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University of Regensburg Working Papers in Business,
Economics and Management Information Systems

Safety first portfolio choice based on financial and sustainability returns

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01.03.2011

Nr. 452

JEL Classification: G11, C61, A13

Key Words: Finance, Socially Responsible Investing, Sustainability Value, Safety First Investor

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Abstract

This paper lays the mathematical foundations of the notion of an investment's sustainability return and investigates three different models of portfolio selection with probabilistic constraints for safety first investors caring about the financial and the sustainability consequences of their investments. The discussion of these chance-constrained programming problems for stochastic and deterministic sustainability returns includes theoretical results especially on the existence of a unique solution under certain conditions, an illustrating example, and a computational time analysis. Furthermore, we conclude that a simple convex combination of financial and sustainability returns – yielding a new univariate decision variable – is not sufficiently general.

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1 Introduction

In recent years, investors' behavior has fundamentally changed. Although the financial return is still important for the investment decision, social or environmental issues of investment opportunities are receiving more and more attention. Military conflicts like the Vietnam war, the apartheid system in South Africa or disasters like the nuclear one in Chernobyl in 1986 or the supertanker Exxon Valdez accident in 1989 are historical reasons for a boosting of this development. The oil platform disaster in the Gulf of Mexico in 2010 is a recent, adverse event that some investors would wish not to be involved in. There are a lot of initiatives that review the development of industrialization and its accompanied dangers. Some investors want to contribute to these issues by putting their money into sustainable investments. The amount of invested capital in sustainable funds has been rapidly increasing over the last few years. In Europe, 4,986 billion Euro

are involved in *socially responsible investments* (SRI) as of 31th of December 2009 (see Eurosif 2010). The growth from 2005 to 2009 in the Euro amount of SRI is 338%.
Our research contributes to the field of SRI in three different ways. Firstly, we provide a comprehensive foundation with axioms and definitions of sustainability in portfolio theory. Secondly, we establish and discuss three general models for generalized portfolio management with probabilistic constraints and conclude conditions for unique optimal solutions. We show that under certain conditions, one model is more restrictive than another and conclude that the aggregation of financial and sustainable returns by convex-combining them has certain disadvantages. Thirdly, we treat the case of deterministic sustainability returns and show that under this assumption two of the three models suggested are equivalent.
The remainder of the paper is organized as follows. Section 1 illustrates the idea of sustainability in investment decisions and gives a literature review. Section 2 covers sustainability ratings and quantifies the notion of an investment's sustainability return based on such ratings. We introduce three different models for downside risk portfolio choice in Section 3 and prove results on the solvability of the constructed models. Section 4 addresses simplifying models with deterministic sustainability return. Section 5 contains the conclusion.

1.1 Characteristics of sustainable interests

This section contains a brief description of an investment's sustainability. Principally, we build on the ideas of SRI. However, we do not regard the sustainability of an investment as an objective issue like the financial return. On the contrary, sustainability of an investment depends on preferences. Every single investor has her individual attitude towards sustainability. Consider nuclear energy as an example. Forty years ago an investor holding shares of a nuclear power plant may have been considered sustainable because she supported clean energy. Today, a contribution to nuclear technologies is a manifested exclusion criterion in the SRI literature and practice. Standardized sustainability definitions are not suited to all investors' preferences because there are still a lot of people who consider nuclear power as very sustainable and clean. In fact, every kind of investment can have non-financial impacts that are desirable for some investors. These impacts are summarized here under the term sustainability. Our approach is not to represent investors' preferences with an inclusion or exclusion criterion for every single asset as screening does, but rather with a scalable quantity capturing the quality of sustainability of an investment as an additional objective variable.

1.2 Literature Review

The established principles of SRI are ethics, governance and environmental interests. Renneboog et al. (2008) serves a comprehensive review of the developments in SRI. They survey research on the performance of socially responsible investments. Actual SRI approaches use screenings of assets in a first step and portfolio selection models with financial objective variables in a second one. Guerard (1997) studies the performance differences of portfolios with various screening criteria. Bello (2005) and Hamilton et al. (1993) compare the performance of sustainable and common funds. Both show that there is no significant under- or overperformance of sustainable funds. Galema et al. (2008) consider the impact of SRI on stock returns. They conclude that SRI has significant impact on the stock returns in particular portfolios that score positively on diversity, environment and product. Bollen (2007) suggests measuring the utility of a portfolio with multi-attributive utility functions. But he still shapes the SRI optimization problem in a binary manner using an indicator function for the fulfillment of SRI attitudes. In contrast to that, Hallerbach et al. (2004) give a practical approach for portfolio selection utilizing multi-attributive preference functions.

Benson & Humphrey (2008) find that SRI fund flow is less sensitive to returns than conventional one and that SRI investors are less concerned about returns than conventional ones. Dupré et al. (2004) discuss the extension of the Markowitz portfolio model about a quantity measuring sustainability.

While all of the references cited above shape the sustainability quantity as deterministic, Dorfleitner et al. (2010) introduce the idea of stochastic social returns and incorporate them into the classical portfolio selection. The notion of μ-σ efficiency is generalized to a concept comprising the expected values and standard deviations of the financial and the sustainability returns as well as their covariance. Taking on these ideas, we principally regard sustainability of an investment as a random number with a finite expected value and a variance. Even if this is not the standard view in SRI, we feel that it is the most realistic assumption since ex ante one never can say to what extent the good intentions the management of a company has will become reality.

Dupré et al. (2004) and Dorfleitner et al. (2010) use variances and covariances as a measure of risk. Several studies of portfolio theory suggest other risk measures than standard deviation. In recent years safety first approaches like the ones presented in Leibowitz & Henriksson (1989), Sortino & Forsey (1996), Haley & Whiteman (2008) – all of them based on the pioneering work of Telser (1955) and Roy (1952) – increasingly gain attention through the related concepts of shortfall constraints and downside risk in practice.

2 Modeling sustainability value and sustainability return

Instead of a two step portfolio selection with sustainability screening first and financial optimization second, we establish models with financial and sustainable real-valued objective variables. In contrast to Dorfleitner et al. (2010), who extend the classical Markowitz framework, we generalize the theory of safety first investors. The basic idea is that an investment is characterized by different quantities, namely the initial wealth V^0 in $t = 0$ and the final wealth V^t and a value of sustainability at the end of the investment period $[0, t]$. Thus, the investor receives three components at maturity t, namely V^0, the financial profit $V^t - V^0$ and a value of sustainability.

2.1 Sustainability ratings

The growing demand for sustainable investments brings up some associated developments. On the one hand, international committees pass standards for sustainability reporting like the AccountAbility 1000 Accountability Principles. On the other hand, sustainable rating agencies come up with creating rankings for sustainability of companies according to these reports and additional information. Most of these rankings are based on positive and negative indicators. Rating agencies score non-monetary values of these different positive indicators for each investment and condense indicators to factors. The scores of factors are aggregated to a number describing the grade of sustainability inherent in an investment. This number can be positive or negative and is often transformed to a relative quantity to ensure a comparison of companies of different size and branches. Therefore, it is appropriate to view this quantity as a "sustainability return". Negative indicators represent the set of exclusion criteria used for negative screenings. Some of the agencies only provide ordinal rankings others compute real returns, representing a cardinal order. The sustainability ratings are based on historical data and upcoming projects. Taking the future actions into account, it is natural to consider the sustainability of a company as a random number.

2.2 Measuring sustainability value and return

In our approach we start with determining the objective sustainability return of every single investment with respect to a set $\mathcal{F}$ of factors, taken from an existing sustainability rating. In a second step an individual investor aggregates these objective values according to her preferences.

We start with the **objective sustainability return** $OSR_j^{[s,t]}(F,\omega)$ of factor F and state ω in investment period $[s,t]$ for investment j, which is directly given by a sustainability rating. In an ex ante view the objective sustainability returns are clearly random variables. Knowing the invested initial wealth V_j^s in investment j, the objective sustainability return can be transformed into an objective sustainability value.

Definition 1 (Objective sustainability value)
The **objective sustainability value** $OSV_j^{[s,t]} : \mathcal{F} \times \Omega \times \mathbb{R} \to \mathbb{R}$ of a factor $F \in \mathcal{F}$ is a real random number with sample space Ω representing the objective non-monetary value that is generated by factor F of an investment j at maturity t. Objective sustainability value depends on initial wealth V_j^s and is defined as

$$OSV_j^{[s,t]}(F,\omega,V_j^s) := V_j^s \cdot OSR_j^{[s,t]}(F,\omega).$$

To shape the investor's preferences, let $\delta \in \mathbb{R}$ be a real number and $F \in \mathcal{F}$ a factor of sustainable interest like environment. Then $\delta(F,\pi)$ denotes the strength of sustainable impact of factor F on investor π. When factor F has a positive impact on the investment decision of investor π, then $\delta(F,\pi) > 0$ holds. If an investor is indifferent with respect to factor F, we define $\delta(F,\pi) = 0$. An investor, who rejects the common interpretation of objective sustainability of factor F, has $\delta(F,\pi) < 0$. Using the notation of Definition 1 we can define sustainability value of an investment j for investor π.

Definition 2 (Sustainability value)
The **sustainability value** $SV_j^{[s,t]} : \Omega \times \Pi \times \mathbb{R} \to \mathbb{R}$ of an investment j is a real random number with sample space Ω representing the non-monetary value of an investment the investor π receives at maturity t. Sustainability value

$$SV_j^{[s,t]}(\omega,\pi,V_j^s) := \sum_{F \in \mathcal{F}} \delta(F,\pi) OSV_j^{[s,t]}(F,\omega,V_j^s)$$

depends on the state ω, the preference π and initial wealth V^s.

Analogously to the objective sustainability return, a preference-dependent sustainability return exists.

Definition 3 (Sustainability return)
The **sustainability return** $SR_j^{[s,t]} : \Omega \times \Pi \to \mathbb{R}$ of investment j to investor π in period $[s,t]$ with sample space Ω and preference space Π is defined by

$$SR_j^{[s,t]}(\omega,\pi) := \frac{SV_j^{[s,t]}(\omega,\pi,V_j^s)}{V_j^s}.$$

An implication of the definitions from above is that the sustainability return can be expressed as a weighted sum over all factors F of the objective sustainability returns with weights $\delta(F,\pi)$. Furthermore, obviously sustainability returns fulfill the property of portfolio additivity.

Lemma 1 (Portfolio Additivity)
Let $w_1,\ldots,w_N$ the weights of N assets with sustainability returns $SR_1^{[s,t]},\ldots,SR_N^{[s,t]}$. If the sustainability value of each factor is additive over different assets held then we have

$$SR_P^{[s,t]} = \sum_{j=1}^{N} w_j SR_j^{[s,t]}.$$

Proof. Lemma 1 follows straight from Definition 2 and Definition 3. □

3 Downside risk portfolio choice based on stochastic financial and sustainability returns

In the following section we present and discuss three models for generalized safety first investors. All considerations below are based on one single period; hence, we drop time and interval indices as well as parameters for the state and the investor's preferences. Let $N \geq 2$ be the number of all assets the portfolio is supposed to be built with. Note that not all of these assets need to have a risky financial return. The weight of asset i is w_i and a well-defined portfolio satisfies $\sum_{i=1}^{N} w_i = 1$. In general, we permit short sales, which are characterized by negative w_i. There has to be at least one risky asset with $w_i \neq 0$ to prevent computations from singularities[1]. Let $R : \Omega \to \mathbb{R}^N$ and $SR : \Omega \to \mathbb{R}^N$ denote the random vectors depicting the financial and sustainability

[1]In investment practice, there may be more than one riskless asset if, due to certain constraints on these assets, it is not possible to perform arbitrage transactions. However, in classical portfolio choice only one riskless asset is used. Contrary to that, it might, in our context, be sensible to invest in different riskless assets with different interest rates and different sustainability returns.

returns of all available assets. The models presented below utilize returns instead of absolute quantities. However, this comes without loss of generality since it still might be the case that the risk aversion depends on initial wealth, which will be represented by the restrictions of the optimization problems. More precisely the thresholds for R and SR introduced below can be considered generally dependent on the initial wealth V^0. The vector SR of sustainability returns is calculated for a fixed but arbitrary investor's preference. The covariance matrices of R and SR are denoted by Σ_R and Σ_{SR}. If asset i is riskless, the ith row and the ith column of both Σ_R and Σ_{SR} are zero vectors.

3.1 General model

This subsection introduces the general structure of a portfolio problem of a generalized safety first investor with financial and sustainable interests. A generalized safety first investor is defined as follows. Let A_i denote a random $J \times 2N$ matrix whose elements are multiples of R and SR. Let be $\mathbb{1}_2 = (1,1)' \in \mathbb{R}^2$, then the Kronecker product $\mathbb{1}_2 \otimes w$ denotes a column vector which is w strung together two times. Let

$$K_i(w) := \mathbf{P}\left(\bigcap_{j=1}^{J_i}\left(A_i^{(j)}(\mathbb{1}_2 \otimes w) \geq c_i^{(j)}\right)\right), \quad i = 1, \ldots, I$$

be a set of probabilities ($K(w) : \mathbb{R}^{2N} \to \mathbb{R}^I$) that depends on portfolio weights w, R and SR with vectors of thresholds $c_i \in \mathbb{R}^{J_i}$ and with $I \in \mathbb{N}$. Some appropriate specifications of $A_i^{(j)}$ and $c_i^{(j)}$ are shown in the following.

Definition 4 (Generalized safety first investor)
An investor with financial and sustainable investment interests is called **generalized safety first investor** (GSFI), if she accepts a portfolio w which satisfies every probability condition $K_i(w) \geq 1 - \alpha_i$ for $i = 1, \ldots, I$ with a default probability $\alpha_i \in \mathbb{R}$.

In general, a safety first investor maximizes a preference-dependent functional $\Psi(w'\mathbf{E}[R], w'\mathbf{E}[SR])$ with $\partial\Psi/\partial w'\mathbf{E}[R] \geq 0$ and $\partial\Psi/\partial w'\mathbf{E}[SR] \geq 0$ at the constraints that all feasible portfolios do not fail with higher probabilities than $\alpha \in \mathbb{R}^I$. A portfolio fails, if its return is below a threshold $c \in \mathbb{R}$. The generalized model for this portfolio problem is

$$\max_{w \in \mathbb{R}^N} \quad \Psi(w'\mathbf{E}[R], w'\mathbf{E}[SR]) \tag{1a}$$

$$\text{s. t.} \quad K_i(w) \geq 1 - \alpha_i \qquad i = 1, \ldots, I \tag{1b}$$

$$w \in \mathcal{W} \tag{1c}$$

with a convex set $\mathcal{W} \in \mathbb{R}^N$ with $\mathcal{W} := \{w | l \leq w \leq u, Bw \propto b, \sum_{i=1}^N w_i - 1 = 0\}$, where $\propto$ stands for $=$, $\leq$ or $\geq$, $B \in \mathbb{R}^{k \times N}$ and $b \in \mathbb{R}^k$. The set $\mathcal{W}$ in (1c) contains all linear constraints on w like lower and upper bounds ($l, u \in \mathbb{R}^N$) for portfolio weights and other budget constraints, where $k \in \mathbb{N}$ is the number of all constraints. The risk preferences of every single investor are conveyed by the thresholds c_i and the default probabilities α_i. In general, problem (1) is a chance-constrained programming problem with random coefficient matrices A_i with not necessarily independent rows and thus a non-convex programming problem. Kall & Mayer (2005, pages 142-143) display the conditions under which problem (1) is an easy-to-solve convex programming problem. The intersection of two convex sets is convex. Thus, it is accurate to consider the probability constraints (1b) in this context, only. It is also a well-known fact that every local optimal solution of a convex optimization problem is a global optimal solution. Below we present specifications of the general model representing three different approaches to deal with the tradeoff between R and SR in a safety first context. We use a convex combination of expected financial and sustainability returns as the objective function and express the preference between both by the scale parameter $\gamma \in (0, 1)$ with $\gamma = 0$ representing the case where only the financial return enters the objective function. We can derive the marginal rate of substitution between financial and sustainable interests following

$$\frac{\frac{\partial \Psi(w'\mathbf{E}[R], w'\mathbf{E}[SR])}{\partial w'\mathbf{E}[R]}}{\frac{\partial \Psi(w'\mathbf{E}[R], w'\mathbf{E}[SR])}{\partial w'\mathbf{E}[SR]}} = \frac{1-\gamma}{\gamma}.$$

This quotient indicates how many units increase in the sustainability return one demands for a loss of one unit of the financial return while the objective function does not change. Note that the sustainability return in general has one degree of freedom since the individual factors $\delta(\omega, F)$ could be multiplied by an arbitrary constant. However, when determining γ according to the preferences, one has to take the general level of SR into consideration and loses the degree of freedom again. Therefore γ can be interpreted most easily if R and SR have about the same range. A value of $\gamma = 1/2$ would then imply that both returns are equally weighted in the objective function. If one then chooses to replace SR by, for instance, $10 \cdot SR$, then γ would have to change from $1/2$ to $1/11$ in order to express the same preferences as before.

3.2 Joint distribution model

The first interpretation of a GSFI utilizes the joint probability distribution of financial and sustainability portfolio returns. A joint distribution type GSFI is defined as follows.

Definition 5 (Joint distribution type GSFI)
An investor with preference functional Ψ is called **joint distribution type GSFI**, if there are thresholds c_R and c_{SR} for financial and sustainability portfolio returns which she allows to underperform with joint probability α.

In this case the parameters of the general problem (1) are $i = 1$, $J = 2$,

$$A = \begin{pmatrix} R_1 & \cdots & R_N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & SR_1 & \cdots & SR_N \end{pmatrix}, c = \begin{pmatrix} c_R \\ c_{SR} \end{pmatrix}$$

and $\alpha \in \mathbb{R}$. For a fixed portfolio P, we can determine the joint probability density function and the joint cumulative distribution function (cdf). The preference functional Ψ is the convex combination of expected returns, where the relation between financial and sustainable quantities is reflected by γ. Hence, our first model for generalized portfolio choice is

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR] \right] \tag{2a}$$

$$\text{s. t.} \quad \mathbf{P}(w'R \geq c_R, w'SR \geq c_{SR}) \geq 1 - \alpha, \tag{2b}$$

where c_R and c_{SR} are the thresholds and the financial and sustainability portfolio returns $w'R$ and $w'SR$ are random numbers. The following theorem treats the uniqueness of the solution if financial and sustainability returns are assumed to be multivariate normally distributed.

Theorem 1 *The deterministic equivalent of chance-constrained programming problem (2) has a unique solution, if R and SR are multivariate normally distributed and the feasible set of the deterministic equivalent is not empty.*

Proof. Solving problem (2) we issue a deterministic equivalent problem. In the spirit of Watanabe & Ellis (1994) we consider a problem with multivariate normally distributed coefficients of matrix A. Therefore, as the deterministic equivalent of problem (2) we receive

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR] \right] \tag{3a}$$

$$\text{s. t.} \qquad \int_{l_{SR}}^{\infty}\int_{l_R}^{\infty} \phi_Z(z)dz_1 dz_2 \geq 1-\alpha, \tag{3b}$$

where

$$\begin{aligned}
l_R(w) &= \frac{c_R - \mu_R' w}{\sqrt{w'\Sigma_R w}} \quad \text{and} \quad l_{SR}(w) = \frac{c_{SR} - \mu_{SR}' w}{\sqrt{w'\Sigma_{SR} w}},\\
z_1(w) &= \frac{r - \mu_R' w}{\sqrt{w'\Sigma_R w}} \quad \text{and} \quad z_2(w) = \frac{s - \mu_{SR}' w}{\sqrt{w'\Sigma_{SR} w}},\\
z &= (z_1, z_2)', \quad \varrho(w) = \begin{pmatrix} 1 & \rho(w'R, w'SR) \\ \rho(w'R, w'SR) & 1 \end{pmatrix} \quad \text{and}\\
\phi_Z(z) &= \frac{1}{2\pi\sqrt{\det\left(\varrho(w)\right)}} \exp\left\{-\frac{1}{2} z' \varrho(w)^{-1} z\right\}.
\end{aligned}$$

The correlation coefficient of financial and sustainability portfolio returns is denoted by $\rho(w'R, w'SR)$ and the probability density function of the bivariate normal distribution with mean $\mu_Z = (0,0)'$ and covariance matrix $\varrho(w)$ by ϕ_Z. Constraint (3b) yields to a convex set because $K(w)$ is quasi-concave due to the attitudes of the bivariate normal distribution. The objective function (3a) is not constant, so it has a local optimum on the bounded feasible set, if this it not empty. Hence this solution is also a global optimal solution. □

3.3 Convolution model

The second approach is probably the most obvious one: We generate a convex combination of financial and sustainability portfolio returns.

Definition 6 (Convolution type GSFI)
An investor with preference functional Ψ is called **convolution type GSFI**, if there is a threshold c for the convex combination of financial and sustainability portfolio returns that she allows to underperform with probability α.

In this context, the problem is to maximize the combination of expected financial and sustainability portfolio returns under the constraint that their convex combination underperforms a threshold c with probability less than α. The parameters are $i = 1$, $J = 1$, $A = ((1-\gamma)R_1, \ldots, (1-\gamma)R_N, \gamma SR_1, \ldots, \gamma SR_N)$, $c \in \mathbb{R}$ and $\alpha \in \mathbb{R}$. The formal notation is given by

$$\max_{w \in \mathcal{W}} \qquad w'\left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{4a}$$

$$\text{s. t.} \qquad \mathbf{P}((1-\gamma)w'R + \gamma w'SR \geq c) \geq 1-\alpha. \tag{4b}$$

If $\gamma = 0$ holds, an investor is not interested in sustainability; we get the standard safety first portfolio optimization problem in accordance with Telser (1955) without sustainable interests. On the contrary, an investor who is only interested in sustainability has $\gamma = 1$. For each investor with $0 < \gamma < 1$ and a fixed threshold c, the constraint (4b) contains the tradeoff between financial and sustainable quantities. Let $R_P^\gamma := (1-\gamma)w'R + \gamma w'SR$ be the convex combination of financial and sustainability portfolio returns with $\gamma \in (0,1)$.

Theorem 2 *For multivariate normally distributed financial and sustainability portfolio returns the deterministic equivalent of problem* (4) *has a unique solution, if the feasible set is not empty.*

Proof. A deterministic equivalent of problem (4) can be derived because due to the normally distributed coefficients of matrix A the sum of $2N$ normally distributed random numbers R_P^γ is normally distributed with parameters

$$\mu_{R_P^\gamma} = w'((1-\gamma)\mu_R + \gamma\mu_{SR})$$
$$\sigma^2_{R_P^\gamma} = (D(\mathbb{1}_2 \otimes w))'\Sigma D(\mathbb{1}_2 \otimes w),$$

where

$$\Sigma = \begin{pmatrix} \Sigma_R & \mathrm{Cov}(R,SR) \\ \mathrm{Cov}(R,SR)' & \Sigma_{SR} \end{pmatrix},$$

and $D \in \mathbb{R}^{2N \times 2N}$ is a diagonal matrix with $D = \mathrm{diag}(1-\gamma, \ldots, 1-\gamma, \gamma, \ldots, \gamma)$, while $\mathrm{Cov}(R,SR) \in \mathbb{R}^{N \times N}$ is the estimated (not necessarily symmetric) covariance matrix between R and SR. For $\alpha \in (0, 0.5)$ we get as the deterministic equivalent of problem (4) the convex programming problem

$$\max_{w \in \mathcal{W}} \qquad w'\left[(1-\gamma)\mathbf{E}[R] + \gamma\mathbf{E}[SR]\right] \tag{5a}$$

$$\text{s. t.} \qquad \Phi^{-1}(\alpha)\sigma_{R_P^\gamma} + \mu_{R_P^\gamma} \geq c, \tag{5b}$$

where $\Phi(\cdot)$ denotes the univariate standard normal cdf. The argumentation for a unique global optimal solution is the same as in proof of Theorem 1. $\square$

3.4 Marginal distributions model

The third model we describe in this paper differs from problems (2) and (4) in the constraints. Again, the objective function is the maximization of a convex combination of expected financial und sustainability portfolio returns. The risk constraints build on the marginal distributions of financial and sustainability portfolio returns.

Definition 7 (Marginal type GSFI)
An investor with preference functional Ψ is called **marginal type GSFI**, if there is a threshold c_R for the portfolio return which she allows to underperform with probability α_R and a threshold c_{SR} for the portfolio sustainability return which she allows to underperform with probability α_{SR}.
The parameters are $i = 2$, $J_1 = J_2 = 1$, $A_1 = \begin{pmatrix} R_1 & \cdots & R_N & 0 & \cdots & 0 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & \cdots & 0 & SR_1 & \cdots & SR_N \end{pmatrix}$, $c = \begin{pmatrix} c_R \\ c_{SR} \end{pmatrix}$ and $\alpha = \begin{pmatrix} \alpha_R \\ \alpha_{SR} \end{pmatrix}$. The formal notation of the third model with fixed α_R, α_{SR}, c_R and c_{SR} is

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{6a}$$

$$\text{s. t.} \quad \mathbf{P}(w'R \geq c_R) \geq 1 - \alpha_R \tag{6b}$$

$$\mathbf{P}(w'SR \geq c_{SR}) \geq 1 - \alpha_{SR}. \tag{6c}$$

Model (6) does not use any correlation between R and SR. The lower bounds c_R and c_{SR} are fixed. Model (6) excludes all portfolios that underperform in one or both dimensions with a higher probability than α_R and α_{SR}.

Theorem 3 *If the marginal distributions of financial and sustainability portfolio returns are marginal normally distributed and the feasible set of the deterministic equivalent is not empty, a unique optimal solution exists.*

Proof. The deterministic equivalent of problem (6) under normality assumptions is the convex programming problem (7)

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{7a}$$

$$\text{s. t.} \quad \Phi^{-1}(\alpha_R)\sqrt{w'\Sigma_R w} + w'\mu_R \geq c_R \tag{7b}$$

$$\Phi^{-1}(\alpha_{SR})\sqrt{w'\Sigma_{SR} w} + w'\mu_{SR} \geq c_{SR}. \tag{7c}$$

with a local, thus global optimal solution, if the feasible set is not empty. □

3.5 Discussion on the GSFI models

In this section we discuss the differences between and similarities of the three models and prove results showing that the marginal distributions model is more restrictive than the joint distribution model for certain conditions.

3.5.1 Model characteristics

A similarity of all models clearly is the identical objective function; the maximization of the convex combination of the expected financial and sustainability portfolio returns. In general, this convex combination could be replaced by a bivariate preference functional with the arguments expected final wealth and expected final sustainability value.
Therefore, the differences of the models lie in the probability constraints. In constrast to the convolution model, the feasible sets of the joint distribution model and the marginal distributions model do not depend on investor's preference displayed by γ. This implies a more flexible shaping. An investor can choose the preference between financial and sustainable quantities in the objective function and the thresholds for both quantities in the probability constraints independently. For example, it is possible in the joint and the marginal distributions model to describe an investor who only maximizes expected financial portfolio return under the constraint, that sustainability portfolio return exceeds a threshold with a certain probability. Figure 1 shows that

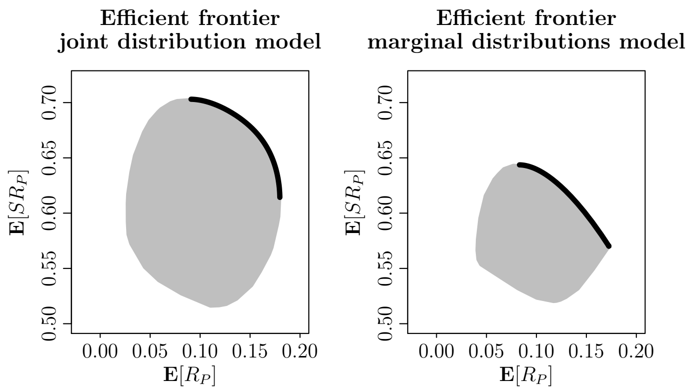

Figure 1: Efficient frontier of joint distribution model and marginal distributions model.

the joint and the marginal distributions model have both an efficient frontier in the $\mathbf{E}[w'R]$-$\mathbf{E}[w'SR]$-space for given parameters c_R, c_{SR}, α or α_R and α_{SR}, respectively. Hence, the optimal investment decision only depends on the choice of γ. Figure 1 outlines the convex hull of the feasible sets for an investor with $c_R = -0.1$, $c_{SR} = 0.5$,

$\alpha = 0.1$ respectively $\alpha_R = 0.051$ and $\alpha_{SR} = 0.051$. We randomly chose three real assets[2] for this illustrating example. We receive the financial data from Thomson Reuters Datastream and the sustainability rating data from the rating agency Inrate. Both, financial and sustainability portfolio returns are supposed to be multivariate normally distributed.

The marginal distributions model is very simple to implement, because the user does not have to estimate correlations between financial and sustainable quantities and no computation of any joint probability functions are necessary. Thereby, the feasible set is the intersection of the sets corresponding to the probability constraints. However, this is a very restrictive way to handle the downside risk; but it is the only model, which guarantees to underperform each single threshold by less than the given default probability.

The probability constraint in the convolution model imitates the convex combination of the objective function. Thus, this model does not have something like an efficient frontier because the corresponding feasible set depends on γ. Basically, every asset has a condensed return, which depends on γ as the performance quantity in the convolution model.

The computational effort of portfolios with 3, 50, 100 and 630 assets is displayed in Table 3.5.1 in relation to the minimal computational time. Obviously, the joint

Number of assets	3	50	100	630
Joint distribution model	7.3	303.4	820.6	31417.0
Convolution model	1.0	25.4	69.0	11913.0
Marginal distributions model	1.0	27.7	73.1	9615.4

Table 1: Computational time analysis. The minimum time is normalized to 1.0. The absolute time for computing the optimal solution with the interior point algorithm for $\gamma = 0.2$ of the convolution model with 3 assets are 0.0658 seconds on a Pentium(R) Dual-Core E5300 2.60 GHz, 3.21 GB RAM. The estimation of the input parameters is not considered.

distribution model is the one with the highest computational effort. But the optimal solution of the portfolio problem with 630 risky assets from our sample, that are

[2] The vector of financial returns is $\mu_R = (0.0265, 0.0688, -0.0400)'$, the vector of sustainability returns is $\mu_{SR} = (0.6360, 0.4060, 0.6860)'$, the covariance matrix of financial returns is $\begin{pmatrix} 0.0108 & 0.0036 & 0.0151 \\ 0.0036 & 0.0090 & 0.0075 \\ 0.0151 & 0.0075 & 0.0257 \end{pmatrix}$ and of sustainability returns is $\begin{pmatrix} 0.0005 & 0.0012 & 0.0010 \\ 0.0012 & 0.0030 & 0.0024 \\ 0.0010 & 0.0024 & 0.0020 \end{pmatrix}$. The portfolio weights w_i are restricted by $-10 \leq w_i \leq 10$ for all i. All quantities are estimated from historical data and display the parameters for a one year investment horizon.

assumed to be multivariate normally distributed, lasts 34.5 minutes. In contrast, the convolution model and marginal distributions model have really low computational costs because there are no multivariate probabilities to handle.

Figure 2 shows the optimal objective function value and the corresponding expected financial and sustainability portfolio returns of the joint distribution model for varying threshold c_{SR} by fixed default probability α and different γ. An increasing threshold

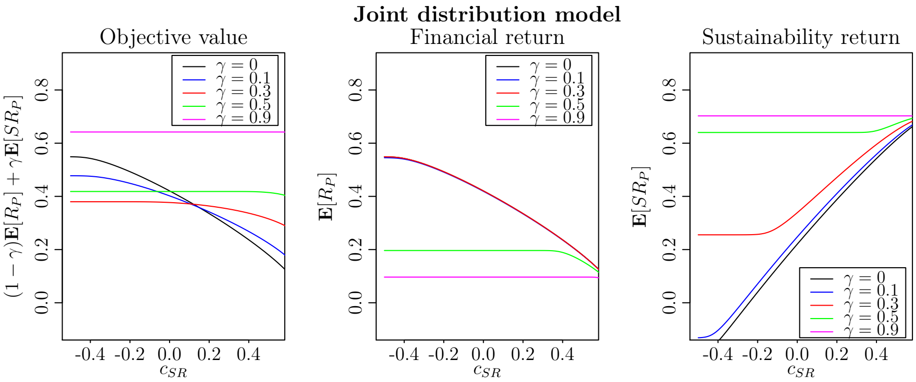

Figure 2: Variations of optimal objective function value, expected financial and expected sustainability portfolio returns of the joint distribution model dependent on c_{SR} and γ. Calculations are based on the parameter specification used throughout this section.

c_{SR} means that the portfolio must satisfy a higher sustainability level. The objective function value decreases with increasing c_{SR} because riskier portfolios with big amounts of short sales are excluded step by step. Portfolio choice with high γ ($\gamma = 0.3, 0.5, 0.9$) – which means high sustainable interests – have segments with constant objective function values, expected financial and sustainability returns, because they mainly maximize the sustainability return and thereby fulfill an even higher threshold c_{SR}. However, the impact of a variation of c_{SR} is higher for investors with small γ – investors with mostly financial aims in the objective function – since these investors mainly maximize the expected financial portfolio return and to a smaller extent the sustainability return in the objective function. This result is consistent with Hong & Kacperczyk (2009) and Fabozzi et al. (2008), who find that sin stock portfolios yield higher financial returns than SRI portfolios. The line for an investor with $\gamma = 0$ shows how strong the expected financial return decreases if the investor increases her sustainable threshold c_{SR}. Hence, the loss in the expected financial return can be viewed as the financial costs of a sustainable investment.

Additionally, when comparing the three models with respect to the optimal portfolios' probabilities of exceeding the thresholds, one can observe that the convolution model may generate outcomes that violate the maximum probabilities very clearly. This is a model consistent phenomenon, implied by the effect that both return dimensions compensate each other in this model. However, for the investor this property might be unfavourable.

3.5.2 Joint versus marginal distributions model

In this section we explore the connection between the feasible sets of the joint distribution model and the marginal distributions model under appreciable assumptions. Both models differ from each other solely in the probability constraints. We prove a next result for positive quadrant dependence.

Theorem 4 *Let the financial and sustainability portfolio returns of the marginal distributions model's optimal solution (6) be distributed according to a bivariate distribution with positive quadrant dependence. Then the marginal distributions model is more restrictive than the joint distribution model if* $(1-\alpha_R)\cdot(1-\alpha_{SR})\geq 1-\alpha$.

Proof. Let $\hat{w}$ be the optimal solution of problem (7), i.e. $\hat{w}$ satisfies the constraints (6b) and (6c). Obviously, $\hat{w}\in\mathcal{W}$ is satisfied for any feasible w in both models. Confirming that problem (6) is more restrictive than problem (2), we have to show that an optimal solution of problem (6) is also feasible for problem (2). Hence, if $\hat{w}$ is the optimal solution of problem (7), we obtain

$$\mathbf{P}(\hat{w}'R\geq c_R)\cdot\mathbf{P}(\hat{w}'SR\geq c_{SR})\geq(1-\alpha_R)\cdot(1-\alpha_{SR}). \tag{8}$$

A joint probability distribution has the attitude of positive quadrant dependence if $\mathbf{P}(X\geq x, Y\geq y)\geq\mathbf{P}(X\geq x)\cdot\mathbf{P}(Y\geq y)$ is satisfied (see Lehmann 1966). Therefore, inequality

$$\mathbf{P}(\hat{w}'R\geq c_R, \hat{w}'SR\geq c_{SR})\geq\mathbf{P}(\hat{w}'R\geq c_R)\cdot\mathbf{P}(\hat{w}'SR\geq c_{SR}) \tag{9}$$

holds and due to (8) and the probability condition $1-\alpha\leq(1-\alpha_R)(1-\alpha_{SR})$, constraint (2b) is satisfied and the result is proven. □

Corollary 1 *Let the financial and sustainability portfolio returns of the marginal distributions model's optimal solution (6) be bivariate normally distributed with correlation* $\rho(\hat{w}'R,\hat{w}'SR)\geq 0$. *Then the marginal distributions model is more restrictive than the joint distribution model if* $(1-\alpha_R)\cdot(1-\alpha_{SR})\geq 1-\alpha$.

Proof. Lehmann (1966) shows that every bivariate normal distribution with positive correlation coefficient is positively quadrant dependent. Then the proof follows straight from Theorem 4. □

The probability constraint of the joint distribution model is less restrictive under the upper conditions compared to the probability constraints of the marginal distributions model. Figure 3 illustrates the relation between the objective values of the three different models for various preferences γ.

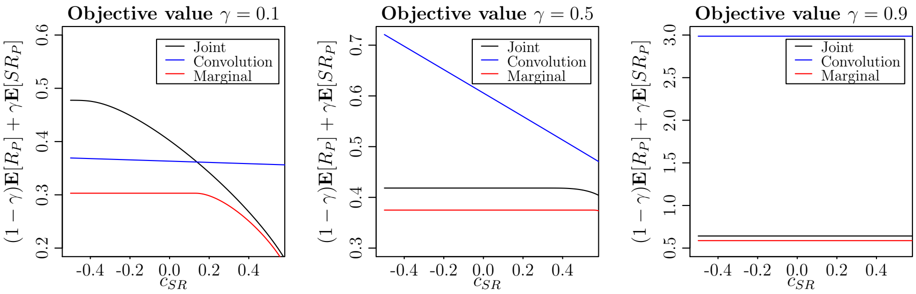

Figure 3: Optimal objective function values of all three models dependent on $c_{SR} \in [-0.5, 0.5]$ and $\gamma \in \{0.1, 0.5, 0.9\}$. Calculations are based on the parameter specification used throughout this section.

4 Shortfall constraint in portfolio choice with deterministic sustainability return

Since the general case of stochastic sustainability returns is not common in established SRI models, we will next outline a special framework for sustainable safety first portfolio choice under the assumption of deterministic sustainability returns. In recent investment and rating practice a deterministic sustainability return is used. Let $F_P(\cdot)$ be the appropriate cdf of the financial portfolio return $w'R$. We consider a setup with a fixed investment period and a confidence level $1-\alpha \in (0.5, 1)$.
There are two different approaches derived from the general models in Section 3. The first one is the counterpart of the convolution model with a deterministic sustainability return, i.e.

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma SR\right] \tag{10a}$$

$$\text{s. t.} \quad 0 \geq c - \gamma w' SR - (1-\gamma) F_P^{-1}(\alpha). \tag{10b}$$

We interpret this model as a sustainable shifted quantile model, which means that the α-quantile of the financial portfolio return distribution is shifted by the sustainability return of the portfolio.
The second model with deterministic sustainability return is deduced from the marginal distributions model and can be regarded as an improved screening portfolio selection. The deterministic model is given by

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma SR\right] \tag{11a}$$

$$\text{s. t.} \quad 0 \geq c_R - F_P^{-1}(\alpha_R) \tag{11b}$$

$$0 \geq c_{SR} - w'SR. \tag{11c}$$

Probability constraint (6c) from the general marginal distributions model is merged to a deterministic linear constraint (11c) and thereby unchanged in the deterministic equivalent. Therefore, a portfolio is feasible for this model if its sustainability return is higher than a given threshold and it satisfies the probability constraint for financial return.
It is easy to show that the joint distributions model is equivalent to (11) in this setting because without variability in the sustainability return the joint distribution degenerates essentially to the marginal distribution of the financial return with a deterministic value in the sustainability return which lies above the required level c_{SR} with probability 0 or 1.

5 Conclusion

We present a mathematical framework for modeling the sustainability return as a computable quantity. Based on ratings from an outside rating agency, one can derive objective sustainability returns of every sustainability dimension of an investment asset. These objective sustainability returns are then aggregated linearly according to the investor's preferences.
Based on these considerations we present a general model for safety first portfolio selection with stochastic financial and sustainability returns and introduce the notion of a generalized safety first investor (GSFI). Whereas the objective function is fixed as a convex combination of the expected financial and sustainability return, the conditions determining the feasible set may vary. We specify this general model in three different forms, namely the convolution type GSFI, the marginal distributions type GSFI and the

joint distribution type GSFI, and show that each model has an optimal solution under the assumption of normally distributed returns. The implementation of the models follows the constructive proofs. We find an efficient frontier in the $\mathbf{E}[w'R]$-$\mathbf{E}[w'SR]$-space for the marginal and the joint distribution type GSFI, representing a set of portfolios on the border of the feasible set. This efficient frontier is independent of the preference in the objective function. Linked with that, both models reveal an interesting property: They allow maximizing only the financial return (and hence avoiding fixing a marginal rate of substitution between both dimensions), but still considering the probability of missing a certain sustainability threshold in the contstraints. However, there is no such efficient frontier for the convolution type GSFI.

The joint distribution model uses most information about the return distributions but is the model with the highest computational effort. The convolution type GSFI model aggregates the financial and sustainability return distributions and generates results which depend on the scale of both returns.

The marginal distributions model has very low computational costs and ensures that the optimal portfolio exceeds the marginal thresholds with the required default probability, but it does not use any correlation. For certain conditions we can prove that the marginal distributions model is more restrictive than the joint distribution model. Furthermore, we can transfer the models considered into the more practice-oriented setup of deterministic sustainability returns and show that the joint and the marginal distributions model are equivalent without any distribution assumptions.

Summarizing, we can give the following recommendations. The convolution type GSFI model is the most straight forward one and hence appealing to some investors. However, especially if the shortfall in one of the both dimension is really important to the investor, this model may not be favourable to many investors because of its partially implausible outcomes in this regard.

The marginal and the joint distribution type models are generally recommendable, the first for less information and computation time available, the latter for good informational and computational resources. However, it can happen that a substantial amount of objective value is lost by implementing the marginal model. The deterministic setup is also easily implementable, but naturally does not account for risk in the sustainability dimension. Hence it may lead to unfavourable outcomes for a safety first investor.

The models presented are suggestions to the real world investors searching for an investment that is 'good' in the financial and the sustainability dimension. Our results only concern properties of these models. It is up to the investors to find those variants describing their views and needs best.

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