Spin Hall Effect

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Abstract

The intrinsic spin Hall effect in semiconductors has developed to a remarkably lively and rapidly growing branch of research in the field of semiconductor spintronics. In this article we give a pedagogical overview on both theoretical and experimental accomplishments and challenges. Emphasis is put on the the description of the intrinsic mechanisms of spin Hall transport in III-V zinc-blende semiconductors, and on the effects of dissipation.

1 Introduction

Starting from about the late 1990s, a pronounced, and partially also renewed, interest in effects of spin-orbit coupling in semiconductors has been emerging in the solid-state community. This development is mainly fueled by the field of spintronics. The latter keyword summarizes an entire plethora of theoretical and experimental efforts towards using the spin degree of freedom of electrons, instead, or in combination with, their charge for information processing, or, even more ambitious, for quantum information processes. Thus, controlling the electron spin in semiconductor structures is a key challenge, and the relativistic effect of spin-orbit coupling is an important, if not indispensable, ingredient for reaching this goal. Among all the rapidly progressing activities in this field, a major and very recent development was the theoretical prediction and subsequent experimental investigation of the spin Hall effect in semiconductor structures. This effect amounts in a spin current, as opposed to a charge current, driven by a perpendicular electric field. In this article we review recent studies on spin Hall transport in semiconductors induced by intrinsic mechanisms.

A brief overview on important aspects and perspectives of the field of spintronics is given in the article by Wolf *et al.* [1]. Selected topics are reviewed in more detail in a volume edited by Awschalom, Loss, and Samarth [2]. A comprehensive review of many parts of spintronics was given by Zutic, Fabian, and Das Sarma [3]. Research directions not covered here include the field of ferromagnetic semiconductors; for a review on this we refer to Refs. [4, 5, 6, 7, 8, 9].

From a historical perspective, the notion of the spin Hall effect in systems of itinerant spinful charge carriers was considered first by Dyakonov and Perel [10] in the early seventies, and in a more recent paper by Hirsch [11]. In these studies the predicted spin Hall effect is due to spin-orbit effects influencing scattering processes upon static impurities. Following the usual terminology of semiconductor physics, this effect is referred to as the *extrinsic* spin Hall effect since it necessarily requires spin-dependent impurity scattering. This is in contrast to the *intrinsic* spin Hall effect which is entirely due to spin-orbit coupling terms in the single-particle carrier Hamiltonian and occurs even in the absence of any scattering process. The story of the intrinsic spin Hall effect starts in summer of 2003, when Murakami, Nagaosa, and Zhang [12], and, almost simultaneously, J. Sinova *et al.* in a collaboration based in Austin (Texas)[13] predicted this phenomenon. The work by Sinova *et al.* considers a two-dimensional electron gas being subject to spin-orbit coupling of the so-called Rashba type, whereas the former paper investigates valence-band holes in three-dimensional bulk systems. Shortly later on, Schliemann and Loss added a study on spin Hall transport of heavy holes confined to a quantum well[14]. Only a few months later, J. Wunderlich *et al.* reported on an experimental study of the latter type of system, confirming the predicted spin Hall effect via optical techniques [15]. The paper by Wunderlich *et al.* was only shortly preceded by another experimental report by Kato *et al.* who detected spin Hall transport in n-doped bulk systems, again using an optical method [16]. In both experiments, the existence of spin Hall transport is signaled by spin accumulation at the boundaries of the sample, which seems to be the easiest method so far to detect this effect.

The theoretical and experimental developments sketched above have generated a still rapidly growing amount of preprints and (subsequent) journal publications, so far mostly theoretical. In this article we give a pedagogical overview on important aspects of intrinsic spin-Hall transport in III-V zinc-blende semiconductors. Emphasis is put on the the description of the intrinsic mechanisms of spin Hall transport induced by spin-orbit coupling,and on the effects of dissipation. This article also addresses researchers who are not particularly specialized in spin phenomena in semiconductors but are interested in this rapidly developing field. A brief review on spin-Hall transport was already given in a conference paper by Murakami [17], and Sinova *et al.* recently provided a short summary of important issues [18].

This article is organized as follows. In section 2 we make a few general comments on spin-orbit coupling in semiconductors and describe its effective contributions to the band structure of both electron and hole doped systems. In section 3 we first make some general remarks on the notion of spin currents before reviewing the particularly rich body of recent theoretical work on spin Hall transport. Our analysis here includes the two-dimensional electron gas as well as p-doped bulk systems and quantum wells. Experimental work and proposed experiments are discussed in section 4. We close with conclusions and an outlook in section 5.

2 Spin-orbit coupling in III-V semiconductors

The coupling between the orbital and the spin degree of freedom of electrons is a relativistic effect described by the Dirac equation and its nonrelativistic expansion in powers of the inverse speed of light c. In second order one obtains, apart from two spin-independent contributions, the following well-known spin-orbit coupling term,

$$\mathcal{H}_{so} = \frac{1}{2m_0c^2}\vec{s}\cdot\left(\nabla V \times \frac{\vec{p}}{m_0}\right) , \tag{1}$$

where m_0 is the bare mass of the electron, $\vec{s}$, $\vec{p}$ its spin and momentum, respectively, and V is some applied external potential. On the other hand, the free Dirac equation, $V = 0$, has two dispersion branches with positive and negative energy,

$$\varepsilon(\vec{p}) = \pm\sqrt{m_0^2c^4 + c^2p^2} \, , \tag{2}$$

which are separated by an energy gap of $2m_0c^2 \approx 1\text{MeV}$. In particular, the nonrelativistic expansion of the Dirac equation quoted above can be seen as a method of systematically including the effects of the negative-energy solutions on the states of positive energy starting from their nonrelativistic limit. Moreover, the large energy gap $2m_0c^2$ appears in the denominator of the right hand side of Eq. (1), suppressing the effects of spin-orbit coupling for weakly bound electrons.

On the other hand, the band structure of zinc-blende III-V semiconductors shows many formal similarities to the situation of free relativistic electrons, while the relevant energy scales are grossly different [19, 20, 21]. For not too large doping of such semiconductors, one can concentrate on the band structure around the Γ point. Here one has a parabolic s-type conduction band and a p-type valence band consisting of the well-known dispersion branches for heavy and light holes, and the split-off band. However, the gap between conduction and valence band is of order 1eV or smaller. This heuristic argument makes plausible that spin-orbit coupling is an important effect in III-V semiconductors which actually lies at the very heart of the field of semiconductor spintronics.

In the following we give an overview on effective model Hamiltonians for conduction-band electrons and valence-band holes in III-V zinc-blende semiconductors in several spatial dimensions. These effective expressions can be obtained via the so-called $\vec{k}\cdot\vec{p}$-theory and related methods, as general references we refer to Refs. [22, 23, 24]. Here we shall just state the results and discuss their main physical implications.

2.1 Conduction-band electrons

Let us first consider three-dimensional bulk systems. For electrons in the s-type conduction band, the contribution to spin-orbit coupling being of lowest order in the electron momentum $\vec{p}$ has been derived by Dresselhaus [25] and reads

$$\mathcal{H}_D^{bulk} = \frac{\gamma}{\hbar^3}\left(\sigma^x p_x\left(p_y^2 - p_z^2\right) + \sigma^y p_y\left(p_z^2 - p_x^2\right) + \sigma^z p_z\left(p_x^2 - p_y^2\right)\right), \tag{3}$$

where $\vec{\sigma}$ is the vector of Pauli matrices describing the electron spin, and γ is an effective coupling parameter. This Hamiltonian is trilinear in the momentum $\vec{p}$ and invariant under all symmetry operations of the tetrahedral group T_d, the point symmetry group of the zinc-blende lattice. As a result, the parameter γ is different from zero because the zinc-blende lattice does not possess an inversion center. Therefore, the Dresselhaus spin-orbit coupling is due to *bulk-inversion asymmetry*.

In a sufficiently narrow quantum well grown along the [001] direction, and at sufficiently low temperatures, one can approximate the operators p_z and p_z^2 by their expectation values $\langle p_z\rangle \approx 0$, $\langle p_z^2\rangle = \hbar^2\langle k_z^2\rangle$. Then neglecting terms of order p_x^2, p_y^2 leads to a spin-orbit coupling term linear in the momentum [26, 27],

$$\mathcal{H}_D = \frac{\beta}{\hbar}\left(p_y\sigma^y - p_x\sigma^x\right) \tag{4}$$

with $\beta = \gamma\langle k_z^2\rangle$. Here k_z is the wave number in the lowest subband of the well. Another important contribution to spin-orbit coupling occurs in quantum wells whose confining potential is lacking inversion symmetry. This contribution due to *structure-inversion asymmetry* is known as the Rashba term [28, 29],

$$\mathcal{H}_R = \frac{\alpha}{\hbar}\left(p_x\sigma^y - p_y\sigma^x\right), \tag{5}$$

where the coupling parameter α is essentially proportional to the potential gradient across the quantum well. Hence, α is in particular tunable by an electric gate and can therefore be varied experimentally.

A both theoretically [30] and experimentally [31, 32] well established value for the Dresselhaus parameter in GaAs is $\gamma = 25\mathrm{eV}\AA^3$. Depending on the width of the quantum well, this leads to values for β being of up to 10^{-11}eVm. Regarding the Rashba coefficient α, values of a few 10^{-11}eVm can be reached in InAs [33, 34, 35, 36, 37, 38, 39], whereas in GaAs this quantity is typically an order of magnitude smaller [40]. Thus, the characteristic energy scales

$$\varepsilon_R = \frac{m\alpha^2}{\hbar^2}\,, \tag{6}$$

$$\varepsilon_D = \frac{m\beta^2}{\hbar^2} \tag{7}$$

for Rashba and Dresselhaus coupling, respectively, can be of order $0.1\ldots1.0$meV, depending on the effective band mass m.

Let us finally briefly discuss the spectrum and eigenstates generated by the above spin-orbit coupling terms. We consider the single-particle Hamiltonian for a two-dimensional electron system

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + \mathcal{H}_R + \mathcal{H}_D\,. \tag{8}$$

The eigenenergies are given by

$$\varepsilon_\pm\left(\vec{k}\right) = \frac{\hbar^2k^2}{2m} \pm \sqrt{(\alpha k_y + \beta k_x)^2 + (\alpha k_x + \beta k_y)^2} \tag{9}$$

with eigenstates

$$\langle\vec{r}|\vec{k},\pm\rangle = \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{A}}\frac{1}{\sqrt{2}}\left(\begin{array}{c} 1 \\ \pm e^{i\chi(\vec{k})}\end{array}\right) \tag{10}$$

where A is the area of the system and

$$\chi(\vec{k}) = \arg(-\alpha k_y - \beta k_x + i(\alpha k_x + \beta k_y))\,. \tag{11}$$

The above spin–orbit coupling terms can be viewed as a momentum-dependent Zeeman field acting on the electron spin. Consequently, the spin state of the electron depends on its momentum, as seen in Eq. (10). Note that for pure Rashba or Dresselhaus coupling, the dispersions form two parabolas being shifted horizontally. This is different form a normal Zeeman field which shifts the dispersion parabolas vertically, i.e. along the energy axis. The case $\alpha = \pm\beta$ is particular [41, 42]. Here a new conserved quantity given by $\Sigma := (\sigma^x \mp \sigma^y)/\sqrt{2}$ arises, and the spin state of the electrons becomes independent of the wave vector. This result for $\alpha = \pm\beta$ is a very general one, it also holds in the presence of any arbitrary scalar potential, or if interactions between electrons are included.

2.2 Valence-band holes

The valence band of III-V zinc-blende semiconductors is of p-type, i.e. it is predominantly composed out of atomic wave functions with angular momentum $l = 1$. Adding this angular momentum to the electron spin $s = 1/2$, we find to multiplets with total angular momentum $j = 3/2$ and $j = 1/2$. The dublett $j = 1/2$ forms essentially an energetically separated so-called split-off band and will not be considered any further. The multiplet $j = 3/2$ consists essentially of the so-called heavy and light hole states which are, to a good degree of approximation, described by Luttinger's Hamiltonian [43],

$$\mathcal{H} = \frac{1}{2m_0}\left(\left(\gamma_1 + \frac{5}{2}\gamma_2\right)\vec{p}^2 - 2\gamma_2\left(\vec{p}\cdot\vec{S}\right)^2\right) . \tag{12}$$

Here m_0 is again the bare electron mass, and $\vec{S}$ are spin-3/2-operators. The dimensionless Luttinger parameter γ_1 and γ_2 describe the valence band of the specific material with effects of spin-orbit coupling being included in γ_2. The eigenstates of the above Hamiltonian can be chosen to be eigenstates of the helicity operator $\lambda = (\vec{k}\cdot\vec{S})/k$, where $\vec{k} = \vec{p}/\hbar$ is the hole wave vector. The heavy holes correspond to $\lambda = \pm 3/2$, while the light holes have $\lambda = \pm 1/2$. From the Hamiltonian (12) one finds the effective band mass for the heavy holes as

$$m_{hh} = \frac{m_0}{\gamma_1 - 2\gamma_2} \tag{13}$$

and for the light holes

$$m_{lh} = \frac{m_0}{\gamma_1 + 2\gamma_2} . \tag{14}$$

Well established values for the Luttinger parameters, among other band structure parameters, can be found in the literature [44]. For example, for GaAs one has $\gamma_1 \approx 7.0$ and $\gamma_2 \approx 2.5$ giving $m_{hh} \approx 0.5m_0$ and $m_{lh} \approx 0.08m_0$.

In a bulk system, the heavy and light hole states are degenerate at the Γ-point $\vec{k} = 0$. This degeneracy is lifted in a quantum well due to size quantization, and for sufficiently narrow wells and low enough temperatures one can concentrate on the lower-lying heavy holes. Moreover, if the quantized wave vector in the growth direction is large enough, i.e. if the well is not too wide, the spin of these heavy holes points predominantly along the growth direction with a projection of $\pm 3/2$. For asymmetric wells these hole states are subject to a spin-orbit contribution due to structure-inversion asymmetry analogous to the Rashba term for electrons in the conduction band. Choosing the growth direction to point along the z-axis, the resulting effective Hamiltonian has the form [45, 46]

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + i\frac{\tilde{\alpha}}{2\hbar^3}\left(p_-^3\sigma_+ - p_+^3\sigma_-\right) , \tag{15}$$

using the notations $p_\pm = p_x \pm ip_y$, $\sigma_\pm = \sigma^x \pm i\sigma^y$. Here the Pauli matrices operate on the total angular momentum states with spin projection $\pm 3/2$ along the growth direction; in this sense they represent a pseudospin degree of freedom rather than a genuine spin 1/2. In the above equation, m is the heavy-hole mass, and $\tilde{\alpha}$ is Rashba spin-orbit coupling coefficient due to structure inversion asymmetry. This Hamiltonian has two dispersion branches given by

$$\varepsilon_\pm(k) = \frac{\hbar^2 k^2}{2m} \pm \tilde{\alpha}k^3 \tag{16}$$

with eigenfunctions

$$\langle\vec{r}|\vec{k},\pm\rangle = \frac{e^{i\vec{k}\vec{r}}}{\sqrt{A}}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \mp i\left(k_x + ik_y\right)^3/k^3 \end{pmatrix} . \tag{17}$$

We note that the Rashba parameter α entering the Hamiltonian (5) for electrons in a quantum well has a different dimension than the parameter $\tilde{\alpha}$ for holes in Eq. (15). In the latter case, the quantity $m\tilde{\alpha}/\hbar^2$ has dimension of length and can reach a magnitude of several nanometers in GaAs samples [47].

3 Spin Hall transport: Theory

In this section we summarize important theoretical results on intrinsic spin Hall effect. We start with some general considerations on spin currents.

3.1 Spin currents: General remarks

A type of current most familiar to physicists is certainly the usual charge or particle current. The particle density in a many-body system is described by the operator

$$\rho(\vec{r}) = \sum_n \delta(\vec{r} - \vec{r}_n) \tag{18}$$

where the index n labels the particles. This density operator is a function of time via the time-dependence of the positions $\vec{r}_n$ entering the argument of the delta-functions. Let this time evolution be generated by an Hamiltonian of the form

$$\mathcal{H} = \sum_n h(\vec{r}_n, \vec{p}_n, \vec{\sigma}_n) + V_{int}\,. \tag{19}$$

Here the term V_{int} describes interaction among the particles and depends only on their spatial coordinates, and the single-particle Hamiltonian h reads

$$h(\vec{r}, \vec{p}, \vec{\sigma}) = \frac{\vec{p}^2}{2m} + \gamma_{ij} p_i \sigma^j + V(\vec{r})\,. \tag{20}$$

Summation over repeated cartesian indices is understood, and the matrix γ parameterizes spin-orbit coupling of the Rashba and Dresselhaus type for electrons in a quantum well. Finally, the static potential $V(\vec{r})$ describes, e.g., static impurities. Now, starting from the Heisenberg equation of motion,

$$\frac{d}{dt}\rho = \frac{i}{\hbar}\left[\mathcal{H}, \rho\right] \tag{21}$$

and performing some elementary algebraic manipulations, one derives the well-known continuity equation for the particle current,

$$\frac{d}{dt}\rho + \nabla \cdot \vec{j} = 0 \tag{22}$$

with the current density operator

$$\vec{j}(\vec{r}) = \frac{1}{2} \sum_n \left\{ \vec{v}(\vec{p}_n, \vec{\sigma}_n), \delta(\vec{r} - \vec{r}_n) \right\}\,. \tag{23}$$

Here $\{A, B\} = AB + BA$ denotes the anticommutator of two operators, and the velocity operator $\vec{v}$ is given by

$$\vec{v}(\vec{p}_n, \vec{\sigma}_n) = \frac{i}{\hbar}\left[\mathcal{H}, \vec{r}_n\right] \tag{24}$$

for each particle n. Note that this operator is in general spin-dependent if spin-orbit coupling is present.

Let us now consider a general observable described by a hermitian single-particle operator A which can be a function of position, momentum, and spin: $A = A(\vec{r}, \vec{p}, \vec{\sigma})$. The density operator corresponding to this physical quantity A is naturally defined as

$$\rho_A(\vec{r}) = \frac{1}{2}\sum_n \{A(\vec{r}_n, \vec{p}_n, \vec{\sigma}_n), \delta(\vec{r} - \vec{r}_n)\} \ , \tag{25}$$

where the symmetrization ensures hermiticity. Now proceeding as above one finds

$$\frac{d}{dt}\rho_A + \nabla \cdot \vec{j}_A = s_A \, , \tag{26}$$

where the current density operator $\vec{j}_A$ is given by

$$\vec{j}_A(\vec{r}) = \frac{1}{4}\sum_n \{A(\vec{r}_n, \vec{p}_n, \vec{\sigma}_n), \{\vec{v}(\vec{p}_n, \vec{\sigma}_n), \delta(\vec{r} - \vec{r}_n)\}\} \ , \tag{27}$$

and the additional source term on the right-hand-side reads

$$s_A(\vec{r}) = \frac{1}{2}\sum_n \left\{ \frac{i}{\hbar}\left[\mathcal{H}, A(\vec{r}_n, \vec{p}_n, \vec{\sigma}_n)\right], \delta(\vec{r} - \vec{r}_n) \right\} \, . \tag{28}$$

Thus, we only obtain the usual from of the continuity equation if the observable A commutes with the Hamiltonian $\mathcal{H}$, which is of course just a restatement of Noether's theorem. For the case electron spin components as observables, the corresponding spin-current densities are given by

$$\vec{j}_i(\vec{r}) = \frac{1}{4}\sum_n \left\{ \frac{\hbar}{2}\sigma^i_n, \{\vec{v}(\vec{p}_n, \vec{\sigma}_n), \delta(\vec{r} - \vec{r}_n)\} \right\} \, , \tag{29}$$

and the source terms are due to spin-orbit coupling being present in the single-particle Hamiltonian. These source terms reflect the fact that magnetization, i.e. the density of magnetic moments, can be altered by two ways: by spatially moving spinful particles, or by manipulating their spin state. The latter process is described by the source terms. For instance, in the case of noninteracting electrons in a quantum well with spin-orbit coupling of the Rashba and Dresselhaus type, the source terms can be expressed via the components of the spin-current densities itself [48, 49],

$$\frac{d}{dt}\rho_x + \nabla \cdot \vec{j}_x = \frac{2m\alpha}{\hbar^2} j^x_z + \frac{2m\beta}{\hbar^2} j^y_z \, , \tag{30}$$

$$\frac{d}{dt}\rho_y + \nabla \cdot \vec{j}_y = \frac{2m\alpha}{\hbar^2} j^y_z + \frac{2m\beta}{\hbar^2} j^x_z \, , \tag{31}$$

$$\frac{d}{dt}\rho_z + \nabla \cdot \vec{j}_z = -\frac{2m\alpha}{\hbar^2}\left(j^x_x + j^y_y\right) - \frac{2m\beta}{\hbar^2}\left(j^x_y + j^y_x\right) . \tag{32}$$

In summary, the definition of the spin current density as given in Eq. (29) is the straightforward generalization of the usual particle current and widely used in the literature. As seen above, this spin current density is, however, not conserved, i.e. it does not fulfill a simple continuity equation. This fact might or might not be seen as a shortcoming of the above definition.

Another peculiarity of this type of current operator was pointed out by Rashba [50] who found that, for the situation of an asymmetric quantum well, the current densities with in-plane spin components, $\vec{j}_x$, $\vec{j}_y$, can have nonzero expectation values even in the absence of an electric field, i.e. in thermal equilibrium. Using periodic boundary conditions and considering an infinite disorder-free system of non-interacting electrons at zero temperature and positive Fermi energy, the full result for the case of both Rashba and Dresselhaus coupling reads [48]

$$\langle j_x^x \rangle = -\langle j_y^y \rangle = \frac{\beta}{6\pi} \left(\frac{m}{\hbar^2} \right)^2 \left(\alpha^2 - \beta^2 \right) , \tag{33}$$

$$\langle j_y^x \rangle = -\langle j_x^y \rangle = \frac{\alpha}{6\pi} \left(\frac{m}{\hbar^2} \right)^2 \left(\alpha^2 - \beta^2 \right) . \tag{34}$$

Note that these equilibrium spin currents vanish in the case $\alpha = \pm\beta$ due to the additional conserved spin operator arising at this point [41]. The findings shown in Eqs. (33),(34), however, certainly depend on the boundary conditions used and are altered in a more realistic description of finite systems [51].

In the recent literature, there are several proposals and discussions on alternative forms of spin currents which possibly fulfill proper continuity equations [52, 53, 54, 55, 56, 57]. However, these issues do not seem to be settled yet. Therefore, in the following we shall concentrate on spin current densities as defined in Eq. (29).

3.2 Conduction-band electrons in two dimensions

We now discuss spin Hall transport of conduction-band electrons in III-V semiconductor quantum wells. As a great simplification used in almost the entire theoretical work so far, we will consider non-interacting electrons. Thus, the system is described by the single-particle Hamiltonian

$$\mathcal{H} = \frac{\vec{p}^2}{2m} + \frac{\alpha}{\hbar} \left(p_x \sigma^y - p_y \sigma^x \right) + \frac{\beta}{\hbar} \left(p_y \sigma^y - p_x \sigma^x \right) , \tag{35}$$

and instead of the many-body spin current density operators (29) we can use the single-particle operator

$$\vec{j}_z = \frac{\hbar}{4} \left(\sigma^z \vec{v} + \vec{v} \sigma^z \right) \tag{36}$$

$$= \frac{\vec{p}}{m} \frac{\hbar}{2} \sigma^z , \tag{37}$$

where we have concentrated on the spin component along the growth direction of the quantum well (chosen as z-axis) and used the anticommutativity of Pauli matrices. To account for effects of disorder and confining boundaries of the system, appropriate potentials should be added to the Hamiltonian (35), as we will discuss in detail below.

The linear response of this spin current to an electric field applied in the plane of the two-dimensional electron gas can be evaluated via the usual Kubo formula [58]. For the off-diagonal (or Hall) components of the response tensor one has [13, 59]

$$\sigma_{xy}^{S,z}(\omega) = \frac{e}{A(\omega + i\eta)} \int_0^\infty e^{i(\omega + i\eta)t} \sum_{\vec{k},\mu} f(\varepsilon_\mu(\vec{k})) \langle \vec{k}, \mu | [j_z^x(t), v_y(0)] | \vec{k}, \mu \rangle . \tag{38}$$

Here A is the volume of the system, e is the elementary charge, and $f(\varepsilon_\mu(\vec{k}))$ is the Fermi distribution function for energy $\varepsilon_\mu(\vec{k})$ at wave vector $\vec{k}$ in the dispersion branch $\mu = \pm$ as given in Eq. (10). The above quantity describes the linear response in terms of a spin current to a perpendicular electric field of frequency ω. In the commutator on the right-hand side the time-dependent spin current operator in the Heisenberg picture enters,

$$\vec{j}_z(t) = e^{i\mathcal{H}t/\hbar}\vec{j}_z e^{-i\mathcal{H}t/\hbar} \,. \tag{39}$$

Moreover, the right-hand side of Eq. (38) has to be understood in the limit of vanishing imaginary part $\eta > 0$ in the frequency argument. This imaginary part in the frequency reflects the fact that the external electric field is assumed to be switched on adiabatically starting from the infinite past of the system, and it also ensures causality properties of the retarded Green's function occurring in Eq. (38). In general, and as we will discuss in more detail below, the limiting process $\eta \to 0$ does not commute with other limits, and, in particular, the dc-limit $\omega \to 0$ has to be taken with care [58]. In the presence of random impurity scattering, the retarded two-body Green's function in Eq. (38) will generically have a frequency argument with positive imaginary part [58]. In this case the limit $\eta \to 0$ is unproblematic, and the imaginary part of the frequency argument is just due to impurity scattering and/or other (many-body) effects. Generically, the imaginary part $\eta > 0$ corresponds to a finite carrier quasiparticle lifetime.

Let us now for simplicity consider the case of Rashba spin-orbit coupling only and assume the electron density to be large enough such that the Fermi energy is positive (which is usually the case in realistic samples). Neglecting all possible disorder effects and concentrating on the case of zero temperature, the Kubo formula (38) can be evaluated straightforwardly, giving a spin Hall conductivity at zero frequency of

$$\sigma_{xy}^{S,z}(0) = -\sigma_{yx}^{S,z}(0) = \frac{e}{8\pi} \,. \tag{40}$$

This result was obtained first by Sinova *et al.* [13]. It is remarkable in the sense that the value of the spin Hall conductivity does not depend on the Rashba parameter α. In particular, even in the limit of vanishing spin-orbit coupling, the above result still predicts a finite spin Hall current. However, no effects of disorder in the system have been included so far, and the infinitesimal parameter η has been put to zero right away. Let us now take into account disorder effects by replacing η with a phenomenological relaxation rate $1/\tau$. Here we find [59]

$$\sigma_{xy}^{S,z}(0) = -\sigma_{yx}^{S,z}(0) = \frac{e}{8\pi} - \frac{e}{32\pi}\frac{\hbar/\tau}{\varepsilon_R}\tan^{-1}\left(4\frac{\varepsilon_R}{\hbar/\tau}\left(1+8\frac{\varepsilon_R\varepsilon_f}{(\hbar/\tau)^2}\right)^{-1}\right) \,. \tag{41}$$

The first term is still the universal expression found in Ref. [13], whereas in the second contribution three energy scales enter: The Fermi energy ε_f, the Rashba energy ε_R, and the energy scale of the scattering by disorder potentials $\hbar/\tau$. Clearly, if the latter quantity dominates over the Rashba coupling, $\hbar/\tau \gg \varepsilon_R$, the second term in Eq. (41) cancels the first one, and the spin Hall conductivity is indeed suppressed by disorder. Analogous results can be found if both the Rashba and the Dresselhaus coupling are included [60]. In this case, the above approach also yields nonvanishing *longitudinal* spin conductivities [60].

Thus, we arrive at an apparently physically satisfactory picture. However, it turns out to be qualitatively incorrect for the following reason: When replacing the infinitesimal parameter

η in Eq. (38) with the inverse of a phenomenological relaxation time τ, one neglects certain contributions in a systematic perturbational expansion with respect to the disorder potentials. These contributions are known as vertex corrections. As is was shown first by Inoue, Bauer, and Molenkamp [61], the full dissipative contribution to the spin-Hall conductivity including the vertex corrections *exactly cancels the universal value, independently of the strength of the disorder potentials and the Rashba coupling.* This result was obtained for an infinitely large system and in lowest perturbational order with respect to the disorder potentials which were modeled by delta-functions. Subsequently, this finding was reproduced an generalized by several other authors [62, 63, 64, 65, 66, 67, 68] using different theoretical methods. The conclusions from these investigations can be summarized as follows: The spin-Hall conductivity for spin polarization along the growth direction in an infinite two-dimensional system with spin-orbit coupling of the Rashba and Dresselhaus type strictly vanishes in the presence of any spin-independent mechanism with forces, via spin-orbit coupling, the electron spins to relax to a constant value. This result is independent of any perturbational expansion with respect to disorder terms and holds also both at finite temperature and in the presence of interactions among the electrons. However, it does in general not hold in the presence of magnetic fields or other spin-dependent contributions in the Hamiltonian.

The proof of this very general statement was worked out by Chalaev and Loss [65], and by Dimitrova [66]; a preliminary version can also be found in Ref. [48]. For the case of both Rashba and Dresselhaus spin-orbit coupling, the argument is as follows: To analyze the electron spin dynamics in an infinite homogeneous system, it is sufficient to consider just the spin operator of a single electron. From the Heisenberg equation of motion we find

$$\frac{d}{dt}\sigma^x = \frac{4m\alpha}{\hbar^3}j_z^x + \frac{4m\beta}{\hbar^3}j_z^y\,, \qquad (42)$$

$$\frac{d}{dt}\sigma^y = \frac{4m\beta}{\hbar^3}j_z^x + \frac{4m\alpha}{\hbar^3}j_z^y\,. \qquad (43)$$

These relations hold also in the presence of any arbitrary spin-independent potential or interaction term in the Hamiltonian. The key observation is that the time derivatives of the spin components can be expressed as linear combinations of the spin current operators itself. This result crucially relies on the fact that the spin-orbit coupling is linear in the electron momentum. Moreover, in the presence of spin-orbit coupling disorder effects can generally be expected to make the electron spins relax to constant values. Thus, in a stationary state, the expectation values of the left-hand sides of Eqs. (42), (43) should vanish. Now it follows immediately that the expectation values of the spin current components j_z^x, j_z^y must also vanish provided $\alpha \neq \pm\beta$. In the case $\alpha = \pm\beta$, however, spin Hall transport is generally absent due to the additional conserved spin quantity which occurs here [60, 69, 41].

The central argument of the above proof is closely related to studies by Erlingsson, Schliemann and Loss [48], and by Dimitrova [66] on the relationship between spin currents and magnetic susceptibilities. For further developments in this direction see also Ref. [70].

Another interesting observation regarding spin Hall transport in n-doped quantum wells was made by Rashba [71] who considered Rashba spin-orbit coupling in the presence of a magnetic field coupling to the orbital degrees of freedom only, neglecting the Zeeman coupling to the electron spin. In this model, the spin Hall conductivity vanishes in the limit of vanishing magnetic field even in the absence of disorder, an effect closely related to the abovementioned vertex corrections [71]. Thus, the case of zero magnetic field (coupling to the orbital degrees of freedom only) is different from the limit of vanishing field. This result certainly relies on the assumption of an infinite system, since the orbital effects of a magnetic field should be small if the typical cyclotron radius is large compared to the system size.

The case of a magnetic field coupling both to the orbital and spin degree of freedom of electrons was investigated by Shen *et al* [72, 73]. Due to the coupling to the spin this situation is not covered by the above general argument. Shen *et al* find a resonant behavior of the spin Hall conductance as a function of the magnetic field when a degeneracy of Landau levels occurs at Fermi level. These studies, however, do not include disorder effects so far [72, 73].

Coming back the case zero external magnetic field, Adagideli and Bauer chose yet another approach to the problem by considering the electron acceleration in the presence of Rashba coupling [74],

$$\frac{d^2}{dt^2}\vec{r} = \frac{4m\alpha^2}{\hbar^4}\vec{j}_z \times \vec{e}_z - \frac{1}{m}\left(e\vec{E} + \nabla V\right) , \tag{44}$$

where $\vec{e}_z$ is the unit vector in the z-direction, $\vec{E}$ is the in-plane electric field, and V is the disorder potential. Since the acceleration should relax to zero in a disordered system, the spin Hall current vanishes if the effects of the electric field and the disorder potential cancel on average. As the authors show, this is indeed the case in the bulk of the system, but not necessarily at its edges. This observation gives rise to the notion of *spin Hall edges* [74].

The above general result on the absence of spin Hall transport in infinite systems with spin-orbit coupling being linear in the electron momentum is the outcome of an intense theoretical discussion in the last about two years. It was also confirmed by numerical studies carried out by Nomura *et al.* [75]. These authors performed a careful numerical evaluation of the Kubo formula for the spin Hall conductivity in the presence of Rashba coupling and delta-function type impurity potentials.

We stress again that the above general conclusion rules out spin Hall transport only in the limit of an infinite system. In fact, several numerical investigations on finite systems have appeared recently [76, 77, 78, 79, 80, 81, 82, 83, 84]. In these studies, the underlying semiconductor structure are described by tight-binding hopping models of finite-size coupled to semi-infinite leads. These hopping Hamiltonians also include local disorder potentials and discrete versions of the spin-orbit contributions in an n-doped quantum well. Transport quantities are then evaluated using the well-established Landauer-Büttiker approach combined with a Green's function treatment of the semi-infinite leads[85]. In summary, it still remains an interesting and unsettled question, whether intrinsic spin Hall transport can be experimentally observed in mesoscopic systems as studied in the above references.

3.3 Spin Hall transport of holes

We now analyze intrinsic spin Hall transport in p-doped III-V semiconductors and start with the case of a three-dimensional bulk system pioneered by Murakami, Nagaosa, and Zhang [12].

3.3.1 Three-dimensional bulk case

We consider valence-band heavy and light holes governed by the Hamiltonian (12) and the conventionally defined spin current operator as described before. In this case a nonzero spin conductivity occurs if the direction of the spin current, its spin polarization, and the driving electric field are mutually orthogonal. For definiteness, let us assume the spin polarization to point alone the z-axis with the electric field being in the xy-plane. From the Kubo formula (38), the zero-frequency spin Hall conductivity can be evaluated as [59, 52, 86]

$$\sigma_{xy}^{S,z}(0) = \frac{e}{4\pi^2}\frac{\gamma_1 + 2\gamma_2}{\gamma_2}\left(k_f^h - k_f^l - \int_{k_f^l}^{k_f^h} dk \frac{1}{1+\left(\frac{2}{\hbar/\tau}\frac{\hbar^2}{m}\gamma_2 k^2\right)^2}\right), \tag{45}$$

where we have again assumed an infinite system at zero temperature, and

$$k_f^{h/l} = \sqrt{\frac{2m}{\hbar^2}\varepsilon_f \frac{1}{\gamma_1 \mp 2\gamma_2}} \tag{46}$$

are the Fermi wave numbers for heavy and light holes, respectively. Similarly to the approach leading to Eq. (41) for conduction-band electrons, disorder effects are taken into account via an effective relaxation time τ. In the case here this approach is justified because vertex corrections can be shown to be absent for scattering potentials described by delta-functions[87]. Thus, for this type of disorder, the result (45) is exact in lowest order Born approximation [58]. The absence of vertex corrections in this case crucially relies on the fact that the underlying Hamiltonian (12) is not linear but of second order in the components of the momentum [87, 88]. The case of impurity potentials of longer spatial range was investigated in Ref. [89].

The remaining integral in Eq. (45) is elementary leading to a rather tedious expression which shall not be given here. However, we see that the value of the above integral is governed by the ratio of energy scale of the impurity scattering $\hbar/\tau$ and the “spin-orbit energy”

$$\varepsilon_{so} := \hbar^2\gamma_2 (k_f^0)^2/m = 2\varepsilon_f \gamma_2/\gamma_1, \tag{47}$$

since

$$k_F^0 = \sqrt{2m\varepsilon_f/\gamma_1\hbar^2} \tag{48}$$

is a typical wave number in the integration interval [59]. If $\hbar/\tau \gg \varepsilon_{so}$ the spin Hall conductivity vanishes as

$$\begin{aligned}\sigma_{xy}^{S,z}(0) &= \frac{e}{\pi^2} 4k_f^0 \left(\frac{\varepsilon_{so}}{\hbar/\tau}\right)^2 \frac{\gamma_2}{\gamma_1} \\ &+ \mathcal{O}\left(\left(\frac{\varepsilon_{so}}{\hbar/\tau}\right)^4, \left(\frac{\varepsilon_{so}}{\hbar/\tau}\right)^2 \left(\frac{\gamma_2}{\gamma_1}\right)^2\right)\end{aligned} \tag{49}$$

where we have also assumed that the ratio γ_2/γ_1 is small as it is usually the case [44]. In the opposite case $\hbar/\tau \ll \varepsilon_{so}$ one finds

$$\begin{aligned}\sigma_{xy}^{S,z}(0) &= \frac{e}{4\pi^2}\frac{\gamma_1 + 2\gamma_2}{\gamma_2}\left[k_f^h - k_f^l + \frac{\left(k_f^0\right)^4}{12}\left(\left(\frac{1}{k_f^h}\right)^3 - \left(\frac{1}{k_f^l}\right)^3\right)\left(\frac{\hbar/\tau}{\varepsilon_{so}}\right)^2\right] \\ &+ \mathcal{O}\left(\left(\frac{\hbar/\tau}{\varepsilon_{so}}\right)^4\right)\end{aligned} \tag{50}$$

Here the the contribution in leading order is the result obtained in Refs. [52, 86] for a disorder-free system (up to some definitorial prefactor [59]). The expression given originally in Ref. [12], however, differs somewhat from the above one due some approximation employed there [12]. Ref. [90] contains calculations of the spin Hall conductivity in band structure models more general than Eq. (12). Numerical results based on *ab initio* band structure calculations were presented in Ref. [91]. A further numerical study of spin Hall transport within the Hamiltonian (12) in the presence of disorder was performed in Ref. [92].

In summary, spin Hall transport in p-doped bulk III-V semicondcutors is robust against disorder of not too large strength, but naturally breaks down if impurity effects are overwhelming the spin-orbit coupling [59].

3.3.2 Heavy holes in a quantum well

Let us now turn to the case of spin Hall transport of heavy holes in p-doped quantum wells which was studied first by Schliemann and Loss [14]. An experimental observation of spin Hall effect in such a system was recently reported by Wunderlich *et al.* [15].

We consider the Hamiltonian (15) and a spin current

$$\vec{j}_z = \frac{\vec{p}}{m}\frac{3\hbar}{2}\sigma^z \tag{51}$$

for heavy holes with spin $\pm 3/2$ polarized along the growth direction of the well. Proceeding as above, one finds for the zero-frequency spin Hall conductivity [14]

$$\sigma_{xy}^{S,z}(0) = -\sigma_{yx}^{S,z}(0) = \frac{e}{\pi}\frac{9}{4}\frac{\hbar^2\tilde{\alpha}}{m}\int_{k_f^+}^{k_f^-} dk \frac{k^4}{(\hbar/\tau)^2 + (2\tilde{\alpha}k^3)^2}\,, \tag{52}$$

where τ is again the momentum relaxation time. Similar to the three-dimensional bulk case, vertex corrections to the spin-Hall conductivity turn out to be zero for delta-function shaped scatterers [88], justifying the above approach. This result is again due to the fact that the spin-orbit coupling is not linear but of higher order in the particle momentum.

The Fermi wave numbers $k_f^{\pm}$ entering Eq. (52) refer to the two dispersion branches (16) and can be expressed in terms of the particle density

$$n = \frac{1}{4\pi}\left(\left(k_f^+\right)^2 + \left(k_f^-\right)^2\right) \tag{53}$$

as[14]

$$\begin{aligned} k_f^{\pm} \quad &= \quad \mp\frac{1}{2}\frac{\hbar^2}{2m\tilde{\alpha}}\left(1-\sqrt{1-\left(\frac{2m\tilde{\alpha}}{\hbar^2}\right)^2 4\pi n}\right) \\ &\quad + \sqrt{-\frac{1}{2}\left(\frac{\hbar^2}{2m\tilde{\alpha}}\right)^2\left(1-\sqrt{1-\left(\frac{2m\tilde{\alpha}}{\hbar^2}\right)^2 4\pi n}\right) + 3\pi n}\,. \end{aligned} \tag{54}$$

Moreover, the *longitudinal* spin conductivities $\sigma_{xx}^{S,z}$, $\sigma_{yy}^{S,z}$ turn out out be identically zero.

The remaining integral in the above expression (52) is elementary; however, it leads to a rather cumbersome expression which shall again not be given here. Analogously to the previous case, the energy scale of impurity scattering $\hbar/\tau$ has to be compared with the "Rashba energy" $\tilde{\varepsilon}_R = \tilde{\alpha}(k_f^0)^3$, where $k_f^0 = \sqrt{2m\varepsilon_f/\hbar^2}$ is the Fermi wave number for vanishing spin-orbit coupling, which is a typical value for k in the integration in Eq. (52). If the impurity scattering dominates over the Rashba coupling, $\hbar/\tau \gg \tilde{\varepsilon}_R$, the spin Hall conductivity vanishes with the leading order correction given by

$$\sigma_{xy}^{S,z}(0) = \frac{e}{\pi}\frac{9}{20}\frac{\tilde{\alpha}}{m}\tau^2\left((k_f^-)^5 - (k_f^+)^5\right) + \mathcal{O}\left(\left(\frac{\tilde{\varepsilon}_R}{\hbar/\tau}\right)^4\right) , \tag{55}$$

where the Fermi wave numbers are given by Eq. (54). In the opposite case $\tilde{\varepsilon}_R \gg \hbar/\tau$, the leading contribution to the spin Hall conductivity reads

$$\sigma_{xy}^{S,z}(0) = \frac{e}{\pi}\frac{9}{16}\frac{\hbar^2}{m\tilde{\alpha}}\left(\frac{1}{k_f^+} - \frac{1}{k_f^-}\right) + \mathcal{O}\left(\left(\frac{\hbar/\tau}{\tilde{\varepsilon}_R}\right)^4\right) . \tag{56}$$

Note that this result for the spin Hall conductivity depends only on the length scale $m\tilde{\alpha}/\hbar^2$ of the Rashba coupling and the total hole density n, but not separately on quantities like the Fermi energy and the effective heavy hole mass. If $m\tilde{\alpha}/\hbar^2$ is small against the inverse square root of the total hole density (but still fulfilling $\tilde{\varepsilon}_R \gg \hbar/\tau$), the spin Hall conductivity approaches a value of

$$\sigma_{xy}^{S,z} = 9\frac{e}{8\pi} . \tag{57}$$

This is the case if $\hbar/\tau \ll \tilde{\varepsilon}_R \ll \varepsilon_f$. This above value should be compared with the universal value of $e/8\pi$ found originally in Ref.[13] for electrons in a fully clean asymmetric quantum well. In this sense the hole spin Hall conductivity is enhanced by a factor of 9 compared to the naive result for electrons, which is partially due to the larger angular momentum of the heavy holes.

Zarea and Ulloa studied the above system in the clean limit but with a perpendicular homogeneous magnetic field coupling to the orbital degrees of freedom of the holes but not to their spin [93], an investigation analogous to the one by Rashba on n-doped quantum wells already mentioned [71]. Again it is found that, for an infinite system, the case of zero magnetic field and the limit of vanishing magnetic field do not coincide [93]. The details of this effect, however, seem to be somewhat different from the observations made in Ref. [71] and need further study. In any case, influence of a magnetic field coupling to the orbital degrees of freedom only should only be appreciable if the field is strong enough to produce typical cyclotron radii being of order of the system size or smaller. Therefore, arbitrarily small fields cannot be expected to have an effect in real experiments. Another study of spin Hall transport in the presence of a perpendicular magnetic field was performed in Ref. [94] where a more general band structure Hamiltonian was used [88]

As seen above, spin Hall transport of heavy holes in a quantum well is robust against disorder effects, differently from the situation for electrons in n-doped wells. This effect is due to the different functional form of the effective spin-orbit coupling and was also confirmed numerically by Nomura *et al.* who performed a careful comparison between these two systems [75]. In a subsequent study, the edge- spin accumulation caused by the spin current was investigated numerically [95]. Further investigations of disorder effects can be found in Ref. [96] where an approach based on nonequilibrium Green's functions was used.

Moreover, several groups have studied numerically tight-binding models for two-dimensional hole systems coupled to semi-infinite leads. [81, 97, 98]. These investigations are analogous in spirit and technical approach to the numerical work on models for n-doped systems mentioned earlier [76, 77, 78, 79, 80, 82, 83, 84]. In particular, the numerical results on heavy-hole systems [81, 98] in the limit of low disorder confirm quantitatively the the enhanced spin conductivity (57) obtained analytically in Ref. [14]. Finally, many of the abovementioned mainly numerical studies on p-doped quantum wells were inspired by the experiments by Wunderlich *et al.* which we will discuss in section 4.

3.4 Spin Hall effect in other systems

Intrinsic mechanisms of spin Hall transport in n-doped bulk III-V semiconductors where investigated by Bernevig and Zhang [99, 100]. Here the leading contribution to spin-orbit coupling is given by the bulk Dresselhaus term (3) being of third order in the electron momentum. On the other hand, Engel, Halperin, and Rashba have studied extrinsic spin Hall effect in such systems [101]. We will discuss these issues in section 4 in the context of the experimental results by Kato *et al.* [16].

Spin Hall effect in graphene, i.e. single planes of graphite, was investigated by Kane and Mele [102]. Using a theoretical picture similar to the edge-state theory of the charge quantum Hall effect, these authors propose a spin current at the edges of a graphene sheet. Work following these theoretical predictions include Refs. [103, 104]. Finally, Shchelushkin and Brataas considered spin Hall transport in normal metals due to extrinsic mechanisms [105].

4 Detection of spin Hall transport: Experiments and Proposals

We now turn to experimental investigations of spin Hall effect in semiconductors. The studies already carried out and many of the experimental proposals found in the literature use the spin accumulation caused by the spin current for detecting spin Hall transport. Using a simple spin diffusion model, this spin accumulation is expected to decay towards the bulk of the sample on a length scale given by the spin diffusion length [106]. The latter quantity is determined by the semiconductor material, but possibly also by further details of the sample and the experimental setup [107]. Further recent theoretical studies on spin accumulation caused by the spin Hall effect include [95, 108, 109, 110, 111, 112].

Kato *et al.* have studied spin Hall transport in n-doped bulk epilayers of GaAs and InGaAs with a thickness of 2μm and 500nm, respectively [16]. The electron density in both samples was $n = 3 \cdot 10^{16}\text{cm}^{-3}$. The spin Hall effect was detected via the optical technique of Kerr rotation microscopy in the presence of an external magnetic field in a Hanle-type setup. The Kerr rotation signal as a function of the applied magnetic field could be fitted well by a Lorentzian where the spin lifetime τ_s entered as a fit parameter. By scanning over the sample, the authors obtained spatial profiles of the spin density along the direction perpendicular to the applied electric field. Fitting this data to solutions of the spin diffusion equation, the spin diffusion length was extracted. The values for this quantity lie between two and four micrometers and are, within the error bars, insensitive to the electric field varying between zero and $25\text{mV}\mu\text{m}^{-1}$. Combining this data with results for the spin lifetime, the authors inferred a spin conductivity of about $0.5\Omega^{-1}\text{m}^{-1}$, where the latter quantity was converted to units of charge transport by multiplying with a factor of $e/\hbar$.

The samples investigated by Kato *et al.* are n-doped and clearly in the bulk regime. Therefore, the intrinsic spin-orbit coupling to conduction-band electrons is dominated by the bulk Dresselhaus term (3) which was studied by Bernevig and Zhang as a possible intrinsic mechanism for spin Hall transport. [100]. The authors apply their results to the experiments by Kato *et al.* [16]. However, the agreement between theoretical predictions and the experimental findings is certainly not very convincing. Moreover, Kato *et al.* find only a very negligible dependence of the spin Hall transport on strain applied to the system. This observation also strongly disfavors an intrinsic mechanism. In fact, Engel, Halperin, and Rashba [101] have developed a theory of extrinsic spin Hall transport in GaAs based on impurity scattering and found reasonable agreement with the results by Kato *et al.* [16].

Further results on this type of experiments were reported by Sih *et al.* from the same research group for the case of n-doped GaAs quantum wells [113] (as opposed to the bulk epilayers discussed so far). The quantum well here has a width of $75\AA$ with a sheet density of $n = 1.9 \cdot 10^{12}\mathrm{cm}^{-2}$ and a mobility of $\mu = 940\mathrm{cm}^2/\mathrm{Vs}$. From the analysis of their Kerr rotation data Sih *et al.* conclude that the signatures of spin Hall transport seen in their experiment are also most likely due to an extrinsic mechanism.

We now turn to spin Hall transport of holes. Wunderlich*et al.* have investigated spin Hall effect in a p-doped triangular quantum well which is part of a p-n junction light emitting diode [15]. The quantum well has a sheet hole density of $n = 2 \cdot 10^{12}\mathrm{cm}^{-2}$ which is for the still low enough, for the given sample geometry, such that only the first heavy hole subband is occupied. Thus, the intrinsic spin-orbit coupling to these heavy holes can be expected to be governed by the Hamiltonian (15), leading to a spin Hall conductivity given by Eq. (52). The spin accumulation at the edges of the well is detected by the circular polarization of the light emitted from the diode. In a subsequent publication, the authors presented further details on this technique [114]. In summary, Wunderlich *at al.* conclude that the spin Hall transport seen in their results is most likely of intrinsic nature, i.e. due to Rashba spin-orbit coupling as described by the Eq. (15), although further work is needed to fully establish this conclusion [15].

Let us now mention some proposals for further possible experimental investigations of spin Hall transport. Hankiewicz *et al.* have considered an H-shape two-dimensional electron system using the Landauer-Büttiker formalism [76]. The sample consists of two, say, horizontal bars connected in their centers by a shorter vertical bar. A voltage applied along one of the long bars should drive a spin current through the shorter connecting bar into the other horizontal bar. There this spin current should manifest itself by a voltage along the bar which can be calculated by inverting the spin conductivity tensor. Numerical calculations support the feasibility of this experimental approach [76].

A scheme to determine the spin Hall conductance purely via measurements of charge transport was put forward by Erlingsson and Loss [115]. The authors study a planar four-terminal setup. Using conventional scattering formalism, they express the spin Hall conductance in terms of voltages, charge conductances, and charge current noise quantities. This allows, in principle, to infer the spin Hall conductance just from electric measurements, avoiding the necessity of any magnetic or optical element [115].

5 Conclusions and outlook

We have given an overview on recent theoretical and experimental developments concerning spin Hall transport in semiconductors. This phenomenon has certainly been over the last years one of the most intensively worked on topics in the solid-state community, and research efforts still continue to grow.

The theoretical situation regarding spin Hall effect in the two-dimensional electron gas with spin-orbit coupling linear in the momentum is by now well settled: There is no spin Hall effect in an infinite system if any kind of dissipation mechanism is present. This is, however, not a statement about finite mesoscopic systems, and further both theoretical and experimental work is to be expected here. On the other hand, p-doped quantum wells, i.e. the two-dimensional hole gas, appears to be a particularly attractive system.

A challenge for future theoretical work is certainly to extend many present results to the case of finite temperature, and, more importantly, include the Coulomb interaction between charge carriers. The last point is addressed by only very few papers so far [116, 117]. A further recent theoretical development are studies on the *zitterbewegung* of electron and hole wave packets due to spin-orbit interaction in semiconductors [118, 119]. These systems offer the possibility to experimentally detect the relativistic phenomenon of *zitterbewegung* which appears to be quite inaccessible in the case of free electrons [118, 119].

Future challenges for experiments on spin Hall transport include the clarification and discrimination of extrinsic and intrinsic mechanisms. Moreover, from both a theoretical and an experimental point of view, it is undoubtedly desirable to develop setups and techniques which allow to detect spin Hall transport independently from spin accumulation.

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