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SEMILEPTONIC INCLUSIVE B–DECAYS AND THE DETERMINATION OF $|V_{ub}|$ FROM QCD SUM RULES

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We calculate the electron spectrum of semileptonic decays of B-mesons into non-charmed hadrons. The shape of the spectrum obtained from QCD sum rules is in general agreement with quark model calculations. At high electron energies the decrease of the spectrum is less steep than predicted by Altarelli et al. Our analysis yields a total decay rate $\Gamma(B \to X_u e\bar{\nu}) = (6.8 \pm 2.0)\, 10^{13}\, |V_{ub}|^2\, \mathrm{s}^{-1}$. For the spectrum integrated over electron energies in the interval 2.4 GeV $\leq E_e \leq$ 2.6 GeV in the laboratory frame we obtain $\Gamma(B \to X_u e\bar{\nu}) = (5.0 \pm 1.6)\, 10^{12}\, |V_{ub}|^2\, \mathrm{s}^{-1}$. Estimating all sorts of theoretical uncertainties, we obtain $|V_{ub}| = 0.003 \pm 0.001$ for the $b \to u$ weak transition matrix element from the new CLEO-data.

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I. INTRODUCTION

Inclusive semileptonic decays of B-mesons have been studied [1,2] for already more than a decade. Lacking a reliable identification of the hadrons in the final state, the data on semileptonic decays provide at present the most accurate information on the determination of the CKM-matrix-element $|V_{ub}|$. The accuracy of the determination of $|V_{ub}|$ is, however, limited to a great extent by the fact that the $b \to u$ transitions can experimentally be separated from the dominating $b \to c$ decays only in a rather narrow kinematical region of high electron energies, close to the kinematical threshold.

Most theoretical predictions for the spectrum are based on a spectator model [3]. This approach makes use of the fact that in the limit of infinite b-quark mass m_b the decay of the meson is reduced to the β-decay of a free b-quark, which at large electron energies is modified by Sudakov-type radiative corrections. In practice, however, the b-quark is not heavy enough, and both the interaction with the spectator quark and hadronization effects cannot be neglected even for the total semileptonic decay rate. The numerical importance of $1/m_b$ corrections becomes apparent already from simple phase space considerations: since the total decay rate is proportional to m_B^5, the replacement of the hadron by the quark mass reduces the decay rate by nearly a factor two. Even for the ratio of semileptonic to hadronic width, where the dimensionful factor m_B^5 cancels, a calculation of the $b \to c$ transition using free quarks yields a value which is (10-15)% below the data [4,5]. The situation gets considerably worse for the electron spectrum near the charm threshold, where the heavy quark expansion breaks down and there is no reason to believe that Sudakov-type effects provide the most important correction. In existing calculations of the spectrum in this region the deviations from a simple free quark decay picture are taken into account in a model-dependent way, introducing constituent quark masses and Fermi-motion of the quarks inside the hadrons by a certain smearing of the momentum [3] or by using the Peterson fragmentation function [6]. The results show that the influence of the spectator quark and hadronization effects become indeed important at large electron energies, and quantitative predictions depend strongly on the model assumptions and on the parameter values. Especially unpleasant is that the kinematical endpoint is not well defined, since it depends on the constituent quark masses used in the model and not on the mass of the hadrons.

In an alternative approach advocated in Refs. [7,8,9] the electron spectrum of the inclusive decay is obtained by an explicit summation of contributions of various exclusive channels, e.g. $B \to \pi e\nu$, $B \to \rho e\nu$, which are calculated in the framework of a quark model. In this method, the results rely heavily on the assumed shape of the exclusive form factors. In addition, it is not possible to take into account exclusive decays with two or more nonresonant mesons (e.g. two pions in an S-state), which, in contrast to the decays to charmed final states, are expected to play a nonnegligible role [10]. Recently, a two-component model has been suggested in Ref. [10], which aims to combine the positive and avoid the negative features of both approaches. However, the justification of this two-component model has been questioned in Ref. [11].

In view of conflicting results in the current literature, we find it important to address the problem from a completely different point of view. In this paper we calculate the electron spectrum of the inclusive semileptonic B-decay within the framework of QCD sum rules.

This method is due to Shifman, Vainshtein and Zakharov [12]. It is essentially a matching procedure between the operator product expansion for a suitable correlation function in the not so deep Euclidean region and the representation of the same correlation function as dispersion integral in terms of contributing hadronic states. Nonperturbative effects are taken into account by nonvanishing vacuum expectation values of gauge invariant operators, the so-called condensates. QCD sum rules have proved to be a very successful tool for calculations of both static and decay properties of light and heavy hadrons. Although they often provide less detailed results than quark model calculations, they have the advantage of being closer to the first principles of QCD. In recent papers we have applied this method to the study of exclusive semileptonic decays [13,14,15]. The application to inclusive decays poses some new problems, which are discussed below. We show that it is possible to use the sum rule technique in order to calculate the electron spectrum up to an electron energy $E_e \simeq 2.1\,\mathrm{GeV}$ in the rest frame of the B-meson, which is slightly below the charm threshold. The shape of the spectrum and a constraint at the endpoint, however, allow an extrapolation to the experimentally accessible region $E_e \gtrsim 2.3\,\mathrm{GeV}$ without introducing a large numerical uncertainty. Predictions for the shape of the spectrum and integrated rates are given and compared to other model calculations.

Our paper is organized as follows. In Sec. II we recollect necessary kinematical formulae. In Sec. III we derive sum rules for the electron spectrum and discuss their region of applicability, which is constrained by the requirements of a sufficiently large interval of duality and the existence of the short-distance expansion. In Sec. IV we discuss the heavy quark limit of our sum rules, Sec. V contains numerical results and conclusions. Technical details of the calculation are given in the appendix.

II. KINEMATICS

The total width Γ of the inclusive decay $B \to X_u e^- \bar{\nu}$, where X_u denotes a charmless hadronic state, can be written in terms of a leptonic tensor $L_{\mu\nu}$ and a hadronic tensor $W_{\mu\nu}$ as

$$\Gamma = \frac{G_F^2 |V_{ub}|^2}{32\pi^5 m_B} \int \frac{d^3p_e}{2E_e}\,\frac{d^3p_\nu}{2E_\nu}\,W_{\mu\nu}\,L^{\mu\nu} \tag{2.1}$$

with

$$L^{\mu\nu} = 2\left(p_e^\mu p_\nu^\nu + p_\nu^\mu p_e^\nu - g^{\mu\nu} p_e p_\nu + i\epsilon^{\mu\nu}{}_{\rho\sigma} p_e^\rho p_\nu^\sigma\right) \tag{2.2}$$

and

$$W_{\mu\nu} = \sum_n \int \prod_{i=1}^{n} \left[\frac{d^3p_i}{(2\pi)^3 2E_i}\right] (2\pi)^3\,\delta^4(p_B - p_e - p_\nu - \sum_i^n p_i)\,\langle\, B \,|\, j_\nu^\dagger(0)\,|\, n\,\rangle\,\langle\, n\,|\, j_\mu(0)\,|\, B\,\rangle. \tag{2.3}$$

The summation includes all possible $n-$particle final hadronic states; $j_\mu = \bar{u}\gamma_\mu(1-\gamma_5)b$ is the weak current mediating the decay $b \to u$, and p_e, p_ν, p_B are the four-momenta of the electron, antineutrino and B-meson, respectively.

The hadronic tensor $W_{\mu\nu}$ can be expressed in terms of invariant functions:

$$\begin{aligned}
W_{\mu\nu} &= -\int \frac{d^4x}{2\pi}\, e^{iqx}\, \langle\, B\,|\, \left[j_\mu(x), j_\nu^\dagger(0) \right] |\, B\,\rangle \\
&= \frac{1}{\pi} \mathrm{Im}\, i \int d^4x\, e^{iqx}\, \langle\, B\,|\, T j_\nu^\dagger(0) j_\mu(x)\,|\, B\,\rangle \\
&= -\, g_{\mu\nu} W_1 + p_{B\mu} p_{B\nu} W_2 - i\epsilon_{\mu\nu\rho\sigma} p_B^\rho q^\sigma W_3 + \ldots
\end{aligned} \tag{2.4}$$

The missing terms proportional to different Lorentz structures do not contribute to the decay rate in the limit of vanishing lepton mass. The structure functions W_i depend on the invariant mass squared of the hadrons in the final state, $s = (p_B - p_e - p_\nu)^2$, and on the invariant mass squared of the lepton pair, $q^2 = (p_e + p_\nu)^2$.

Integrating over the neutrino momentum, we obtain from (2.1):

$$\begin{aligned}
\Gamma = \frac{G_F^2 |V_{ub}|^2}{32\pi^3 m_B^2} \int dE_e\, dq^2\, ds \Bigg\{ & q^2 W_1 + m_B^2 \left(\frac{E_e}{m_B} [m_B^2 + q^2 - s] - 2E_e^2 - \frac{q^2}{2} \right) W_2 \\
& + m_B q^2 \left(\frac{m_B^2 + q^2 - s}{2m_B} - 2E_e \right) W_3 \Bigg\}
\end{aligned} \tag{2.5}$$

with the electron energy E_e in the rest-frame of the B-meson. The limits of integration are given by:

$$0 \le E_e \le \frac{m_B}{2}, \tag{2.6a}$$

$$0 \le q^2 \le 2 m_B E_e, \tag{2.6b}$$

$$0 \le s \le (m_B - 2E_e)(m_B - q^2/(2E_e)). \tag{2.6c}$$

The subprocess with the lowest possible value of s is the production of a single pion in the final state. Hence the structure functions W_i vanish for s less than m_π^2 and the accessible phase space is restricted to the values depicted in Fig. 1. The shaded area indicates the region of phase space where $s \le 1$ GeV^2, the dashed-dotted lines bound regions with $s \le 5, 10, 15, 20\,\mathrm{GeV}^2$ (from top to bottom). Since in the present paper the calculations are carried out in the chiral limit $m_\pi \to 0$, we will in the following stick to the limits of integration given in (2.6a).

III. DERIVATION OF THE SUM RULE

Our objective is to evaluate the structure functions W_i entering (2.5) by means of QCD sum rules. To this end we consider the four-point function

$$\begin{aligned}
T_{\mu\nu} &= i^3 \int d^4x\, d^4y\, d^4z e^{-iq_1x - ip_1y + ip_2z} \langle\, 0\,|\, T j_B(z) j_\nu^\dagger(0) j_\mu(x) j_B^\dagger(y)\,|\, 0\,\rangle \\
&= T_g g_{\mu\nu} + T_p \frac{1}{4}(p_1 + p_2)_\mu (p_1 + p_2)_\nu + T_\epsilon \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} (p_1 + p_2)^\rho q_1^\sigma + \ldots
\end{aligned} \tag{3.1}$$

where $j_B = \bar{q} i \gamma_5 b$ is the interpolating field of the B-meson and $j_\mu = \bar{u}\gamma_\mu(1-\gamma_5)b$ is the weak current mediating the $b \to u$ decay.

The four-point function in (3.1) can be calculated in perturbation theory in the deep Euclidean region, i.e. for large negative values of the virtualities s, $(p_1^2 - m_b^2)$ and $(p_2^2 - m_b^2)$. When the physical region is approached, nonperturbative effects become important. The idea underlying the QCD sum rules method is that at virtualities of $\sim -1\,\mathrm{GeV}^2$ the most important nonperturbative corrections can be taken into account by nonvanishing vacuum expectation values of operators occurring in the operator product expansion of nonlocal matrix elements like the correlation function (3.1).

In our calculation, we take into account the leading perturbative contribution, which is given by the box-diagram shown in Fig. 2(a), and the contributions of the quark and the mixed condensate, examples of which are given in Fig. 2(b).

The correlation function (3.1) at $t = (p_1 - p_2)^2 = 0$ and $q^2 := q_1^2 = (p_2 - p_1 - q_1)^2$ is related to hadronic matrix elements via dispersion relations. We express the invariant functions $T_i(s, p_1^2, p_2^2, q^2)$ by the dispersion relation (see e.g. [16])

$$T_i(s, p_1^2, p_2^2, q^2) = \int\limits_{m_\pi^2}^{\infty} d\tilde{s}\, \frac{W_i(\tilde{s}, p_1^2, p_2^2, q^2)}{\tilde{s} - s} + \int\limits_{-\infty}^{s_u} d\tilde{s}\, \frac{R_i(\tilde{s}, p_1^2, p_2^2, q^2)}{\tilde{s} - s}\,. \tag{3.2}$$

The dispersive part for $s > m_\pi^2$ is due to the insertion of intermediate states in the direct channel, $\langle\, 0 \,|\, j_B\, j_\nu^\dagger \,|\, Z_s \,\rangle \langle\, Z_s \,|\, j_\mu\, j_B^\dagger \,|\, 0 \,\rangle$, whereas the dispersive part below s_u comes from the crossed channel $\langle\, 0 \,|\, j_B\, j_\mu \,|\, Z_u \,\rangle \langle\, Z_u \,|\, j_\nu^\dagger\, j_B^\dagger \,|\, 0 \,\rangle$. The lowest lying intermediate state Z_u contains two heavy quarks und thus

$$s_u = p_1^2 + p_2^2 + 2q^2 - (m_B + m_{B^*})^2\,. \tag{3.3}$$

In the heavy quark limit $m_Q \to \infty$, $q^2/m_Q^2 < 1$, the contribution of the crossed channel vanishes, and thus the structure functions can be related directly to the expansion at short distances [5]. On the other hand, for finite quark masses the two cuts may come very close to each other for on-mass-shell B-mesons and $q^2 \to m_B^2$. It is specific to the QCD sum rule approach, however, that calculations are done for sufficiently off-shell values of p_i^2: $p_i^2 - m_B^2 \lesssim -1$ GeV^2. In this way the singularities of the direct and the crossed channel are well separated. Furthermore, the application of this technique requires q^2 to stay several GeV^2 below m_b^2, the quark mass squared, and hence the branching point s_u lies at least 6 GeV^2 below the the physical region of the B-decay. We shall see later that in our approach the contribution of the crossed channel can be neglected (see below Eq. (3.8)). The required information on the structure functions $W_i(s, q^2)$ of the B-decay is contained in the terms with poles in both the variables p_1^2 and p_2^2 of the functions $W_i(s, p_1^2, p_2^2, q^2)$ in Eq. (3.2):

$$W_i(s, p_1^2, p_2^2, q^2) = \frac{|\langle\, 0 \,|\, j_B \,|\, B \,\rangle|^2}{(p_1^2 - m_B^2)(p_2^2 - m_B^2)}\, W_i(s, q^2) + \ldots \tag{3.4}$$

Here the dots stand for contributions of higher resonances and the continuum.

In order to estimate the contribution of higher states in (3.4) we use an approximation which is standard in the QCD sum rule approach and assume that all contributions of higher mass states are eliminated by expressing the four-point correlation function in (3.1) by a double dispersion relation in p_1^2 and p_2^2 and retaining only the contribution below a certain threshold s_0. The interval of duality, i.e. the value of the continuum threshold s_0, is estimated by the criterium of stability of the sum rules.

Following [12], we subject the correlation function (3.1) to a Borel transformation in both the variables p_1^2 and p_2^2. For an arbitrary function of the Euclidean momentum, $f(P^2)$ with $P^2 = -p^2$, this transformation is defined as

$$\widehat{B}\, f = \lim_{\substack{P^2\to\infty, N\to\infty \\ P^2/N=M^2 \text{ fixed}}} \frac{1}{N!}(-P^2)^{N+1}\frac{d^{N+1}}{(dP^2)^{N+1}}\, f \tag{3.5}$$

where M^2 is a new variable, called Borel parameter. The objective of this step is to reduce the dependence of the sum rules on the unknown higher-order terms in the operator expansion and on the continuum model. Indeed, the application of the Borel transformation to a typical contribution yields

$$\widehat{B}\, \frac{1}{(p^2-m^2)^n} = \frac{1}{(n-1)!}(-1)^n\frac{1}{(M^2)^n}e^{-m^2/M^2}, \tag{3.6}$$

and thus contributions of vacuum condensates of high dimension (which contain high powers of $(p^2 - m^2)$ in the denominator) are suppressed by factorials. In addition, the contributions of higher mass states become exponentially suppressed, since the Borel transformed expressions for the hadron propagators $(p^2-m_1^2)^{-1}$ and $(p^2-m_2^2)^{-1}$ differ by an exponential factor $\exp(-(m_2^2-m_1^2)/M^2)$. It is important to note that a single Borel transformation in the variable $p_1^2 = p_2^2 = p^2$ is not sufficient for our purposes, since it does not suppress terms containing a pole in one variable only – either in p_1^2 or in p_2^2.

Since, according to Eq. (3.2), we are further interested in comparing $T_{\mu\nu}$ to the dispersion integral in s in terms of the structure functions W_i and R_i, we express the amplitudes T_i in form of triple dispersion integrals in s, p_1^2 and p_2^2. This step requires some care, since in the region of positive q^2 the condition $t = (p_1 - p_2)^2 = 0$ enforces $p_1^2 = p_2^2$. Thus, we start from negative values of q^2, where p_1^2 and p_2^2 are independent variables and write

$$T_i(p_1^2, p_2^2, s, q^2, t=0) = \int d\tilde{s}\, ds_1\, ds_2 \frac{\rho_i(s_1, s_2, \tilde{s}, q^2)}{(s_1-p_1^2)(s_2-p_2^2)(\tilde{s}-s)} + \ldots \tag{3.7}$$

As can be shown, (3.7) defines an analytic function of q^2 and can be continued to positive values of q^2. As pointed out in [13], for positive q^2 non-Landau singularities may generally enter the game. These additional contributions are, however, numerically unimportant in the present analysis and thus have been omitted, cf. the appendix for details.

Due to the condition $t = 0$, the support of the spectral function ρ_i in (3.7) is restricted to the line $s_1 = s_2$, i.e. $\rho_i(s_1, s_2, \tilde{s}, q^2) = \rho_i(s_1, \tilde{s}, q^2)\delta(s_1 - s_2)$, which means that the four-point function does not receive any contribution from non-diagonal transitions like $B \to B'$. All contributions to the four-point function with a single pole only in p_1^2 or p_2^2 cannot be expressed as double dispersion relation in both p_1^2 and p_2^2 and are related to subtraction terms in one of the variables. All these subtractions are eliminated by the double Borel transformation.

Putting together (3.1), (3.2) and (3.7) we get, after performing the Borel transformation:

$$|\langle\, 0\,|\, j_B\,|\, B\,\rangle|^2\, e^{-2m_b^2/M^2} \left\{ \int\limits_{m_\pi^2}^{\infty} d\tilde{s} \frac{W_i(\tilde{s},q^2)}{\tilde{s}-s} + \int\limits_{-\infty}^{s_u} d\tilde{s} \frac{R_i(\tilde{s},q^2)}{\tilde{s}-s} \right\} = \int d\tilde{s} \int\limits_0^{s_0} ds_1 \frac{\rho_i(s_1,\tilde{s},q^2)}{\tilde{s}-s}\, e^{-2s_1/M^2} \tag{3.8}$$

where $R_i(s, q^2)$ is defined analogously to $W_i(s, q^2)$ in (3.4).

Staying strictly in the framework of the operator product expansion we cannot discriminate distributions from the direct and the crossed channel. We, therefore, have to invoke a concept of duality and assume that the relation (3.8) holds true for the spectral densities themselves, if the latter are smeared over a sufficiently large interval of s. The leading contributions to the operator product expansion of (3.7) do not contribute to the discontinuities R_i of the crossed channel; for this reason the crossed channel singularities do not occur in our calculations.

Existing calculations of deep inelastic structure functions by QCD sum rules [18,19] use the approximation of local duality, equating in the expressions analogous to (3.8) the structure functions and the spectral densities pointwise in s. Since we are interested in the electron spectrum and integrate over the neutrino momentum, we automatically are led to an integration over s, cf. Eq. (2.5). Therefore, in this paper we only make the much weaker assumption, that the physical spectral density coincides to the one obtained by the operator product expansion upon the integration over a sufficiently large interval of duality Δs as mentioned before. Since the lowest mass physical state is the pion, we demand s_{max} to be larger than the interval of duality in the QCD sum rules for the pion $\Delta s \simeq 1\,\mathrm{GeV}^2$ [12]. Since at $E_e \approx E_{max}$ the physical region in s shrinks to a point, the requirement of having a sufficient interval of duality excludes a certain fraction of the kinematical region, which we show as a gray area in the Dalitz-plot Fig. 1. For small values of E_e it covers only a small fraction of the integration region in q^2 and is heavily suppressed by phase space, so that this limitation is not restrictive. More important constraints on the accessible values of q^2 follow for higher electron energies. However, the major part of the excluded region in the Dalitz plot is not accessible to the sum rules for a different reason as well: the value of $q^2 - m_b^2$ should be sufficiently large and negative in order the operator product expansion to be justified. Taken altogether, these requirements lead to an upper bound for accessible electron energies of 2.1 GeV. At this energy requiring an interval of duality larger than $\Delta s \sim 1\,\mathrm{GeV}$ excludes 4.7% of the whole phase space in s and q^2. At lower electron energies it is an even smaller fraction.

Expanding (3.8) in powers of energy one may relate the moments of structure functions, $\int ds\, s^{-n} W_i(s)$, to corresponding integrals over the spectral densities, and, in turn, to matrix elements of local operators over the B-meson, which appear in the short-distance expansion of the T-product of weak currents. The operator expansion of this product is the starting point in approaches to the inclusive B-decays based on the $1/m_b$ expansion [5], or the parton model [17]. It is worthwile to note that the integration over s in the differential decay rate at fixed E_e in (2.5) is constrained by the value

$$s_{max} = (m_B - 2E_e)\left(m_B - \frac{q^2}{2E_e}\right) \tag{3.9}$$

which for not too large E_e is less than the actual support of the structure functions. Thus, the knowledge of the first few moments of the structure functions W_i is not sufficient for the calculation of the inclusive spectrum, and we cannot reduce our task to the calculation of three-point instead of four-point functions.

Writing the lepton decay constant $\langle 0 \,|\, j_B \,|\, B \rangle$ as $m_B^2 f_B/m_b$, we get from (3.8) the following sum rules for the structure functions W_i:

$$W_i(s,q^2) = \frac{m_b^2}{m_B^4 f_B^2} \int_0^{s_0} ds_1\, \rho_i(s_1,q^2,s)\, e^{-2(s_1-m_B^2)/M^2}. \tag{3.10}$$

These equations should be understood in the sense that the physical W_i coincide with the ones given in (3.10) after smearing over an interval of duality in s larger or of the order of 1 GeV^2.

The spectral densities ρ_i receive contributions from perturbation theory and the vacuum condensates

$$\rho_i = \rho_i^{pert} + \rho_i^{\langle 3\rangle} \langle\, \bar{q}q \,\rangle + \rho_i^{\langle 5\rangle} \langle\, \bar{q}\sigma g G q \,\rangle. \tag{3.11}$$

A calculation of the box graph in Fig. 2(a) yields (see appendix for the details)

$$\rho_1^{pert} = -\frac{3}{4\pi^2}\frac{m_b^2 - s_1}{q^2}\left(m_b^2 + \frac{2q^2 s + (m_b^2-q^2)(s+q^2-s_1)}{\lambda^{1/2}}\right)\theta(s_1 - s_1^L), \tag{3.12a}$$

$$\rho_2^{pert} = \frac{3}{\pi^2}\,(m_b^2 - s_1)\,\frac{2q^2 s + (m_b^2-q^2)(s+q^2-s_1)}{\lambda^{3/2}}\theta(s_1 - s_1^L), \tag{3.12b}$$

$$\rho_3^{pert} = \frac{3}{2\pi^2}\frac{m_b^2 - s_1}{\lambda^{1/2}}\theta(s_1 - s_1^L), \tag{3.12c}$$

where

$$s_1^L = m_b^2 + \frac{s m_b^2}{m_b^2 - q^2}. \tag{3.13}$$

Note that ρ_1^{pert} is regular in the limit $q^2 \to 0$.

The contribution of the quark condensate is given by

$$\rho_1^{\langle 3\rangle} = -\,2\, m_b(s + m_b^2 - q^2)\, \delta(s_1 - m_b^2)\, \delta(s), \tag{3.14a}$$

$$\rho_2^{\langle 3\rangle} = -8 m_b\, \delta(s_1 - m_b^2)\, \delta(s), \tag{3.14b}$$

$$\rho_3^{\langle 3\rangle} = 4 m_b\, \delta(s_1 - m_b^2)\, \delta(s), \tag{3.14c}$$

and the contribution of the mixed condensate reads

$$\rho_1^{\langle 5\rangle} = m_b \Big\{ -\left[\frac{1}{3}\delta(s) + (m_b^2 - q^2)\delta'(s)\right]\delta(s_1 - m_b^2) - \Big[2(m_b^2-q^2)\delta(s) - \frac{1}{2}(m_b^2-q^2)^2\delta'(s)\Big]\,\delta'(s_1 - m_b^2) + \frac{m_b^2(m_b^2-q^2)}{2}\delta(s)\delta''(s_1 - m_b^2) \Big\}, \tag{3.15a}$$

$$\rho_2^{\langle 5\rangle} = m_b \Big\{ -\frac{8}{3}\delta'(s)\,\delta(s_1-m_b^2) + \Big[-4\delta(s) + 2(m_b^2-q^2)\delta'(s)\Big]\,\delta'(s_1-m_b^2)$$

$$+\, 2m_b^2\delta(s)\,\delta''(s_1-m_b^2)\Big\}\,, \tag{3.15b}$$

$$\rho_3^{\langle 5\rangle} = m_b \Big\{ \delta'(s)\,\delta(s_1-m_b^2) + \Big[3\delta(s) - (m_b^2-q^2)\delta'(s)\Big]\;\delta'(s_1-m_b^2)$$

$$-\, m_b^2\delta(s)\,\delta''(s_1-m_b^2)\Big\}\,. \tag{3.15c}$$

Note that $\delta(s)$ has to be understood as the limit of $\delta(s-m_u^2)$ where we have put m_u, the mass of the u-quark, to zero. Thus there is no ambiguity in the integration over s within the range specified by (2.6a). Contributions of the u-quark condensate $\langle\bar{u}u\rangle$ and of the corresponding mixed condensate $\langle\bar{u}\sigma gGu\rangle$ vanish. Note that the contributions containing $\delta'(s)$ give rise to terms $\sim \delta(2E_e(m_B - m_u^2/(m_B-2E_e)) - q^2)$ in the double differential rate $d^2\Gamma/(dE_e dq^2)$.

The leptonic decay constant f_B entering Eq. (3.10) is determined by a two-point sum rule [20] where the same approximations are made and the same input parameters are used as for the sum rule (3.10), except for the value of the Borel parameter M_{2pt}^2, which is taken two times smaller, $M_{2pt}^2 = M^2/2$ [13]:

$$f_B^2 m_B^4\, e^{(m_b^2-m_B^2)/M_{2pt}^2} = \tag{3.16}$$

$$= \frac{3m_b^2}{8\pi^2}\int_{m_b^2}^{s_0} ds_1 \frac{(s_1-m_b^2)^2}{s_1}\, e^{(m_b^2-s_1)/M_{2pt}^2} - m_b^3\,\langle\,\bar{q}q\,\rangle + \frac{m_b^3}{2}\left(\frac{m_b^2}{2M_{2pt}^4} - \frac{1}{M_{2pt}^2}\right)\langle\,\bar{q}\sigma gGq\,\rangle.$$

Considering the ratio of sum rules instead of fixing f_B at a certain value considerably reduces the dependence of the results on the input value of the b-quark mass and on the value of the Borel parameter, since the effect of a change in these parameters is cancelled between the numerator and the denominator to a great extent.

Radiative corrections to the sum rule in (3.10) may in general be important, cf. [21,22]. In the limit of large m_b, however, all radiative corrections factorize into corrections to f_B, which are cancelled by taking the ratio of the four-point and two-point sum rule, and Sudakov-type radiative corrections to the spectrum of the decay of a free b-quark, which we take into account as a multiplicative factor, see next section. A full account for the radiative correction requires a laborous calculation of the α_s-correction to the box graph and is beyond the tasks of this paper. On formal grounds, all corrections which we do not take into account are suppressed by a power of the heavy quark mass.

IV. THE HEAVY QUARK LIMIT

We consider the region where the invariant mass of the lepton pair q^2 and the electron energy $2E_e$ constitute a finite fraction of the heavy quark mass, i.e.

$$m_b^2 - q^2 = \mathcal{O}(m_b^2),$$

$$m_b - 2E_e = \mathcal{O}(m_b). \tag{4.1}$$

It is convenient to introduce the scaling variable [3]

$$x = 2E_e/m_B \tag{4.2}$$

in terms of which the last of the conditions in (4.1) reads $1 - x = \mathcal{O}(1)$. In this kinematical range the sum rules (3.10) simplify drastically in the limit of infinite heavy quark mass and yield the results of the free quark decay.

To show this, we use the common assumption that all excitation energies remain finite in the heavy quark limit, and hence s_0, the continuum threshold, scales as

$$s_0 - m_b^2 = (m_B + E_{exc})^2 - m_b^2 \simeq \mathcal{O}(m_b). \tag{4.3}$$

The working region in the Borel parameter is determined by the requirement that the exponential suppression factor for the contributions of excited states, $\exp[-(s_0 - m_B^2)/M^2]$, remains finite in that limit. Therefore, also the Borel parameter M^2 should be taken at values of order $\mathcal{O}(m_b)$. Next, we note that the support of the structure functions is restricted to an interval $0 < s < \mathcal{O}(m_b)$. As far as the perturbative contribution to the sum rules is concerned, this statement follows from the kinematical restriction for the double discontinuity of the box graph, $s_1 > s_L$, in (3.13), where from

$$s \leq (s_1 - m_b^2)\frac{m_b^2 - q^2}{m_b^2} \leq \mathcal{O}(m_b). \tag{4.4}$$

The last inequality is a consequence of Eq. (4.3). Therefore s can be neglected against s_1, which is of $\mathcal{O}(m_b^2)$. The nonperturbative contributions generally form a series in δ-functions of s and its derivatives, which should in principle produce a certain smooth function upon summation.

Retaining the leading terms in the heavy-quark mass only, we find from (3.10):

$$\begin{aligned}\frac{f_B^2 m_B^4}{m_b^2} W_1(s, q^2) = {} & \frac{3}{4\pi^2}\int_{s_1^L}^{s_0} ds_1\, e^{2(m_B^2 - s_1)/M^2}(s_1 - m_b^2) \\ & - 2\langle\, \bar{q}q\,\rangle m_b (m_b^2 - q^2) e^{2(m_B^2 - m_b^2)/M^2}\delta(s) \\ & + 2\langle\, \bar{q}\sigma g G q\,\rangle \frac{m_b^3}{M^4}(m_b^2 - q^2) e^{2(m_B^2 - m_b^2)/M^2}\delta(s)\end{aligned} \tag{4.5}$$

In order to obtain the inclusive electron spectrum, we integrate (4.5) over s. Changing the order of integration in s and in s_1, we find the integration intervals

$$\begin{aligned} & m_b^2 \leq s_1 \leq s_0, \\ & 0 \leq s \leq min\left(\frac{m_b^2 - q^2}{m_b^2}(s_1 - m_b^2),\ (m_b - 2E_e)(m_b - \frac{q^2}{2E_e})\right). \end{aligned} \tag{4.6}$$

Due to (4.4), the second term in $min(\ldots,\ldots)$ is parametrically large as compared to the first one. Hence the full support of W_1 in s is covered, exemplifying the general statement that in the heavy quark limit the knowledge of the first moment $\int ds\, W_1(s)$ is sufficient for the calculation of the electron spectrum in the kinematical range given in (4.1), cf. Refs. [5]. Thus, we obtain:

$$\frac{f_B^2 m_B^4}{m_b^2}\int ds\, W_1(s,q^2) = 2(m_b^2-q^2)\left\{\frac{3}{8\pi^2}\int_{m_b^2}^{s_0} ds_1\, e^{2(m_B^2-s_1)/M^2}\frac{(s_1-m_b^2)^2}{m_b^2}\right.$$

$$\left. -\langle\,\bar q q\,\rangle m_b e^{2(m_B^2-m_b^2)/M^2} + \langle\,\bar q\sigma g G q\,\rangle\frac{m_b^3}{M^4}e^{2(m_B^2-m_b^2)/M^2}\right\}. \qquad (4.7)$$

Letting $M^2 = 2M_{2pt}^2$, one realizes that the expression in curly brackets coincides (apart from an overall factor m_b^2) with the leading term of the two-point sum rule (3.16) in the limit of infinite quark mass. The same factorization takes place in the other two structure functions. Thus, in the heavy quark limit we end up with the following expressions:

$$\int ds\, W_1(s,q^2) = 2(m_b^2-q^2), \qquad (4.8a)$$

$$\int ds\, W_2(s,q^2) = 8, \qquad (4.8b)$$

$$\int ds\, W_3(s,q^2) = 4, \qquad (4.8c)$$

which coincide with the structure functions for the decay of a free heavy quark. The corresponding electron spectrum is [3]

$$\frac{d\Gamma}{dx} = \frac{G^2|V_{ub}|^2 m_b^5}{96\pi^3}x^2(3-2x), \qquad (4.9)$$

where $x = 2E_e/m_b$, and the total decay rate equals

$$\Gamma = \frac{G^2|V_{ub}|^2 m_b^5}{192\pi^3}. \qquad (4.10)$$

Note that to this accuracy there is no distinction between the quark and the meson mass.

The results presented above do not take into account radiative corrections $\propto \alpha_s(m_b)$. As it is well known, such corrections can in general get enhanced by large logarithms of the heavy quark mass. The corresponding corrections to the spectrum for the decay of a free quark have been calculated in [3]. They result in a modification of the uncorrected spectrum (4.9) by a multiplicative factor:

$$\frac{d\Gamma}{dx} = \frac{d\Gamma^{(0)}}{dx}\left[1-\frac{2\alpha_s}{3\pi}\widetilde{G}(x)\right]\exp\left[-\frac{2\alpha_s}{3\pi}\ln^2(1-x)\right], \qquad (4.11)$$

where the Sudakov-like exponential has been factored out and the function $\widetilde{G}$ is given by

$$\widetilde{G}(x) = \frac{1}{x}\left\{\ln(1-x)\left[\frac{3}{2}\ln\frac{1+2x}{3}+\frac{11}{6x}+\frac{1+4x}{3}\right]+2x\,\mathrm{Li}\,(x)\right.$$

$$\left. +\frac{3}{2}\,\mathrm{Li}\left[\frac{2}{3}(1-x)\right]-\frac{3}{2}\,\mathrm{Li}\left(\frac{2}{3}\right)+x\left(\frac{2}{3}\pi^2-\frac{19}{12}\right)+\frac{11}{6}\right\}. \qquad (4.12)$$

In numerical calculations, the results of which are presented below, we take into account this Sudakov-type correction, which is the leading one in the kinematical region in (4.1). All the radiative corrections due to the spectator quark are suppressed by a power of m_b. It should be noted, however, that for the region of high electron energy $1 - x \sim 1/m_b$ the application of (4.11) is not justified, and the true effect of radiative corrections may be different, as indicated by the qualitative change of the Sudakov-type exponential behaviour with the account of the constituent mass of the u-quark [3].

V. RESULTS AND DISCUSSION

The results of the numerical calculations given below have been obtained using the following values of the vacuum condensates:

$$\langle \bar{q}q \rangle(1\,\mathrm{GeV}) = (-0.24\,\mathrm{GeV})^3,$$
$$\langle \bar{q}\sigma g G q \rangle(1\,\mathrm{GeV}) = 0.8\,\mathrm{GeV}^2\,\langle \bar{q}q \rangle(1\,\mathrm{GeV}). \tag{5.1}$$

These values have been rescaled to a higher normalization point $\mu^2 = M^2$, which is the relevant short distance expansion parameter in the sum rule (3.10). We have varied the value of the pole mass of the b-quark m_b within the range $(4.6 - 4.8)$ GeV, which covers the values given in literature [23,24]. The continuum threshold s_0 is determined from the requirement of stability of the two-point sum rule in (3.16) and depends on the quark mass. We use the values $s_0(m_b = 4.6$ GeV$) = 36$ GeV^2, $s_0(4.7$ GeV$) = 35$ GeV^2 and $s_0(4.8$ GeV$) = 34$ GeV^2 [21]. These values provide a good stability of the two-point sum rule in a large range of Borel parameters M^2_{2pt}. Note that we use the same value of s_0 in both the four-point and the two-point sum rules. Similarly, the range of appropriate values of the Borel parameter M^2 in the four-point sum rule is fixed from the two-point sum rule (3.16) by the requirement that both the contribution of the continuum and of nonperturbative corrections do not exceed 40% each. In this way we come to the range 3.5 GeV$^2 \lesssim M^2_{2pt} \lesssim 5$ GeV^2. Similar to the case of the three-point function [13], the Borel parameter of the four-point function, M^2, should be taken two times larger than M^2_{2pt}, i.e. we arrive at the interval of values 7 GeV$^2 \lesssim M^2 \lesssim 10$ GeV^2.

The QCD sum rule predictions for the differential spectrum $d\Gamma/dx$ in the rest frame of the B-meson are presented in Fig. 3 in dependence on the scaling variable $x = 2E_e/m_B$ for different values of the Borel parameter, and for input values $m_b = 4.8$ GeV and $s_0 = 34$ GeV. The curves are calculated by means of Eq. (2.5) up to the electron energy 2.1 GeV ($x = 0.80$), where the upper limit of integration in q^2 is still small enough as to ensure the validity of the operator product expansion (cf. Sect. III). For higher electron energies we did a smooth interpolation to the end point of the spectrum at $x = 1$, where $d\Gamma/dx = 0$ owing to vanishing phase space. The position of the maximum of the spectrum is obtained at values $x \sim 0.75$, which are still within the region of validity of the QCD sum rule calculation, and due to that the uncertainty introduced by the degree of the interpolating polynomials is negligible: it turns out to be smaller than 1% in the experimentally interesting region 2.2 GeV $\leq E_e \leq$ 2.4 GeV when changing from a cubic to an order five polynomial. We find that the variation of the Borel parameter within the range given above induces a ±15%

uncertainty in the absolute normalization, while the shape of the spectrum is much more stable.

Fig. 4 shows the separate contributions to the differential spectrum coming from the perturbative graph and from the nonperturbative corrections. The contributions of the vacuum condensates are large by themselves, but partly cancel in the sum. Up to the very end of the spectrum the relative weights of the perturbative and nonperturbative contributions remain practically constant.

A variation of the mixed condensate in the range $0.6\,\mathrm{GeV}^2 \leq m_0^2 \leq 1\,\mathrm{GeV}^2$ leads to a change of the decay rate by $\pm 10\%$.

The influence of the mass of the heavy quark, which is large in the case of the sum rule for f_B [21,25], is eliminated to a great extent by compensations between the numerator and the denominator in (3.10). A variation of m_b from 4.8 to 4.6 GeV reduces the decay rate by 8%. A similar compensation occurs for the dependence on the continuum threshold s_0: variation between 30 and $38\,\mathrm{GeV}^2$ results in a change by $\pm 4\%$. Variations of other parameters within the standard limits have only marginal effect.

The radiative corrections in (4.11) reduce the decay rate by 20%. Since the main effect is however achieved at large values of x, where neither the derivation of (4.11) nor the QCD sum rules approach are justified, we add an additional error of 10% to our results, which corresponds to an uncertainty in the radiative corrections of 50%.

It is worthwhile to mention that major part of uncertainties of the QCD sum rule calculation result in an uncertainty in the overall normalization factor, but not affect the shape of the spectrum.

Combining all sorts of errors, we obtain the total decay rate:

$$\begin{aligned} \Gamma &= (4.5 \pm 1.3)\, 10^{-11} |V_{ub}|^2\, \mathrm{GeV}, \\ &= (6.8 \pm 2.0)\, 10^{13}\, |V_{ub}|^2\, \mathrm{s}^{-1}. \end{aligned} \tag{5.2}$$

In order to be able to compare to the experimental results which were obtained for B-mesons created on the $\Upsilon(4S)$ resonance, we have to boost to the laboratory frame,

$$\frac{d\Gamma^{lab}}{dE_e} = \int\limits_{-v_B E_e}^{+v_B E_e} du\, \frac{1}{2 v_B E_e}\, \frac{d\Gamma^{rest}}{dE_e}(E_e + u)\,, \tag{5.3}$$

where the electron energy E_e is measured in the laboratory frame and $v_B = 0.065$ is the velocity of the B-meson. The change induced by the boost is negligible for the total rate, but modifies the high energy part of the spectrum. Thus, in Table I we give the values of the integrated spectrum in energy bins of 0.1 GeV from 2.1 GeV to 2.7 GeV both in the rest and the lab-frame.

For the integrated value of the decay rate in the interval of electron energies $2.4\,\mathrm{GeV} \leq E_e \leq 2.6\,\mathrm{GeV}$ we get:

$$\Gamma^{lab} = (5.0 \pm 1.6)\, 10^{12}\, |V_{ub}|^2\, \mathrm{s}^{-1}. \tag{5.4}$$

Combining the latter value with the new experimental result of CLEO [2] for the branching ratio in the same region of energies,

$$B = (0.53 \pm 0.14 \pm 0.13)\, 10^{-4}, \tag{5.5}$$

we obtain the following value for the CKM-matrix element:

$$|V_{ub}| = (0.003 \pm 0.001) \left(\frac{1.3\,\mathrm{ps}}{\tau_B}\right)^{1/2} , \tag{5.6}$$

where τ_B is the lifetime of the B-meson. The error takes into account the accuracy in the calculated decay rate, a conservative estimate for the uncertainty induced by the extrapolation and the experimental error, where theoretical and experimental error are roughly of the same size.

The comparison with the other existing calculations of absolute rates is given in Table I, and the comparison of the predicted shape of the spectrum is shown in Fig. 5. The QCD sum rule calculation of the electron spectrum for $2.1\,\mathrm{GeV} < E_e^{lab} < 2.5\,\mathrm{GeV}$ lies inbetween the predictions of the free [3] and the modified [3,10] and the direct summation of the contributions of exclusive channels [9], while for higher energies our results coincide with [3] and [10] within the errors. At such high energies, however, the extrapolation procedure used might not be adequate. All calculations of the shape of the spectrum are in a good agreement at low values of x, while starting from $x \sim 0.5$ the discrepancies amount (10–15)%. The calculations using a certain modification of the free quark model have an endpoint which might differ from the physical one, since the quark masses enter in this approach instead of the hadron ones. The slope of all curves near the end is, however, not so much different, the one of [9] being the smallest.

Concluding we remark that the sum rule method allows a determination of the semileptonic decay rate which is on a theoretically sound basis for electron energies below 2.1 GeV. Though at first sight it may seem unsatisfactory that the most interesting range of electron energies can be reached only by a constrained extrapolation, this does not induce a major error. This is due to the fact, that the electron spectrum can be calculated to energies larger than the position of the maximum and that the endpoint of the spectrum, where it vanishes, is determined by hadron, and not quark masses in our approach. In the determination of the CKM matrix element V_{bu} the uncertainty of the lifetime is still the most significant error.

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THE TRIPLE DISPERSION RELATION OF THE BOX-DIAGRAM

In order to be able to subtract the contribution of the continuum from our sum rules and to take the imaginary part in s, the invariant mass squared of the final state hadrons, we need to represent the box-diagram Fig. 2(a) by a triple dispersion relation. Since the forward scattering amplitude is relevant to our problem, we consider the kinematical configuration $t = (p_1 - p_2)^2 = (q_1 - q_2)^2 \equiv 0$. In addition to that, we want to keep $p_1^2 \neq p_2^2$ whilst

$q_1^2 \equiv q_2^2 \equiv q^2$ which implies $q^2 < 0$. Thus, in our case the independent variables determining the box-diagram are p_1^2, p_2^2, q^2 and s. The external momenta of the interpolating field of the B-meson, p_1 and p_2, are considered in the Euclidean region $p_1^2, p_2^2 < 0$.

We infer the existence of such a triple dispersion relation for the fermionic case from the analysis of the scalar box-diagram for $q^2 < 0$

$$
\begin{aligned}
S &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{((p_1+k)^2 - m_b^2)((p_2+k)^2 - m_b^2)(p_1+q_1)^2 k^2} \\
&= \frac{i}{16\pi^2} \frac{1}{m_b^2 - q^2} \int d\tilde{s} \frac{1}{\tilde{s} - s} \int ds_1 \int ds_2 \frac{\delta(s_1 - s_2)}{(s_1 - p_1^2)(s_1 - p_2^2)} \\
&= \frac{i}{16\pi^2} \frac{1}{m_b^2 - q^2} \int_0^\infty d\tilde{s} \frac{1}{\tilde{s} - s} \int_{s_1^L}^\infty ds_1 \frac{1}{(s_1 - p_1^2)(s_1 - p_2^2)}
\end{aligned} \tag{7}
$$

with

$$
s_1^L = \frac{\tilde{s} m_b^2}{m_b^2 - q^2} + m_b^2. \tag{8}
$$

The right hand side of (7) can be continued analytically to $q^2 > 0$ with p_1^2 and p_2^2 remaining independent variables. We have checked this issue also explicitly by comparing with the corresponding Feynman-parameter integral. For arbitrary values of q^2, Eq. (7) can equally well be obtained from the scalar triangle graph T with external momenta squared p_1^2, q^2 and s as

$$
S = \frac{d}{dm_b^2} T(p_1^2, s, q^2) \Big|_{(s_1 - p_1^2)^2 \to (s_1 - p_1^2)(s_1 - p_2^2)} \tag{9}
$$

which follows immediately from the expressions for the loop-integrals.

As pointed out in [13], a naïve continuation of the double dispersion relation for the triangle graph to the region of positive values of q^2 is not possible because of the existence of non-Landau singularities. Technically, the problem arises because the double spectral function of the triangle graph T with respect to p_1^2 and s, $\rho_T(p_1^2, q^2, s)$, contains a factor $1/\lambda^{1/2}$, where $\lambda = p_1^4 + s^2 + q^4 - 2p_1^2 s - 2p_1^2 q^2 - 2sq^2$. For positive values of q^2, λ can vanish and ρ_T can become singular at the boundary of the integration region as given by the Landau equations. This allows the logarithmic branching cut on the unphysical sheet of the square root to dive through the square root cut into the physical sheet. The proper treatment of that additional singularity is described in detail in [13]. As follows from Eq. (7), for the scalar box-graph the non-Landau singularities are absent, but are present for fermions. They play, however, no role in our analysis, because it turns out that for physically relevant q^2 they lie above the continuum threshold or are numerically negligible due to phase space suppression.

It proves convenient to apply the above method of taking the derivative of some triangle diagram with appropriate vertices with respect to m_b^2 likewise to the fermionic case with open Lorentz-indices. From the double spectral functions for the tensor-integrals given in [26] we obtain (for $q^2 < 0$)

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{((p_1+k)^2-m_b^2)((p_2+k)^2-m_b^2)(p_1+q_1)^2k^2} =$$

$$= P_+ \frac{1}{2}(p_1+p_2)_\mu + P_- \frac{1}{2}(p_1-p_2)_\mu + Q_1 q_{1\mu}. \qquad (10)$$

The only invariant needed is P_+ for which we obtain:

$$P_+ = -S + \frac{i}{16\pi^2} \int_0^\infty d\tilde{s} \frac{1}{\tilde{s}-s} \int_{s_1^L}^\infty ds_1 \frac{1}{\lambda^{1/2}(s_1-p_1^2)(s_1-p_2^2)} \qquad (11)$$

with $\lambda = s_1^2 + \tilde{s}^2 + q^4 - 2s_1\tilde{s} - 2s_1q^2 - 2\tilde{s}q^2$.

The integral with two open Lorentz-indices is decomposed as

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{((p_1+k)^2-m_b^2)((p_2+k)^2-m_b^2)(p_1+q_1)^2k^2} =$$

$$= A g_{\mu\nu} + B \frac{1}{4}(p_1+p_2)_\mu (p_1+p_2)_\nu + \dots \qquad (12)$$

with

$$A = -\frac{i}{16\pi^2} \frac{1}{4q^2} \int_0^\infty d\tilde{s} \frac{1}{\tilde{s}-s} \int_{s_1^L}^\infty ds_1 \frac{(m_b^2-q^2)\lambda^{1/2} + 2q^2\tilde{s} + (m_b^2-q^2)(\tilde{s}+q^2-s_1)}{\lambda^{1/2}(s_1-p_1^2)(s_1-p_2^2)} \qquad (13a)$$

$$B = -\frac{i}{16\pi^2} \int_0^\infty d\tilde{s} \frac{1}{\tilde{s}-s} \int_{s_1^L}^\infty ds_1 \frac{1}{(s_1-p_1^2)(s_1-p_2^2)}$$

$$\times \left\{ -\frac{1}{m_b^2-q^2} + \frac{2}{\lambda^{1/2}} + \frac{(m_b^2-q^2)(q^2+\tilde{s}-s_1) + 2\tilde{s}q^2}{\lambda^{3/2}} \right\} \qquad (13b)$$

which are the only invariants needed.

The expressions given above (including non-Landau singularities for $q^2 > 0$) have been checked for arbitrary values of q^2 by comparison with the corresponding Feynman-parameter integrals and by an independent calculation of the double spectral densities using the decomposition of momenta in light-cone variables.

By means of the above expressions we obtain the perturbative double spectral functions given in Sec. III.

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FIGURES

FIG. 1. Dalitz-plot of the phase space in the plane of invariant lepton mass squared, q^2, and electron energy, E_e. The solid border-line where $s = m_\pi^2$ can only be reached with a single pion in the final hadronic state. The shaded area shows the region of phase space with $s \leq 1$ GeV^2. The dashed-dotted lines denote curves of constant $s \geq 5, 10, 15, 20$ GeV^2 (from top to bottom).

FIG. 2. Feynman diagrams contributing to the operator product expansion of (3.1): (a) perturbative contribution, (b) contribution of the quark and part of the contribution of the mixed condensate. Lines ending in a cross denote vacuum expectation values.

FIG. 3. The spectrum $|V_{ub}|^{-2} d\Gamma/dx$ as function of x with $m_b = 4.8$ GeV and $s_0 = 34$ GeV^2 for different values of the Borel parameter, $M^2 = 7, 8, 9, 10$ GeV^2 (from top to bottom). Solid lines: evaluation using (3.10), dashed lines: interpolation to zero at $x = 1$.

FIG. 4. Relative contributions of the separate terms in the operator product expansion to the spectrum as function of x ($M^2 = 8$ GeV^2, $m_b = 4.8$ GeV, $s_0 = 34$ GeV^2). Solid line: perturbative contribution, long dashes: contribution of the quark condensate, short dashes: contribution of the mixed condensate.

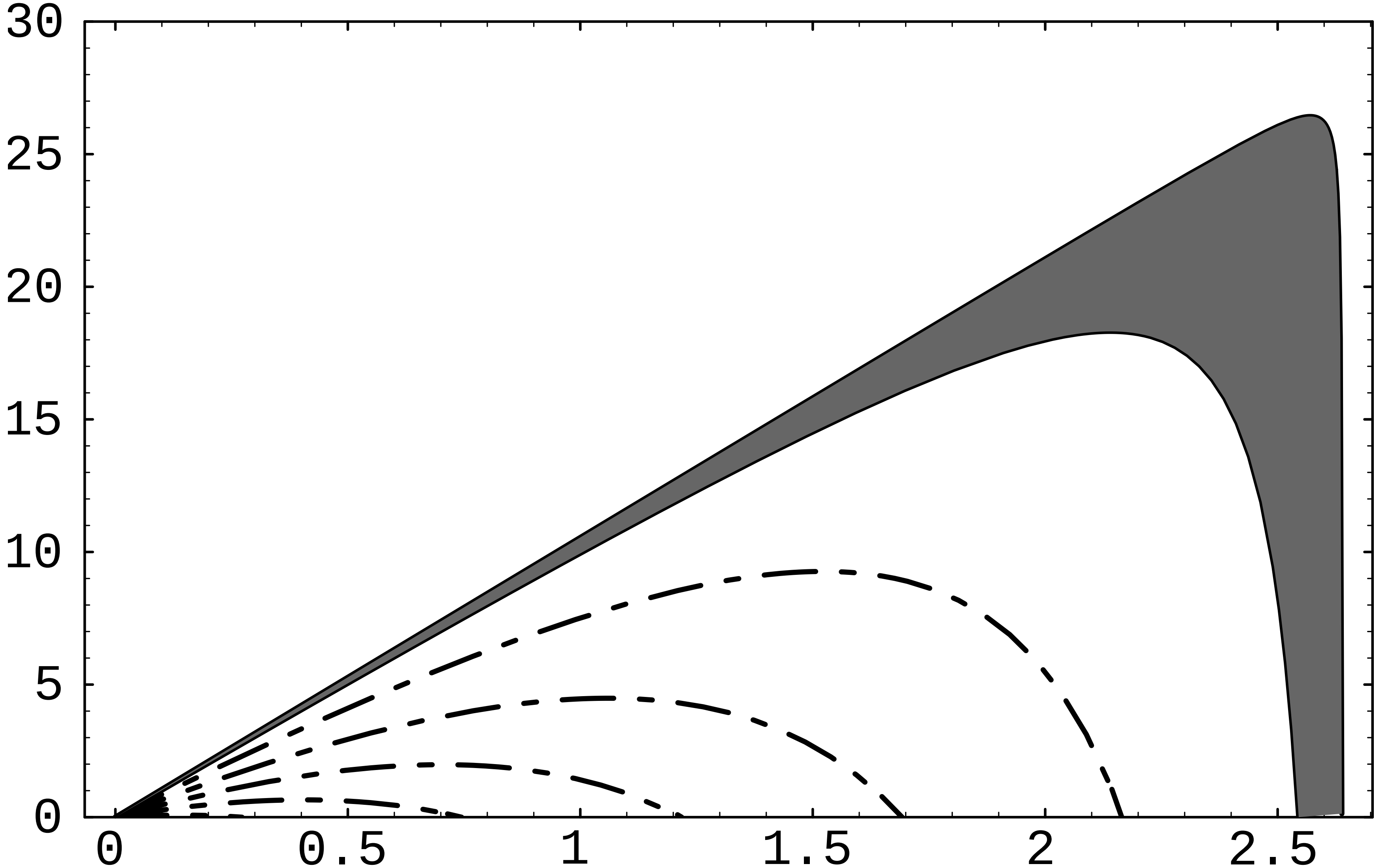
FIG. 5. The spectrum $|V_{ub}|^{-2} d\Gamma/dx$ as function of x with $m_b = 4.8$ GeV, $s_0 = 34$ GeV^2 and $M^2 = 8$ GeV^2. Solid line: the spectrum using (3.10), long dashes: the same with inclusion of radiative corrections for the free-quark decay acc. to (4.11).

FIG. 6. $1/\Gamma\, d\Gamma/dx$ as function of x for the free quark decay and in different model-calculations. The chosen normalization emphasizes the differences in shape. ACCM: [3], GISW: [9], RDB: [10], BBD: this paper.

TABLES

TABLE I. Central values for the inclusive semileptonic $B \to X_u\, e\, \nu$ decay rate from this paper and from different other calculations. Free: free quark decay with QCD corrections [3], Eq. (25), $\alpha_s = 0.24$, $m_b = 5.0\,\mathrm{GeV}$; ACCM: decay including bound-state corrections [3], Eq. (27), $m_s = 0.16GeV,\ p_f = 0.15GeV, \alpha_s = 0.24$; GISW: [9], Fig. 6; RDB: [10], Fig. 2, cut-off 1.5 GeV; BBD: this paper. We give values for the total rate Γ in $|V_{ub}|^2 10^{14}\,\mathrm{s}^{-1}$ and for the spectrum integrated over electron energies in bins of 0.1 GeV in the rest and in the lab frame. In the lab frame the B mesons are assumed to have a momentum of 0.34 GeV and isotropic angular distribution.

Frame	Model	Γ	2.1–2.2	2.2–2.3	2.3–2.4	2.4–2.5	2.5–2.6	2.6–2.7
Rest	BBD	0.68	0.053	0.050	0.044	0.034	0.019	0.002
	Free	0.85	0.066	0.067	0.062	0.045	0	0
	ACCM	0.89	0.070	0.071	0.068	0.052	0.019	0
	GISW	–	0.033	0.030	0.023	0.013	0.003	0
	RDB	0.92	0.071	0.070	0.066	0.043	0.009	0
Lab	BBD	0.68	0.052	0.049	0.042	0.032	0.018	0.008
	Free	0.85	0.066	0.066	0.058	0.037	0.017	0.0015
	ACCM	0.89	0.069	0.069	0.063	0.045	0.025	0.008
	GISW	–	0.032	0.029	0.022	0.013	0.006	0.002
	RDB	0.92	0.071	0.069	0.059	0.039	0.019	0.005



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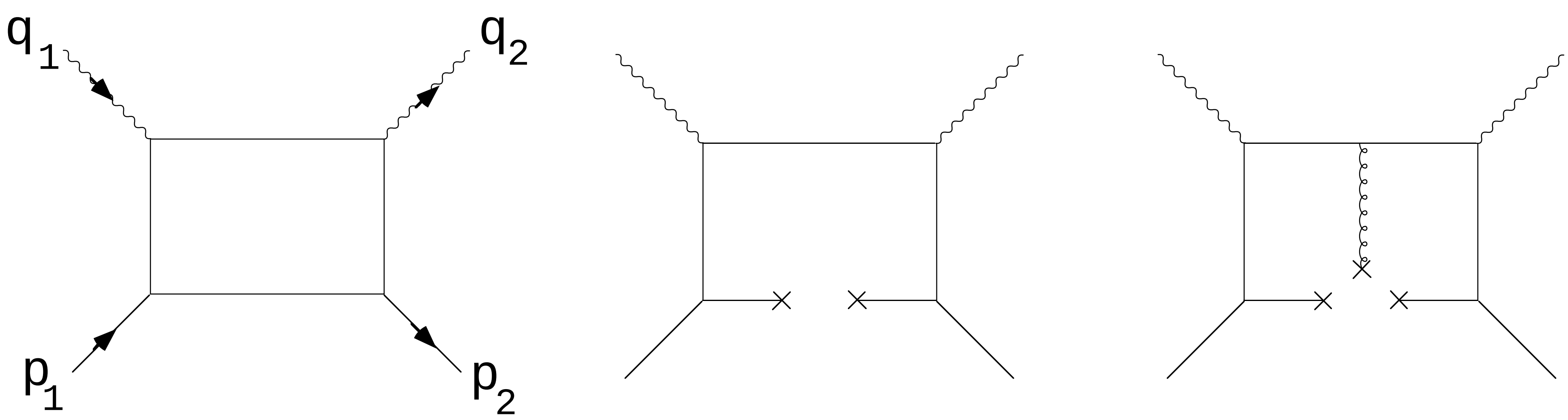
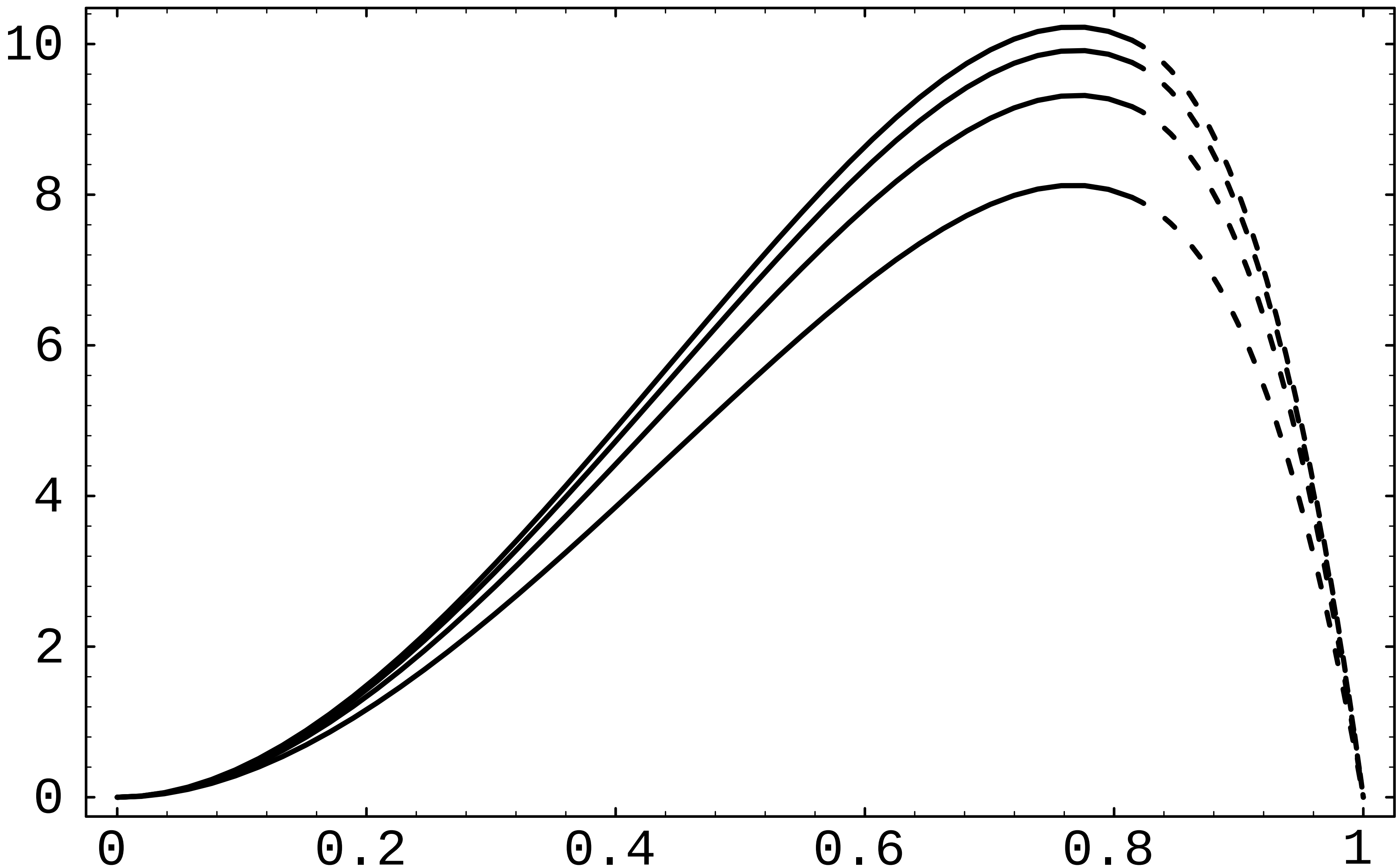
1
1.5
2
2.5



(a) (b)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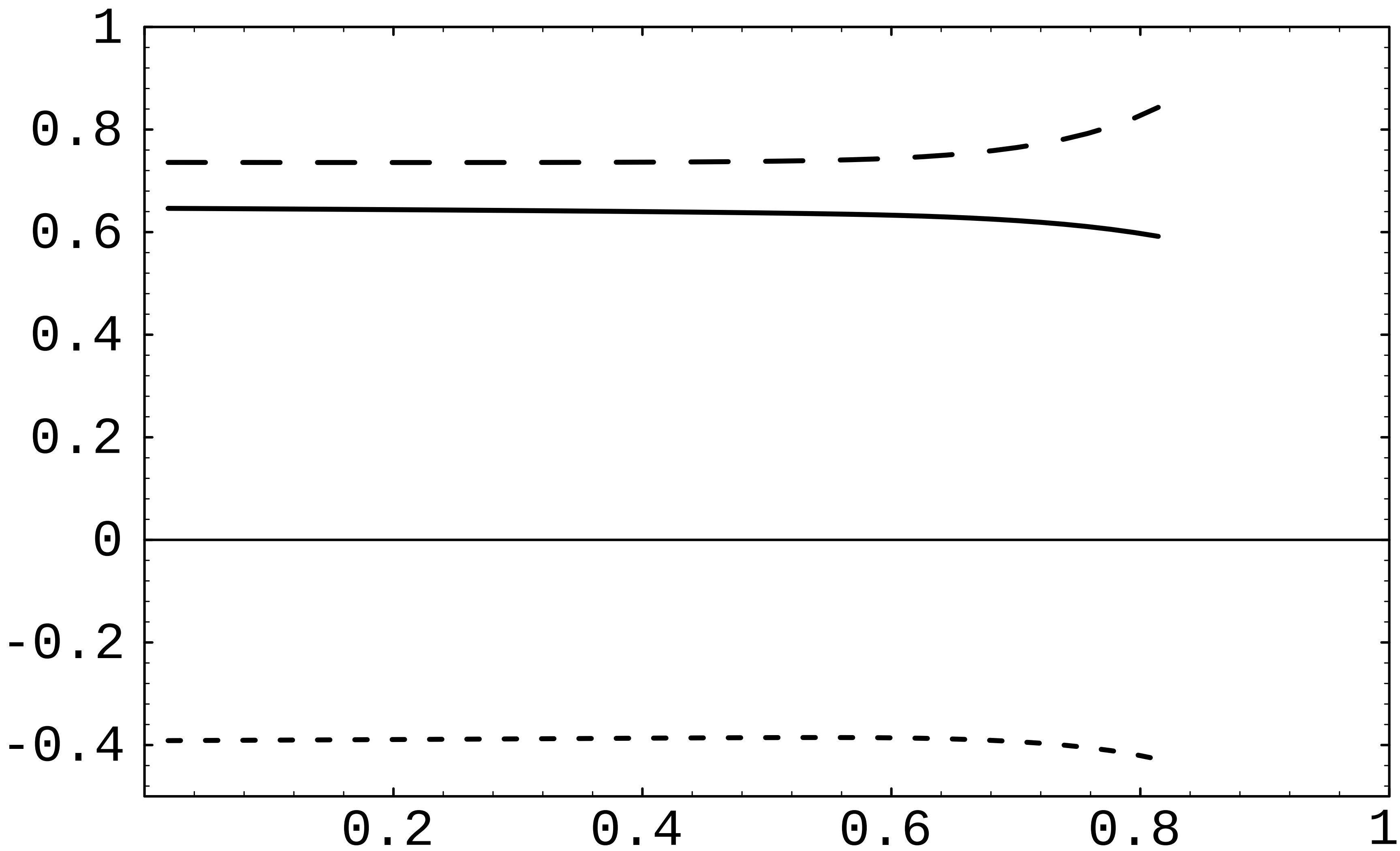
0.4
0.6
0.8
1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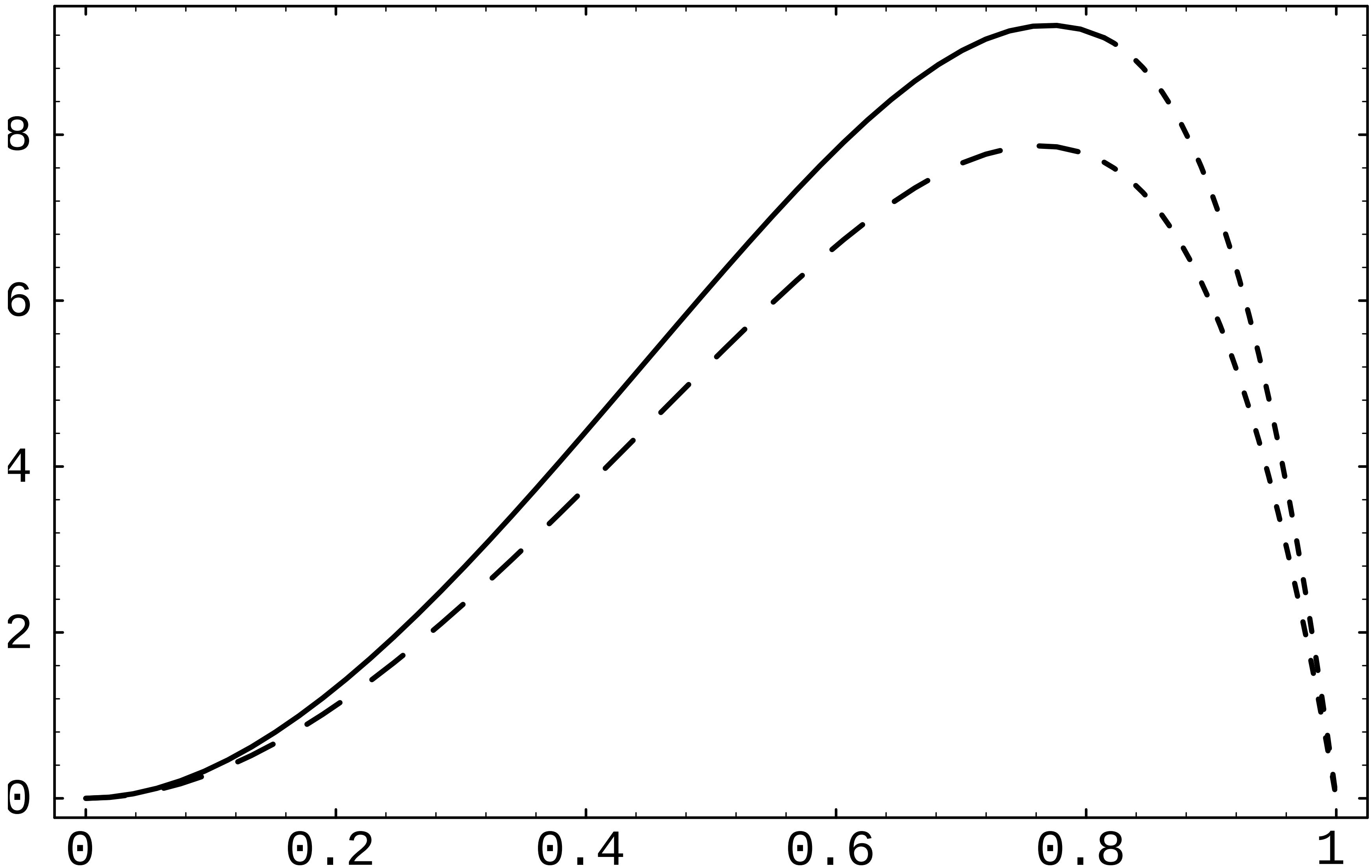
1



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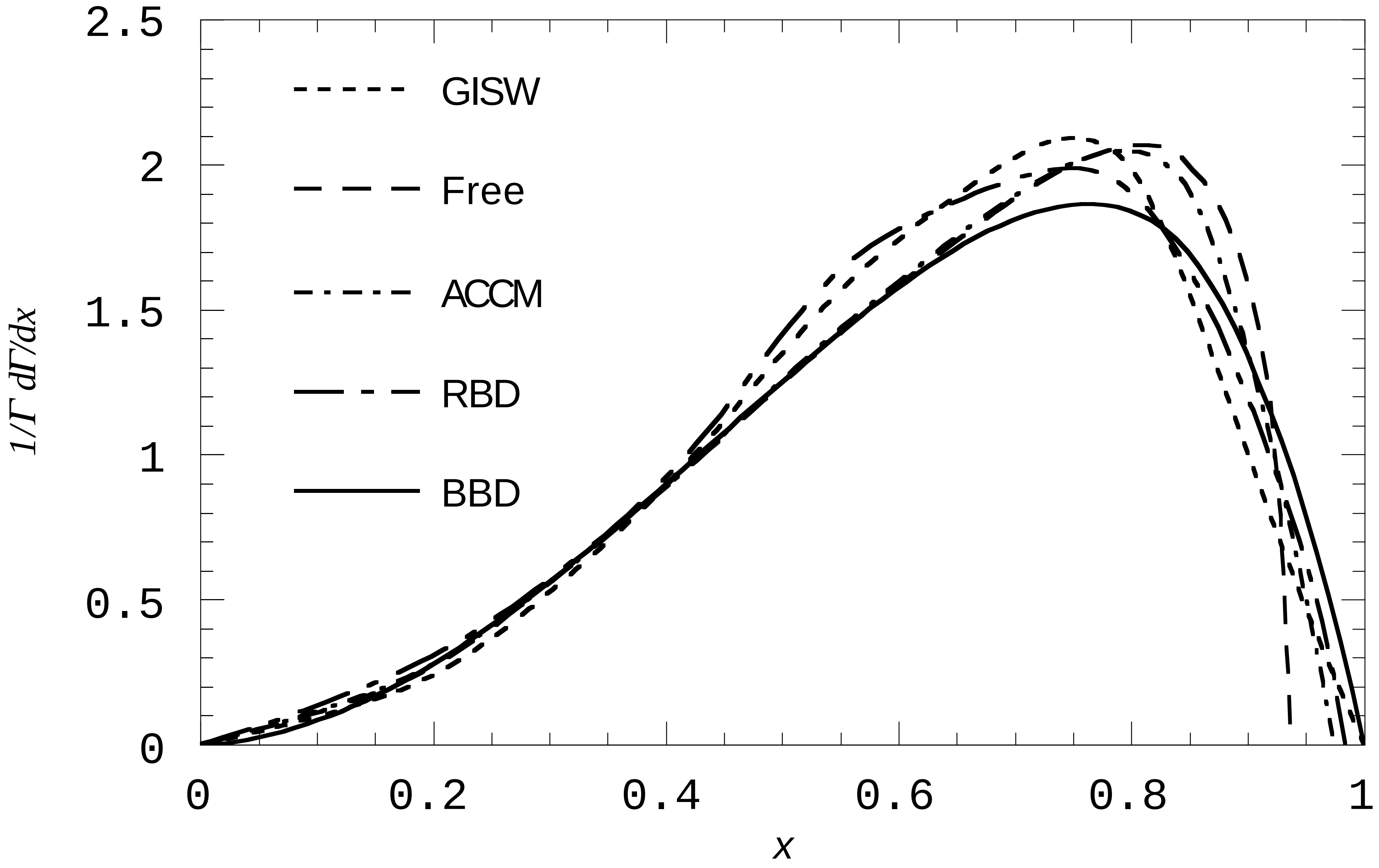
0.6
0.8
1



GISW
Free
ACCM
RBD
BBD
2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
1/Γ dΓ/dx
x