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Instanton-induced contributions to structure functions of deep inelastic scattering

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Abstract:

We identify and calculate the instanton-induced contributions to deep inelastic scattering which correspond to nonperturbative exponential corrections to the coefficient functions in front of parton distributions of the leading twist.

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1. The deep inelastic lepton-hadron scattering at large momentum transfers Q^2 and not too small values of the Bjorken scaling variable $x = Q^2/2pq$ is studied in much detail and presents a classical example for the application of perturbative QCD. The celebrated factorization theorems allow one to separate the Q^2 dependence of the structure functions in coefficient functions $C_i(x, Q^2/\mu^2, \alpha_s(\mu^2))$ in front of parton (quark and gluon) distributions of leading twist $P_i(x, \mu^2, \alpha_s(\mu^2))$

$$F_2(x, Q^2) = \sum_i C_i(x, Q^2/\mu^2, \alpha_s(\mu^2)) \otimes P_i(x, \mu^2, \alpha_s(\mu^2)), \quad (1)$$

where

$$C(x) \otimes P(x) = \int_x^1 \frac{dy}{y} C(x/y) P(y), \quad (2)$$

the summation goes over all species of partons, and μ is the scale, separating "hard" and "soft" contributions to the cross section. At $\mu^2 = Q^2$ the coefficient functions can be calculated perturbatively and are expanded in power series in the strong coupling

$$C(x, 1, \alpha_s(Q^2)) = C_0(x) + \frac{\alpha_s(Q^2)}{\pi} C_1(x) + \left(\frac{\alpha_s(Q^2)}{\pi} \right)^2 C_2(x) + \dots \quad (3)$$

whereas their evolution with μ^2 is given by famous Gribov-Lipatov-Altarelli-Parisi equations. Going over to a low normalization point $\mu^2 \sim 1\text{GeV}$, one obtains the structure functions expressed in terms of the parton distributions in the nucleon at this reference scale. The parton distributions absorb all the information about the dynamics of large distances and are fundamental quantities extracted from the experiment. Provided the parton distributions are known, all the dependence of the structure functions on the momentum transfer is calculable and is contained in the coefficient functions C_i . Corrections to this simple picture come within perturbation theory from the parton distributions of higher twists and are suppressed by powers of the large momentum Q^2 .

Objective of this letter are possible exponential corrections to the r.h.s. of eq.(3) of the form $F(x) \exp[-4\pi S(x)/\alpha_s(Q^2)]$. Since the experimental data are becoming more and more precise, it is of acute interest to find a boundary for a possible accuracy of the perturbative approach, set by nonperturbative effects. Our study has also been fuelled by recent findings of an enhancement of instanton-induced effects at high energies in a related problem of the violation of baryon number in the electroweak theory [1, 2], and by an indication [3] that in the case of QCD the instanton-induced effects may be numerically large at high energies, despite the fact that they correspond formally to a contribution of a very high fractional twist $\exp(-4\pi S(x)/\alpha_s(Q^2)) \sim (\Lambda_{QCD}^2/Q^2)^{bS(x)}$. Instanton-induced contributions may have a direct relation to the contributions to structure functions of high orders of perturbation theory [4, 5].

In this letter we consider the contribution to the structure functions coming from the instanton-antiinstanton pair. Contributions of single instantons are only present for scattering from polarized targets and for higher-twist terms in the light-cone expansion.

A Ringwald-type enhancement [1] of the instanton-induced cross sections at high energy can compensate the extra semiclassical suppression factor $\exp(-2\pi/\alpha)$ accompanying instanton-antiinstanton contributions compared to single-instanton ones. In such case the $\bar{I}I$ terms become the leading ones owing to a bigger power of the coupling in the preexponent.

As it is well known, the instanton contributions in QCD are in general infrared-unstable, the integrals over the instanton size are strongly IR-divergent. Our starting point is the observation that this problem does not affect calculation of instanton contributions to the coefficient functions. Let us introduce for a moment an explicit IR cutoff Λ_{IR} to regularize the integrals over the instanton size. Then the contribution of the instanton-antiinstanton pair to the cross section can be written schematically as

$$\sigma(Q^2) \sim (\Lambda_{IR}/\Lambda_{QCD})^{2b} + (\Lambda_{QCD}/Q)^{2bS(x)}. \quad (4)$$

The second term in (4) gives an IR-protected contribution. It depends in a nontrivial way on the external large momentum and is identified unambiguously with a contribution to the coefficient function. The first term contributes to the parton distribution. To be precise, one should separate in the first term the contributions coming from instanton sizes above and below the reference scale μ , and to add the contribution of small-size instantons to the coefficient function. Schematically, one has in this way

$$(\Lambda_{IR}/\Lambda_{QCD})^{2b} = (\mu/\Lambda_{QCD})^{2b} + [(\Lambda_{IR}/\Lambda_{QCD})^{2b} - (\mu/\Lambda_{QCD})^{2b}]. \quad (5)$$

However, this reshuffling of the Q^2 -independent contribution between the coefficient and the parton distribution does not affect the observable cross section. It is analogous to an ambiguity in the separation between contributions to the coefficient function and to the parton distribution in perturbation theory, induced by possibility to use different regularization schemes (e.g. \overline{MS} instead of MS , etc.). Hence, we can concentrate on contributions of the second type in (4), which are IR-protected.

2. The distinction between the instanton-induced contributions to the coefficient functions, which are given by convergent integrals over the instanton size, and the contributions to parton distributions, given in general by IR-divergent integrals, becomes especially transparent for an example of the cross section of hard gluon scattering from a real gluon, see Fig.1a, considered in detail in [3]. In this case one needs to evaluate the contribution to the functional integral coming from the vicinity of the instanton-antiinstanton configuration (and amputate external gluon legs afterwards). Each hard gluon is substituted by the Fourier transform of the instanton field at large momentum, and brings in the factor [6]

$$A_\nu^I(q) \simeq \frac{i}{g}(\sigma_\nu \bar{q} - q_\nu) \left\{ \frac{8\pi^2}{Q^4} - (2\pi)^{5/2} \frac{\rho^2}{2Q^2} (\rho Q)^{-1/2} e^{-\rho Q} \right\} \quad (6)$$

The first term in (6) produces a power-like divergent integral over the instanton size ρ . All dependence on the hard scale comes in this case through the explicit power of Q^2 in front of the divergent integral. This is a typical contribution to the parton distribution — in the present case to the probability to find a hard gluon within a soft gluon. The second term gives rise to a completely different behavior. The cross section is given in this case by the following integral over the common scale of the instanton and antiinstanton $\rho_I \sim \rho_{\bar{I}}$ and over their separation R in the c.m. frame:

$$\int \rho d\rho \int dR_0 \exp \left\{ -2Q\rho + ER_0 - \frac{4\pi}{\alpha_s(\rho)} S(\xi) \right\}. \quad (7)$$

Three important ingredients in this expression are: the factor $\exp(-2Q\rho)$, which comes from the two hard gluon fields, the factor $\exp(ER_0)$, which is obtained from the standard exponential factor $\exp(i(p+q)R)$, $E^2 = (p+q)^2$ by the rotation to Minkowski space, cf. [7], and the action $S(\xi)$ evaluated on the instanton-antiinstanton configuration. The normalization is such that $S(\xi) = 1$ for an infinitely separated instanton and antiinstanton, and ξ is the conformal parameter [8]

$$\xi = \frac{R^2 + \rho_1^2 + \rho_2^2}{\rho_1 \rho_2}. \quad (8)$$

Writing the action as a function of ξ ensures that the interaction between instantons is small in two different limits: for a widely separated $I\bar{I}$ pair, and for a small instanton put inside a big (anti)instanton, which are related to each other by the conformal transformation. In the limit of large ξ the expansion of $S(\xi)$ for the dominating maximum attractive $I\bar{I}$ orientation reads [8]

$$S(\xi) = \left(1 - \frac{6}{\xi^2} + O(\ln(\xi)/\xi^4) \right) \quad (9)$$

where the $1/\xi^2$ term corresponds to a slightly corrected dipole-dipole interaction. In general, one can have in mind a certain smooth function which turns to zero at $R \rightarrow 0$.

To the semiclassical accuracy the integral in (7) is evaluated by a saddle-point method. The saddle-point equations take the form [3]

$$\begin{aligned} Q\rho_* &= \frac{4\pi}{\alpha_s(\rho_*)} (\xi_* - 2) S'(\xi_*) + b S(\xi_*), \\ E\rho_* &= \frac{8\pi}{\alpha_s(\rho_*)} \sqrt{\xi_* - 2} S'(\xi_*), \end{aligned} \quad (10)$$

where $S'(\xi)$ is the derivative of $S(\xi)$ over ξ , and ρ_* , ξ_* are the saddle-point values for the instanton size and the conformal parameter, respectively.

Neglecting in (10) the terms proportional to $b = (11/3)N_c - (2/3)n_f$, which come from the differentiation of the running coupling and only produce a small correction, one finds

$$\xi_* = 2 + \frac{R_*^2}{\rho_*^2} = 2 \frac{1+x}{1-x},$$

$$Q\rho_* = \frac{4\pi}{\alpha_s(\rho_*)} \frac{12}{\xi_*^2} \quad (11)$$

A numerical solution of the saddle-point equations in (10) for the particular expression of the action $S(\xi)$ corresponding to the conformal instanton-antiinstanton valley is shown in Fig.2.[†] Note that the difference between the hard scale Q^2 and the effective scale for nonperturbative effects ρ_*^{-2} is numerically very large. This is a new situation compared to calculations of instanton-induced contributions to two-point correlation functions, see e.g. [6, 9, 10], where the size of the instanton is of order of the large virtuality, and indicates that the instanton-induced contributions to deep inelastic scattering may turn out to be nonnegligible even at values $Q^2 \sim 1000 GeV^2$, which are conventionally considered as a safe domain for perturbative QCD.

In the case of hard gluon-gluon scattering it is easy to collect all the preexponential factors (to the semiclassical accuracy). The result for the scattering of a transversely polarized hard gluon from a soft gluon reads [3][‡]

$$2E^2\sigma_{\perp} = \frac{4}{9}d^2 \frac{(1-x)^2 + x^2}{x^2(1-x)^2} \pi^{13/2} \left(\frac{2\pi}{\alpha(\rho_*)}\right)^{21/2} \exp\left[-\left(\frac{4\pi}{\alpha(\rho_*)} + 2b\right)S(\xi_*)\right]. \quad (12)$$

It is expressed in terms of the saddle-point values of ρ and ξ . Here $d \simeq 0.00363$ (for $n_f = 3$) is a constant which enters the expression for the instanton density

$$d = \frac{1}{2}C_1 \exp[n_f C_3 - N_c C_2], \quad (13)$$

$C_1 = 0.466, C_2 = 1.54, C_3 = 0.153$ in the \overline{MS} scheme. At $\alpha_s(\rho_*) \simeq 0.3 - 0.4$ and $S(\xi_*) \simeq 1/2$ the expression on the r.h.s. of (12) is of order $10^{-2} - 10^0$, which means that at $Q^2 \sim 100 - 1000 GeV^2$ and $x < 0.25 - 0.40$ the nonperturbative contribution is significant.

3. Similar contributions are present in the structure functions of deep inelastic lepton-hadron scattering, but the calculation turns out to be much more involved. The situation appears to be simpler for the case of deep inelastic scattering from a real gluon. To this end we need to evaluate

$$\begin{aligned} T_{\mu\nu} &= i \int d^x e^{iqx} \langle A^a(p), \lambda | T \{ j_{\mu}(x) j_{\nu}(0) \} | A^a(p), \lambda \rangle \\ W_{\mu\nu} &= \frac{1}{\pi} \text{Im} T_{\mu\nu} = \\ &= \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) F_L(x, Q^2) + \left(\frac{p_{\mu}p_{\nu}}{pq} - \frac{p_{\mu}q_{\nu} + q_{\mu}p_{\nu}}{q^2} + g_{\mu\nu} \frac{pq}{q^2} \right) F_2(x, Q^2) \end{aligned} \quad (14)$$

[†] Numerical results are strongly sensitive to the particular value of the QCD scale. We use the two-loop expression for the coupling with three active flavors, and the value $\Lambda_{\overline{MS}}^{(3)} = 290 MeV$ which corresponds to $\Lambda_{\overline{MS}}^{(4)} = 240 MeV$ [11]. The corresponding value of the coupling is $\alpha_s(1 GeV) = 0.41$.

[‡] In difference to [3] we give the result for scattering of gluons with fixed color and polarization.

We have found it simpler to do the calculations in the coordinate space. Full details will be published elsewhere [12], and below we only indicate the main steps.

The true small parameter in our calculation is the value of the coupling constant at the scale Q^2 , which ensures that the effective instanton size is sufficiently small. We are trying to be accurate to collect to the semiclassical accuracy all the dependence on ρ^2/R^2 in the exponent, having in mind the valley method [13], in which all the dependence on the $\bar{I}I$ separation is absorbed in the action $S(\xi)$ on the $\bar{I}I$ configuration. However, in this letter we do not take into account corrections of order ρ^2/R^2 in the preexponent, and to this accuracy need the first nontrivial term only in the cluster expansion of the quark propagator at the $\bar{I}I$ background [6]: $\langle x|\nabla_{\bar{I}\bar{I}}^{-2}\bar{\nabla}_{\bar{I}\bar{I}}|0\rangle = \int dz \langle x|\nabla_1^{-2}\bar{\nabla}_1|z\rangle\sigma_\xi\frac{\partial}{\partial z\xi}\langle z|\bar{\nabla}_2\nabla_2^{-2}|0\rangle$.

The leading contribution to the gluon matrix element of the T-product of the electromagnetic currents in (15) is given by the following expression

$$\begin{aligned}
& \langle A^a(p), \lambda|T\{j_\mu(x)j_\nu(0)\}|A^a(p), \lambda\rangle^{\bar{I}I} = \\
& = \sum_q e_q^2 \int dU \int \frac{d\rho_1}{\rho_1^5} d(\rho_1) \int \frac{d\rho_2}{\rho_2^5} d(\rho_2) \int d^4R \int d^4T \\
& \times \frac{1}{8} \lim_{p^2 \rightarrow 0} p^4 \epsilon_\alpha^\lambda \epsilon_\beta^\lambda \text{Tr} \left\{ A_{\alpha}^{\bar{I}}(p) A_{\beta}^I(-p) \right\} \exp \left[-\frac{16\pi^2}{g^2} S^{\bar{I}I} \right] \\
& \times (a^\dagger a)^{n_f-1} \left\{ a \bar{\phi}_0(0) \bar{\sigma}_\nu \langle 0|\nabla_2^{-2}\nabla_2\bar{\partial}\nabla_1\nabla_1^{-2}|x\rangle \bar{\sigma}_\mu \phi_0(x) \right. \\
& \left. + a^\dagger \bar{\kappa}_0(x) \sigma_\mu \langle x|\nabla_1^{-2}\bar{\nabla}_1\partial\bar{\nabla}_2\nabla_2^{-2}|0\rangle \sigma_\nu \kappa_0(0) + (\mu \leftrightarrow \nu, x \leftrightarrow 0) + \dots \right\} \quad (15)
\end{aligned}$$

which corresponds to the diagram shown in Fig.1b. The full expression contains many more terms [12] which are not shown because we have found that all of them are of order $O(\alpha_s(Q^2))$ compared to the expression in (15). Here and below the subscript '1' refers to the antiinstanton with the size ρ_1 and the position of the center $x_{\bar{I}} = R + T$, and the subscript '2' refers to the instanton with the size ρ_2 and the center at $x_I = T$. We use conventional notations $\nabla = \nabla_\mu \sigma_\mu$ and $\bar{\nabla} = \nabla_\mu \bar{\sigma}_\mu$, etc, where $\sigma_\mu^{\alpha\dot{\alpha}} = (-i\sigma, 1)$, $\bar{\sigma}_{\mu\dot{\alpha}\alpha} = (+i\sigma, 1)$, and σ are the standard Pauli matrices. The expressions for quark propagators at the one-instanton (antiinstanton) background are given in [14].

The quark zero modes are written in terms of two-component Weil spinors $\psi_0 = \begin{pmatrix} \kappa_0 \\ \phi_0 \end{pmatrix}$, $\psi_0^\dagger = \begin{pmatrix} \bar{\phi}_0 & \bar{\kappa}_0 \end{pmatrix}$, explicit expressions for which in case of the instanton (antiinstanton) with the center at the origin are

$$\begin{aligned}
\kappa_{\dot{\alpha}}^k(x) &= \epsilon^{\alpha\beta} (U_I)_\beta^k \frac{\bar{x}_{\dot{\alpha}\alpha}}{2\pi^2 x^4} \frac{2\pi\rho_2^{3/2}}{\Pi_{2x}^{3/2}}, & \bar{\phi}_k^{\dot{\alpha}}(x) &= \epsilon_{\beta\alpha} (U_I^\dagger)^\beta_k \frac{x^{\alpha\dot{\alpha}}}{2\pi^2 x^4} \frac{2\pi\rho_2^{3/2}}{\Pi_{2x}^{3/2}} \\
\phi^{k\alpha}(x) &= \epsilon_{\dot{\alpha}\dot{\beta}} (U_{\bar{I}} u_0)^{k\dot{\beta}} \frac{x^{\alpha\dot{\alpha}}}{2\pi^2 x^4} \frac{2\pi\rho_1^{3/2}}{\Pi_{1x}^{3/2}}, & \bar{\kappa}_{k\alpha}(x) &= \epsilon_{\dot{\alpha}\dot{\beta}} (\bar{u}_0 U_{\bar{I}}^\dagger)_{\dot{\beta}k} \frac{\bar{x}_{\dot{\alpha}\alpha}}{2\pi^2 x^4} \frac{2\pi\rho_1^{3/2}}{\Pi_{1x}^{3/2}} \quad (16)
\end{aligned}$$

where $\Pi_{1x} = 1 + \rho_1^2/x^2$ and $\Pi_{2x} = 1 + \rho_2^2/x^2$, respectively. The corresponding overlap

integrals are equal to

$$a = - \int dx (\bar{\kappa} \partial \kappa) = 2\xi^{-3/2} \text{Tr } O \quad , \quad a^\dagger = - \int dx (\bar{\phi} \partial \phi) = 2\xi^{-3/2} \text{Tr } O^\dagger \quad (17)$$

where O is the left upper 2×2 corner of the matrix $U_I^\dagger U_{\bar{I}} u_0$ of the relative $I\bar{I}$ orientation. The integration over the $I\bar{I}$ orientations in (15) can be done in the saddle-point approximation. Assuming the standard orientation dependence of the action corresponding to the dipole-dipole interaction, we get [3] (for $N_c = 3$):

$$\int dU e^{-\frac{16\pi^2}{g^2} S(\xi, U)} = \frac{1}{9\sqrt{\pi}} \left(\frac{\alpha_s \xi^2}{2\pi \cdot 4} \right)^{7/2} e^{-\frac{16\pi^2}{g^2} S(\xi)}. \quad (18)$$

The saddle point is achieved at the maximum attractive $I\bar{I}$ orientation corresponding to the choice $U_I^\dagger U_{\bar{I}} = 1$, $u_0 = R/\sqrt{R^2}$. The factor coming from soft gluons is

$$\begin{aligned} \frac{1}{8} \lim_{p^2 \rightarrow 0} p^4 \epsilon_\alpha^\lambda \epsilon_\beta^\lambda \text{Tr} \left\{ A_\alpha^{\bar{I}}(p) A_\beta^I(-p) \right\} &= -\frac{\pi^4}{g^2} \rho_1^2 \rho_2^2 e^{ipR} \epsilon_\alpha^\lambda \epsilon_\beta^\lambda \text{Tr} \left\{ \sigma_\beta \bar{p} u_0 \bar{\sigma}_\alpha p \bar{u}_0 \right\} \\ &= \frac{\pi^3}{\alpha_s} (pR)^2 \frac{\rho_1^2 \rho_2^2}{R^2} e^{ipR} \end{aligned} \quad (19)$$

Main complication comes from the quark propagator. Using explicit expression for the zero modes in (16) and the propagators from [14] we find

$$\begin{aligned} \bar{\kappa}_0(x) \sigma_\mu \langle x | \nabla_1^{-2} \bar{\nabla}_1 \partial \bar{\nabla}_2 \nabla_2^{-2} | 0 \rangle \sigma_\nu \kappa_0(0) &= \\ &= -\frac{1}{4\pi^6} \int dz \frac{(\rho_1 \rho_2)^{3/2}}{\Pi_{1x}^2 \Pi_{20}^2 (x-R-T)^4 T^4} \frac{1}{\sqrt{R^2}} \text{Tr} \left\{ \frac{\bar{\sigma}_\nu z \bar{\sigma}_\xi (x-z) \bar{\sigma}_\mu}{(x-z)^4 z^4} \right. \\ &\times \left. \left[(x-R-T) + \rho_1^2 \frac{(z-R-T)}{(z-R-T)^2} \right] \bar{R} \frac{1}{\sqrt{\Pi_{1z}}} \frac{\partial}{\partial z_\xi} \frac{1}{\sqrt{\Pi_{2z}}} \left[T - \rho_2^2 \frac{(z-T)}{(z-T)^2} \right] \right\} + \dots \end{aligned} \quad (20)$$

Omitted terms have turned out to be of order $O(\alpha_s)$.

We have found that in order to pick up the leading contribution in the strong coupling one can have in mind the following mnemonic rules for the essential regions of integration:

$$\begin{aligned} z^2 &\sim (z-x)^2 \sim x^2 \\ (x-R-T)^2 + \rho_1^2 &\sim T^2 + \rho_2^2 \sim x^2 \\ (z-R-T)^2 + \rho_1^2 &\sim (z-T)^2 + \rho_2^2 \sim x^2/\alpha_s \\ T^2 &\sim R^2 \sim \rho_1^2 \sim \rho_2^2 \sim x^2/\alpha_s \end{aligned} \quad (21)$$

Note that all the calculation is done in Euclidian space, and the evaluation of integrals by means of the analytical continuation effectively corresponds to going over to negative values of ρ^2 .

Hence the integration over z in (21) can be done in the "light-cone" approximation:

$$\int dz \frac{F(z)}{(x-z)^4 z^4} = \frac{\pi^2}{x^4} \int_0^1 d\gamma \gamma^{-1} \bar{\gamma}^{-1} F(\gamma x) + O(\sqrt{\alpha_s}), \quad (22)$$

where $F(z)$ is an arbitrary function containing all other possible denominators like $(z - R - T)^2 + \rho_1^2$ etc. Here and below we use the notation

$$\bar{\gamma} = 1 - \gamma.$$

etc. To this accuracy a simpler expression for (21) can be written:

$$-\frac{1}{2\pi^4 x^4} \int_0^1 d\gamma \frac{(\rho_1 \rho_2)^{3/2}}{[(x-R-T)^2 + \rho_1^2][T^2 + \rho_2^2] \sqrt{R^2}} \quad (23)$$

$$\times \text{Tr} \left\{ \bar{\sigma}_\nu x \bar{\sigma}_\mu \left[\bar{\gamma} x \frac{1}{\sqrt{\Pi_{1\gamma}}} + (\gamma x - R - T) \sqrt{\Pi_{1\gamma}} \right] \bar{R} \frac{d}{d\gamma} \left[\gamma x \frac{1}{\sqrt{\Pi_{2\gamma}}} - (\gamma x - T) \sqrt{\Pi_{2\gamma}} \right] \right\}$$

where $\Pi_{1\gamma} = 1 + \rho_1^2/(\gamma x - R - T)^2$ and $\Pi_{2\gamma} = 1 + \rho_2^2/(\gamma x - T)^2$. To do the remaining integrals, we expand the exponential factor corresponding to the dipole-dipole $I\bar{I}$ interaction

$$\exp \left[\frac{24\pi}{\alpha_s} \frac{\rho_1^2 \rho_2^2}{R^4} \right] = \sum_{n=0}^{\infty} \frac{1}{n!} (6b)^n \left(\frac{\rho_1^2 \rho_2^2}{R^4} \right)^n \left(\ln \frac{1}{\rho_1 \rho_2 \Lambda^2} \right)^n. \quad (24)$$

The logarithms come from the running of the coupling. To take them into account, one needs to replace $n \rightarrow n + \epsilon$, do all the integrations, and pick up the n -th term of the expansion in ϵ at the end. At the cost of this expansion the dependence on ρ_1 and on ρ_2 factorize from each other. The corresponding integrals are divergent and must be understood in the sense of analytical continuation. For definiteness, one can have in mind the following representation

$$\int_0^{\infty} d\rho^2 \frac{(\rho^2)^{\mu+n-1} \Gamma(\lambda)}{(T^2 + \rho^2)^\lambda} = \frac{\Gamma(\lambda)}{2i \sin[\pi(\lambda - \mu - n)]} \int_{-\infty}^0 d\rho^2 (-\rho^2)^{\mu+n-\lambda-1}$$

$$\times \left[\left(\frac{\rho^2 + i\epsilon}{T^2 + \rho^2 + i\epsilon} \right)^\lambda - \text{c.c.} \right] = \frac{\Gamma(\lambda - \mu - n) \Gamma(\mu + n)}{(T^2)^{\lambda - \mu - n}} \quad (25)$$

The master formula for doing the remaining integrations reads

$$\sum_{n=0}^{\infty} \left(\frac{24\pi}{\alpha_s} \right)^n \frac{1}{n!} \int dT \int \frac{d\rho_1^2}{\rho_1^2} (\rho_1^2)^{\mu_1+n} \int \frac{d\rho_2^2}{\rho_2^2} (\rho_2^2)^{\mu_2+n} \int dR \frac{e^{ipR}}{R^{4n+2}} \frac{\Gamma(\lambda_1)}{[(x-R-T)^2 + \rho_1^2]^{\lambda_1}}$$

$$\times \frac{\Gamma(\lambda_2)}{[T^2 + \rho_2^2]^{\lambda_2}} \frac{[(\gamma x - R - T)^2]^{-\sigma_1}}{[(\gamma x - R - T)^2 + \rho_1^2]^{\nu_1}} \frac{[(\gamma x - T)^2]^{-\sigma_2}}{[(\gamma x - T)^2 + \rho_2^2]^{\nu_2}} N(x, R, T)$$

$$= \pi^3 \int_0^1 du (2u)^{\theta-3} \exp \left[ipxu + \frac{3\pi}{2\alpha_s} \frac{\bar{u}^2}{u^2} \right] \left(\frac{3\pi}{2\alpha_s} \frac{\bar{u}^2}{u^2} \right)^{\mu_1 + \mu_2 - \nu_1 - \nu_2 - \sigma_1 - \sigma_2 - 2} \left(\frac{\bar{u}}{2\bar{\gamma}} \right)^{\nu_1} \left(\frac{\bar{u}}{2\bar{\gamma} - \bar{u}} \right)^{\sigma_1}$$

$$\times \left(\frac{\bar{u}}{2\bar{\gamma}} \right)^{\nu_2} \left(\frac{\bar{u}}{2\bar{\gamma} - \bar{u}} \right)^{\sigma_2} \int dR e^{ipR} \frac{\Gamma(\theta - 1)}{[R^2 + \bar{u}u x^2]^{\theta-1}} N(x, R + ux, \frac{1}{2}(\bar{u}x - R)), \quad (26)$$

where $\theta = \nu_1 + \nu_2 + \sigma_1 + \sigma_2 + \lambda_1 + \lambda_2 - \mu_1 - \mu_2$ and the function $N(x, R, T)$ collects the factors in the numerator. The final integration over R is elementary.

After considerable algebra we obtain the following result, where we have added the symmetric to (15) contribution with the instanton replaced by the antiinstanton:

$$\begin{aligned} \langle A^a(p), \lambda | T \{ j_\mu(x) j_\nu(0) \} | A^a(p), \lambda \rangle^{I\bar{I}} &= \\ &= i (x_\mu p_\nu + x_\nu p_\mu - \delta_{\mu\nu} (px)) \sum_q e_q^2 \frac{2}{9} \pi^{5/2} d^2 \left(\frac{2\pi}{\alpha_s} \right)^{19/2} \\ &\times \int_0^1 du \cos(ux) \frac{u}{\bar{u}^2} \left(\frac{16}{\xi^3} \right)^{n_f-3} \frac{\Gamma[-bS(\xi)]}{x^4} J(u) \exp \left[-\frac{4\pi}{\alpha_s(\tilde{\rho}^2)} S(\xi) \right] \end{aligned} \quad (27)$$

The effective scale of the coupling under the exponential is equal to

$$\tilde{\rho}^2 = x^2 \frac{\bar{u}}{2} \frac{6}{\xi^2} \frac{4\pi}{\alpha_s(\tilde{\rho}^2)} \quad (28)$$

and the function $J(u)$ is defined as

$$\begin{aligned} J(u) &= 4 \int d\gamma \operatorname{Re} \left\{ \frac{1 - i \cot[\pi b S(\xi)/2]}{\sqrt{1 - \bar{u}/(2\gamma) + i\epsilon}} \right\} \frac{d}{d\gamma} \operatorname{Re} \left\{ \frac{1 - i \cot[\pi b S(\xi)/2]}{\sqrt{1 - \bar{u}/(2\gamma) + i\epsilon}} \right\} \\ &= 1 + O(\bar{u}) \end{aligned} \quad (29)$$

To the accuracy of our approximation, which was in taking into account the dipole-dipole term only in the $\bar{I}I$ interaction, we obtain $\xi = 4u/\bar{u}$, and $S(\xi) = 1 - 6/\xi^2$. It is easy to trace that writing the interaction in the conformal form (8),(9) amounts to the substitution $\xi \rightarrow 2 + 4u/\bar{u} = 2(1+u)/(1-u)$, cf.(11). The integrand in (27) is exact for $1 - u \ll 1$ and to leading accuracy in $\alpha_s(x^2)$.

Making a Fourier transformation, going over to the Minkowski space and taking the imaginary part (15), we obtain the following answer for the instanton-antiinstanton contribution to the structure function of a real gluon:

$$\begin{aligned} F_1^{(G)}(x, Q^2) &= \sum_q e_q^2 \frac{1}{9\bar{x}^2} \frac{d^2 \pi^{9/2}}{bS(\xi_*) [bS(\xi_*) - 1]} \left(\frac{16}{\xi_*^3} \right)^{n_f-3} \\ &\times \left(\frac{2\pi}{\alpha_s(\rho_*^2)} \right)^{19/2} \exp \left[-\left(\frac{4\pi}{\alpha_s(\rho_*^2)} + 2b \right) S(\xi_*) \right] \end{aligned} \quad (30)$$

where the expressions for ρ_* and ξ_* coincide to the ones given in (11), which correspond to the direct evaluation of the integrals in the momentum space by the saddle-point method. Encouraged by this coincidence, we have put by hands an additional factor $\exp(-2bS(\xi_*))$ to the r.h.s. of (30), which arises from taking into account the running of the coupling in the saddle-point equations in (10) and which is difficult to trace starting from the

coordinate space. To our accuracy, we find that the instanton- induced contributions obey the Callan-Gross relation $F_2(x, Q^2) = 2xF_1(x, Q^2)$.

The expression in (30) presents main result of this letter. It gives the exponential correction to the coefficient function in front of the gluon distribution of the leading twist in (3). The exponential factor is exact to the accuracy of (9). Taking into account further terms in the expansion of the action in inverse powers of the conformal parameter, one should in principle add the contributions of rescattering in the initial state (the so-called hard-hard corrections [15]). The preexponential factor in (30) is calculated to leading accuracy in the strong coupling and up to corrections of order $O(1-x)$. The corresponding to (30) contribution to the structure function of the nucleon is obtained in a usual way, making a convolution of (30) with a distribution of gluons in the proton at the scale ρ_*^2 . Leading contribution to (30) in the perturbation theory is due to the mixing with the flavor-singlet quark distribution and is given by the box graph:

$$F_1^{(G)}(x, Q^2)_{pert} = \sum_q e_q^2 [x^2 + \bar{x}^2] \frac{\alpha_s(Q^2)}{2\pi} \ln \left[\frac{Q^2 \bar{x}}{\mu^2 x} \right], \quad (31)$$

where in order to compare to the instanton contribution in (30) one should choose the scale μ to be of order ρ_* .

The instanton-antiinstanton contribution to the structure function of a quark contains a similar contribution shown in Fig.1c. The difference to the graph in Fig.2b considered above is in trivial factors only, and the answer for this contribution reads

$$F_1^{(q)}(x, Q^2) = \left[\sum_{q' \neq q} e_{q'}^2 + \frac{1}{2} e_q^2 \right] \frac{128}{81 \bar{x}^3} \frac{d^2 \pi^{9/2}}{bS(\xi_*) [bS(\xi_*) - 1]} \left(\frac{16}{\xi_*^3} \right)^{n_f - 3} \\ \times \left(\frac{2\pi}{\alpha_s(\rho_*^2)} \right)^{15/2} \exp \left[- \left(\frac{4\pi}{\alpha_s(\rho_*^2)} + 2b \right) S(\xi_*) \right] \quad (32)$$

However, in this case additional contributions exist of the type shown in Fig.1d. They are finite (the integral over instanton size is cut off at $\rho^2 \sim x^2/\alpha_s$), but the instanton-antiinstanton separation R does not stay large at sufficiently small energies (large values of x), but always appears to be of order ρ . This probably means that the structure of nonperturbative contributions to quark distributions is more complicated. This question is under study. The answer given in (32) presents the contribution of the particular saddle point in (11).

4. The region of validity of the above expressions for the $\bar{I}I$ contributions to structure functions is restricted by the requirement that the effective value of the instanton size is not too large, say $\rho_* < 1\text{GeV}^{-1}$. This sets a boundary for the lowest possible value of Q^2 of order 100GeV^2 , see Fig.2. The instanton-induced contribution to the structure function of a gluon in (30) is shown as a function of Bjorken x for different values of

$Q \sim 10 - 100 GeV$ in Fig.3. The contribution of the box graph in (31) is plotted by dots for comparison. It is seen that the \bar{II} contribution is rising very rapidly with the decrease of x , the effect being due mainly to the decrease of the \bar{II} action $S(\xi_*)$. We take $S(\xi_*) = 1/2$ as a reasonable boundary for which our calculation is justified, which translates to the condition that $x > 0.3 - 0.35$. We expect that at $S(\xi_*) \simeq 0.5$ the accuracy of (30) is within one order of magnitude. Thus, instantons produce a well-defined and calculable contribution to the cross section of deep inelastic scattering for sufficiently large values of x and large $Q^2 \sim 100 - 1000 GeV^2$, which turns out, however, to be rather small — of order $10^{-2} - 10^{-5}$ compared to the perturbative cross section. This means that the accuracy of standard perturbative analysis is sufficiently high (in this region of x), and that there is not much hope to observe the instanton-induced contributions to structure functions experimentally. However, instantons are likely to produce events with a very specific structure of the final state, and such peculiarities may be subject to experimental search. A detailed discussion of the hadron distribution in the final state in the instanton-induced events goes beyond the tasks of this letter. On general grounds one may expect a resonance-like production of a fireball of quark and gluon minijets in a narrow region of Bjorken x of order $0.25 - 0.40$ (for photon-gluon scattering), and Q^2 of order $10^2 - 10^3 GeV^2$. Such events may be a direct analog of baryon-number violating processes induced by instantons in the electroweak theory [1]. However, it is not clear whether in the theory with quark confinement like QCD the structure of the distributions in the final state can be obtained from the saddle-point evaluation of functional integrals in the Euclidian space.

Continuation of the calculation of the instanton-induced contributions to lower values of Bjorken x is a difficult problem. Main open question is the behavior of the exponential suppression factor. The situation may be quite different in QCD compared to the electroweak theory, in which case the unitarization corrections due to multiinstantons [16] most likely stop the exponential growth of the instanton-induced cross sections at about one half of the original semiclassical suppression. The reason is that owing to the IR divergences instantons do not produce point-like vertices in QCD, and it is the external virtuality only, which makes possible the separation of well-defined finite contributions. The multiinstanton contributions of Maggioro-Shifman type [17] are absent in QCD, since the virtuality of the photon is able to cut the integrations over the size of the first and the last instantons in the chain only. A more detailed discussion will be given in [12]. Further theoretical analysis of high-energy behavior of the instanton-induced cross sections would be extremely welcome and could well jeopardize application of perturbative methods to the study of deep inelastic scattering at low values of x .

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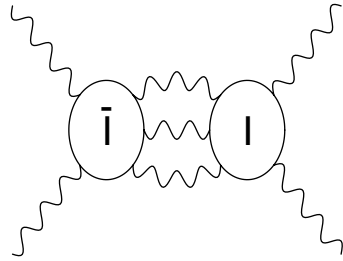
Captions

- Fig. 1** The contribution of the instanton-antiinstanton pair to the cross section of hard gluon-gluon scattering (a), structure function of a gluon (b) and of a quark (c,d). Wavy lines are (nonperturbative) gluons. Solid lines are quark zero modes in the case that they are ending at the instanton (antiinstanton), and quark propagators at the $\bar{I}I$ background otherwise.
- Fig. 2** The non-perturbative scale in deep inelastic scattering (instanton size ρ_*^{-1}), corresponding to the solution of saddle-point equations in (10) as a function of Q and for $S(\xi_*) \sim 0.5 - 0.6$ ($\xi_* \sim 3 - 4$).
- Fig. 3** Nonperturbative contribution to the structure function $F_1(x, Q^2)$ of a real gluon (30) as a function of x for different values of Q (solid curves). The leading perturbative contribution (31) is shown for comparison by dots. The dashed curves show lines with the constant effective value of the action on the $\bar{I}I$ configuration.

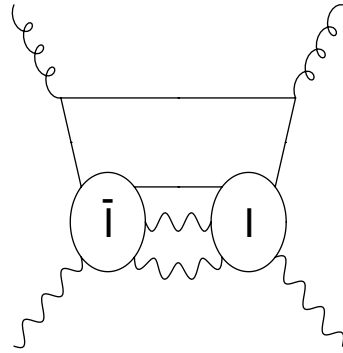
Figure 1: The contribution of the instanton-antiinstanton pair to the cross section of hard gluon-gluon scattering (a), structure function of a gluon (b) and of a quark (c,d). Wavy lines are (nonperturbative) gluons. Solid lines are quark zero modes in the case that they are ending at the instanton (antiinstanton), and quark propagators at the $\bar{I}I$ background otherwise.

Figure 2: The non-perturbative scale in deep inelastic scattering (instanton size ρ_*^{-1}), corresponding to the solution of saddle-point equations in (10) as a function of Q and for $S(\xi_*) \sim 0.5 - 0.6$ ($\xi_* \sim 3 - 4$).

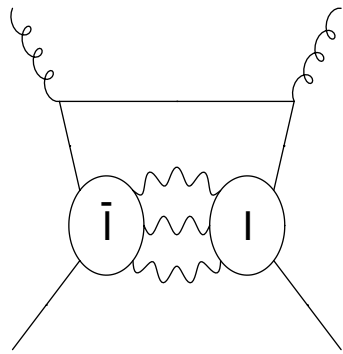
Figure 3: Nonperturbative contribution to the structure function $F_1(x, Q^2)$ of a real gluon (30) as a function of x for different values of Q (solid curves). The leading perturbative contribution (31) is shown for comparison by dots. The dashed curves show lines with the constant effective value of the action on the $\bar{I}I$ configuration.



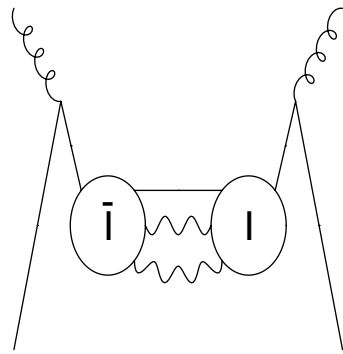
a)



b)



c)



d)

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