

# Deep Inelastic Scattering Beyond Perturbation Theory

V.M. Braun \*

Max-Planck-Institut für Physik  
– Werner-Heisenberg-Institut –  
P.O.Box 40 12 12, Munich (Fed. Rep. Germany)

## Abstract:

I discuss possibilities to observe the instanton-induced contributions to deep inelastic scattering which correspond to nonperturbative exponential corrections to the coefficient functions in front of parton distributions of the leading twist.

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\* On leave of absence from St.Petersburg Nuclear Physics Institute, 188350 Gatchina, Russia

1. The deep inelastic lepton-hadron scattering at large momentum transfers  $Q^2$  and not too small values of the Bjorken scaling variable  $x = Q^2/2pq$  is studied in much detail and presents a classical example for the application of perturbative QCD. The celebrated factorization theorems allow one to separate the  $Q^2$  dependence of the structure functions in coefficient functions  $C_i(x, Q^2/\mu^2, \alpha_s(\mu^2))$  in front of parton (quark and gluon) distributions of leading twist  $P_i(x, \mu^2, \alpha_s(\mu^2))$

$$F_2(x, Q^2) = \sum_i C_i(x, Q^2/\mu^2, \alpha_s(\mu^2)) \otimes P_i(x, \mu^2, \alpha_s(\mu^2)), \quad (1)$$

where

$$C(x) \otimes P(x) = \int_x^1 \frac{dy}{y} C(x/y) P(y), \quad (2)$$

the summation goes over all species of partons, and  $\mu$  is the scale, separating "hard" and "soft" contributions to the cross section. At  $\mu^2 = Q^2$  the coefficient functions can be calculated perturbatively and are expanded in power series in the strong coupling

$$C(x, 1, \alpha_s(Q^2)) = C_0(x) + \frac{\alpha_s(Q^2)}{\pi} C_1(x) + \left( \frac{\alpha_s(Q^2)}{\pi} \right)^2 C_2(x) + \dots \quad (3)$$

whereas their evolution with  $\mu^2$  is given by famous Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations. Going over to a low normalization point  $\mu^2 \sim 1\text{GeV}$ , one obtains the structure functions expressed in terms of the parton distributions in the nucleon at this reference scale. The parton distributions absorb all the information about the dynamics of large distances and are fundamental quantities extracted from the experiment. Provided the parton distributions are known, all the dependence of the structure functions on the momentum transfer is calculable and is contained in the coefficient functions  $C_i$ . Corrections to this simple picture come within perturbation theory from the parton distributions of higher twists and are suppressed by powers of the large momentum  $Q^2$ . These higher-twist contributions are relatively well understood theoretically, and, unfortunately, very poorly experimentally. I have nothing to say about them in this talk.

The picture described above presents a part of the common wisdom about hard processes in the Quantum Chromodynamics, and in a more or less detailed presentation can be found in any textbook. Less widely known, is the fact that from the theoretical point of view this picture is not complete. An indication that some contributions may be missing, comes from the asymptotic nature of the perturbative series in (3). This series is non-Borel-summable, which means that any attempt to attribute a quantitative meaning to the *sum* of the series in (3) would produce an exponentially small imaginary part  $\sim i \exp\{-\text{const} \cdot \pi/\alpha_s(Q^2)\}$ , which is to be cancelled by the imaginary part coming from nonperturbative contributions. Thus, separation between perturbative and nonperturbative pieces in the cross section as the ones which contribute to the coefficient function and the parton distribution, respectively, cannot be rigorous. A modern discussion of the asymptotical properties of the perturbation series in QCD can be found in [1, 2].

In addition to *imaginary* exponential corrections which must cancel identically against the corresponding nonperturbative contributions, the coefficient functions may acquire also *real* exponential corrections, which potentially produce observable effects. In this talk I shall report on recent results by I. Balitsky and myself [3, 4], indicating that these

corrections are indeed present. We have found that the deep inelastic cross section indeed possesses exponential contributions of the form  $F(x) \exp[-4\pi S(x)/\alpha_s(Q^2)]$ , where  $S(x)$ ,  $F(x)$  are certain functions of Bjorken  $x$ , which we are able to calculate in a certain kinematical domain. Since the experimental data are becoming more and more precise, it is of acute interest to find a boundary for a possible accuracy of the perturbative approach, which is set by nonperturbative effects. Our study has been fuelled by recent findings of an enhancement of instanton-induced effects at high energies in a related problem of the violation of baryon number in the electroweak theory [5, 6]. In the case of QCD the instanton-induced effects could turn out to be numerically large at high energies, despite the fact that they correspond formally to contributions of a very high fractional twist  $\exp(-4\pi S(x)/\alpha_s(Q^2)) \sim (\Lambda_{QCD}^2/Q^2)^{bS(x)}$ .

To be precise, in this talk I consider the contribution to the structure functions coming from the instanton-antiinstanton pair. Contributions of single instantons are only present for higher-twist terms in the light-cone expansion and are less interesting. A Ringwald-type enhancement [5] of the instanton-induced cross sections at high energy can compensate the extra semiclassical suppression factor  $\exp(-2\pi/\alpha)$  accompanying instanton-antiinstanton contributions compared to single-instanton ones. In such case the  $\bar{I}I$  terms become the leading ones owing to a bigger power of the coupling in the preexponent.

As it is well known, the instanton contributions in QCD are in general infrared-unstable. In a typical situation integrations over the instanton size are strongly IR-divergent. Our starting point is the observation that this problem does not affect calculation of instanton contributions to the coefficient functions. Let us introduce for a moment an explicit IR cutoff  $\Lambda_{IR}$  to regularize the integrals over the instanton size. Then the contribution of the instanton-antiinstanton pair to the cross section can be written schematically as

$$Q^2\sigma(Q^2) \sim (\Lambda_{IR}/\Lambda_{QCD})^{2b} + (\Lambda_{QCD}/Q)^{2bS(x)}. \quad (4)$$

The second term in (4) gives an IR-protected contribution. It depends in a nontrivial way on the external large momentum and is identified unambiguously with a contribution to the coefficient function. The first term contributes to the parton distribution. To be precise, one should separate in the first term the contributions coming from instanton sizes above and below the reference scale  $\mu$ , and to add the contribution of small-size instantons to the coefficient function. Schematically, one has in this way

$$(\Lambda_{IR}/\Lambda_{QCD})^{2b} = (\mu/\Lambda_{QCD})^{2b} + [(\Lambda_{IR}/\Lambda_{QCD})^{2b} - (\mu/\Lambda_{QCD})^{2b}]. \quad (5)$$

However, this reshuffling of the  $Q^2$ -independent contribution between the coefficient and the parton distribution does not affect the observable cross section. It is analogous to an ambiguity in the separation between contributions to the coefficient function and to the parton distribution in perturbation theory, induced by possibility to use different regularization schemes (e.g.  $\overline{MS}$  instead of  $MS$ , etc.). Hence, we can concentrate on contributions of the second type in (4), which are IR-protected.

2. The distinction between the instanton-induced contributions to the coefficient functions, which are given by convergent integrals over the instanton size, and the contributions to parton distributions, given in general by IR-divergent integrals, becomes

Figure 1: The contribution of the instanton-antiinstanton pair to the cross section of hard gluon-gluon scattering (a), structure function of a gluon (b) and of a quark (c,d). Wavy lines are (nonperturbative) gluons. Solid lines are quark zero modes in the case that they are ending at the instanton (antiinstanton), and quark propagators at the  $\bar{I}I$  background otherwise.

especially transparent for an example of the cross section of hard gluon scattering from a real gluon, see Fig.1a, considered in detail in [3]. This process is not directly physically relevant (e.g., the cross section is not gauge invariant), but it serves as a good toy model. Following Zakharov [7], we calculate the cross section by means of the optical theorem. The trick is to evaluate the contribution to the functional integral coming from the vicinity of the instanton-antiinstanton configuration in Euclidian space, and calculate the cross section by the analytical continuation to Minkowski space and by taking the imaginary part. Each hard gluon is substituted by the Fourier transform of the instanton field in the singular gauge at large momentum, and brings in the factor [8]

$$A_\nu^I(q) \simeq \frac{i}{g}(\sigma_\nu \bar{q} - q_\nu) \left\{ \frac{8\pi^2}{Q^4} - (2\pi)^{5/2} \frac{\rho^2}{2Q^2} (\rho Q)^{-1/2} e^{-\rho Q} \right\} \quad (6)$$

The first term in (6) produces a power-like divergent integral over the instanton size  $\rho$ .

$$\sigma \sim \frac{1}{Q^2} \int d\rho \sigma(\rho), \quad \sigma(\rho) \sim \rho^b \quad (7)$$

All dependence on the hard scale comes in this case through the explicit power of  $Q^2$  in front of the divergent integral. This is a typical contribution to the parton distribution — in the present case to the probability to find a hard gluon within a soft gluon. The second term gives rise to a completely different behavior. The cross section is given in this case by the following integral over the common scale of the instanton and antiinstanton  $\rho_I \sim \rho_{\bar{I}}$  and over their separation  $R$  in the c.m. frame [9, 3]:

$$\int \rho d\rho \int dR_0 \exp \left\{ -2Q\rho + ER_0 - \frac{4\pi}{\alpha_s(\rho)} S(\xi) \right\}. \quad (8)$$

Three important ingredients in this expression are: the factor  $\exp(-2Q\rho)$ , which comes from the two hard gluon fields, the factor  $\exp(ER_0)$ , which is obtained from the standard exponential factor  $\exp[i(p+q)R]$ ,  $E^2 = (p+q)^2$  by the rotation to Minkowski space, cf. [7], and the action  $S(\xi)$  evaluated on the instanton-antiinstanton configuration. The normalization is such that  $S(\xi) = 1$  for an infinitely separated instanton and antiinstanton, and  $\xi$  is the conformal parameter [10]

$$\xi = \frac{R^2 + \rho_1^2 + \rho_2^2}{\rho_1\rho_2}. \quad (9)$$

Writing the action as a function of  $\xi$  ensures that the interaction between instantons is small in two different limits: for a widely separated  $I\bar{I}$  pair, and for a small instanton put inside a big (anti)instanton, which are related to each other by the conformal transformation. In the limit of large  $\xi$  the expansion of  $S(\xi)$  for the dominating maximum attractive  $I\bar{I}$  orientation reads [10]

$$S(\xi) = \left(1 - \frac{6}{\xi^2} + O(\ln(\xi)/\xi^4)\right) \quad (10)$$

where the  $1/\xi^2$  term corresponds to a slightly corrected dipole-dipole interaction. Thus, the action  $S(\xi)$  decreases with the distance between instantons, so that the instanton and the instanton effectively attract each other. This attraction results in the exponential increase of the cross section — the effect found by Ringwald [5]. Further terms in the expansion of the action can be obtained by the so-called valley method [11], and a typical solution (conformal valley) [10] gives a monotonous function of the conformal parameter, which turns to zero at  $R \rightarrow 0$ . In the traditional language, the valley approach corresponds to the summation of all so called "soft-soft" corrections arising from the particle interaction in the final state. Main problem is in the evaluation of "hard-hard" corrections [12], which come from particle interaction in the initial state. These corrections are likely to decrease the cross section, and in physical terms must take into account an (exponentially small) overlap between the initial state, which involves a few hard quanta, with the semiclassical final state [13]. Thus, the instanton-antiinstanton action is substituted by an effective "holy grail" function, which determines the leading exponential factor for the semiclassical production at high energies, and which received a lot of attention in recent years. Unitarity arguments [14, 15] suggest that the decrease of the action will stop at values of order  $S(\xi) \simeq 0.5$ . In a recent preprint [16] Diakonov and Petrov argue that  $S(\xi)$  indeed decreases up to the value  $1/2$  at a certain energy of order of the sphaleron mass, and then starts to increase, so that the semiclassical production cross section is resonance-like. The question seems to us to be not settled finally. In this study, we have taken the value  $S = 1/2$  as a reasonable guess for the residual suppression, and assumed that the behavior of the "true" function  $S(\xi)$  for  $S(\xi) > 1/2$  is close to that given by the conformal valley [10]. The latter assumption is supported by numerical studies, e.g. in [16].

To the semiclassical accuracy the integral in (8) is evaluated by a saddle-point method. The saddle-point equations take the form [3]

$$Q\rho_* = \frac{4\pi}{\alpha_s(\rho_*)}(\xi_* - 2)S'(\xi_*) + bS(\xi_*),$$

Figure 2: The non-perturbative scale in deep inelastic scattering (instanton size  $\rho_*^{-1}$ ), corresponding to the solution of saddle-point equations in (11) as a function of  $Q$  and for  $S(\xi_*) \sim 0.5 - 0.6$  ( $\xi_* \sim 3 - 4$ ).

$$E\rho_* = \frac{8\pi}{\alpha_s(\rho_*)} \sqrt{\xi_* - 2S'(\xi_*)}, \quad (11)$$

where  $S'(\xi)$  is the derivative of  $S(\xi)$  over  $\xi$ , and  $\rho_*, \xi_*$  are the saddle-point values for the instanton size and the conformal parameter, respectively.

Neglecting in (11) the terms proportional to  $b = (11/3)N_c - (2/3)n_f$ , which come from the differentiation of the running coupling (and produce a small correction), one finds

$$\begin{aligned} \xi_* &= 2 + \frac{R_*^2}{\rho_*^2} = 2 \frac{1+x}{1-x}, \\ Q\rho_* &= \frac{4\pi}{\alpha_s(\rho_*)} \frac{12}{\xi_*^2} \end{aligned} \quad (12)$$

A numerical solution of the saddle-point equations in (11) for the particular expression of the action  $S(\xi)$  corresponding to the conformal instanton-antiinstanton valley is shown in Fig.2. Note that the difference between the hard scale  $Q^2$  and the effective scale for nonperturbative effects  $\rho_*^{-2}$  is numerically very large. This is a new situation compared to calculations of instanton-induced contributions to two-point correlation functions, see e.g. [8, 17, 18], where the size of the instanton is of order of the large virtuality. The effect is that the instanton-induced contributions to deep inelastic scattering may turn out to be non-negligible at the values  $Q^2 \sim 1000 GeV^2$ , which are conventionally considered as a safe domain for perturbative QCD.

In the case of hard gluon-gluon scattering it is easy to collect all the preexponential factors (to the semiclassical accuracy). The result for the scattering of a transversely

polarized hard gluon from a soft gluon reads [3]

$$2E^2\sigma_{\perp} = \frac{4}{9}d^2\frac{(1-x)^2+x^2}{x^2(1-x)^2}\pi^{13/2}\left(\frac{2\pi}{\alpha(\rho_*)}\right)^{21/2}\exp\left[-\left(\frac{4\pi}{\alpha_s(\rho_*)}+2b\right)S(\xi_*)\right]. \quad (13)$$

It is expressed in terms of the saddle-point values of  $\rho$  and  $\xi$ . Here  $d \simeq 0.00363$  (for  $n_f = 3$ ) is a constant which enters the expression for the instanton density

$$d = \frac{1}{2}C_1 \exp[n_f C_3 - N_c C_2], \quad (14)$$

$C_1 = 0.466, C_2 = 1.54, C_3 = 0.153$  in the  $\overline{MS}$  scheme.

Note that the preexponential factor in (13) is calculated to the leading semiclassical accuracy, i.e. in the limit  $x \rightarrow 1$ , while in the exponential factor  $\exp\{-(4\pi/\alpha_s + 2b)S(\xi_*)\}$  we used the expression for  $S(\xi)$  suggested by the valley method, which is likely to be a good approximation provided  $S(\xi) > 1/2$ , alias  $x > 0.25 - 0.3$ , cf. (12). With this restriction, and at  $\alpha_s(\rho_*) \simeq 0.3 - 0.4$ , the expression on the r.h.s. of (13) reaches values of order  $10^{-2} - 10^0$ , which means that at  $Q^2 \sim 100 - 1000 GeV^2$  and  $x < 0.25 - 0.40$  the nonperturbative contribution appears to be significant.

3. Similar contributions are present in the structure functions of deep inelastic lepton-hadron scattering, but the calculation turns out to be much more involved [4]. The situation proves to be somewhat simpler for the case of deep inelastic scattering from a real gluon. To this purpose we need to evaluate

$$\begin{aligned} T_{\mu\nu} &= i \int d^x e^{iqx} \langle A^a(p), \lambda | T \{ j_{\mu}(x) j_{\nu}(0) \} | A^a(p), \lambda \rangle \\ W_{\mu\nu} &= \frac{1}{\pi} \text{Im} T_{\mu\nu} = \\ &= \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) F_L(x, Q^2) + \left( \frac{p_{\mu}p_{\nu}}{pq} - \frac{p_{\mu}q_{\nu} + q_{\mu}p_{\nu}}{q^2} + g_{\mu\nu} \frac{pq}{q^2} \right) F_2(x, Q^2) \end{aligned} \quad (15)$$

There are two technical problems to be solved. First of all, the separation of the finite contribution to the coefficient function under the background of a divergent contribution to the parton distribution is no longer given by a simple formula in (6). Instead, we extract the contribution of interest by making an analytic continuation of integrals over the instanton size  $\rho$  from *negative* values of  $\rho^2$ . For a typical integral we write down, e.g.

$$\begin{aligned} \int_0^{\infty} d\rho^2 \frac{(\rho^2)^{\mu+n-1} \Gamma(\lambda)}{(T^2 + \rho^2)^{\lambda}} &= \frac{\Gamma(\lambda)}{2i \sin[\pi(\lambda - \mu - n)]} \int_{-\infty}^0 d\rho^2 (-\rho^2)^{\mu+n-\lambda-1} \\ &\times \left[ \left( \frac{\rho^2 + i\epsilon}{T^2 + \rho^2 + i\epsilon} \right)^{\lambda} - \text{c.c.} \right] = \frac{\Gamma(\lambda - \mu - n) \Gamma(\mu + n)}{(T^2)^{\lambda - \mu - n}} \end{aligned} \quad (16)$$

The second, and main complication, comes from the necessity to consider the quark propagator in the  $\bar{I}I$  background, see the diagram in Fig.1b. Neglecting corrections of order  $\rho^2/R^2$  in the preexponential factor, we can make use of the cluster expansion [8], and keep the first nontrivial term only:

$$\langle x | \nabla_{\bar{I}\bar{I}}^{-2} \bar{\nabla}_{\bar{I}\bar{I}} | 0 \rangle = \int dz \langle x | \nabla_1^{-2} \bar{\nabla}_1 | z \rangle \sigma_{\xi} \frac{\partial}{\partial z_{\xi}} \langle z | \bar{\nabla}_2 \nabla_2^{-2} | 0 \rangle.$$

Here and below the subscript '1' refers to the antiinstanton with the size  $\rho_1$  and the position of the center  $x_{\bar{I}} = R + T$ , and the subscript '2' refers to the instanton with the size  $\rho_2$  and the center at  $x_I = T$ . We use conventional notations  $\nabla = \nabla_\mu \sigma_\mu$  and  $\bar{\nabla} = \nabla_\mu \bar{\sigma}_\mu$ , etc, where  $\sigma_\mu^{\alpha\dot{\alpha}} = (-i\sigma, 1)$ ,  $\bar{\sigma}_{\mu\dot{\alpha}\alpha} = (+i\sigma, 1)$ , and  $\sigma$  are the standard Pauli matrices. The expressions for quark propagators at the one-instanton (antiinstanton) background are given in [19].

We have found that to the leading accuracy in the strong coupling the instanton contribution in Fig.1b comes from the following integration regions in the coordinate space:

$$\begin{aligned} z^2 &\sim (z - x)^2 \sim x^2 \\ (x - R - T)^2 + \rho_1^2 &\sim T^2 + \rho_2^2 \sim x^2 \\ (z - R - T)^2 + \rho_1^2 &\sim (z - T)^2 + \rho_2^2 \sim x^2/\alpha_s \\ T^2 &\sim R^2 \sim \rho_1^2 \sim \rho_2^2 \sim x^2/\alpha_s \end{aligned} \quad (17)$$

We remind that all the calculation is done in Euclidian space, and the evaluation of integrals by means of the analytical continuation effectively corresponds to the integration over negative values of  $\rho^2$ , see (16).

Hence the integration over  $z$  in the cluster expansion of the quark propagator can be done in the "light-cone" approximation:

$$\int dz \frac{F(z)}{(x-z)^4 z^4} = \frac{\pi^2}{x^4} \int_0^1 d\gamma \frac{F(\gamma x)}{\gamma(1-\gamma)} + O(\sqrt{\alpha_s}), \quad (18)$$

where  $F(z)$  is an arbitrary function containing all other possible denominators like  $(z - R - T)^2 + \rho_1^2$  etc. This is a major simplification compared to the general case.

After a considerable algebra, we obtain the following answer for the  $\bar{I}I$  contribution to the structure function of a real gluon:

$$\begin{aligned} F_1^{(G)}(x, Q^2) &= \sum_q e_q^2 \frac{1}{9\bar{x}^2} \frac{d^2\pi^{9/2}}{bS(\xi_*)[bS(\xi_*) - 1]} \left(\frac{16}{\xi_*^3}\right)^{n_f-3} \\ &\times \left(\frac{2\pi}{\alpha_s(\rho_*^2)}\right)^{19/2} \exp\left[-\left(\frac{4\pi}{\alpha_s(\rho_*^2)} + 2b\right)S(\xi_*)\right] \end{aligned} \quad (19)$$

where the expressions for  $\rho_*$  and  $\xi_*$  coincide to the ones given in (12). To our accuracy, we find that the instanton- induced contributions obey the Callan-Gross relation  $F_2(x, Q^2) = 2xF_1(x, Q^2)$ .

The expression in (19) presents our main result. It gives the exponential correction to the coefficient function in front of the gluon distribution of the leading twist in (3). The exponential factor is exact to the accuracy of (10). The preexponential factor is calculated to leading accuracy in the strong coupling and up to corrections of order  $O(1-x)$ . The corresponding contribution to the structure function of the nucleon is obtained in a usual way, making a convolution of (19) with a distribution of gluons in the proton at the scale  $\rho_*^2$ .

In the language of the operator product expansion, the answer in (19) can be rewritten as the exponential nonperturbative contribution to the high  $n \sim \pi/\alpha_s$  moment of the structure function

$$M_1^{(G)}(n, Q^2) = \int_0^1 dx x^n F_1^{(G)}(x, Q^2) \quad (20)$$

Taking  $n = \hat{n} \cdot 4\pi/\alpha_s(\rho_*)$  with  $\hat{n}$  of order unity, one can evaluate the integral in (20) with  $F_1^{(G)}(x, Q^2)$  given in (19) by the saddle point method. The saddle-point equation reads, approximately

$$\hat{n} = S'(\xi_*)(\xi_*^2 - 4)/4 \quad (21)$$

where  $\xi_*$  is related to the saddle-point value of  $x$  by (12). The lowest possible value  $\xi_* \simeq 3$ , corresponding within the valley-method approximation to  $S(\xi_*) \simeq 1/2$ ,  $S'(\xi_*) \simeq 0.27$ , thus yields  $\hat{n} \simeq 0.3$ . Hence the expression in (19) is applicable to the evaluation of instanton-induced corrections to the coefficient functions in front of local operators starting from  $n \simeq 10 - 12$ .

Leading contribution to (19) in the perturbation theory is due to the mixing with the flavor-singlet quark distribution and is given by the box graph:

$$F_1^{(G)}(x, Q^2)_{pert} = \sum_q e_q^2 [x^2 + \bar{x}^2] \frac{\alpha_s(Q^2)}{2\pi} \ln \left[ \frac{Q^2 \bar{x}}{\mu^2 x} \right], \quad (22)$$

where in order to compare to the instanton contribution in (19) one should choose the scale  $\mu$  to be of order  $\rho_*$ .

The instanton-antiinstanton contribution to the structure function of a quark contains a similar contribution shown in Fig.1c. The answer reads

$$F_1^{(q)}(x, Q^2) = \left[ \sum_{q' \neq q} e_{q'}^2 + \frac{1}{2} e_q^2 \right] \frac{128}{81 \bar{x}^3} \frac{d^2 \pi^{9/2}}{b S(\xi_*) [b S(\xi_*) - 1]} \left( \frac{16}{\xi_*^3} \right)^{n_f - 3} \\ \times \left( \frac{2\pi}{\alpha_s(\rho_*^2)} \right)^{15/2} \exp \left[ - \left( \frac{4\pi}{\alpha_s(\rho_*^2)} + 2b \right) S(\xi_*) \right] \quad (23)$$

However, in this case additional contributions exist of the type shown in Fig.1d. They are finite (the integral over instanton size is cut off at  $\rho^2 \sim x^2/\alpha_s$ ), but the relevant instanton-antiinstanton separation  $R$  is small, of order  $\rho$ . This probably means that the structure of nonperturbative contributions to quark distributions is more complicated. This question is under study. The answer given in (23) presents the contribution of the particular saddle point in (12).

4. The instanton-induced contribution to the structure function of a gluon in (19) is shown as a function of Bjorken  $x$  for different values of  $Q \sim 10 - 100 GeV$  in Fig.3. The contribution of the box graph in (22) is shown by dots for comparison. The low boundary for possible values of  $Q$  is determined by the condition that the effective instanton size is not too large. At  $Q = 10 GeV$  we find  $\rho_* \simeq 1 GeV^{-1}$ , cf. Fig.2. This value is sufficiently small, so that instantons are not distorted too strongly by large-scale vacuum fluctuations. Another limitation is that the valley approach to the calculation of the "holy grail" function  $S(\xi)$  is likely to be justified at  $S(\xi) \geq 1/2$ , which translates to the condition

Figure 3: Nonperturbative contribution to the structure function  $F_1(x, Q^2)$  of a real gluon (19) as a function of  $x$  for different values of  $Q$  (solid curves). The leading perturbative contribution (22) is shown for comparison by dots. The dashed curves show lines with the constant effective value of the action on the  $\bar{I}I$  configuration.

that  $x > 0.3 - 0.35$ . Numerical results are strongly sensitive to the particular value of the QCD scale parameter. We use the two-loop expression for the coupling with three active flavors, and the value  $\Lambda_{\overline{MS}}^{(3)} = 290 MeV$  which corresponds to  $\Lambda_{\overline{MS}}^{(4)} = 240 MeV$  [20]. The corresponding value of the coupling at the scale of  $\tau$ -lepton mass is  $\alpha_s(m_\tau) = 0.29$ . The new ALEPH data [21] indicate a somewhat higher value  $\alpha_s(m_\tau) = 0.33 \pm 0.05$ . Since the dependence on the coupling is exponential, the 20% increase of  $\alpha_s(\rho_*)$  induces the increase of the cross section by an order of magnitude! Together with uncertainties in the function  $S(\xi)$  and in the preexponential factor, this indicates that the particular curves given in Fig.3 should not be taken too seriously, and rather give a target for further theoretical (and experimental?) studies to shoot at.

To summarize, we have found that instantons produce a well-defined and calculable contribution to the cross section of deep inelastic scattering for sufficiently large values of  $x$  and large  $Q^2 \sim 100 - 1000 GeV^2$ , which turns out, however, to be rather small — of order

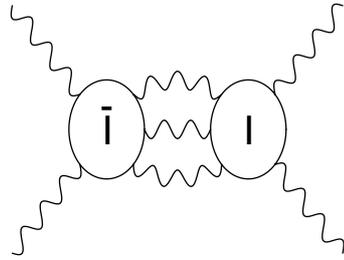
$10^{-2} - 10^{-5}$  compared to the perturbative cross section. This means that the accuracy of standard perturbative analysis is sufficiently high, and that there is not much hope to observe the instanton-induced contributions to the total deep inelastic cross section experimentally. However, instantons are likely to produce events with a very specific structure of the final state, and such peculiarities may be subject to experimental search. The dominating Feynman diagrams in our calculation correspond to  $2\pi/\alpha(\rho_*) \sim 15$  gluons and  $2n_f - 1 = 5$  quarks in the final state, with the low virtuality of order  $\rho_*^{-1} \sim 1 \text{ GeV}$ , and which are produced in the S-wave in the c.m. frame of the colliding partons. It is not likely that these can be resolved as minijets, and we rather expect a spherically symmetric production of final state hadrons. The effect is likely to be resonance-like, that is present in a narrow interval of values of Bjorken  $x$  of order 0.25–0.35 (in the parton-parton collision). In any case, finding of an instanton-induced particle production at high energies is a challenging problem, and further theoretical efforts are needed to put it as a practical proposal to experimentalists.

5. I would like to thank Ian Balitsky for a very rewarding collaboration. It is a pleasure to acknowledge also useful discussions with M. Beneke, M.A. Shifman and V.I. Zakharov.

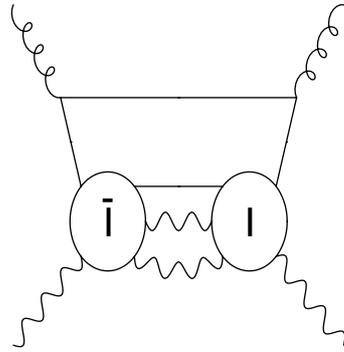
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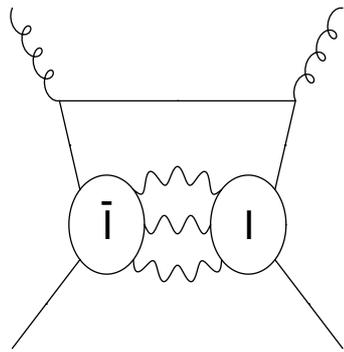
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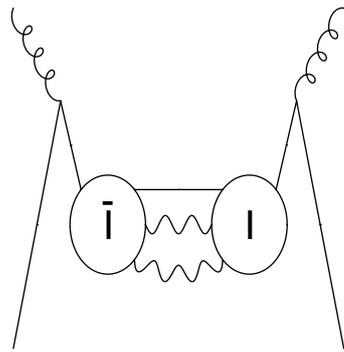
a)



b)



c)



d)

