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Gluon form factor of the proton from QCD sum rules

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We discuss the possibility to estimate hadronic matrix elements of gluonic operators in the framework of the QCD sum rule approach and consider in detail the particular example of the gluon form factor of the proton, normalized at $Q^2=0$ to the fraction of the proton momentum carried by gluons. To this end we evaluate the proton coupling to a suitable quark–quark–quark–gluon current and calculate the three-point correlation function of a pair of such currents and the traceless part of the gluonic contribution to the energy–momentum tensor. We obtain a rather slow Q^2-dependence of this form factor, which corresponds to the radius of the gluon distribution $R\simeq 0.3$–0.35 fm and is much smaller than the electromagnetic radius.

1. In the last years there has been an increasing interest in the evaluation of hadronic matrix elements and form factors of gluonic operators. This task turns out to be difficult because phenomenological models of the structure of hadrons typically avoid the introduction of explicit gluonic degrees of freedom. As a result, even making an order-of-magnitude estimate becomes non-trivial. For example, we can recall an intensive discussion of the *CP*-odd three-gluon operator $GG\tilde{G}$ [1], which arises in various models of the *CP*-violation and might induce the electric dipole moment of the neutron on the level of current experimental limitations. Existing estimates of the matrix element of this operator over the nucleon are based on simple dimension counting and are controversial. Gluonic form factors of the proton have been discussed in connection with the diffraction dissociation processes at large momentum transfers and high energies [2]. Quantitative estimates are in this case absent.

The QCD sum rules [3] have proved to provide a rather reliable instrument for the calculation of both the static and the dynamic properties of hadrons and have the advantage of being close to the first principles of the theory. The calculation of a certain form factor within this framework proceeds essentially in two steps. At the first step one finds a suitable current, which couples to the hadron of interest and calculates the value of this coupling from the sum rule for the corresponding two-point correlation function. At the second step one considers the three-point function of two such currents and the operator of interest, and finds the hadron form factor using the value of the coupling obtained from the analysis of the two-point function to fix the normalization. Such calculations have been initiated in ref. [4] and have become standard. Practically all calculations of nucleon matrix elements use the particular three-quark current introduced by Ioffe [5]

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$$\eta(x) = [u^a(x) C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu d^c(x)\epsilon^{abc}, \qquad (1)$$

which has proved to be the most suitable, for the reasons that we briefly discuss below.

When addressing the matrix elements of gluonic operators, the QCD sum rules seemingly face the same difficulties as discussed above, since a typical contribution to the three-point correlation function looks in this case as shown in fig. 1a and suffers from additional divergences that are not removed by the Borel transformation. An escape, which we suggest in this paper, is to consider a more complicated current, which contains explicitly a gluon field instead of the one in (1), and simplify drastically, at this cost, the treatment of three-point functions; see a typical graph in fig. 1b.

The idea to consider quark–gluon operators as interpolating currents is in itself not new and has often been used in studies of mesons and baryons with exotic quantum numbers, see e.g. refs. [6,7]. The ρ-meson coupling to the quark–gluon current has been calculated in ref. [8]. The main objective of our study is to find out whether such a program may provide an acceptable accuracy for the evaluation of the form factors of the nucleon and work out the necessary machinery. We propose a suitable quark–quark–quark–gluon current and calculate its coupling to the proton from the corresponding two-point sum rules. This current is further used to calculate the gluon form factor of the proton, normalized at $Q^2=0$ to the fraction of the proton momentum carried by the gluons. The sum rule prediction turns out to be quite reasonable both in the shape and in the absolute normalization. The main result is that the gluon form factor decreases very slowly with Q^2. The slope in the region of $Q^2 \sim 1$–3 GeV^2 appears to be several times less than the slope of the electromagnetic form factor and the radius of the gluon distribution is estimated to be $R_\theta \simeq 0.3$–0.35 fm. The results are quite satisfactory and indicate a sufficiently good accuracy of this method.

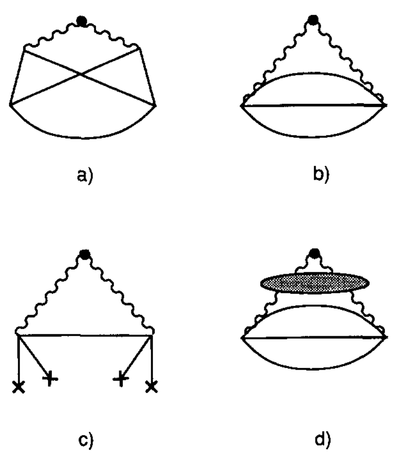

Fig. 1. Contributions to the gluon form factor of the proton in the QCD sum rule approach.

2. The QCD sum rule approach formulated by Shifman, Vainstein and Zakharov [3] is essentially a matching procedure between two representations for the correlation function of suitable currents at euclidean momenta of the order of $p^2 \sim -1$ GeV^2. First, one evaluates the correlation function in perturbation theory and takes into account non-perturbative corrections induced by non-zero vacuum expectation values of quark and gluon condensates. Then, one writes down the representation for the same correlation function as the dispersion integral over hadron states, separating the contribution of the lowest-lying state, and higher resonances and the continuum. The two representations are then equated to each other in the region of intermediate euclidean distances: not too large, so that taking into account a first few terms in the operator expansion gives already a reasonable approximation, and not too small, so that the contribution of the lowest-mass state is distinguishable from higher-mass excitations.

To make such a matching meaningful one should try to minimize the influence of unknown high-order terms in the operator product expansion and high-mass excitations. This is partly achieved by the Borel transformation, which suppresses both sources of uncertainties, and by choosing the interpolating currents of lowest possible dimension. In simple cases, it is possible to show that the "working region" in the range of values of the Borel parameter indeed exists, where both the expansions – from small and from large distances – are under control. In more complicated problems, one normally uses the uncertainties induced by the particular choice of the Borel parameter and by the model for the continuum to estimate the accuracy of the calculation. The large experience with calculations shows that the results obtained in this way are reliable. On the other hand, the QCD sum rules typically cannot be used in a kind of iterative procedure; an improvement of the accuracy by including further corrections (both perturbative and non-perturbative) is not possible.

Although academically one can build the sum rules using arbitrary currents with the required quantum numbers, an inadequate choice may considerably enhance the contributions of higher-mass states and decrease drastically the actual accuracy of the calculation. Thus, the choice of a suitable current always provides an important part of the game. This question has been studied in much detail for the nucleon: in this case one more current with the proper non-relativistic limit exists [9] in addition to that in (1). The current in (1) has proved able to provide more stable sum rules and has the advantage that the four-fermion operators, which arise in the operator product expansion, turn out to be always of the type $(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$, or $(\bar{q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma_\mu\gamma_5 q)$ for which the estimation of the vacuum expectation value by the vacuum saturation is reliable.

We do not address here the problem of the classification of all possible $qqqG$ operators with nucleon quantum numbers, which is rather of academic interest. Instead of that, we note that all the nice features of the current in (1) are preserved by making it *non-local*, e.g. by shifting the d-quark to the point $x+\epsilon$ (and adding the required gauge factor to preserve gauge invariance). Expanding two times in ϵ and averaging over the directions in euclidean space $\epsilon_\mu\epsilon_\nu \to \frac{1}{4}\delta_{\mu\nu}\epsilon^2$ we arrive at the current

$$\eta_G(x) = [u^a(x)C\gamma_\mu u^b(x)]\gamma_5\gamma_\mu\sigma_{\alpha\beta} \times [gG_{\alpha\beta}(x)d(x)]^c\epsilon^{abc}\,, \qquad (2)$$

which we use throughout this paper. The above current does not have definite isospin and couples both to isospin-$\frac{1}{2}$ and -$\frac{3}{2}$ states. The latter do not contribute to our sum rule in a significant way since their masses are greater than the continuum threshold in the isospin-$\frac{1}{2}$ channel. It is easy to project out this unwanted part of $\eta_G(x)$ and to work with the resulting isospin-$\frac{1}{2}$ current which has the structure $\frac{2}{3}[(uu)\sigma Gd-(ud)\sigma Gu]$. This makes the calculations slightly more cumbersome, however. We have checked explicitly that the main results of this work remain virtually unchanged by this projection. Details will be presented in a future publication. We would like to stress here that the technique of QCD sum rules does not require the use of "the best" current from all the possible ones, it is only necessary that the current is not too bad in order that the contribution of interest is not suppressed by some special reason.

The next task is to evaluate the proton coupling to the current in (2). We define

$$\langle 0|\eta|p,s\rangle = \lambda u(p,s)\,, \qquad (3)$$

$$\langle 0|\eta_G|p,s\rangle = m_N^2\lambda_G u(p,s)\,, \qquad (4)$$

and consider the following set of correlation functions:

$$\mathrm{i}\int \mathrm{d}x\,\exp(\mathrm{i}px)\langle 0|\mathrm{T}[\eta(x)\bar{\eta}(0)]|0\rangle = \not{p}\Pi(p^2)+\ldots\,, \qquad (5)$$

$$\mathrm{i}\int \mathrm{d}x\,\exp(\mathrm{i}px)\langle 0|\mathrm{T}[\eta_G(x)\bar{\eta}(0)]|0\rangle = \not{p}\Pi_G(p^2)+\ldots\,, \qquad (6)$$

$$\mathrm{i}\int \mathrm{d}x\,\exp(\mathrm{i}px)\langle 0|\mathrm{T}[\eta_G(x)\bar{\eta}_G(0)]|0\rangle = \not{p}\Pi_{GG}(p^2)+\ldots\,. \qquad (7)$$

In this paper we consider one of the two possible Lorentz structures which has proved [5] to provide more stable sum rules. Proceeding in the standard way we derive the following set of sum rules:

$$2(2\pi)^4\lambda^2\exp(-m_N^2/M^2) = M^6E_3+\tfrac{1}{4}bM^2E_1+\tfrac{4}{3}a^2\,, \qquad (8)$$

$$2(2\pi)^4 m_N^2\lambda\lambda_G\exp(-m_N^2/M^2) = \frac{6}{5}\frac{\alpha_s}{\pi}M^8E_4+\tfrac{1}{2}bM^4E_2-\frac{4}{3}\frac{\alpha_s}{\pi}a^2M^2E_1+\tfrac{2}{3}m_0^2a^2\,, \qquad (9)$$

$$2(2\pi)^4 m_N^4\lambda_G^2\exp(-m_N^2/M^2) = \frac{4}{15}\frac{\alpha_s}{\pi}M^{10}E_5+\frac{16}{9}\frac{\alpha_s}{\pi}a^2M^4E_2 -\tfrac{1}{6}cM^4E_2-\frac{26}{9}\frac{\alpha_s}{\pi}m_0^2a^2M^2E_1+\tfrac{4}{3}\pi\alpha_s m_0^4a^2\,, \qquad (10)$$

where

$$E_n = 1 - \exp(-s_0/M^2) \times \left[1 + \frac{s_0}{M^2} + \ldots + \frac{1}{(n-1)!}\left(\frac{s_0}{M^2}\right)^{n-1}\right], \quad (11)$$

$a = -(2\pi)^2\langle\bar{\psi}\psi\rangle \simeq 0.67\ \mathrm{GeV}^3$ is the quark condensate, and $m_0^2 = 0.65\ \mathrm{GeV}^2$ is the ratio of the mixed quark-gluon and the quark condensate: $m_0^2 = \langle\bar{\psi}g\sigma G\psi\rangle / \langle\bar{\psi}\psi\rangle$. The given values correspond to the standard ITEP values of condensates at a low normalization point $\mu \sim 500$ MeV $\langle\bar{\psi}\psi\rangle = -(240\ \mathrm{MeV})^3$ and $m_0^2 = 0.8\ \mathrm{GeV}^2$, rescaled to the normalization point $\mu^2 \sim M^2 \sim 1\ \mathrm{GeV}^2$ with the appropriate anomalous dimensions. The other entries are $b = \langle g^2G^2\rangle = 0.47\ \mathrm{GeV}^4$, and $c = \langle g^3fG^3\rangle = 0.046\ \mathrm{GeV}^6$. The influence of the results on the particular values of gluon condensates is very weak. Note that the contribution of the two-gluon condensate to the correlation function in (7) vanishes. The strong coupling at 1 GeV is taken to be $\alpha_s = 0.37$ ($\Lambda = 150$ MeV).

The sum rule in (8) is just the standard sum rule considered by Ioffe and we use it for the normalization. Taking the ratio of the sum rules in (9) and (10) to the one in (8) considerably improves the stability and reduces the uncertainties in input values of the condensates.

A general drawback of sum rules for correlation functions of high dimension is that they are strongly affected by values of vacuum condensates of operators of high dimension, which are badly known. This issue is seen clearly on the example of the sum rule in (10), which is dominated by the contribution of the operator of dimension 10. In addition, this sum rule is more sensitive to the model of the continuum (the value of the continuum threshold s_0). In effect one can only use this sum rule for a crude estimate. On the other hand, the sum rule in (9) should be rather precise. Note that one should not try to fix the continuum threshold anew from each of the sum rules, but rather conform to the standard value $s_0 = (1.5\ \mathrm{GeV})^2$ which has been found from the analysis of (8). The same is true for the "working region" of the Borel parameter, which has been found in ref. [5] to be $M^2 \sim 1$–$1.5\ \mathrm{GeV}^2$.

The ratio of the sum rules in (9) and (8) is shown as a function of the Borel parameter in fig. 2. This ratio turns out to be very stable and practically insensitive to the choice of parameters (e.g. to the particular value of the continuum threshold). We find

$$\frac{\lambda_G}{\lambda} \simeq 0.35 \quad (12)$$

with a small error. Using the conventional value $2(2\pi)^4\lambda^2 \simeq 2.5\ \mathrm{GeV}^6$ (at 1 GeV) we arrive at the absolute value of the coupling

$$2(2\pi)^4\lambda_G^2 \simeq 0.30\ \mathrm{GeV}^6\ (\text{at 1 GeV}). \quad (13)$$

We estimate the accuracy of the value in (13) not to be worse than (20–30)%.

To check the self-consistency of the procedure it is necessary to make sure whether the same or a close value of the coupling may be obtained using the diagonal sum rule in (10). The ratio of the sum rule in (9) to the square root of the product of the sum rules in (10) and (8) is plotted in fig. 3. This ratio should equal 1, provided the separation of the nucleon contribution is exact, and in practice turns out to be ~ 0.5–0.7. This result is encouraging, since it indicates that the error made in the estimation of the vacuum expectation value of the operator of dimension 10 in eq. (10) by the vacuum saturation is not too large, of a factor 2. We recall that the current in (2) is deliberately chosen so as to make the estimation of vacuum averages by the vacuum saturation more reliable.

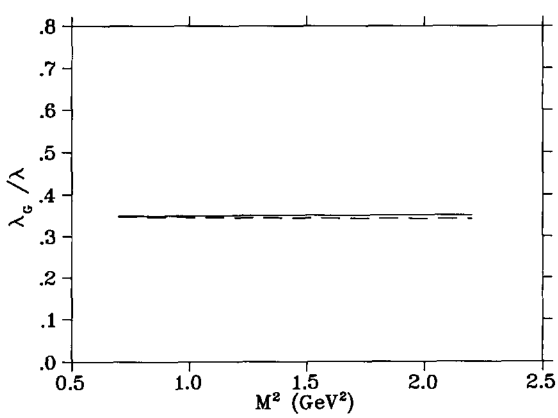

Fig. 2. The ratio of the couplings λ_G/λ from the ratio of sum rules in (9) and in (8). Solid and dashed lines correspond to the choice of the continuum threshold $\sqrt{s_0} = 1.4$ and 1.6 GeV, respectively.

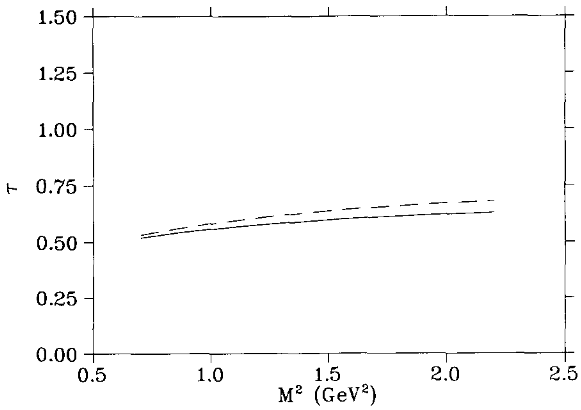

Fig. 3. The ratio of the sum rule in (9) to the square root of the product of the sum rules in (10) and in (8). Solid and dashed lines correspond to the choice of the continuum threshold $\sqrt{s_0}=1.4$ and 1.6 GeV, respectively.

3. Now we use the above results to calculate the form factor of the proton induced by the traceless part of the gluonic contribution to the energy–momentum tensor

$$\theta^G_{\mu\nu}=G^A_{\mu\xi}G^A_{\xi\nu}-\text{traces}\,, \tag{14}$$

and to this end consider the three-point function

$$\begin{aligned} &i^2\int d^4x\exp(iqx)\int d^4y\exp(iqy)\\ &\quad\times\langle 0|T[\eta_G(x)\theta^G_{\mu\nu}(y)\bar\eta_G(0)]|0\rangle\\ &\quad=p_\mu p_\nu \not p\, T(p^2,(p+q)^2,q^2)+\dots\,. \end{aligned} \tag{15}$$

The form factor of interest is defined as

$$\begin{aligned}\langle p+q|\theta^G_{\mu\nu}|p\rangle &=(2p_\mu p_\nu-\tfrac{1}{2}g_{\mu\nu}m_N^2)F_\theta(Q^2)\\ &\quad+\dots\,,\end{aligned} \tag{16}$$

where $Q^2=-q^2>0$, and is normalized at $Q^2=0$ to the fraction of the proton momentum carried by gluons, i.e. to the contribution of gluons to the second moment of the structure function $F_2(x,Q^2)$,

$$F_\theta(Q^2=0)=\int_0^1 dx\,F_2(x,Q^2)=\int_0^1 dx\,xG(x)\,.$$

Here $G(x)$ is the probability to find a gluon–parton in the proton, carrying the fraction x of the total momentum, see e.g. ref. [10]. The contribution of the form factor F_θ to the correlation function in (15) equals

$$\frac{2\lambda_G^2 m_N^4 p_\mu p_\nu \not p}{[(p+q)^2-m_N^2](p^2-m_N^2)}F_\theta(Q^2)+\dots\,. \tag{17}$$

On the other hand, following the procedure of ref. [4] we can write the correlation function in (15) in form of the double dispersion relation

$$\begin{aligned}&T(p^2,(p+q)^2,q^2)\\ &\quad=\int\frac{ds_1}{s_1-p^2}\int\frac{ds_2}{s_2-(p+q)^2}\rho(s_1,s_2,Q^2)\,.\end{aligned} \tag{18}$$

Making the standard assumption that the contributions of higher resonances and the continuum are removed by setting an upper bound s_0 in both the integrals over s_1 and s_2 (which coincides with the continuum threshold in the corresponding two-point sum rules), and making the Borel transformation in both momenta associated with proton currents, we arrive at the sum rule

$$\begin{aligned}&2\lambda_G^2 m_N^4 F_\theta(Q^2)\exp(-m_N^2/M^2)\\ &\quad=\int_0^{s_0}ds_1\int_0^{s_0}ds_2\exp[-(s_1+s_2)/2M^2]\\ &\quad\times\rho(s_1,s_2,Q^2)\,,\end{aligned} \tag{19}$$

in which the value of the Borel parameter should be taken in the same interval of values as in the two-point sum rules in (8)–(10). The calculation of the double spectral density $\rho(s_1,s_2,q^2)$ turns out to be straightforward but tedious. We take into account the perturbative graph and the contribution of the four-quark condensate, shown in figs. 1b, 1c. The results are

$$\rho_{\langle\bar\psi\psi\rangle^2}=\frac{128\alpha_s}{9\pi}\langle\bar\psi\psi\rangle^2Q^6s_1s_2\left(\frac{5s_1s_2}{R^{7/2}}+\frac{1}{R^{5/2}}\right), \tag{20}$$

$$\rho_{\rm pert}=\frac{\alpha_s Q^4}{840\pi^5}\left(\frac{4R^{1/2}}{5}-\frac{8Q^2+7(s_1+s_2)}{10}\right.$$
$$-\frac{Q^2(s_1+s_2)+(s_1-s_2)^2-12s_1s_2}{10R^{1/2}}$$
$$+\frac{s_1s_2[Q^2(s_1+s_2)+(s_1-s_2)^2-4s_1s_2]}{5R^{3/2}}$$
$$-\frac{s_1^2s_2^2[3Q^2(s_1+s_2)+3(s_1-s_2)^2-4s_1s_2]}{5R^{5/2}}$$
$$\left.+\frac{2s_1^3s_2^3[Q^2(s_1+s_2)+(s_1-s_2)^2]}{R^{7/2}}\right), \qquad (21)$$

where

$$R(s_1,s_2,Q^2)=(s_1+s_2+Q^2)^2-4s_1s_2\,. \qquad (22)$$

It turns out that the contribution of the perturbative graph is numerically small compared with the contribution of the four-quark condensate over all the interval of momentum transfers. The reason for the smallness of the perturbative contribution is purely kinematical and can be traced to the strong suppression by the phase space factor for the production of the three-quark jet in the intermediate state. This suppression can easily be understood, since the calculated graphs contribute to the so-called Feynman mechanism for the hadron form factors [11], in which the large momentum flows through the hadron wave function. The Feynman-type contribution picks up the high-momentum component in the hadron wave function with nearly all the initial momentum (in the proton infinite momentum frame) carried by the gluon. The momentum carried by the quarks is small, and the main contribution comes from the quark condensate. The mechanism of hard rescattering which is dominating at asymptotically large values of Q^2 corresponds, within the framework of QCD sum rules, to the radiative corrections to the leading-order graphs in figs. 1b, 1c, which we do not consider in this paper.

We have not taken into account the contribution of the gluon condensate, which suffers from the same kinematical suppression as the perturbative graph and is expected to be small.

The resulting behavior of the form factor is shown in fig. 4. The QCD sum rule predictions are shown by dots, and we have made fits of them in the interval $0.5<Q^2<3$ GeV^2 using the expression

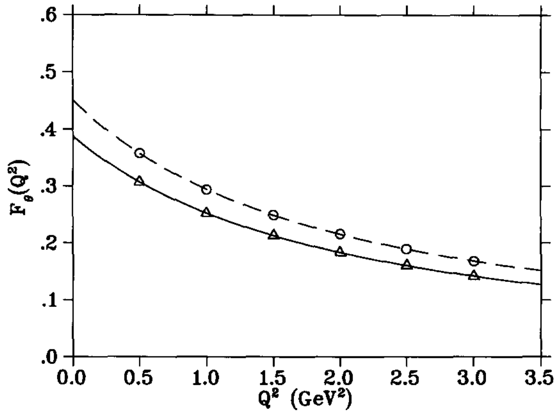

Fig. 4. The gluon form factor (16) of the proton. QCD sum rules prediction for $M^2=1$ GeV^2 (triangles) and for $M^2=2$ GeV^2 (circles). The value of the continuum threshold is $\sqrt{s_0}=1.5$ GeV. The curves present results of fits using the parametrization in (23).

$$F_\theta(Q^2)=\frac{F_\theta(0)}{[1+Q^2/(\alpha\mu^2)]^\alpha}\,. \qquad (23)$$

Taking $\alpha=3$ as suggested by the quark counting rule we get

$$F_\theta(0)=0.36-0.42,\quad \mu^2\simeq 2.6\ \mathrm{GeV}^2,\quad R_\theta\simeq 0.3\ \mathrm{fm}\,, \qquad (24)$$

where R_θ is the radius of the corresponding gluon distribution $R_\theta^2=6/\mu^2$, defined in the usual way as $\mathrm{d}F_\theta(Q^2)/\mathrm{d}Q^2|_{Q^2=0}=-\frac{1}{6}R_\theta^2F_\theta(0)$. The given interval of values of $F_\theta(0)$ corresponds to the choice of the Borel parameter within the interval $1<M^2<2$ GeV^2. Varying the value of the continuum threshold within the limits $1.4<\sqrt{s_0}<1.6$ GeV yields additional 10% uncertainty in the absolute normalization. On the other hand, taking the power α as a free parameter, we get $\alpha=1.2$–1.3 and

$$F_\theta(0)=0.38-0.46,\quad \mu^2\simeq 2.0\ \mathrm{GeV}^2,\quad R_\theta\simeq 0.35\ \mathrm{fm}\,. \qquad (25)$$

The results of this fit are shown in fig. 4 by the solid and the dashed curves, which correspond to values $M^2=1$ and $M^2=2$ GeV^2, respectively.

The functional form in (23) should not be taken too seriously, since the QCD sum rules are only applicable in a rather narrow region of moderate Q^2.

The restriction from the side of large Q^2 is due to the contributions of hard rescattering and to power corrections of higher order, which tend to increase with the rise of the momentum transfer. An explicit estimate of the upper bound of Q^2 would require a cumbersome calculation of the power correction of dimension 8; this seems not to be worthwhile in this paper, which mainly aims at the methodical questions. An experience of existing calculations shows that one should expect their effect to be a factor of 2–3 smaller than the contribution of the four-quark condensate. Note that unlike the case of two-point correlation functions considered above, the coefficient functions in front of operators of dimension 6 and 8 contain the same number of loops (one), so that the contribution of the latter is not enhanced by an extra factor $(2\pi)^2$. In the case of a pion form factor, the "working region" in Q^2 extends to values of the order of 2–3 GeV^2 and may further be enlarged by introducing the non-local quark condensates [12].

The restriction from the side of small values of Q^2 is due to a new type of power corrections, which come into the game in the case that the relevant distances in the t-channel become large [13], see the graph in fig. 1d. In our case, these corrections are proportional to the correlation function at $Q^2 \to 0$

$$\Pi_{\mu\nu\alpha\beta} = \mathrm{i} \int \mathrm{d}^4x \exp(iqx) \langle 0 | \mathrm{T}[\theta^G_{\mu\nu}(x) \theta^G_{\alpha\beta}(0)] | 0 \rangle$$

$$= (g_{\mu\alpha} g_{\nu\beta} + g_{\nu\alpha} g_{\mu\beta}) \langle\langle \Pi \rangle\rangle + \dots , \qquad (26)$$

which has been studied in ref. [14] in connection with tensor gluonia. By an explicit calculation, we have found that the corresponding bilocal power correction to the three-point correlation function in (15) is numerically small for the perturbative graph, and vanishes identically for the case of the $\langle \bar{\psi}\psi \rangle^2$ contribution. The corresponding correction to $F_\theta(0)$ turns out to be of the order of 0.02 with a large error. Thus, we argue that the sum rule in (19) can be extrapolated to $Q^2 = 0$ without significant corrections.

The value of $F_\theta(0)$ has a direct phenomenological relevance, since it should give the fraction of the proton momentum carried by gluons and is measured in experiments on deep inelastic lepton scattering as the contribution of gluons to the second moment of the structure function F_2. Our value corresponds to a low normalization point of the order of 1 GeV and agrees with the experimental value ~ 0.4. The agreement is even better than expected since the typical accuracy of the QCD sum rules for the three-point functions is about 30%. Earlier calculations of the fraction of the proton momentum carried by gluons in the framework of QCD sum rules have used the momentum sum rule, relating this quantity to the momentum carried by quarks, and have yielded values ~ 0.2 [15] and ~ 0.35 [16] (at 1 GeV) which are somewhat below our result.

4. The objective of this paper is to point out a possibility to obtain, within the QCD sum rule approach, the semiquantitative estimates of matrix elements and form factors of gluon operators by considering a more complicated interpolating current for the proton, which contains explicitly the gluon field. A suitable current is proposed and its coupling to the proton is calculated. We check the potential accuracy of this approach by considering the gluon form factor of the proton, which is normalized at $Q^2 = 0$ to the fraction of the proton momentum carried by gluons and find good agreement with experimental value. We estimate the radius of the corresponding gluon distribution as $R_\theta = 0.3$–0.35 fm, which confirms the conventional expectations about clustering of glue in the proton. This value turns out to be in remarkable agreement with the estimations for the size of the constituent quark which have been obtained from the condition of self-consistency of the additive quark model [17] and follow from the instanton models of the QCD vacuum [18].

The possibilities for further applications of this method are rather extensive. Among the most interesting ones we can mention the calculation of the form factor normalized to the fraction of the proton spin carried by gluons, estimates of higher-twist contributions to moments of deep inelastic structure functions, and the evaluation of the nucleon expectation value of the three-gluon operator [1] discussed above.

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