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EPISTEMIC INTERPRETATION OF CONDITIONALS

The presence of a connection between conditionals and conditional probabilities has been pointed out by several authors (the first, to my knowledge, was E. W. Adams in his (1965).) The epistemic interpretation of conditionals given here will not refer to conditional probability directly, but is based on the logic of conditional belief. Conditional beliefs, however, can be derived from conditional subjective probabilities.

The connection between this form of Epistemic Logic and the logic of conditionals is threefold: (a) conditional belief constitutes a model of the axioms of Conditional Logic; (b) an epistemic interpretation of conditionals is in a sense natural; (c) as R. C. Stalnaker points out in his paper (1970), an epistemic interpretation of conditionals may give us further insight into their formal properties, about which there seems to be far less agreement than there is about the logic of probability. Following a few introductory remarks in Section I, we shall develop a logic of conditional belief, in Section II; then go on to show it to be a model of the axioms of Conditional Logic, in Section III; discuss the consequences of such an interpretation of conditionals for sentences with iterated applications of "if-then", in Section IV, and finally, in Section V, argue to the effect that an epistemic interpretation of conditionals is adequate.

I

Epistemic Logic is the logic of believing and knowing. Since knowing can be defined in terms of believing and being right, its basic concept is that of belief. There are several such concepts. First we have to distinguish between *descriptive* and *rational* concepts of belief. A descriptive concept "Person a believes that p" might roughly be defined by "If a understands the question 'Is it the case that p?' and answers truthfully, he will answer 'Yes'". There is no logic of such a concept, since a might believe the most absurd things. He might believe logically false propositions; he might believe A and B, but not $A \wedge B$, etc. Just as Formal Logic is not interested in describing how people actually argue, and the logic of subjective

A. Kasher (ed.), Language in Focus, 487–501.

probability is not interested in describing the degrees of probability people actually assign to propositions, so is Epistemic Logic interested only in rational principles of believing, and could therefore be used only with completely rational people for descriptive purposes. To emphasize the distinction we shall also express the rational concept of belief by saying *a has reason to believe that p* instead of *a believes that p*.

Among the rational concepts of belief there are classificatory, comparative and metric concepts. Since a metric concept of belief has to be based upon a comparative concept, and classificatory concepts can be defined by comparative concepts, but not vice versa, a comparative concept of belief suggests itself as the basic notion of Epistemic Logic. There is, however, one further distinction to be made: There are absolute (2-place) and conditional (4-place) relations of comparative belief (and therefore of classificatory and metric belief). Since the absolute concept can again be defined by the conditional concept, full generality is achieved by starting from the 4-place relation as basic concept.

Since there are no logical principles that tell us what one person, a, has reason to believe if another person, b, has reason to believe such-and-such, the basic sentences can be written in the form $A, B \leqslant. \ C, D$: *For the person or persons referred to B is at most as much reason to believe that A, as D is reason to believe that C*; i.e., it is not necessary to mention explicitly the person or persons referred to, just as in Probability Logic we write $p(A) = r$ instead of $p_a(A) = r$, where r is the (subjective) probability assigned to A by person a. $A, B \leqslant. \ C, D$ can also be read as "On condition that B, A is at most as subjectively probable as C, on condition that D". The logic of such statements is therefore nothing else than the logic of comparative conditional subjective probability.

Since our aim here is to interpret conditionals by classificatory statements of belief, we shall not discuss the logic of sentences $A, B \leqslant. \ C, D$.[1] We may define our classificatory concept of conditional belief by $B(A, C) := C, C \leqslant. \ A. \ C$, but we shall take the operator B as a basic constant here and refer to the concept $\leqslant.$ only briefly in justifying the semantics given for B.

II

Let LB be the language containing all sentences of predicate logic and the sentence $B(A, C)$ where A and C are sentences.

D1. An *interpretation* of LB is to be a quadruple $\langle U, I, b, \Phi\rangle$ such that:

(1) U is a non-empty set of objects (the universe of discourse);
(2) I is a non-empty set of worlds;
(3) $b(i, X)$ is, for all $i \in I$ and all $X \subset I$, a subset of I such that
(a) $b(i, X) \subset X$,
(b) $X \subset Y \wedge b(i, X) \neq \Lambda \supset b(i, Y) \neq \Lambda$,[2]
(c) $X \subset Y \wedge b(i, Y) \cap X \neq \Lambda \supset b(i, X) = b(i, Y) \cap X$,
(d) $i \in S_i$, where $S_i := \bigcup_X b(i, X)$,
(e) $j \in b(i, I) \supset b(i, Y) = b(j, Y)$ for all $Y \subset I$ and $j \in I$;
(f) $j \in S_i \supset S_j = S_i$;
(4) For all $i \in I$, Φ_i is a function from the set of sentences of LB into $\{t, f\}$ such that:
(a) $\Phi_i(a) = \Phi_j(a)$ for all individual constants a of LB and all $j \in I$;
(b) Φ_i fulfills the conditions for interpretations set down in Predicate Logic;
(c) $\Phi_i(B(A, C)) = t \equiv b(i, C) \subset [A]$

where $[A] := \{i \in I : \Phi_i(A) = t\}$ and $b(i, C) := b(i, [C])$.

We define necessity by

D2: $NA := B(A, \neg A)$,
and unconditional belief by

D3: $BA := B(A, T)$,
where T is a tautology.

This concept of an interpretation of LB is based on the following considerations:

(1) U is to be taken as a set of possible objects. The different worlds in I may have different subsets U_i of U of existing objects. If E is a 1-place predicate constant of LB and we set $\Phi_i(E) = U_i$, we can define universal quantification over actual instead of possible objects by $\Lambda.xA[x] := \Lambda x(Ex \supset A[x])$. However, in the following, we shall not be concerned with existence.

(2) $b(i, A)$ is to be the set of worlds, so that the person a referred to believes in i that in case of A, one of the elements of it is the real world, but he cannot say which one. So A for a is reason to believe that B iff $b(i, A) \subset [B]$.

Condition (3a) is evident: If a believes that A, then all the worlds in

$b(i, A)$ are A-worlds. $b(i, A)$ is to be empty iff A is considered impossible in i. Since $S_i = U_X b(i, X)$, S_i is the set of worlds that for a in i might be the real world under some condition, and S_i is a good candidate for the set of worlds considered possible by a in i.

(3d) then says that i is considered possible in i. And we have

(α) $b(i, A) = \Lambda \equiv S_i \subset [\neg A]$:

From the definition of S_i and (3a) we have $b(i, A) = \Lambda$ if $S_i \subset [\neg A]$. And if $S_i \cap [A] \neq \Lambda$ then there is a B such that $b(i, B) \cap [A] \neq \Lambda$, so according to (3$a$, c) $b(i, A \wedge B) = b(i, B) \cap [A] \neq \Lambda$, and in view of (3b) $b(i, A) \neq \Lambda$.

From D2 it follows that A is necessary in i iff $S_i \subset [A]$. For if $b(i, \neg A) \subset$ $\subset [A]$, by (3a) we have $b(i, \neg A) = \Lambda$, so $S_i \subset [A]$ by (α). And if $S_i \subset [A]$ then by (3a) $b(i, \neg A) = \subset [A]$.
And if $S_i \subset [A]$ then by (3a) $b(i, \neg A) = \Lambda \subset [A]$.

Finally we have by the definition of S_i

(β) $S_i \subset Y$ iff for all X $b(i, X) \subset Y$,

i.e. a proposition Y is necessary iff every proposition X is reason to believe Y.

Now (3b) says that if a proposition X is not impossible then Y is not impossible for $X \subset Y$. (3c) is equivalent to $b(i, B) \cap [A] \neq \Lambda \supset b(i, A \cap B) =$ $= b(i, B) \cap [A]$, i.e. if B is not reason to believe that $\neg A$, $A \wedge B$ is reason to believe that C iff B is reason to believe that $A \supset C$.

All these postulates of D1 can be derived from the semantics of $A, B \leqslant . C, D$. Since such a comparative concept, restricted to sentences A, B, C, D without the operator $\leqslant .$, can be metricized under a plausible assumption in the form of a conditional probability p with $p(A, B) \leqslant p(C, D) \equiv$ $\equiv A, B \leqslant . C, D$, we have $b(i, X) = \bigcup_Y (p(Y, X) = 1)$, from which we obtain the conditions D1, (3a), (3b), (3c) for b by the well-known axioms for p.

(3) The postulate (3e) in D1 gives a connection between the beliefs of a in different worlds. This postulate derives from the two principles

$$B(A, C) \supset BB(A, C)$$
$$\neg B(A, C) \supset B \neg B(A, C).$$

If C is (not) reason for a to believe that A, then a has reason to believe that this is (not) so. For unconditional descriptive belief we have the principles $BA \supset BBA$ – if a believes that A, then a believes that he believes that A – he could scarcely doubt that – and analogously $\neg BA \supset B\neg BA$. These principles carry over to rational belief and also to conditional belief.

(4) (3f) implies that we interpret necessity in the manner of C. I. Lewis' system S5.

(5) Condition (4a) of D1 implies that we interpret all individual constants of L as standard names.

D4. An interpretation $\mathfrak{M}=\langle U, I, g, \Phi\rangle$ *satisfies* a sentence A in $i\in I$ iff $\Phi_i(A)=t$. A is *valid* in $\mathfrak{M}$ iff $\mathfrak{M}$ satisfies A for all $i\in I$. And A is D-true (*doxastically true*) iff A is valid in all interpretations $\mathfrak{M}$.

We can formulate an axiom system $\mathfrak{B}$ of the logic of conditional belief by adjoining the following axioms and rules to predicate logic:

B1	$B(A, A)$
B2	$NA \supset B(A, C)$
B3	$N(A \supset C) \wedge B(A, D) \supset B(C, D)$
B4	$B(A, C) \wedge B(D, C) \supset B(A \wedge D, C)$
B5	$\neg B(\neg A, C) \supset (B(D, A \wedge C) \equiv B(A \supset D, C)$
B6	$NA \supset A$
B7	$NA \supset NNA$
B8	$\neg NA \supset N\neg NA$
B9	$B(A, C) \supset BB(A, C)$
B10	$\neg B(A, C) \supset B\neg B(A, C)$
B11	$\wedge x B(A[x], C) \supset B(\wedge x A[x], C)$
BR1	$A \vdash NA$.

$\mathfrak{B}$ can be shown to be semantically consistent and complete.

III

An indicative conditional is a sentence of the form "If it is the case that A, then it is the case that D" – symbolically $C(D, A)$.

According to D. Lewis (1973) we can formulate the semantics and the logic of conditions in the following way:
If LC is like LB with C instead of B, then an interpretation of LC is like an interpretation of LB with C substituted for B in D1, (4c), with the postulate

(3d′) $i \in b(i, I)$

instead of (3d) and with (3e) omitted. (3d′) implies (3d) and

(γ) $i \in X \supset i \in b(i, X)$ for all $i \in I$,

since from (3c) and (3d′) we obtain $i \in X \supset b(i, X) = b(i, I) \cap X$, and therefore $i \in b(i, X)$.

We obtain a logic of conditionals $\mathfrak{C}$ corresponding to this concept of an *LC*-interpretation from $\mathfrak{B}$ by reading *C* everywhere for *B*, omitting B9 and B10 and substituting

(B6′) $C(A, B) \supset (B \supset A)$

for B6. B6′ entails B6.[3]

According to the ideas of Stalnaker in (1968) and Lewis in (1973) $b(i, A)$ in the case of conditionals is to be interpreted as the set of *A*-worlds most similar to *i*. They start from comparative relations $j \leqslant_i k$ on *I* for all $i \in I$ – *world j is at most as similar to i as world k*. Then under the *Limit-Assumption*[4] that for all *i* and *A* there is an *A*-world that is most similar to *i* $b(i, A)$ may be defined in this sense. The Limit-Assumption is not very plausible, as Lewis points out, but at least it makes no difference for the resulting logic. If we start from the relation $j \leqslant_i k$, however, it is very plausible that *i* is more similar to itself than to any other world. This gives us

(γ′) $i \in X \supset b(i, X) = \{i\}$

instead of (γ), i.e. in Lewis' terminology *strong* instead of *weak* centering. From (γ′) we obtain $A \wedge B \supset C(A, B)$. This is harmless for counterfactuals $C(A, B)$ which are normally only used with the presupposition $\neg B$, but not for indicative conditionals and it is quite unacceptable for causal statements "*It is the case that A, since it is the case that B*" since it would make them all true. Therefore, if $\mathfrak{C}$ is to be a general logic for conditionals, weak centering, i.e. (γ) is advocated. Since this is implausible with the interpretation of $b(i, A)$ given above, I have proposed a different interpretation of this set in my paper (1974) which does not refer to a comparative relation of similarity for worlds. $C(A, B)$ is interpreted there as "*Under condition that B it is necessary that A*". We then have to distinguish weak necessity from strong necessity. *Weak necessity* of *A* means roughly that *prima facie* A is normally the case. Weak necessity is defined by

D5 $CA := C(A, T)$, where T is a tautology.

$C(A, B)$ says that A is (weakly) necessary or normally the case on condition that B.

Strong necessity is weak necessity under any circumstances and can be defined by $NA := C(A, \neg A)$.

Then $b(i, A)$ is the set of worlds (weakly) possible on condition that A, from the standpoint of worlds i.

An interpretation of $b(i, A)$ by a comparative relation of similarity between worlds suggests no principles relating the sets $b(i, A)$ and $b(j, A)$ for $i \neq j$, i.e. no principles for iterated applications of the operator C. Besides local uniformity – our condition D1, (3f) – Lewis in (1973, p. 120) discusses only

(a) *Local Absoluteness*: $j \in S_i \supset b(i, X) = b(j, X)$ for all $i, j \in I$ and $X \subset I$, which is too strong since it gives $C(A, B) \equiv NC(A, B)$, $\neg C(A, B) \equiv N \neg C(A, B)$, $CA \equiv NA$ and $C(A, B) \equiv N(B \supset A)$,

(b) *Universality:* $S_i = I$, for all $i \in I$;

(c) *Weak triviality:* $b(i, X) = I$ for all $i \in I$;

and

(d) *Triviality:* $I = \{i\}$.

Universality makes no difference as to validity in view of Uniformity. With Triviality, weak or otherwise, Conditional Logic collapses.

Interpreting conditionals as statements about conditional necessity suggests conditional analoga to B7 and B8. Substituting C for B in B9 and B10 gives the most likely candidates. $C(A, B) \supset NC(A, B)$ and $\neg C(A, B) \supset \supset N \neg C(A, B)$ would imply local absoluteness; $C(A, B) \supset C(C(A, B), B)$ and $\neg C(A, B) \supset C(\neg C(A, B), B)$ do not seem right, since if on condition that B, A is necessary or not necessary, this does not depend on condition B.

In view of (3d′) and B6′ not every concept of belief is a model of $\mathfrak{C}$. But we can define *correct* beliefs by the condition $B(A, C) \supset (C \supset A)$ for all sentences A and C. We call an *LB*-interpretation *correct* iff (3d′) holds, since $B(A, C) \supset (C \supset A)$ is valid in all such interpretations.

Correct beliefs, therefore, constitute a model for $\mathfrak{C}$, and since for correct beliefs we have $B(A, C) \supset K(A, C)$, where we define *conditional knowledge* $K(A, C)$ – *C is reason to know that A* – by

D6 $K(A, C) := B(A, C) \wedge (C \supset A)$
in analogy to $KA := BA \wedge A$, we can speak of an *epistemic* instead of a *doxastic* model of $\mathfrak{C}$.

IV

In the introduction we noted that an epistemic interpretation of conditionals may give us new insight into the formal properties of conditionals. This presupposes, of course, that the epistemic interpretation of conditionals is a natural one. The logic of conditional obligation has, for instance, the same formal structure as that of conditional belief, but we would not regard deontic interpretations of conditionals as natural and would not therefore try to justify postulates for conditionals by principles from deontic logic. The question of how natural is an epistemic interpretation of conditionals is postponed to the next section. Let us take a positive answer for granted, at the moment.

$\mathfrak{B}$, as a logic of *correct* belief with B6′ instead of B6, contains only two principles that do not occur in the better known logics of conditionals[5]: B9 and B10; and even they are suggested, as we have seen, by the interpretation of conditionals as statements about conditional necessity. *The* epistemic interpretation of conditionals therefore does not produce any new principles; still, it is a welcome confirmation of these two axioms. How plausible are these axioms?

From B9 and B10 we obtain

(δ) $CC(A, B) \vee C\neg C(A, B)$

and with B5

(ε) $\neg C\neg A \supset (C(C(B, D), A) \equiv C(B, D))$ and

(ζ) $\neg C\neg C(A, B) \supset (C(D, C(A, B)) \equiv CD)$.

Take the following examples:

(1) *If John will come, then if Jack will come too, it will be a lively party.*

(2) *If in case John will come, Jack will come too, it will be a lively party.*

According to (ε), if it is (weakly) possible that John will come, then (1) is equivalent to

(3) *If Jack will come, it will be a lively party.*

And according to (ζ), if it is (weakly) possible that if John comes, Jack will come too, then (2) is equivalent to

(4) *(Prima facie) it will be a lively party.*

This is intuitively not very convincing. We should rather have expected (1) to be equivalent with

(5) *If John and Jack will come, it will be a lively party,*

under the condition that Jack's coming is (weakly) possible if John is coming (so that Jack is not disposed to stay away if John comes). Under this condition ($\neg C(\neg D, A)$) we have $C(C(B, D), A) \supset C(B, D \wedge A)$, but no equivalence.

I think, however, that these two examples – or other examples from ordinary language – are not convincing counter-examples for B9 and B10, either. Just as we have no reliable intuition concerning iterated modalities, we have no intuitive criteria of truth for ordinary language sentences of the form $C(A, C(B, D))$, $C(C(A, B), D)$ or $C(C(A, B), C(D, E))$. Such sentences are very rare, and we are at a loss to explain the difference in meaning, for instance, between (1), (2) and (5). It therefore seems best either to exclude such sentences altogether or to accept principles that allow the elimination of iterated "if-then"s in as many instances as possible, just as in S5.

All this is no strong evidence that an epistemic interpretation of conditionals produces interesting new principles of Conditional Logic. But if we want to have axioms like B9 or B10, every additional evidence thereof is welcome.

V

A conditional $C(A, D)$ may be asserted by a person a in a subjectively correct way iff $B_a(A, D)$ (i.e. $B(A, D)$ holds with respect to a). If for a D is reason to believe that A, then a is justified in asserting that A if D. And if for a D is not a reason to believe that A, then a may not assert that A if D. This, however, does not imply that $C(A, D)$ may be interpreted to mean that $B_a(A, D)$. This is obvious if we refer to the beliefs of individual persons; first, the parameter a in $B_a(A, D)$ does not occur in $C(A, D)$, and a definition $C(A, D) := B_a(A, D)$ would therefore be incorrect and give rise to contradictions, $\Lambda xy(B_x(A, D) \equiv B_y(A, D))$ not being valid. Secondly, $B_a(A, D)$ may have a truth-value different from $C(A, D)$.

But let us assume that in the community P speaking the language L, to which the sentences $C(A, D)$ belong, there are common rational and correct beliefs, shared by almost all, which are expressed by the operator B. Then $C(A, D)$ may be asserted in P iff $B(A, D)$. But does this imply that $C(A, D)$ means the same as $B(A, D)$? Assertibility conditions have to be distinguished from truth conditions. Truth conditions, but not assertibility conditions determine the meaning of a sentence.

The distinction between truth conditions and conditions of assertibility is of course systematically important. But in the present case this distinction collapses. If we accept the principles of assertibility – formulated as principles of belief –

(1) $B(A, D) \supset BC(A, D)$ and $\neg B(A, D) \supset B \neg C(A, D)$

and the postulate of correctness of B

(2) $B(A, D) \supset (D \supset A)$, and therefore $BA \supset A$,

we obtain

(3) $B(A, D) \equiv C(A, D)$,

i.e., an epistemic interpretation of conditionals.

It might be said that (3) still does not imply that $C(A, D)$ and $B(A, D)$ have the same meaning, but in the framework of our semantics of B and C, we have been talking about meanings only in the approximation of intensions, and we are therefore quite satisfied with an intensional equivalence of $B(A, D)$ and $C(A, D)$.

Generally speaking, the distinction between conditions of assertibility and truth conditions is not so clear as it seems to be. "There is no distinction of meaning so fine as to consist in anything but a possible difference of practice" said Peirce, and we say that the meaning of a sentence A is determined by the conventions for its use in a linguistic community P. A convention for (the use of) A, according to D. Lewis (1969), is a regularity in the behavior of the members of P. This behavior depends on what they believe. Thus, even if they understand this convention so that A may be uttered iff it is the case that B, i.e. if the use of A, for them, is governed by truth conditions, how they actually use A does not directly depend on the truth value of B but on what they believe about B. If, for instance, the members of P understand the convention for A so that A may be uttered iff $B_1 \wedge B_2$, but they all believe that in all instances of B_2 B_1 is normally the case, then a person not belonging to P who does not believe this, will observe that they use A in case of B_2 (independently of B_1) and will therefore understand A as expressing B_2 to be the case. For him that is the truth condition for A. Therefore, as long as we do not refer to the beliefs of single subjects but to the common beliefs in P, the linguistic behavior and the use of the language are determined by the conditions of assertibility; these determine its meaning, and the distinction between the truth conditions of a sentence and its assertibility conditions in P will become void, if we have to say that the truth conditions for A in P are truth conditions relative to the common beliefs of the members of P.

There is still another reason to take a more lenient view of an epistemic interpretation of conditionals. Let us first take a look at another better known case: The subjective interpretation of probability. The truth conditions of a statement $P(F)=r$ – the objective probability of an event of type F equals r – have never been adequately stated in a purely objectivist fashion. The interpretation of $p(F)$ by Mises as the limit of the relative frequencies $r(n)/n$ of F's in a series of n trials has been criticized on many points, one of them being that infinite series of trials are never realised – not only are they unobservable, but they are physically impossible. We may therefore merely say that $P(F)$ is the limit of the relative frequencies that would be reached, if an infinite series of trials could be realized. In that case, however, the value of $P(F)$ is an object not of physical inquiry but of our belief. As Peirce remarked, the question of what would occur under circumstances which do not actually arise is not a matter of fact but only of our systematisation of facts. Instead of truth conditions we have, however, conditions of assertibility that determine the use of the concept of objective probability on which we are well agreed, for instance, the (weak) law of great numbers

$$(4) \qquad \Lambda\varepsilon(\varepsilon < 0 \supset \lim_{n\to\infty} p(\{x:|h_n(F, x) - P(F)| > \varepsilon\} = 1),$$

where p is a subjective probability measure, the x's are infinite series of possible results of a trial[6] and $h_n(F\,x)$ is the relative frequency of F's in the first n members of x.

The subjective probability p here apparently cannot be taken to express just anyone's opinions. Let p' be any subjective probability for which the events $Fa_1, Fa_2, \ldots$ are *interchangeable* i.e. $p'(Fa_{i1} \wedge \ldots \wedge Fa_{i_n}) = p'(Fa_{j_1} \wedge \ldots \wedge Fa_{j_n})$ for all n-tuples $(i_1, \ldots, i_n)$ and $(j_1, \ldots, j_n)$. We then can define $p'(F) := p'(Fa_i)$ for any i. Then, as B. de Finetti has shown, there is a function $h(F, x)$ so that $p'(\{x: \lim_{n\to\infty} h_n(F, x) = h(F, x)\}) = 1$, i.e., practically for all sequences x there is a limit of the relative frequencies of F's in x. If $\Phi(F, z) := p'(\{x: h(F, x) \leqslant z\})$ and if p' is regular, i.e., if $d\Phi(F, z) \neq 0$ everywhere so that no value of $h(F, x)$ is *a priori* considered impossible according to p', then $\lim_{n\to\infty}(p'(Fa_{n+1}, A_n^r) - {}^{r}_{n}) = 0$, where A_n^r says that, in the events $Fa_1, \ldots, Fa_n$ r F's have occurred.

This implies that for all p' if the F's are interchangeable with respect to p' and p' is regular, the functions $p'(B, A_n^r)$ converge with increasing numbers n of observations of F's, so that they have a common value $p(F)$. $p(F)$ then is a sort of "epistemic" probability, on which we agree if we have enough common experience concerning the outcomes of F's – a subjective probability objectivized by experience. And, in view of $p(F) = = z \equiv \Lambda\varepsilon(\varepsilon > 0 \supset \lim_{n\to\infty} p(\{x: |h_n(F, x) - z| < \varepsilon\}) = 1$, we have $p(F) = P(F)$ according to (4). There is therefore an epistemic interpretation for objective probability, and since two persons agreeing as to (4) use $P(F)$ in the same way it is questionable whether we can construe a difference of meaning between them. However, we can at least say, that the "epistemicist" is better off than the "objectivist", since he knows what he is talking about when talking about objective probabilities.

The problem with conditionals is somewhat similar. There seems to be no way to explain the truth conditions for statements about necessity, conditional or otherwise, in "objectivist" notions. First we do not have an operative definition of C. In view of the principle $C(A, B) \supset (B \supset A)$ we can falsify such statements by observations. But we cannot thereby justify them. As Hume has pointed out, there is nothing in an observation of the fact that A which would characterize it as necessary or contingent. And how could we justify a statement that A holds in all possible worlds, or in all worlds possible under condition that B?

A possible world is no distant cosmos, which we would have to visit to ascertain by observation whether A is true there or not. Our world, according to Wittgenstein, is the set of facts, i.e., of propositions which hold true in our world. A world, then, is a set W of propositions that is consistent and maximal (so that if p is not in W then $\neg p$ is). In our definition, D1 the "set of worlds" I is a set of *indices* for worlds, and the proposition p is represented as the set of indices $i \in I$ of worlds W_i so that $p \in W_i$. It is then not a question of fact – whether a sentence A is true in i – but of the definition of W_i. If we have defined the sets $b(i, B)$ it is, consequently, not problematic if $b(i, B) \subset [A]$. But if we want to know whether $C(A, B)$ is true in *our* world, we should know which of the worlds W_i with $i \in I$ is our world; having only limited information about our world, all we can say is that it belongs to some set $I' \subset I$, which will still be very inclusive. Only if for all $j \in I'$ $b(j, B) \subset [A]$ could we say, on the basis of our information, that $C(A, B)$ is true in our world.

The empirical problem of justifying a statement $C(A, B)$ therefore refers only to our world, the real world – there is no question of metaphysical observations – but it is such that we cannot claim, except in trivial cases, that $C(A, B)$ is true on the basis of limited information about our world.

The problem of ascertaining by observation whether $C(A, B)$ is true, however, is not the same problem as furnishing objective truth conditions for $C(A, B)$. In the case of general statements $\Lambda x F(x)$ there is a corresponding Humean problem of justifying such statements by only finite observations, but there we usually see no difficulty in furnishing objective truth conditions.[7]

Conditional necessity is an objectivist notion if we can define or explain the sets $b(i, A)$ by other objectivist concepts. Let us interpret $b(i, B)$ as the set of B-worlds most similar to i, in the fashion of R. Stalnaker and D. Lewis. This similarity is not defined with respect to some properties of the worlds – that all or not all of them contain certain propositions – but it is an overall similarity. Lewis compares our faculty to diagnose such overall similarities of worlds with our faculty to detect overall similarities in such sizable and variegated objects as cities or people. But statements such as "San Francisco is more similar to Los Angeles than to Boston" or "Swedes are more similar to Turks than to Spaniards", without further specifications as to the aspect of comparison, are too vague to be informative or have a definite truth value. Conditional statements would then presumably be as totally vague as are those about the overall similarity of other worlds with ours, of which we enjoy no overall knowledge. While statements about the similarity of people can be rendered more precise by stating the area of their asserted similarity, i.e., by foregoing the pretense of overall similarity for which there are no criteria, this is explicitly excluded in the case of conditionals on Lewis' analysis. Also, worlds, unlike cities or people, are abstract entities which do not impress us as having a certain total character.

The similarity of two worlds *i* and *j*, according to Lewis, is determined not only by the wealth of details which they share but also by the *importance* of the propositions common to both[8]. Now importance is not only a "highly volatile matter", as Lewis (quoting Goodman) remarks but also a highly subjective term. We generally call something "important" with respect to some person or persons or their aims, and we would be at a loss to understand a statement that something was important *per se*, irrespective of its importance for anyone.

If we choose the notion of relative necessity as basic notion instead of similarity, I cannot see any way how this concept may be explained in an objectivist way, just as I do not see that unconditional (strong) necessity, discussed for centuries, has ever been accounted for in objectivist terms. We have a fairly good understanding of what might be the case if something else were the case. But this seems to be based solely on our common expectations, on our systematisations of the facts and our preferences, according to which some of them are more firmly established, more central, more important than others. As in the case of objective probability, we can again say: Since the "epistemicist" and the "objectivist" agree in their use of conditionals, it is doubtful that a difference may be construed in the way they understand them. And as long as no "objectivist" interpretation of conditional necessity is forthcoming, the "epistemicist" has the advantage over the "objectivist" of knowing what he says when asserting a conditional.

I expect that conditionals are only one example out of a larger class of intensional statements which, though objective in their content and not explicitly involving beliefs and preferences, have to be interpreted in the light of common beliefs and preferences. But that is another story.

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NOTES

[1] A logic of comparative conditional probability was first formulated in B. O. Koopman (1940). Koopman also proved that such a comparative structure can be metricized by a conditional probability p obeying the usual axioms, so that $p(A, B) \leqslant p(C, D) \equiv \equiv A, B \leqslant . C, D$. For a brief exposition of epistemic logic, cf. Kutschera, forthcoming, chapter 4.

[2] For the sake of brevity we use the logical operators of our object-language *LB* also as metalinguistic symbols.

[3] So far, aside from B7, B8 and B11, $\mathfrak{C}$ is equivalent to D. Lewis' system *VW*; cf. Lewis (1973, pp. 132 seq.)

[4] Cf. Lewis (1973, pp. 57 seq.)

[5] Lewis' system (1973) is formulated for propositional logic only, so B11 is missing there too. In Stalnaker's system (in 1968) there is the additional principle $C(A, B) \wedge \wedge C(\neg A, B)$, which is too strong. Cf. Lewis' criticism in (1973, p. 80.)

[6] There is no presupposition here that such infinite sequences could ever be realized by sucessive trials.

[7] In traditional philosophy, however, generality was a modal notion closely connected with necessity. A fresh look at this conception, I think, would be rewarding.

[8] Cf. Lewis (1973, pp. 91 seq.)

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