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Oscillations and Chaotic Current Fluctuations in n-GaAs.

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Abstract. – Chaotic current fuctuations have been observed in the negative differential conductivity region of n-GaAs at 4.2 K under static external conditions. Power spectra, Poincaré sections and the fractal dimension of the attractor were determined as functions of the applied voltage for various magnetic-field strengths B. For $B = 0$ only limit cycles were observed. For $B > 0$ our results indicate successive Hopf bifurcations and a Ruelle-Takens-Newhouse transition to chaos and point to a crucial role of the magnetic field.

In high-purity semiconductors at low temperatures breakdown in the current-voltage characteristics occurs due to impact ionization of shallow impurities yielding a rapid increase of the current at a critical voltage [1, 2]. There is a nonequilibrium transition from a weakly conducting to a strongly conducting steady state, whose critical behaviour was investigated experimentally in n-GaAs by methods of far-infra-red photoconductivity [3] and described in terms of nonlinear carrier generation-recombination kinetics [4]. Associated with the impurity breakdown, spontaneous oscillations and chaotic current fluctuations have recently been discovered in several semiconductors [5-9]. Upon varying the bias voltage across the samples the onset of chaotic behaviour was found either just below the threshold of the instability [5, 6] or in the post-breakdown regime [7]. It was shown that a periodic driving current in the presence of impact ionization can give rise to chaotic fluctuations [10]. The physical mechanisms underlying spontaneous chaotic fluctuations in the absence of a periodic driving, however, are not well understood. In n-GaAs AOKI *et al.* [5] have obtained evidence that weak optical interband excitations are responsible for chaotic current fluctuations in the pre-breakdown regime.

In view of the present uncertainty about physical origins, we have attempted to characterize the chaotic transition as far as possible from a detailed analysis of the time dependence of the chaotic signals using methods of nonlinear dynamics. We have investigated current fluctuations in high-purity n-GaAs epitaxial layers subject to a

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magnetic field at liquid-helium temperature without any other external stimulations. The power spectra and the Poincaré sections were derived as functions of the external voltage and the fractal dimension of the strange attractor was determined. Chaotic fluctuations were observed only for finite magnetic fields in the post-breakdown regime, whereas at zero magnetic field solely relaxation oscillations were found. In contrast to the case studied by AOKI *et al.* in the pre-breakdown regime, our observation is independent of optical interband excitations. With increasing voltage the fractal dimension rises stepwise from 0 to 1.1 and to 2.0 indicating successive Hopf bifurcations. The formation of a torus and the existence of two incommensurate frequencies are also reflected in the Poincaré surface of section and the power spectrum. At the transition to chaos the fractal dimension rises to 2.7. We thus conclude that the mechanism of the transition follows the Ruelle-Takens-Newhouse scenario.

The measurements presented here were carried out on a high-purity n-GaAs epitaxial layer of 14.2 μm thickness grown by liquid-phase epitaxy on a Cr-compensated substrate. The carrier concentration and the electron mobility were $n = 6.5 \cdot 10^{13}$ cm^{-1} and $\mu = 1.4 \cdot 10^5$ $vs.$ cm^{-1} at 77 K which corresponds to a donor concentration of $N_D = 2.7 \cdot 10^{14}$ cm^{-3} at 70% compensation. Ohmic contacts were formed by alloying Au-Sn strips on the sample surface separated by 1.5 mm. The samples were immersed in liquid helium and shielded against visible and infra-red radiation. Using a superconducting solenoid a magnetic field B was applied perpendicular to the sample surface and parallel to the [100] crystallographic direction. The sample was biased in series with a load resistor. Under these conditions the current-voltage characteristic showed a S-type behaviour at $B = 0$ and in the whole range of the applied magnetic-field strength. The breakdown voltage of the sample increased from 0.5 V at $B = 0$ T to 0.9 V at $B = 1$ T. Time series of the voltage across the sample were digitally recorded with sampling intervals Δt ranging from 25 to 500 ns. We have analysed the recorded signals to determine power spectra, phase portraits, Poincaré sections and the fractal dimension of the current fluctuations as functions of the voltage across the sample and a load resistor. The bias voltage of the sample itself could not be taken as a control parameter because of the strong fluctuations in the chaotic regime. The power spectra were obtained using the segment averaging method. For each spectrum 20 segments obtained from 10 independent time series were averaged.

Without an external magnetic field regular oscillations are found, which set in just at the breakdown voltage. Their frequency increases with rising voltage and decreases with increasing magnitude of the load resistor as is expected for RC-relaxation oscillations. These oscillations remain coherent up to voltages well above breakdown. No other spectral features could be observed.

In the presence of an external magnetic field exceeding about 0.1 T, the spectral characteristics of the fluctuation change drastically. Again current oscillations occur at breakdown. With increasing voltage, however, additional characteristic frequencies appear and finally the fluctuations become totally aperiodic. The experimental results for the power spectra are summarized in fig. 1 for $B = 0.83$ T. The fundamental frequency of the oscillation is $f_1 = 91$ kHz just above breakdown occurring at 3 V for the applied load resistor. In contrast to the case of $B = 0$, f_1 slightly decreases upon raising the voltage. Above about 5 V a lower-lying oscillation of frequency $f_2 = 5$ kHz appears, which is also observed in side bands of the f_1 fundamental and its harmonics (fig. 1*b*)). On further increase of the voltage an independent third oscillation of frequency $f_3 = 41$ kHz occurs, which, within a very small voltage interval, merges with broad band noise of aperiodic chaotic fluctuations due to a drastic increase of the noise background. We have verified that the third frequency in fig. 1*c*) cannot be expressed as $f_3 = nf_1 + mf_2$ for $|n|$, $|m| < 5$.

Above 6 V the noise background decreases again, the f_1 oscillation is still present,

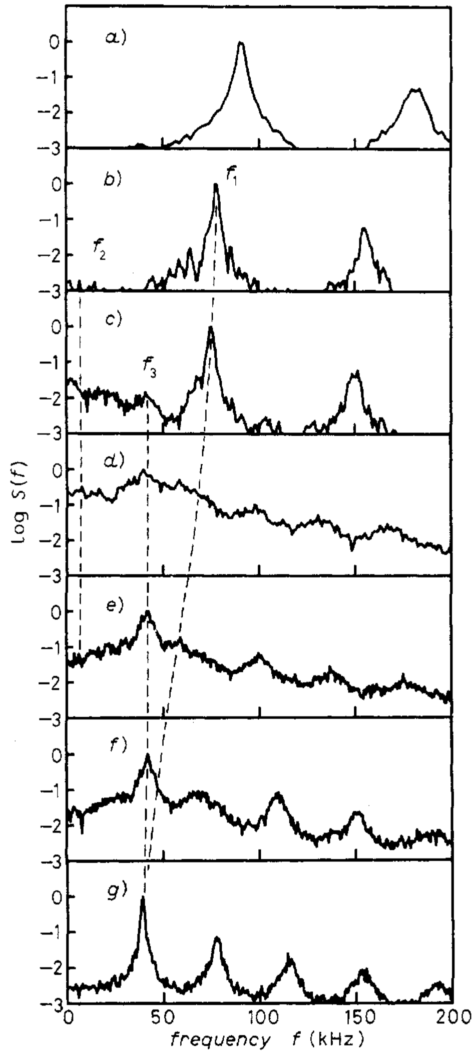

Fig. 1. – Sequence of power spectra as a function of the voltage (*a*) 4.5 V, *b*) 5.3 V, *c*) 5.6 V, *d*) 5.9 V, *e*) 6.2 V, *f*) 6.8 V, *g*) 7.8 V) across the load resistor and the sample for $T = 4.2$ K and $B = 0.83$ T. Broken lines indicate the shifts of the three incommensurate frequencies.

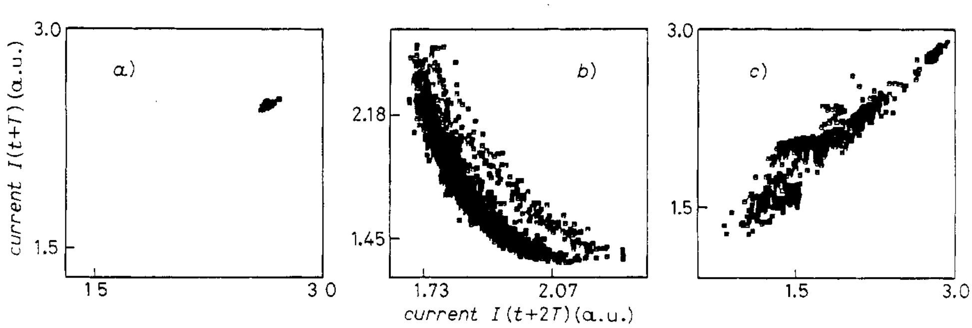

Fig. 2. – Poincaré sections for three different voltages (*a*) 4.5 V, *b*) 5.3 V, *c*) 5.9 V) for $B = 0.83$ T and 4.2 K. T is the time delay.

however, the f_3 oscillation dominates the spectra. The frequency f_1 continuously approaches f_3 with rising voltage and finally both frequencies coincide. This behaviour indicates the entrainment of two oscillations, which is observed for the first time in a semiconductor. Above about 7 V the spectra exhibit the f_3 fundamental and its overtones. As the appearance of two incommensurate frequencies in the power spectra above 5 V points to the existence of a torus attractor, we have tried to reconstruct this attractor from the measured signal. We span a phase space using as co-ordinates the measured signal $I(t)$ and its delayed versions $I(t+T)$, $I(t+2T)$ with a suitable delay time T. In this way one can obtain phase portraits of an attractor embedded into R^3. For more details of this method see *e.g.* ref. [11]. Figure 2 shows Poincaré sections of the reconstructed attractors for planes perpendicular to the first co-ordinate $I(t)$. Between 3 V and 4.6 V the attractor appears as a simple limit cycle and is indicated by a point with some noisy broadening in the Poincaré section (fig. 2*a*)). Above 5 V we find a section of a torus, indicating motion on a torus attractor, which is obviously associated with the incommensurate frequencies f_1 and f_2 (fig. 2*b*)). For voltages above 5.75 V the torus disappears, giving rise to a less ordered Poincaré section of the attractor (fig. 2*c*)).

The results presented so far suggest a Ruelle-Takens-Newhouse transition to chaos. We, therefore, have determined the fractal dimension of the attractors using the method developed by GRASSBERGER and PROCACCIA [12]. The correlation integral $C^m(r)$ was obtained according to

$$C^m(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j}^{N} \theta(r - |X_i^m - X_j^m|) \,,$$

where $\theta(X)$ is the Heavyside function and $X_i^m = \{I(t_i), I(t_i+T), \ldots I(t_i+(m-1)T)\}$ with $t_i = i\,\Delta t$ denotes a point on the attractor embedded in a m-dimensional phase space as described before. Assuming that the correlation integral scales as $C^m(r) \sim r^d$ for small r, an estimate of the fractal dimension is obtained as

$$d = \lim_{r\to 0} \frac{\ln C^m(r)}{\ln r} \,,$$

if d becomes independent of the embedding dimension m and constant with increasing m.

Figure 3 shows the dimension d as a function of voltage obtained in this way with embedding dimensions up to $m = 14$. In the absence of a magnetic field d jumps from zero to one at 0.5 V and remains constant for the entire range of voltages investigated. This corresponds to the observed relaxation oscillation and shows that chaotic current fluctuations do not occur for zero magnetic field. In contrast to this simple case, for $B = 0.83$ T the fractal dimension increases stepwise in the post-breakdown regime from

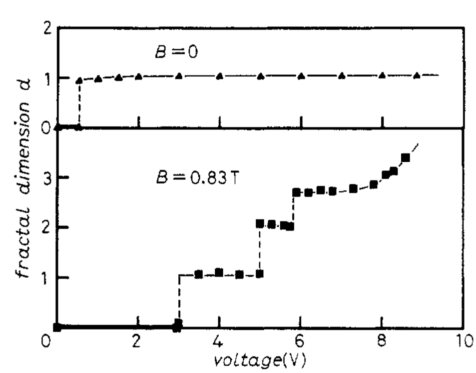

Fig. 3. – Fractal dimension for $B = 0$ and $B = 0.83$ V as a function of the voltage.

$d = 0$ to 1.1, to 2.0 and to 2.7. The latter value indicates the existence of a strange attractor with fractal properties. The method of Grassberger and Procaccia has been applied to other cases of chaotic fluctuations in semiconductors by AOKI *et al.* [13] and HELD *et al.* [14].

These results give strong evidence for a transition to chaos according to the Ruelle-Takens-Newhouse scenario [15]. In this scenario two successive Hopf bifurcations give rise to periodic motion and to a 2-torus. A 3-torus following a third Hopf bifurcation can become unstable against the formation of a strange attractor. This scenario seems to be realized here, as can be observed in the power spectrum near the onset of the f_3-oscillation and also in the other figures.

In the periodic regime the spectral peaks are relatively broad, a fact which deserves some explanation. We attribute the broadening to the influence of external (*e.g.* thermal) noise. This is because in our experiments we have observed that, near the onset of periodic oscillations, the system is very sensitive to external fluctuations. With increasing voltage the stability of the oscillation is expected to increase and accordingly the spectral peaks become narrower again.

Having cleared up the mathematical mechanism for the transition to chaos in our case, we now come to a discussion of the physical origins of the chaotic fluctuations. TEITSWORTH and WESTERVELT have demonstrated that a periodic driving current in the presence of impact ionization can give rise to chaotic fluctuations [10]. AOKI *et al.* have observed that the period-doubling route to chaos is cut short by the inclusion of a Joule heating effect due to thermal ionization [16]. While there exists a good deal of theoretical and experimental work for the periodically *driven* cases, attempts to explain the nature of chaotic fluctuations of semiconductors in the *autonomous* cases are still in a beginning stage. In n-GaAs, in particular, AOKI *et al.* have analysed current fluctuations arising under weak photoexcitation and pulsed voltage [5, 13]. We have studied n-GaAs in a different situation, where chaotic fluctuations arose in the absence of photoexcitation and pulsed voltage. Our results shed some light on the physical mechanism in this autonomous situation. We have demonstrated that in n-GaAs it crucially involves the magnetic field; in the absence of a magnetic field spontaneous oscillations were found, but never showed a transition to chaos.

At present we only know of one mechanism that could possibly explain our observation. Recently SCHÖLL has studied a model that leads to self-sustained chaotic current fluctuations in high-purity extrinsic semiconductors at low temperatures [17]. It is based on the coupling between multilevel impact ionization of shallow donors and dielectric relaxation. The presence of impact ionization of more than one impurity bound state appears to be essential for the occurrence of chaos in the autonomous case. If the model is simplified to a single impurity bound state, chaos only occurs under external periodic driving. In Schöll's analysis chaos is related to a longitudinal current instability, which sets in when the dielectric relaxation time exceeds the generation-recombination lifetime of free carriers. In other simulations the dielectric relaxation time even had to be large compared to the latter [16]. The assumptions and results of Schöll's model suggest a mechanism for the magnetic-field dependence of the transition to chaos. The excited shallow donor states lie very close to the conduction band edge with binding energies of 1.3 meV or less. The magnetic field increases the separation of some of those states from the conduction band edge and supresses impurity conduction due to the shrinking of the wave function normal to the magnetic field. Thereby excited donor states become effective in impact ionization and trapping processes. In addition the magnetoresistance effect of the magnetic field raises the dielectric relaxation time due to the reduction of the conductivity. In this way the dielectric relaxation time may exceed the free-carrier lifetime as required in ref. [17]. The latter assumes relatively large values itself of the order of 1 μs or more in the vicinity of the instability [18].

SCHÖLL has reported a period-doubling transition to chaos, but has also observed a variety of other nonlinear and chaotic phenomena, depending upon the numerical parameters [17]. It is customary that nonlinear systems can exhibit several routes to chaos. One may thus expect that, in further investigations of the model, also the Ruelle-Takens-Newhouse scenario can be found, which we have reported here.

Other theoretical studies have shown that nonlinear coupling of current filaments favour Hopf bifurcations [16]. Magnetic fields are also crucial for chaos in an electron-hole plasma in Ge [19]. This explanation can be ruled out in our case, as holes were not injected by the contacts used here. Finally, it has recently been argued that the spontaneous current oscillations in p-Ge are due to moving space charge waves caused by a negative differential rate of the coefficient of impact ionization [20]. Further theoretical and experimental work is needed to decide whether this mechanism could play a role for n-GaAs in a magnetic field.

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REFERENCES

[1] KOENIG S. H., *Phys. Rev.*, **110** (1958) 986.
[2] OLIVER P. I., *Phys. Rev.*, **127** (1962) 1045.
[3] OBERMAIER R., BÖHM W., PRETTL W. and DIRNHOFER P., *Phys. Lett. A*, **105** (1984) 149; WEISPFENNING M., HOESER I., BÖHM W., PRETTL W. and SCHÖLL E., *Phys. Rev. Lett.*, **55** (1985) 754.
[4] SCHÖLL E., *Z. Phys. B*, **46** (1982) 23 and 48 (1983) 153.
[5] AOKI K., KOBAYASHI T. and YAMAMOTO K., *J. Phys. Soc. Jap.*, **51** (1982) 2373; AOKI K. and YAMAMOTO K., *Phys. Lett. A*, **98** (1983) 72.
[6] TEITSWORTH S. W., WESTERVELT R. M. and HALLER E. E., *Phys. Rev. Lett.*, **51** (1983) 825.
[7] PEINKE J., MÜHLBACH A., HUEBENER Z. P. and PARISI J., *Phys. Lett. A*, **108** (1985) 407; PEINKE J., RÖHRICHT B., MÜHLBACH A., PARISI J., NÖLDEKE CH., HUEBENER R. P. and RÖSSLER O. E., *Z. Naturforsch. A*, **40** (1985) 562.
[8] SEILER D. G., LITTLER C. L., JUSTICE R. J. and MILONNI P. W., *Phys. Lett. A*, **108** (1985) 462.
[9] TEITSWORTH S. W. and WESTERVELT R. M., *Phys. Rev. Lett.*, **56** (1986) 516.
[10] TEITSWORTH S. W. and WESTERVELT R. M., *Phys. Rev. Lett.*, **53** (1984) 2587.
[11] GEISEL T., *Nature*, **298** (1982) 322 and GEISEL T. in *Chaos in Astrophysics*, edited by J. R. BUCHLER, J. PERDANG and E. A. SPIEGEL (D. Reidel, Dordrecht) 1985, p. 165.
[12] GRASSBERGER P. and PROCACCIA I., *Phys. Rev. Lett.*, **50** (1983) 346.
[13] AOKI K., IKEZAWA O., MUGIBAYASHI N. and YAMAMOTO K., *Physica B*, **134** (1965) 288.
[14] HELD G. A., JEFFRIES C. D. and HALLER E. E., *Proceedings of the XVII International Conference on the Physics of Semiconductors*, edited by J. D. CHADI and W. A. HARRISON (Springer, New York) 1985, p. 1289.
[15] NEWHOUSE S., RUELLE D. and TAKENS F., *Commun. Math. Phys.*, **64** (1978) 35.
[16] AOKI K., MUGIBAYASHI N. and YAMAMOTO K., *XVIII International Conference on the Physics of Semiconductors.*
[17] SCHÖLL E., *Proceedings of the IV International Conference on Hot Electrons in Semiconductors, Physica $B+C$*, **134** (1985) 271.
[18] KOENIG S. H., *International Conference on Solid State Physics*, edited by M. DESIRANT (Academic Press, London) 1960, p. 422.
[19] HELD G. A., JEFFRIES C. and HALLER E. E., *Phys. Rev. Lett.*, **52** (1984) 1037; HELD G. A. and JEFFRIES C., *Phys. Rev. Lett.*, **55** (1985) 887.
[20] TEITSWORTH S. W. and WESTERVELT R. M., to be published.