu \{1/\cosh^2(E_\nu/2T)\} \tag{17}$$

In the case of $\Delta = 0$, the quasi-particle energies E_ν in Eqs (14) (17) are replaced by the single-particle energies $\epsilon_\nu - \lambda$.

In Fig.4, some results of numerical calculations are presented which demonstrate the role of the shell structure and pairing for the specific case of N = 144 with the Nilsson potential. The quantities introduced above are plotted against the deformation of the Nilsson potential well for different values of the temperature T. (In applications to real processes, the temperature would, of course, also change with deformation.) Fluctuations with respect to deformation are apparent. However, the correlation between the fluctuations and the shell structure is not simple.

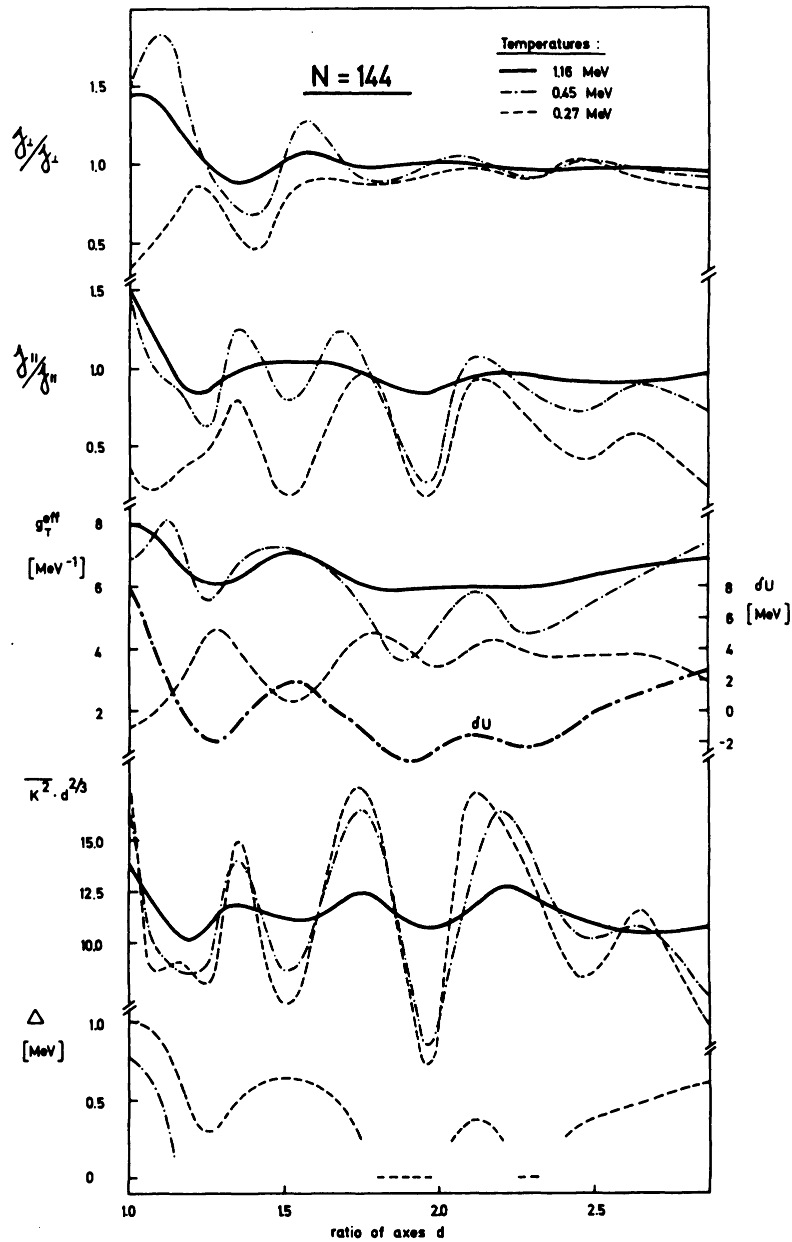

FIG.4. Moments of inertia and related quantities for N = 144 at three different temperatures are shown as functions of the deformation d which is defined as the ratio of the axes in the ellipsoidal Nilsson potential. The moments of inertia $\mathscr{J}_{\perp}$ and $\mathscr{J}_{\parallel}$ are expressed in units of the corresponding values $\tilde{\mathscr{J}}_{\perp}$ and $\tilde{\mathscr{J}}_{\parallel}$ for a rigid body with the same shape. The quantities g_T^{eff} and $\overline{K^2}$ are defined by Eqs (16) and (17), respectively. The gap Δ is calculated by taking $\tilde{\Delta}$ equal to 0.6 MeV. For T = 0.45 MeV, the gap Δ vanishes when the deformation is larger than 1.2 and is not shown in the figure. For the value of $\hbar\omega_0$ we used 55 $A^{-1/3}$ MeV which is obtained by setting the average of r^2 near the Fermi level equal to 3 $R_0^2/5$.

The gap parameter Δ is known to increase when the local density of the single-particle states near the Fermi energy increases. This can lead to a reversed effect in the quasi-particle density (16) which is exponentially decreasing with increasing Δ. Therefore, it is expected that at higher excitations, maxima and minima of $\mathscr{I}_{\parallel}$ should approximately correspond to those of the single-particle level density, while at low excitations maxima of $\mathscr{I}_{\parallel}$ correspond to minima of $g_{s.p.}$ (i.e. to maxima of the quasi-particle level density), and vice versa. While this is approximately valid for high excitations, the actual correlation at low excitations is more complicated because there is also a shell effect in the averaged value of K^2. Indeed, for low temperatures, only few states contribute to the averaging of K^2. It is known that the energies of the single-particle states with higher K values go up with deformation while those with lower K values go down. Therefore, the average value of K^2 oscillates about as frequently as that of the energy shell correction but with a different phase, see Fig.4. The value of K^2 should have the WKB values in the middle of a shell but higher and lower values at other deformations depending on the number of nucleons N. The result for our case of N = 144 at low temperature is that for small deformations $\mathscr{I}_{\parallel}$ fluctuates in nearly the opposite phase as the energy shell correction.

For $\mathscr{I}_{\perp}$, and at low temperatures, it is known that an increase in Δ leads to a decrease in $\mathscr{I}_{\perp}$ [4]. However, this is not the only effect due to shell structure as the matrix elements of j_x are also affected. The situation is simpler at high temperatures as contributions to $\mathscr{I}_{\perp}$ come from matrix elements between states in a large energy interval. Therefore, as is the case with $\mathscr{I}_{\parallel}$, the quantity $\mathscr{I}_{\perp}$ is also correlated with the density of the single-particle states at the Fermi energy.

In any case, the shell structure influences rather strongly the moments of inertia. The fluctuation nevertheless decreases with increasing temperature. One of the problems is therefore to find the critical temperature T_s^* when the shell fluctuations become small. This is important to know for the analysis of the fission anisotropy at low excitations above the fission barrier. In these considerations, the dependence of Δ on the temperature should be taken into account in the usual way.

In Fig.5, the quantities $\mathscr{I}_{\mathrm{eff}}$, $\mathscr{I}_{\parallel}$ and $\mathscr{I}_{\perp}$ are shown as a function of the temperature T, evaluated for the most interesting shapes of the nucleus ^{236}U, namely the ground-state deformation, the second minimum and the two barriers. It can be seen that for the first and the second barriers, the rigid-body values of the moments of inertia are essentially reached at a nuclear temperature of $T_s^* \simeq 0.8 - 1.0$ MeV. This value is close to that for the critical temperature at which the shell-structure effects in the level density disappear as was found earlier [13]. It is also higher than the critical temperature T_c^*, at which the pair correlation effects disappear ($T_c^* \simeq 0.4 = 0.5$ MeV).

The evaluated moments of inertia are applied to the analysis of the angular anisotropy data in the neutron induced fission at lower excitations. The angular distribution of the fragments is described, in this case, approximately by [8]

$$1 + \frac{5E_n}{8T\mathscr{I}_{\mathrm{eff}}} \cos^2\theta$$

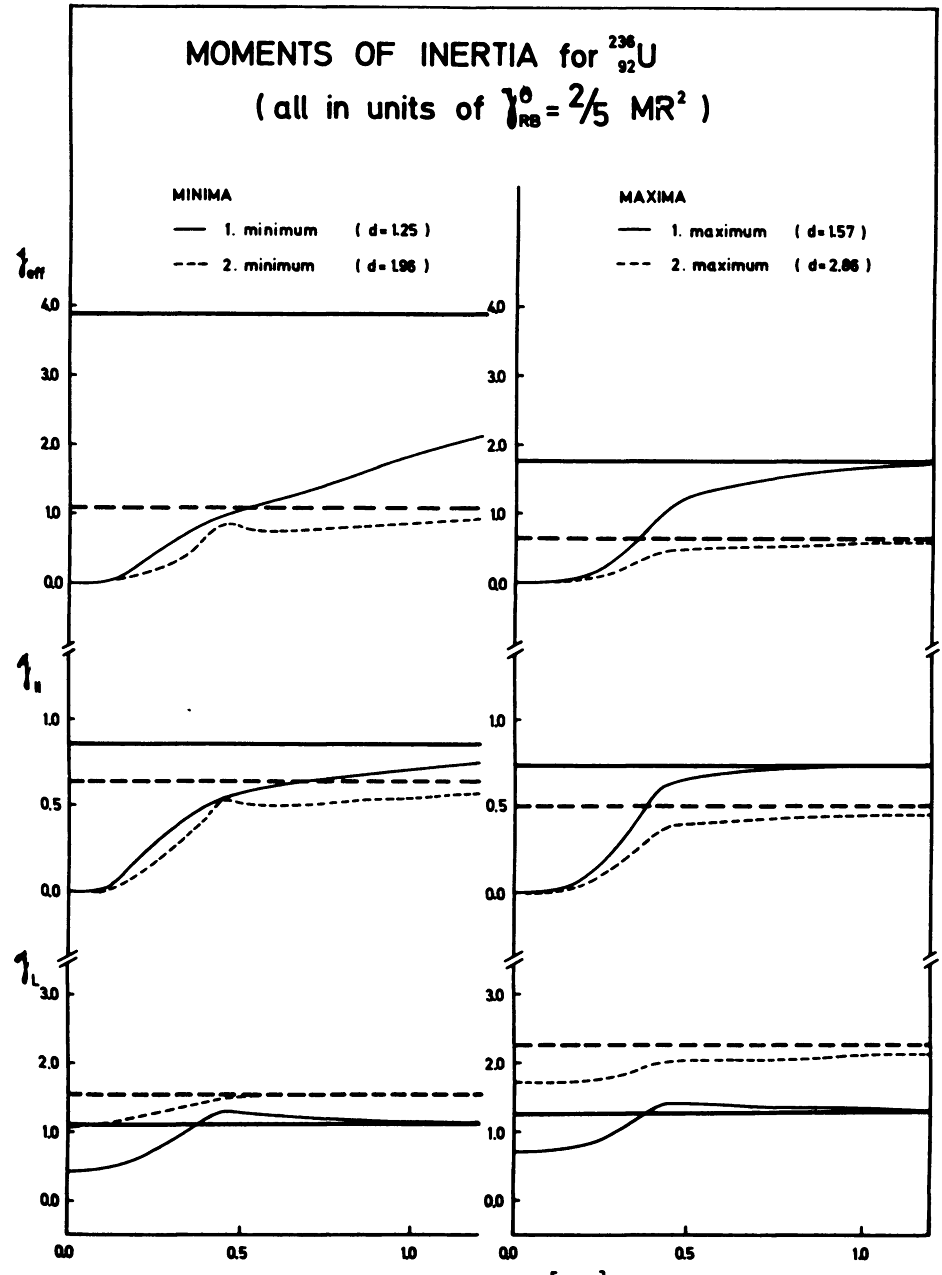

FIG. 5. The moments of inertia $\mathcal{J}_{eff}$, $\mathcal{J}_{\perp}$ and $\mathcal{J}_{\parallel}$ for ^{236}U are shown as a function of the nuclear temperature for the deformations at the ground state (d = 1.25), the second minimum (d = 1.86), the first barrier (d = 1.5) and the second barrier (d = 2.86). All the moments of inertia are expressed in units of the rigid-body value for a sphere, $\mathcal{J}_{RB}$. The thick horizontal lines, in all cases, represent the corresponding moments for a rigid rotator of the same shape. These values are reached at high temperatures.

For the specific case of the reaction $^{235}U(n, f)$ with 3 MeV neutrons, the value of the temperature is found to be equal to 0.28 MeV if the barrier shape was assumed to be the same as that for the first barrier and T = 0.33 MeV for the second barrier (with the rigid-body values these would be equal to completely unreasonable values of 0.05 MeV and 0.14 MeV, respectively).

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REFERENCES

[1] INGLIS, D., Phys. Rev. 96 (1954) 1059.
[2] SOBICZEWSKI, A. et al., Nucl. Physics A131 (1967) 67; NILSSON, S.G. et al., Nucl. Physics A131 (1969) 1.
[3] STRUTINSKY, V.M., PAULI, H.C., these Proceedings.
[4] BOHR, A., MOTTELSON, B., Nuclear Structure, 3, to be published by W.A. Benjamin, Inc. (New York).
[5] BES, D.R., Mat. Fys. Medd. Dan. Vid. Selsk. 33 2 (1961).
[6] STRUTINSKY, V.M., Nucl. Physics A112 (1968) 1.
[7] DAMGAARD, J., PAULI, H.C., PASHKEVICH, V.V., STRUTINSKY, V.M., (to be published in Nucl. Physics).
[8] HALPERN, I., STRUTINSKY, V.M., Int. Conf. peaceful Uses atom. Energy, (Proc. Conf. 1955) Geneva 15 (1958) 408.
[9] STRUTINSKY, V.M., LJASHENKO, N.Y., POPOV, N.A., Nucl. Physics 46 (1963) 639.
[10] BATE, G.L., CHAUDRY, R., HUIZENGA, J.R., Phys. Review 131 (1963) 722.
[11] MUZYCKA, Yu.A., OGANESSIAN, Y.T., PUSTYLNIK, B.I., FLEROV, G.N., Yad. Fiz. 6 (1968) 306.
[12] OGANESSIAN, Yu.Ts., JINR E7-3942 preprint (1968).
[13] PAVLINCHUK, V.A., private communication (1967).

DISCUSSION

L. WILETS (Chairman): Have you been able to follow the deformation to sufficiently large values so that you could see the mass parameter asymptoting to the separated fragment value, namely the reduced mass?

H.C. PAULI: We have not got so far yet in our calculations. The fact that the quantity $B_\rho \rho$ remains on the average a constant seems, however, to indicate such behaviour. One should also remember that the data presented hold good for spheroidal shapes only. For large deformations one should use other shapes. This work is under way.

P. von BRENTANO: Could you comment on the impact of your considerations on the calculation of the life-times of fission isomers?

H.C. PAULI: The life-times of the fission isomers should be affected as well as the spontaneous fission life-time. Increased mass, as compared to the average, at the barriers will also increase the life-time.

A. SOBICZEWSKI: You mentioned the discrepancy between the values of the mass parameter B obtained in your calculations and in ours (cited in your paper as Ref.[2]). You suggest that it may come from cutting off the levels from the $N \geq 10$ shells. I agree that the poorly convergent term Σ_1, which enters into B in our approach and is not needed in your calculation, may be lowered by this cut-off but I do not think this effect could account for the whole of the discrepancy obtained. This concerns especially the low deformations for which the levels cut off lie very far from the Fermi level.

H.C. PAULI: I agree in so far as the agreement, on the average, is much better at smaller than at larger deformations.

E.R.H. HILF: Did I understand correctly that you wanted to study a nucleus of finite temperature but actually studied a cold nucleus, applying Fermi statistics of some finite temperature? So you started with the single-particle energy level density for $T = 0$ and filled in the nucleons, using a Fermi distribution of finite temperature. If so, you missed one of the two effects that compete with each other, being of the same order of magnitude, and come into play in a shell-model calculation for finite temperature, i.e. the temperature dependence of the level density itself. This is due to the fact that heating the nucleus (a little bit, otherwise the summation of single-particle energies becomes increasingly useless because of the interaction energies) leads to wave functions of higher energy and orbital momentum, which have a different radial distribution too, and this in due course leads to a higher concentration of high-energy nuclei at the surface. In a self-consistent calculation this would lead to a change of the potential and to a rise of the level density. In your non-self-consistent calculation you can take care of this effect approximately by some single ansatz, say a first-step calculation, since the wave functions are available to you.

H.C. PAULI: We use the energy levels of the cold system but do not expect the structure to be completely destroyed by a temperature of 1 MeV.