

## INTENSIONAL SEMANTICS AND NATURAL LANGUAGE

F. v. Kutschera

I have been asked to give a brief survey of the connections between intensional semantics and semantics for natural languages, i.e. between a branch of logic and one of linguistics. Since these connections are the result of a still very active development, this survey can only be concerned with the general outlines and so is not addressed to the specialists in the field. My aim is just to give those of you who have not studied intensional logic an inkling of what it is about, and how its language is employed in the analysis of natural languages. I shall, however, have to presuppose some basic knowledge of elementary logic.

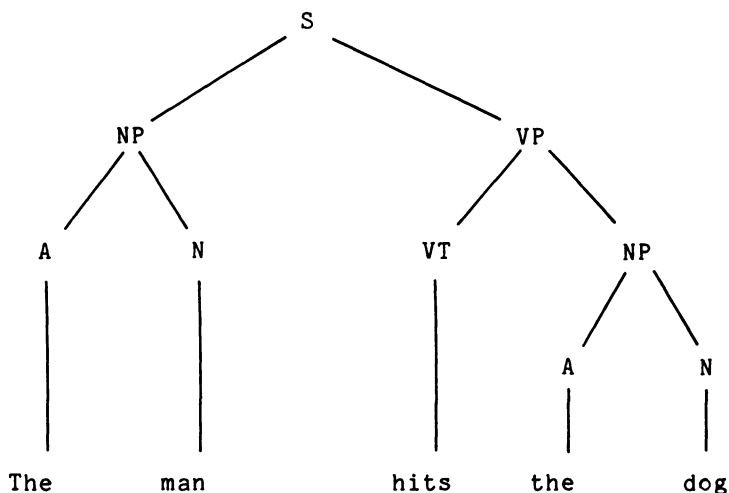
I

First, let me briefly sketch the development in theoretical linguistics that has been leading up to today's close cooperation with logic.

Logic first gained influence in linguistics when its standards of preciseness for the syntactical description of languages were taken over by linguists. It is, among others, the merit of Y. Bar-Hillel and of N. Chomsky to

have firmly implanted this idea in modern grammar. Modern logic from its beginning - essentially since Frege's "Begriffsschrift" (1879) - has been using artificial languages that are syntactically and, since Tarski's paper on the concept of truth of 1931, also semantically built up in a rigorous manner. The, so to speak, idealized laboratory conditions under which such artificial languages are constructed allow an exactness of their grammatical rules, and therefore of linguistic analysis that contrasts very positively with the vague concepts and the assertions of doubtful generality in traditional grammar. Clearly, natural languages, evolving from long historical developments, are much more complex and difficult to describe by exact rules than constructed languages, that are simply defined by such rules. But if the property of well-formedness of the sentences of natural language L is decidable, as it should be, then on Church's thesis on the mathematical definability of the concept of decidability, and in view of the development of general systems for generating decidable sets of expressions in metamathematics, there must be such systems for generating the sentences of L. Generative grammar mostly uses Semi-Thue-systems. These, in effect, are the Chomsky-grammars Professor Lindenmayer mentions in his paper. If "S" (for "sentence"), "NP" (for "noun phrase"), "VP" (for "verb phrase"), "A" (for "article"), "N" (for "noun"), "VT" (for "transitive verb"), etc. are grammatical symbols, and the expressions from the lexicon of L provide the terminal vocabulary, the well-formed sentences of L can (in a first approximation) be described as the expressions derivable from the symbol S by applications of the rules of the system. These rules are of the form  $X\sigma Y \rightarrow X\tau Y$ , where  $\sigma$  is a grammatical symbol and  $\tau$  such a symbol of a terminal expression. With such rules we can derive, for instance, the sentence

"The man hits the dog" in the following way:



This model has the advantage of being familiar for linguistics: the sentences of a language are analysed into a linearly concatenated sequence of constituents and this parsing operation can be performed at various levels of generality to yield a hierarchical branching-diagram.

There are many complications involved in this grammatical model that I shall not dwell upon here. Let me just say that the end-expressions of such derivations represent only the deep-structures of the sentences of L which in many cases do not coincide with their surface structures i.e. their normal grammatical forms. These have to be derived from the deep structures by transformation rules which rearrange the expressions, take care of congruence, mode, number, etc.

But even if you count the theory of Semi-True-Systems as a logical theory, this is not a syntactical analysis of the sentences of L that could be termed "logical" in any

stricter sense, since it is based on the categories "verb phrase" etc. of traditional grammar. So this was a step in the direction of a logical analysis of natural language but it did not carry very far.

The first attempt at a generative semantics as made by Fodor and Katz in (63) was even less successful. They tried to coordinate semantical rules to the syntactical ones, but since the basic type of their semantical projection rules was only that of forming a conjunction of one-place attributes, this attempt ended in failure.

The failure, however, of these projects to integrate logical ideals into the framework of traditional grammar cleared the way to linguistic analyses that are logical in a deeper sense. The idea seemed more and more attractive to depart from the categories of traditional grammar and use logical categories instead, as developed by K. Ajdukiewicz, St. Łeśniewski, Y. Bar-Hillel, H.B. Curry and others, and to represent the deep-structure of the sentences by formulae of a logical language. Syntactically, this idea was not very revolutionary since a lot of the complications of natural languages were already deferred to the transformational part of the grammar, which now could be left essentially unchanged. The only syntactical problem was not to make the deep structure too different from the surface structure of a sentence which it will be if the usual logical representation is used.

Semantics, however, at first presented the difficulty that natural languages are full of non-extensional contexts, while logic, till about 15 years ago, had only extensional semantics to offer and then till about the end of the sixties only intensional semantics for elementary types of language.

## II

W.V. Quine in his paper "The Problem of Meaning in Linguistics" (51) and in other papers since has argued that, while the theory of reference, i.e. of the extensions of expressions is, thanks to the work of Tarski and others, a sound and rigorous discipline, the theory of meaning is still in a desolate state since it has not even been able to define its basic notions, as those of proposition, attribute, synonymity, analyticity, etc. Neither, according to Quine, was it ever likely to attain the state of a sound discipline since these concepts cannot be rigorously defined. To vary a Wittgensteinian dictum, Quine thought that all that can be said clearly can be said in an extensional language, and whereof we cannot speak clearly, we should be silent.

In his book "Meaning and Necessity" (47), however, R. Carnap had already shown the way to a rigorous definition of these concepts in the same set-theoretical framework extensional semantics uses. His idea was roughly this: If we know the meaning of A we know in which possible worlds it is true. The inversion of this principle is not so obvious: Do we know the meaning of a sentence if we know under which conditions it would be true? But we can at least define a concept of intension as a first approximation to that of meaning by postulating that this inversion holds. Then we have for two sentences A and B: The intension of A is identical with that of B iff they have the same truth value in all possible worlds.

And we can define the intension of A by abstraction to be that function  $f$ , such that for every world  $i$   $f(i)$  is the truth value of A in  $i$ . This can be generalized for other types of expressions: The intension of an expression E is that function which assigns to every world  $i$  the extension of E in  $i$ .

A possible world is no distant cosmos on whose existence we speculate, but, as our world can be defined, according to Wittgenstein, as the set of all facts, a possible world, or simply a world can be defined as a set of propositions that is consistent and maximal, i.e. a "complete novel".

As two <sup>2</sup>logically equivalent sentences like " $2 + 2 = 4$ " and " $dx/dx = 2x$ " have identical intensions but different meanings - meanings are to be defined as that two expressions, that are identical in meaning, may be substituted for each other in all contexts salva veritate - intensions are but approximations to meanings. They are, however, good approximations since it is possible, as we shall see, to define meanings with the help of intensions.

Carnap's ideas were first put to use in modal logic by S. Kripke and others, although with a slight modification of the basic idea: instead of sets of worlds they used sets of interpretations. The language  $L$  is that of propositional or of first-order predicate logic with an additional sentential operator  $N$  for necessity, and a model of  $L$  is a set of function  $\Phi_i$  with  $i \in I$  that have the properties of the usual extensional interpretations while  $\Phi_i(NA)$  depends not only on  $\Phi_i(A)$  but also on the values  $\Phi_j(A)$  with  $j \neq i$ . A model for propositional logic for instance is a triple  $\langle I, S, \Phi \rangle$ , so that

- a)  $I$  is a non-empty set of worlds (or of indices for interpretations).
- b) For all  $i \in I$ ,  $S_i$  is a subset of  $I$  with  $i \in S_i$ .  $S_i$  is to be the set of all worlds accessible from  $i$ . Different concepts of necessity may be obtained by different stipulations for these sets  $S_i$ .

c) For all  $i \in I$ ,  $\Phi_i$  is a function from the set of sentences into the set  $\{t, f\}$  of truth-values so that

c1)  $\Phi_i$  satisfies the conditions for extensional propositional interpretations, and

c2)  $\Phi_i(NA) = t$  iff  $S_i \subseteq [A]$ ,

where  $[A]$  is the set  $\{j \in I: \Phi_j(A) = t\}$  of A-worlds. Such intentional models made it possible for the first time to define the formal properties of the intuitive notions of necessity exactly and to prove the soundness and completeness of systems of modal logics with respect to such notions. Up to Kripke's work there was a host of competing axiomatic systems of modal logic, while nobody could justify his intuition that his axioms should make up an adequate system, nor say exactly how his notion of necessity compared with others. Different concepts of necessity could now be distinguished semantically as we pointed out, by the conditions they impose on  $S_i$ . For instance, if  $S_i = I$  for all  $i \in I$  then  $N$  expresses the notion of analytical necessity - i.e.  $NA$  holds iff  $A$  is true in all possible worlds, or in all circumstances. On the other hand, if we determine  $S_i$  as the set of all worlds in which the same natural laws hold as in  $i$  then  $NA$  is true iff  $A$  is a consequence of the laws of nature.

There has been a lot of fruitful research in modal logic in the wider sense since, including for instance deontic, epistemic and conditional logic. Instead of sets  $S_i$  families of sets or families of sets of sets were used. But all this did not give the general framework for the application of this sort of semantics to natural languages. What was needed was a richer language than that of first-order predicate logic, and a simple and general characterization for the different types of intensional functors.

### III

This was provided at the end of the sixties in several papers, foremost in R. Montague's "Universal Grammar" (70). Let me briefly sketch his language, call it  $M$ , in an extensional and an intensional interpretation, so that we get a better notion of what intensional semantics is like.

First we define categories:

D1: a)  $\sigma$  and  $\nu$  are categories (of sentences and proper names).

b) If  $\tau$  and  $\rho$  are categories,  $\tau(\rho)$  is a category (of functors which applied to expressions of category  $\rho$  produce expressions of category  $\tau$ ).

So a one-place predicate is a functor of the category  $\sigma(\nu)$ , since it produces sentences if applied to proper names; and the operator  $\iota x$  for definite descriptions is a functor of category  $\nu(\sigma(\nu))$ , since it produces a proper name if applied to a one-place predicate. (We can write  $\sigma(\nu, \nu, \dots, \nu)$  for  $\sigma(\nu)(\nu)\dots(\nu)$ , i.e. for the categories of  $n$ -place predicates).

$M$  is to contain the symbols  $\lambda$  (for functional abstraction),  $\equiv$  (for identity), brackets and an infinite supply of constants and variables for each category.

The well-formed expressions of  $M$  are called terms of  $M$ :

D2: a) All constants of  $M$  of category  $\tau$  are terms of category  $\tau$ .

b) If  $F$  is a term of category  $\tau(\rho)$  and  $t$  a term of category  $\rho$   $F(t)$  is a term of category  $\tau$ .

c) If  $A[b]$  is a term of category  $\tau$  and  $b$  a constant and  $x$  a variable (not occurring in  $A[b]$ ) of category  $\rho$ , then  $\lambda x A[X]$  is a term of category  $\tau$ .

$\tau(\rho)$ .

- d) If  $s$  and  $t$  are terms of the same category,  $(s \equiv t)$  is a term of category  $\sigma$ .

For the interpretation of  $M$  we first define the sets of possible extensions of terms of category  $\tau$  relative to the universe of discourse  $U$ :

$$D3: E_{\nu, U} = U$$

$$E_{\sigma, U} = \{t, f\}_{E_{\tau, U}}$$

$$E_{\tau(\rho), U} = E_{\tau, U}$$

where  $A^B$  is the set of functions from  $B$  into  $A$ .

D4: An extensional interpretation of  $M$  over  $U$  is a function  $\Phi$  such that

a)  $\Phi(a) \in E_{\tau, U}$  for all constants  $a$  of category  $\tau$ .

b)  $\Phi(F(t)) = \Phi(F)(\Phi(t))$ .

c)  $\Phi(\lambda x A[x])$  is the function  $f \in E_{\tau(\rho), U}$  so that for all  $\Phi'$  with  $\Phi' \stackrel{b}{=} \Phi$   $f(\Phi'(b)) = \Phi'(A[b])$  (where the constant  $b$  does not occur in  $\lambda x A[x]$  and  $\Phi' \stackrel{b}{=} \Phi$  says that  $\Phi'$  and  $\Phi$  coincide with the possible exception of the values  $\Phi(b)$  and  $\Phi'(b)$ ).

d)  $\Phi(s \equiv t) = t$  iff  $\Phi(s) = \Phi(t)$ .

$M$  is a type-theoretical language with predicates treated as truth value functions as Frege proposed in his "Funktion und Begriff" (1891) and two and more-place functions treated as one-place functions as in combinatory logic. As Tarski has shown we can define the usual logical operators,  $\neg, \vee, \wedge, \in$  in  $M$ .

Intensional interpretations of  $M$  may then be defined

thus: We supplement the alphabet of M by two new symbols  $\mu$  and  $\delta$ .  $\mu t$  is to be an expression whose extension is the intension of  $t$ .  $\mu t$  occurring instead of  $t$  signifies that  $t$  stands in an indirect or non-extensional context, where its extension, according to Frege, is its usual intension. We need then new categories for such expressions and incorporate into D1 the condition:

D1c: If  $\tau$  is a category then  $\iota(\tau)$  is a category (of expressions of the form  $\mu t$ ).

and into D3 the definition:

$$E_{\iota(\tau), U} = E_{\tau, U}^I,$$

so that extensions of expressions of category  $\iota(\tau)$  are intensions of expressions of category  $\tau$ .

$\delta$  is to be an operator such that  $\delta\mu t \equiv t$ . The operator  $\delta$  de-intensionalizes  $t$ , therefore. D2 is then supplemented by two stipulations:

e) If  $t$  is a term of category  $\tau$ ,  $\mu t$  is a term of category  $\iota(\tau)$ .

f) If  $t$  is a term of category  $\iota(\tau)$ ,  $\delta t$  is a term of category  $\tau$ .

D5: An intensional interpretation of M over U and I (a non-empty set of worlds) is a function  $\Phi$  such that for all  $i \in I$ :

a)  $\Phi_i$  satisfies the conditions for extensional interpretations of M over U according to D4.

b)  $\Phi_i(\mu t) = \lambda_j^* \Phi_j(t)$  (where  $\lambda^*$  is a metalinguistic symbol for functional abstraction).

$$c) \Phi_i(\delta a) = \Phi_i(a)(i).$$

Instead of NA we have to write  $N\mu A$  now;  $N$  is a functor of category  $\sigma(i(\sigma))$ , producing sentences if applied to terms expressing intensions of sentences. The extension of  $N$  in a world  $i$  is that function  $g_i$  from

$$E_{\sigma(i(\sigma))} = \{w, f\}^{(i)} \quad (\{w, f\}^I)$$

such that for all functions  $h$  from  $\{w, f\}^I$   $g_i(h) = w$  iff for all  $j \in I$ , we have  $j \in S_i \supset h(j) = w$ . The intension of  $N$  is that function from  $E_{\sigma(i(\sigma))}^I$  which coordinates to every

$i \in I$  the function  $g_i$ . The operator  $\delta$  is used only in a few cases; for instance in case you want to quantify over intensions of names as in  $\lambda x N\mu F(\delta(x))$  instead of over their extensions as in  $\lambda y N\mu F(y)$ .

Condition (c) of  $D4$  now is to be modified so that  $\Phi'$  is an interpretation with  $\Phi'_i(b) = \Phi'_j(b)$  for all  $j \in I$ : We want to quantify over  $E_{\rho, U}^i$  if  $b$  is of category  $\rho$ , and since there are more functions in  $E_{\rho, U}^i$  than objects of  $E_{\rho, U}^i$ , and since  $\Phi'_i(A[b])$  may depend on values  $\Phi'_j(b)$  for  $j \neq i$ , we must restrict the  $\Phi'$ 's accordingly. If  $\Phi'_j(A[b])$  does not depend on values  $\Phi'_j(b)$  for  $j \neq i$ , then the  $i$  nature of the restriction does not matter; if it does, then  $\lambda x A[x]$  may make no sense - that was Quine's argument against quantifying into modal contexts - and in that case again any restriction will do. If we interpret individual constants  $b$  as standard names, however, so that  $\Phi'_i(a) = \Phi'_j(a)$  for all  $j \in I$  - and S. Kripke has given good reasons for that in "Naming and Necessity" (72) - then quantification over individuals into modal contexts makes sense, the same sense as our interpretation of expressions of the form  $\lambda x A[x]$ .

Quine's argument, briefly, was this: The principle  $NF(a) \rightarrow \forall x NF(x)$  does not hold, since  $\forall x NF(x)$  means "There is one identical object to which the predicate  $F$  applies in all the worlds accessible from ours", while, if a names different objects in different accessible worlds,  $NF(a)$  does not imply the existence of such an object. Generally speaking, for modal contexts  $A$  the principle

$A[a] \rightarrow \forall x A[x]$ , which is a truth of extensional logic, holds only for standard names  $a$ ; for other names we have only  $A[a] \vee \forall x (x=a) \rightarrow \forall x A[x]$ .

A word may be in order on the much discussed problem whether all the worlds in the set  $I$  should contain the same individuals, as we have stipulated, following Montague, or not, and how trans-world-identity is to be understood, or if there can only be correspondences, counterpart-relations as D. Lewis suggests in (68) e.g. but no identities.

First the objects in  $U$  are to be possible objects. For each world  $i \in I$  we may introduce sets  $U_i \subset U$  of objects existing in  $i$  and these sets may be  <sup>$i$</sup> different for different  $i$ 's. If  $E$  is a constant of category  $\sigma(v)$  and  $\Phi_i(E) = U_i$  we may define quantification over existing instead of possible objects in the manner of Free Logic by  $\Lambda x A[x] := \Lambda x (E(x) \supset A[x])$ . Second we can take the identity of objects as a basic notion that need not be defined for each world by the Leibniz-principle of coincidence of properties, or for different worlds by a restricted Leibniz-principle of coincidence of "essential" properties or something of that sort. Introducing counterpart-relations in the sense of D. Lewis certainly makes for higher generality, but I know of no cases where this increase in generality is fruitful, and therefore I prefer simplicity.

Since non-extensional contexts are very frequent in natural languages the use of the  $\mu$ -operator is somewhat tedious. Therefore we might either treat all functors as correlating extensions to intensions, or assign intensions to the expressions directly. But as we want to distinguish, for instance, between quantification over extensions and that over intensions, between quantification over individuals and quantification over individual concepts, we have to mark the difference syntactically any way so that we cannot hope to get off

much cheaper by such approaches than in languages of the Montague-type.

#### IV

If  $L$  is a natural language and  $M$  an interpreted Montague-language then a logical grammar for  $L$  is defined by an analyzing relation  $R(A,B)$  on  $T(M) \times T(L)$ , where  $T(M)$  is the set of well-formed expressions of  $M$  and  $T(L)$  this set for  $L$ , such that

1) For all  $B \in T(L)$  there is an  $A$  with  $R(A,B)$ .

2) If  $R(A,B)$  then the meaning of  $A$  is a possible meaning of  $B$ . If  $R$  is explicitly defined, all essential grammatical concepts for  $L$  can be defined from this relation.

If  $R(A,B)$ , then the expression  $A$  represents the deep-structure of  $B$  with constants of  $M$  in place of words or morphemes of  $L$ . There is no need now to supply analyses of deep-structures in the form of their derivations, since the structure of the terms of  $M$  is unambiguous.  $R$  may be taken to contain the rules of substitution of the terminal vocabulary of  $L$  for grammatical symbols in Generative Grammar as well as its transformational part.

#### V

Analyzing relations have been given only for very small fragments of natural languages. There are numerous difficulties to overcome if they are to be defined for larger and more interesting parts of language. I shall only mention some to convey an impression of the complexity of a logical analysis of natural language:

1) First there is the syntactical problem that logical deep structure, i.e. the structure of the terms of M, is often very different from the surface structure of the terms of L. This makes for very complicated transformations, and therefore is an incentive to change the usual logical representation. Take the following two examples:

a) Quantifiers like "everybody", "somebody", and "nobody" are treated in English like proper names in the sentences Joe sings, Everybody sings, Nobody sings. Instead of representing those sentences in the usual form  $G(a)$ ,  $\forall xG(x)$  and  $\exists xG(x)$ , there have been attempts therefore, to assimilate proper names to quantifiers by treating them as functors of category  $\sigma(\sigma(v))$ , or by treating quantifiers ("a man", "all men", "no man"), as well as proper names, as names for bundles of properties (the "universal-generic man" having those properties that all men have, the "existential-generic man" having the properties that some man has etc.). Cf. Lewis (70), e.g.

b) In the German sentences

α) Fritz singt laut (Fritz sings loudly)

β) Fritz singt gern (Fritz likes to sing)

γ) Fritz singt wahrscheinlich (Probably Fritz sings)

the adverbs have the same function in surface structure though logically they are to be treated quite differently: "wahrscheinlich" is applied to the proposition that Fritz sings, "laut" characterizes the verb, and "gern" has itself the function of a verb, as becomes apparent in the English translations. The usual logical representations of the three sentences would look something like this  $\forall F(S(f) \wedge f(a) \wedge L(f))$  ("There is an action of singing that Fritz performs and that has the property of being loud"),  $F(a,g)$ , and  $P(f(a))$ . "singt" occurs in (α) as a 2nd-order predicate, in (β) and (γ) as a 1st-order predicate.

These two examples show that we should look for non-standard logical representations of ordinary language sentences close to their syntactical surface structure.

2) Generally speaking, there is a variability and plasticity of the terms of natural languages quite unparalleled in logic. The same term of L often has to be coordinated by the analyzing relation R to many categorially and semantically different terms of M. The task of getting along with a minimum of morphemes without ending up with ambiguity in too many cases is solved much better by natural languages, it seems, than by logic. It is quite an interesting problem whether we could not do better in logic even if we hold on, as we should, to the principle of unambiguity in all cases.

3) Besides the syntactical problems of natural languages analysis there are semantic problems which call for generalizations of the concept of an interpretation of M defined in D5. While we usually only consider eternal sentences in logic, many sentences of L contain index-expressions like "I", "you", "here", "now", "yesterday", "this" etc., whose extensions vary for different utterances of the same sentence. Therefore extensions and intensions must be defined for utterances, i.e. pairs  $\langle A, j \rangle$  of a sentence A and an occurrence of A. If J is a set of n-tuples of parameters, specifying speaker, audience, time, place, indicated things etc., i.e. a set of points of reference, then we may introduce in D5 besides  $i$  another index  $j$  for  $\Phi$  so that  $\Phi_{i,j}(A)$  is the extension,  $\lambda^* i \Phi_{i,j}(A)$  the intension of the utterance  $\langle A, j \rangle$  of A, while  $\lambda^* j \Phi_{i,j}(A)$  is the extension and  $\lambda^* i j_{i,j}(A)$  the intension of the sentence A.

There  $i,j$  is, however, no obvious limitation of the parameters in j, so that we should perhaps take j just as an index for a space-time-point in i where A was uttered, as suggested by D.Lewis in (69). The idea is that if we

know this point  $j$  and world  $i$  we know all the relevant details of the utterance: speaker, listener, object(s) spoken about etc. The meaning of an utterance may also depend, for instance, on the facts obvious for speaker and audience in the situation of its occurrence as in the sentence "I shall now go (which may mean: walk, drive, go by train, fly) to Boston".

4) In ordinary language there are well-formed but meaningless expressions, as "17 laughs", "The King of Bavaria is sitting in the audience", "If you were alive, you could read this paper", etc. Most empirical predicates are not defined for all syntactically admissible arguments and many sentences for being meaningful presuppose that something is the case which in fact may not be the case at all. Therefore we should, following D. Scott in (70), define the sets of possible extensions for the non-basic categories by

$$E_{\tau(\rho),U} = E_{\tau,U}^{(E_{\rho,U})} \quad \text{and} \quad E_{\iota(\tau),U} = E_{\tau,U}^{(I)}$$

where  $A^{(B)}$  is the set of functions from subsets of  $B$  into  $A$ .

5) Besides syntactical ambiguity (As "Flying planes can be dangerous") and pragmatic ambiguity (as in "The problem I mentioned above was first noted by Quine"). As semantic ambiguity is often eliminated by the context ("Peter is going to the bank to cash a cheque"), we should not represent all ambiguous words by different constants of  $M$ . Instead we might assign classes of extensions to expressions and formulate the conditions in D5 thus:

- a)  $\Phi_i(a) \subset E_{\tau,U}$  for all constants  $a$  of category  $\tau$ .
- b)  $\Phi_i(F(t)) \stackrel{\text{def}}{=} \{\gamma \in E_{\tau,U} : \forall \alpha (\alpha \in \Phi_i(F) \wedge \beta \in \Phi_i(t) \wedge \alpha(\beta) = \gamma)\}$

- c)  $\Phi_i(\lambda x A[x])$  is that class of functions  $f \in E_{\tau(\rho), U}$  such that for all  $\Phi'$  with  $\Phi'_i = \Phi$ ,  $\Phi'_i(b) = \{\alpha\}$  and  $\Phi'_j(b) = \Phi'_i(b)$  for all  $j \in I$  there is a  $\beta \in \Phi'_i(A[b])$  with  $f(\alpha) = \beta$ .
- d)  $\Phi_i(s \equiv s') \equiv \{v \in E_{\sigma, U} : \forall \alpha \beta (\alpha \in \Phi_i(s) \wedge \beta \in \Phi_i(s') \wedge (\alpha = \beta \wedge \gamma = t) \vee (\alpha = \beta \wedge \gamma = f))\}$ .
- e)  $\Phi_i(\mu t) = \{f \in I_{\tau, U} : \bigwedge j \forall \alpha (\alpha \in \Phi_i(t) \wedge f(j) = \alpha)\}$ .

Then an expression  $t$  is unambiguous in  $i$  iff  $\Phi_i$  is a unit-class. We may then also abandon  $i$  partial interpretations as considered under (4), since we can represent a function  $f = \Phi(F) \in E_{\tau, U}(E_{\rho, U})$  which is defined on the subset  $E' \subseteq E_{\rho, U}$  by the set of functions from  $E_{\tau, U}$  coinciding on  $E'$  with  $f$ . "Laugh", for instance, may be so interpreted, that its extension is a set of functions from  $E_{\sigma(v), U}$ , which have the same values only for human beings as arguments. So the sentence "Peter laughs" is unambiguous and has only one truth value as extension, while the sentence "17 laughs" has as its extension the set of both truth values.

6) Not all differences in meaning can be represented by differences in extension. The two sentences "Jack believes that  $2+2=4$ " and "Jack believes, that  $dx^2/dx=2x$ " may have different truth-values though " $2+2=4$ " and " $dx^2/dx=2x$ " have the same intensions, as we saw. There is one approach to meaning, first taken by S. Kripke in his completeness proofs for the modal systems S1, S2 and S3, envisaging abnormal worlds, in which not all logically true sentences hold. This has the advantage of formal simplicity, but there is no way of determining what sort of absurd worlds we should assume to account for the logical incapacibilities of all possible people in all our possible worlds.

Another approach is this: We introduce indices  $k \in K$  for

the terms of M. Let  $k(A)$  be the index of the term A. Then we define  $\Phi_{i,k}$  as in D5 and introduce an operator  $\lambda$  such that  $\Phi_{i,k}(\lambda t) = \lambda * i \Phi_{i,k}(t)$ . This way we assign a term t an intension for every context A, represented by  $k(A)$ , in which it occurs.

$\Phi_{i,k}$  can, for instance, be defined so that  $\Phi_{i,k}(\lambda s) = \Phi_{i,k}(s)$  iff t is obtained from s by substituting constants with the same intensions. Then this concept of meaning coincides with Carnap's notion of intensional isomorphism in (47). Carnap suggested as a criterion for identity of meaning for the terms of a formal language, that they be constructed from intensionally identical expressions in the same syntactical way.

7) Besides descriptive sentences natural languages also contain questions, imperatives, exclamations, guesses, suggestions etc. As has been emphasized especially by J.L. Austin in (55) and J.R. Searle in (70) a semantics of natural language has also to account for these illocutionary modes of sentences or utterances.

We may, however, assign the question "Is Tom coming?" as an utterance, addressed by John to Jack, the (descriptive) meaning of the assertion "John asks Jack, whether Tom is coming" as its performatory meaning. And the question "Is Tom coming?", as a sentence, can be assigned the descriptive meaning of the predicate "to ask, whether Tom is coming". In this way, which is essentially identical with what D. Lewis proposed in (70), we can, with the help of illocutionary verbs like "order", "ask", "promise" etc., define the semantics for other illocutionary modes in the framework of a semantics for assertions.

So the attempt at a logical analysis of natural language suggests quite a few syntactical and semantical modifications of the language M. Besides such specific problems encountered in logical grammar for natural languages we should also mention some fundamental objections that have been raised against the whole project:

1) Natural languages are vague in many respects, syntactical and semantical. Analysing such languages, it has been said, by assigning them exact logical descriptions is therefore inadequate in principle since it projects on them a higher degree of precision than they actually have and is therefore a modification rather than a description. It is not the task of a grammar of a language L to transform L into a precise language in the sense of logic, but to mirror faithfully the properties L actually has.

This is not just the difficulty of how to derive the properties of a natural language from observations of how it is used, as D. Lewis suggests in (69), but as a natural language it is not something precise but fuzzy all over. Instead of a well-defined class of well-formed expressions there are degrees of grammaticalness; instead of predicates with well-defined domains there are predicates more or less well-defined for different arguments; instead of a well-defined class of possible interpretations of a term t there is a class of more or less possible or natural interpretations of t.

In view of this John R. Ross in (73) gave the advice to grammarians: "You have to get yourself thinking the fuzzy way!" Now, for logicians at least, this cannot mean thinking the vague or unprecise way, but only thinking

the comparative instead of the classificatory way. This means that, after the more fundamental difficulties of logical grammar are overcome, we should think of defining notions like "Expression  $s$  is more well-formed than expression  $t$ ", " $\Phi$  is a more typical (or normal) interpretation of  $t$  than  $\Phi'$ " and " $s$  is less vague than  $t$ ". In that way we may also define comparative concepts of synonymy and analyticity, as advocated by Quine. If, just to give an example, we have a relation of comparative similarity of worlds, as employed for instance by R. Stalnaker in (68) and D. Lewis in (73) in their analyses of conditionals, we might say that sentence  $A$  is at most as analytical as  $B$  iff  $A$ -worlds are at least as similar to the real world as  $B$ -worlds. Such comparative concepts certainly make for higher complexity, but I see no a priori reasons why logic should not be able to mirror the fuzziness of natural languages this way.

2) Accounting for vagueness in this way would also solve another fundamental problem, pointed out by Quine: The interpretation of  $M$  - and if we analyze a natural language  $L$  with the help of  $M$  also that of  $L$  - depends on the set  $I$  of possible worlds. Now we cannot take  $I$  to be the set of all logically possible worlds, since the analytic sentences of  $L$  are to hold in all worlds of  $I$ . If, on the other hand, we determine  $I$  as the set of worlds in which all analytic sentences of  $L$  hold, then  $I$  is not well-defined since, as Quine has convincingly shown, the set of analytic sentences is not well defined. There is no firm boundary between analytic and synthetic truths, and with a little ingenuity you can always think of bizarre words, where the validity of supposedly analytic statements becomes doubtful. But if we admit partial interpretations, vagueness and a comparative concept of analyticity, we can take  $I$  to be the set of all logically possible worlds, 5-dimensional ones and those with married bachelors included, but with the non-

logical terms (almost) undefined there.

3) The most fundamental objection against intensional semantics, at last, comes to this: The whole approach of this semantics is based on the realistic idea, that we confer extensions, intensions, and meanings on linguistic expressions by coordinating extra-linguistic entities, concrete things, attributes, propositions etc. to them. That way we can abstract semantics from pragmatics, semantic coordination from the use of the expressions in accordance with these correlations. But this idea has been questioned with, as I believe, very sound arguments from Peirce onward. The slogan of today's Philosophy of Language is: "The meaning of a word is determined by its use". Use, therefore, comes before, not after meaning, and therefore pragmatics, not semantics, is the fundamental discipline. Though we can certainly distinguish and identify many properties and facts without the use of language, a large and important class of concepts and propositions is defined only with the help of linguistic distinctions. In this sense Wittgenstein said: "How do I know that this color is red?" - An answer would be: "I have learned English" ((53), 381). Semantics, therefore, is not a theory of correlations of words with meanings, defined independently of language, but it has to be based on a theory of linguistic behavior.

In his introduction to "Word and Object" (60), p.IX Quine said: "Language is a social art. In acquiring it we have to depend entirely on intersubjectively available cues as what to say and when. Hence there is no justification for collating linguistic meanings, unless in terms of men's dispositions to respond overtly to socially observable stimulations."

His "hence", however, is a non sequitur: Every semantics that is useful for the analysis of linguistic phenomena

is thereby practically justified, no matter what theoretical constructs it employs - if it makes no pretense of being able to explain the fundamental facts of language. That, however, has never been the aim of intensional semantics. A deeper, philosophical analysis of meaning has to start from linguistic conventions in the sense of D. Lewis in (69) or from non-natural meaning in the sense of P. Grice. But it can also be shown that the descriptions of meanings in the framework of intensional semantics may be based upon descriptions of such conventions.

To sum up this brief survey we can say then that intensional semantics for natural languages, though still facing a lot of problems, has proved to be a very effective instrument for linguistic analyses. From a logical point of view its interest lies in the fact that a closer look at the phenomena of natural languages is giving new stimulations to logical developments. Logic, in the process of its cooperation with linguistics has become more interesting again - at least for philosophically minded people.