Optimal Product Diversity in an Economy with Search Unemployment*

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Abstract

Optimum product diversity is analysed within a model of monopolistic competition and search unemployment. We amend the Pissarides (2000) search model by Dixit-Stiglitz (1977) preferences. Thus, entrepreneurs do not only have to decide whether to offer a vacancy, but also whether to produce quantities of an established or of a new variety. We derive equilibrium in the decentral case and compare it to the social optimum. We show that in general variety may be too large or too small. In addition, decentral unemployment rates are not optimal, even when assuming the Hosios (1990) condition to hold.

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1 Introduction

The question of the optimal number of firms in an industry has been the focus of a vast amount of economics literature. This is due to the obvious importance of the question. A thorough analysis increases our understanding of the impact of barriers to entry, but it also guides competition policy how to regulate (if at all) industries which are characterised by, for example, economies to scale (as one reason for entry barriers, see Weizsäcker (1980)).

The early literature that researched into the optimality of free entry when an industry is characterised by increasing returns to scale argued that there will be excessive entry (see, for example Chamberlin (1933), Robinson (1933) and Kaldor (1934)). The argument for this is that (monopoly) profits which are generated by increasing returns, attract too many entrepreneurs such that firms will become too small. The economy (and as such consumers) cannot take full advantage of the scale economies.

This argument, however, only concerns the production side of the economy. More recently, the literature included the fact that consumers value variety, see Spence (1976) and Dixit and Stiglitz (1977). In these situation it is ambiguous whether there will be excess entry or not.

Building on this latter strand of literature, Mankiw and Whinston (1986) present a general model that clearly identifies the forces at work which cause the deviation of the social optimum from the private one with free entry. Two externalities drive a wedge between the private and social evaluation of free entry: a business stealing effect (which is negative) and a product variety effect (which is positive). Depending on which effect dominates, the economy is characterised by too much or too little diversity (see for example Vives (1999)).

A crucial assumption in the literature on the optimal number of variety (and free entry) is that firms face frictionless input markets. Thus, the trade-off between the number of firms and firm output is given by the resource restriction of the economy. In real world economies, however, firms face input markets which are characterised by a number of frictions. One of these is the search and matching friction in the labour market. Before firms are able to start producing (and hence can enter the industry), they have to search for suitable workers. It is well known that this generates equilibrium unemployment, see for example Pissarides (2000).

Far from clear is the relation between search unemployment and the optimal number of firms. The question comes up whether for example free entry would on the one hand result in a suboptimal number of firms, but on the other hand would decrease unemployment or whether an increase in monopoly profits to promote variety would come at the cost of additional unemployment. Without taking unemployment into account results concerning free entry and the optimal number of firms seem to be flawed.

In this paper, we amend a standard search unemployment model (see for example, Diamond (1982), Mortensen (1986), Mortensen and Pissarides (1999) and Pissarides (2000)) by a goods market structure which is characterised by monopolistic competition and Dixit-Stiglitz (1977) preferences.

Only few papers analyse search unemployment models which are characterised by monopolistic competition. Our model is close to Ebell and Haefke (2004) who analyse the impact of product market deregulation and endogenous wage bargaining institutions to explain the different US European unemployment experience and to Ziesemer (2005) who analyses the effects of the digital revolution on unemployment. None of these papers, however, addresses the issue of optimal product diversity and both have to rely on the notion of multiple worker firms. This assumption indirectly implies that firms are already equipped with optimal production capacity when entering the market (see, for example Ebell and Haefke (2004)), but this seems to be restrictive. Usually, new founded firms enter the market and simultaneously start looking for a suitable workforce. Thus, the assumption suppresses channels through which the search friction may affect the choice of firms to enter the market.

In our framework, we stick to the standard Pissarides (2000) assumption of one worker firms. The presence of monopolistic competition only offers an additional choice to firms. Either they they use their (fixed) production capacity to produce existing varieties or they sink some fixed costs and produce a new variety. With this, the number of varieties as well as the quantity per variety is endogenous.[1] We can solve for the decentral equilibrium number of varieties, the quantity produced per variety and the unemployment rate and compare this to the social planer choice. Our main results are as follows:

1. Product diversity, i.e. the number of varieties, is either too small or too large. This confirms the results found in the literature with frictionless input markets.

[1] Due to the one-worker firm assumption, the number of firms is not equivalent to the number of varieties as true in models without search frictions.

2. The condition for the unemployment rate to be efficient, the so-called Hosios (1990) condition, does not hold with an imperfectly competitive goods market. However, we can show that the condition is nested in our model for a competitive goods market.

3. Numerical simulations of the model suggest that production per variety is too large. This result suggests that there will be, independently of aggregate employment, over-employment in any variety producing sector. This is due to a wage externality. Similar results are also found in Ebell and Haefke (2004) and Smith (1999).

Section 2 presents the basic structure of the model. Section 3 derives the decentralised equilibrium and some comparative static results. Section 4 determines the social planner equilibrium given the labour market friction. Eventually, section 5 compares the decentral and the central equilibrium and analyses the question of optimal product diversity. Section 6 concludes.

2 The Model

2.1 The Labour Market

The modelling of the labour market closely follows the standard assumptions in the literature, see e.g. Pissarides (2000) or Cahuc and Zylberberg (2004). Firms offering vacancies and unemployed workers have to invest time to find each other. Matching (i.e. finding the right counterpart) is driven by the following (Cobb-Douglas) matching function:

$$M = U^{\eta} V^{1-\eta} \tag{2.1}$$

where M denotes matches, U is the number of unemployed and V is the number of vacancies. The probability for an entrepreneur/firm to find a suitable worker is:

$$\frac{M}{V} = \left(\frac{U}{V}\right)^{\eta} = \theta^{-\eta}, \tag{2.2}$$

where $\theta \equiv \frac{V}{U}$, which is referred to as labour market tightness (as seen from an entrepreneur's point of view). The probability for an unemployed agent to find a job is:

$$\frac{M}{U} = \left(\frac{V}{U}\right)^{1-\eta} = \theta^{1-\eta}. \tag{2.3}$$

Filled jobs (=employment) are destroyed at an exogenous rate s. The number of all filled jobs in the economy is $(\bar{L} - U)$, where $\bar{L}$ denotes the labour force. Thus, the inflow into unemployment is given by $s(\bar{L} - U)$. The outflow of unemployment is $\theta^{1-\eta}U$. The differential equation governing the dynamics of the unemployment rate is given by:

$$\frac{dU}{dt} \equiv \dot{U} = s(\bar{L} - U) - \theta^{1-\eta}U \tag{2.4}$$

Steady state unemployment is given by:

$$U = \frac{s\bar{L}}{s + \theta^{1-\eta}} \tag{2.5}$$

2.2 Households

Households are characterised by the following love-of-variety utility function:

$$w = x_0 + \left(\sum_{i=1}^{n} x_i^{\alpha} \right)^{\frac{1}{\alpha}}, \tag{2.6}$$

where x_i denotes the amount of consumption of some variety i and x_0 is some outside good which is the numéraire, hence $p_0 = 1$.[2] Households are endowed with one unit of labour and with one unit of the outside good.[3] The budget constraint of the household reads:

$$I = x_0 + \sum_{i=1}^{n} p_i x_i,$$

where I is the income stream of the household. Solving this maximisation problem (see the appendix) yields the demand for a variety i of the monopolistic good

$$x_i = \frac{I - x_0}{q} \left[\frac{p_i}{q} \right]^{1/(\alpha-1)}, \tag{2.7}$$

[2]Modelling the household utility in this quasi-linear way has become common in the literature, see for example Pflüger (2004) and the literature therein.

[3]Think of the outside good as being land. This shortcut simplifies our analysis since we do not need a production sector for the outside good. We could also model this production sector and all results would carry over as long as we assume labour only employed in the monopolistic sector of the economy.

where $q \equiv \left(\sum_1^n p_i^{\alpha/(\alpha-1)}\right)^{(\alpha-1)/\alpha}$, which is the minimum expenditure for the CES bundle of heterogeneous goods.[4]

Using this solution to the household's optimisation problem, we can show that utility of the household is linear in income. Hence, we are able to determine the value of having a job E and the value of participating in the labour market, but being unemployed and searching for a job S. The (steady state) Bellman equation for having a job is given by:

$$\rho E^i = \omega^i + s(S - E^i), \tag{2.8}$$

where $i \in E, I$ depending on having a job in the established or the innovative sector and ω^I is the respective wage. There is no job specific human capital. Every unemployed household has the same probability of finding a job. With a zero unemployment benefit, the value of being unemployed is:

$$\rho S = \theta^{1-\eta}(E - S), \tag{2.9}$$

2.3 Firms

Firms consist of an entrepreneur who offers one vacancy. After having filled this vacancy, the entrepreneur produces a fixed amount of output a of a variety of the monopolistic good and sells this to the consumers.

Before the firm is able to produce it has to invest a flow amount ϕ of the outside good in order to search for a suitable worker. Once the firm has found the worker, it has to sink fixed costs in order to start production. The amount of fixed costs depends on the choice of the firm in which "sector" to engage. The firm can sink an amount R (for example to buy a blueprint) and start producing a new variety of the monopolistic good. In this case the firm enjoys monopoly profits for one period. After the elapse of this period other firms can produce additional amounts of this new variety. These firms have to sink an amount δR (with $0 \leq \delta \leq 1$) to produce quantities of existing varieties. Figure 1 offers a graphical illustration of the structure of the labour

[4]We do not consider any intertemporal aspects of the choice of households. However, the same demand structure would occur in an intertemporal setting as long as households are risk neutral, see for example Grossman and Helpman (1991) or Shapiro and Stiglitz (1984). Obviously, matters change in case the households are not risk neutral anymore, see Acemoglu and Shimer (1999).

market and of firm behaviour.[5]

To determine the behaviour of entrepreneurs, we have to look at the values of offering a vacancy, having a filled job and producing quantities of an existing variety or being the monopolist and producing a new variety. Let us turn to the value of offering a vacancy. Denoting this value W, the Bellman equation determining this value is given by:

$$\rho W = -\phi + \theta^{-\eta} \left(J^I - W \right), \tag{2.10}$$

when planning to produce a new variety or

$$\rho W = -\phi + \theta^{-\eta} \left(J^E - W \right), \tag{2.11}$$

when entering an established industry. J^I and J^E denote the value of having a filled job and producing a new variety or producing existing varieties respectively. The value of offering a vacancy is the search costs plus the probability weighted "gain" of turning a vacancy into a filled job (i.e. the option value). Since there is free entry into the labour market, vacancies will be offered until W is zero. Using this it is easy to see that the following holds:

$$J^I = J^E = \frac{\phi}{\theta^{-\eta}} \tag{2.12}$$

Thus, free entry for offering vacancies establishes a no-arbitrage situation. Since every entrepreneur can freely choose in which sector to go, the values of having jobs in either sector must be identical. By equation (2.12), the lower bound for the value of having a job is given by expected search costs.

In general, the value of having a filled job slot are given by the following Bellman equations:

$$(1+\rho) J^I = p_i(a)a - \omega^I + (1-s) \left(J^E \right) - R, \tag{2.13}$$

$$\rho J^E = p_i(x_i + a)a - \omega^E + s \left(-J^E \right) - \delta R. \tag{2.14}$$

The producer of a new variety is the one period monopolist and gets the higher monopoly price. However, once his patent expires (and the job is not destroyed by the exogenous shock), the variety becomes established and other firms can produce additional quantities. Thus, the former monopolist

[5]Note that with a downward sloping demand curve for any variety i the revenue of producing a new variety is always larger than producing an existing one. However, fixed costs are also higher making the production of an existing variety an attractive option.

is in the same situation as a firm that has chosen to produce quantities of an additional variety in the first place.

Solving these equations for J^I and J^E gives:

$$J^I = \frac{p_i(a)a - \omega^I - R}{1+\rho} + \frac{1-s}{1+\rho}\frac{p_i(x_i + a)a - \omega^E - \delta R}{\rho + s} \tag{2.15}$$

$$J^E = \frac{p_i(x_i + a)a - \omega^E - \delta R}{\rho + s} \tag{2.16}$$

2.4 Wage Bargaining

The wage will be endogenously determined by a bargain between the household and the entrepreneur. The entrepreneur is only able to produce once a wage agreement is settled. If no solution to the bargaining problem is found, the household and the entrepreneur split up and both continue searching.[6]

Timing is of crucial importance for the bargaining result. When offering the vacancy, the entrepreneur is free to choose in which sector she/he is going to produce in the case the vacancy is filled. After meeting the suitable worker (and before the bargaining starts), the firm invests the fixed costs depending on the choice whether to produce quantity or variety. Once an agreement is reached the worker-firm-pair can start to produce. If no agreement is reached the entrepreneur can sell its equipment on a perfect second hand market and continues searching.

Let us first consider the wage bargain between in the case the entrepreneur has chosen to enter the established sector. Thus, the entrepreneur and the household bargain the wage ω^E. The solution to this bargaining is determined by the following Nash product:

$$\Omega^E = (E^E - S)^\zeta (J^E)^{1-\zeta}, \tag{2.17}$$

where ζ denotes bargaining power of the worker. The bargained wage in the established sector will be given by (see the appendix A.2):

$$\omega^E = \zeta(p_i(x_i + a)a - \delta R + \theta\phi). \tag{2.18}$$

[6] Note that due to the assumption of the single worker firm, we do not have to consider intrafirm bargaining (and the problems associated with that) as e.g. analysed by Stole and Zwiebel (1996) or Cahuc and Wasmer (2004).

The wage for the situation in which the firm produces a new variety is found by maximising the following Nash product:

$$\Omega^I = (E^I - S)^\zeta (J^I)^{1-\zeta}. \tag{2.19}$$

We assume bargaining power of workers to be identical across sectors. This is due to the fact that bargaining power is determined by the institutional framework of the whole economy (organisation of the labour market etc.) which is independent of whether the firm produces a new variety or only additional quantities. Firms which produce new varieties pay the following wage:

$$\omega^I = \zeta(p_i(a)a - R + \theta\phi) \tag{2.20}$$

The bargained wage in both "sectors" of the economy distributes rents between worker and entrepreneur. These rents are the saved search costs of the firm and the monopoly rents when producing and selling output. These rents, however, need not to be identical between firms which produce established goods or new varieties. Thus, there might exist a wage differential which is given by:

$$\omega^E - \omega^I = \zeta((1-\delta)R - (p_i(a) - p_i(x_i + a))a). \tag{2.21}$$

This wage differential is decreasing in the price differential and increasing in the fixed cost differential. The effect of an increase in workers' bargaining power ζ depends on the sign of the net rent differential.

3 Decentralised Equilibrium of the Economy

In this section we solve for the decentral equilibrium in the economy and present and discuss some comparative statics results. We focus on a symmetric equilibrium. This means that every variety is produced by the identical number m of firms such that the price of every variety is identical. Thus, we have to solve for five endogenous variables: the amount of production of a variety x, the number of varieties n, the labour market tightness θ (as a proxy for offered vacancies), the price for any symmetric variety p and the wage differential $\omega^E - \omega^I$. For the ease of exposition we will concentrate in the following on the number of firms m instead of the amount of production (although there is a one to one relationship of $x = ma$ between these two variables).

We solve the model in three steps. First we equate the expression for J^I and J^E. With this, we are able to determine the amount of production in a sector of the economy.

$$\frac{p(a)a - \omega^I - R}{\rho + s} = \frac{p(ma + a)a - \omega^E - \delta R}{\rho + s}$$
$$\Leftrightarrow \omega^E - \omega^I = p(ma + a)a - p(a)a + (1 - \delta)R. \tag{3.1}$$

Plugging in the wage differential given by equation (2.21) (remembering that we focus on a symmetric equilibrium in which $x_i = x = ma$ and $p_i = p$) into the no-arbitrage relation gives:

$$(p(a) - p(ma + a))a = (1 - \delta)R, \tag{3.2}$$

which says that in an arbitrage free equilibrium, the revenue difference between producing a new variety and an established one will be identical to the difference in (sunk) costs. Equation (3.2) implicitly gives a relation between the number of firms which produce quantity and the number of varieties. This relation does not depend on labour market parameters such as bargaining power, ζ, or alike. Labour market imperfection affects firms in a symmetric way irrespective of whether they engage as an established or an innovative producer, thus it does not affect the equilibrium allocation of resources between the two uses.

Using the demand relations (A.11) and (A.12), this equation gives (see the appendix A.1):

$$an^{\frac{1-\alpha}{\alpha}} m^{1-\alpha}(1 - (1 + m)^{\alpha-1}) = (1 - \delta)R. \tag{3.3}$$

The right-hand side of equation (3.3) is the additional cost of producing a new variety and the left-hand side is the additional profit of being the monopolist and producing a new variety. The latter is the revenue when being the monopolist $an^{\frac{1-\alpha}{\alpha}} m^{1-\alpha}$ adjusted for the price difference when compared to the other sectors with m firms. Note that the excess profit when being the monopolist is increasing in n as well as in m. This is due to the fact that both increases consumer demand and hence, the price. Additionally, a larger m will increase the incentive to be the monopolist, because it increases competition in the established goods sector.

The no-arbitrage condition determines the allocation of entrepreneurs with filled jobs between the two alternatives of producing some existing variety or producing a new one. In addition to this, we also have to determine the resource base which can be allocated between these two uses. The resource base is determined by the unemployment rate and thus, by the amount of vacancies which will be offered in the economy. The latter can be determined by the following considerations.

Entering the market for varieties is free. As such, entrepreneurs will continue offering vacancies until the expected profit of doing so is zero. This will be the case as long as the value of a filled job, e.g. J^E (or J^I, remember that both will be identical) equals expected search costs $\frac{\phi}{\theta^{-\eta}}$. The free-entry assumption constitutes the following condition (using for example J^E):

$$\frac{p(ma+a)a - \omega^E - \delta R}{\rho + s} = \frac{\phi}{\theta^{-\eta}}. \tag{3.4}$$

Plugging in equation (2.20) for the endogenous wage and using the expression for the price given by the demand function yields:

$$\begin{aligned} &\frac{p(ma+a)a - \zeta(p(ma+a)a - \delta R + \theta\phi) - \delta R}{\rho + s} = \frac{\phi}{\theta^{-\eta}} \\ &\Leftrightarrow n^{\frac{1-\alpha}{\alpha}}(\frac{1}{m}+1)^{\alpha-1}a = \frac{\rho+s}{1-\zeta}(\phi\theta^{\eta}) + \frac{\zeta}{1-\zeta}\phi\theta + \delta R. \end{aligned} \tag{3.5}$$

Equation (3.5) determines the equilibrium relation between the number of existing product varieties (which governs the price which can be earned in the monopolistic sector), the number of firms in every sector and labour market tightness, θ. The relation between the number of firms (either producing quantity or variety) and labour market tightness is positive. More firms imply more production and thus an increase in households demand. This in turn increases the profit of opening up vacancies.

The last equation which constitutes the equilibrium in the economy is the resource restriction. This reads $(1-u)\bar{L} = mn$, where the left-hand side is de facto endowment of the economy and the right-hand side is the usage of resources. Using the expression for the steady state unemployment rate, equation (2.5), the resource restriction reads:

$$nm = \left(1 - \frac{s}{s+\theta^{1-\eta}}\right)\bar{L}. \tag{3.6}$$

The three equations (3.3), (3.5) and (3.6) determine the three endogenous variables n, m and θ of the system. Solving the model for the changes in the

	dR	dδ	da	dρ	ds	dζ	dϕ	dL
dm	?	?	?	+	+	+	+	-
dn	?	?	?	-	-	-	-	+
dθ	?	?	?	-	?	-	-	+

Table 1: Comparative Static Results

endogenous variables gives the comparative statics results shown in Table 1 (see appendix A.3.4 for the detailed solution). Let us briefly comment on some of these comparative static results. An increase in the available resource base $\bar{L}$ will on impact evenly increase the n and m. This gives rise to a "demand push" such that profit increases which leads entrepreneurs to open up more vacancies. In addition to this producing new variety becomes more attractive. Hence, employment (which is larger than before) will be allocated towards producing more variety and less quantity. An increase in the present value of search costs (increase in ϕ and ρ) decrease the number of offered jobs and thus, decreases the de facto resource base. On impact variety and quantity decrease evenly. However, the decrease in m makes producing quantity relatively more attractive. Equilibrium quantity increases, whereas variety decreases. Similar arguments hold true for an increase in worker's bargaining power ζ. This increases employment, but this is not evenly allocated between quantity and variety. On the contrary, variety declines and quantity increases.

An increase in the fixed costs of opening up a new firm (an established firm), R (δ), decreases the number of entrepreneurs and hence, the number of vacancies. Thus, variety and quantity would have to decrease. However, the increase in R (δ) increases the incentive for producing quantity (variety). It is ambiguous which one of these two effects dominates.[7] The intuition for the effects of a change in productivity, a, are quite similar albeit with different signs.

The comparative static results reveal unemployment affects the quantity-variety choice of the economy. An increase in unemployment does not lead to

[7]The effect of a reduction in R or δ has also ambiguous effects on labour market tightness θ and as such on unemployment. This is especially interesting since some authors (for example Dulleck et al. (2004) or Fonseca et al. (2001)) argue that reducing start-up costs would lead to a decrease in unemployment. This needs not to be true when taking the reallocation effects into account.

a decline in both, quantity and variety. In fact, direct changes in the labour market are transmitted very asymmetric into the goods market.

4 Centralised Equilibrium in the Economy

The decentral equilibrium which was derived in the previous section is characterised by a number of externalities in the goods and in the labour market:

1. Congestion Externality: Additional entrepreneurs entering the market and offering vacancies have a negative effect on the matching probability of the already searching entrepreneurs.

2. Thick Market Externality: Additional entrepreneurs that offer vacancies increase the probability of unemployed agents to find a job.

3. Business Stealing Externality: Entrepreneurs with production capacity that choose to produce already existing varieties ceteris paribus decrease the profit of the following entrepreneurs who want to produce quantities of this good.

4. Love-of-Variety Externality: Entrepreneurs with production capacity that choose to produce a new variety exert an externality on households by increasing welfare and on other entrepreneurs by increasing the demand for their products.

The existence of these externalities drives a "wedge" between the social optimal and the decentral equilibrium. Up to now, the litearture has exclusively focussed either on the effects of the labour market externalities (congestion and thick market) or the effects of the goods market externalities (business stealing and love-of-variety), see for example Mankiw and Whinston (1984) and Hosios (1990). Our model offers an integrative framework to analyse these externalities simultaneously. By analysing this model, we are in a position to better understand how goods market and labour market effects interact with each other.

The social planner maximises the following social welfare function:[8]

$$SW = \int_0^\infty e^{-\rho t} \left(\left(\sum_{i=1}^{n} x_i^\alpha \right)^{\frac{1}{\alpha}} + x_0 \right) dt. \tag{4.1}$$

Every household owns one unit of the outside good. There are $\bar{L}$ households in the economy, thus the endowment of the economy with the numéraire resource is given by $\bar{L}$. As such, the following resource restriction must hold:

$$\bar{L} = x_0 + nR + m\delta R + \phi\theta\bar{L}u.$$

Using this, social welfare in a symmetric equilibrium reads:

$$SW = \int_0^\infty e^{-\rho t} \left(n^{\frac{1}{\alpha}} ma + \bar{L} - nR - m\delta R - \phi\theta u\bar{L} \right) dt. \tag{4.2}$$

The control variables of the social planner in order to maximise welfare are the number of varieties, n, the amount of production in one sector, m and the number of vacancies (or the labour market tightness), θ. When choosing these control variables, the planner has to take two restrictions into account. First, the dynamic restriction concerning the evolution of unemployment (unemployment is the state variable of the economy) and second the resource constraint which constitutes a trade-off between quantity and variety. The first restriction is given by:

$$\dot{U} = s(\bar{L} - U) - \theta^{1-\eta}U$$

which is identical to (2.4). Noting that $\dot{U} = \dot{u}\bar{L}$ and doing some simplifications, this reads:

$$\dot{u} = s(1-u) - \theta^{1-\eta}u \tag{4.3}$$

The latter restriction reads:

$$(1-u)\bar{L} = nm. \tag{4.4}$$

The central planner faces an intertemporal problem of controlling the path of unemployment and a static problem of allocating a given resource pool

[8]Note that we write the social planner problem as that of maximising an infinite utility stream. We could also have done this for the households problem without changing any of the previous results.

between quantity and variety producing. The Lagrangean for solving this problem reads (e.g. Kamien and Schwartz (1991), sec. 10):

$$\mathcal{L} = n^{\frac{1}{\alpha}}ma + Bu\bar{L} - nR - m\delta R - \phi\theta u + \mu_2(s(1-u) - \theta^{1-\eta}u) + \mu_3((1-u)\bar{L} - nm), \tag{4.5}$$

where μ_2 and μ_3 are the marginal value of unemployment (which turns out to be negative) and the marginal value of an additional resource, respectively. With this, the first order conditions for this maximisation problem are:

$$\frac{\partial \mathcal{L}}{\partial \theta} = -\phi u\bar{L} - \mu_2(1-\eta)\theta^{-\eta}u = 0, \tag{4.6}$$

$$\frac{\partial \mathcal{L}}{\partial u} = B\bar{L} - \phi\theta\bar{L} - \mu_2(s + \theta^{1-\eta}) - \mu_3\bar{L} = -\dot{\mu}_2 + \rho\mu_2, \tag{4.7}$$

$$\frac{\partial \mathcal{L}}{\partial n} = \frac{1}{\alpha}n^{\frac{1-\alpha}{\alpha}}ma - R - \mu_3 m = 0, \tag{4.8}$$

$$\frac{\partial \mathcal{L}}{\partial m} = n^{\frac{1}{\alpha}}a - \delta R - \mu_3 n = 0, \tag{4.9}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_2} = s(1-u) - \theta^{1-\eta}u = 0, \tag{4.10}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_3} = (1-u)\bar{L} - nm = 0. \tag{4.11}$$

Using these conditions, we can derive three equations which resemble the equilibrium determining equations in the decentralised economy (see appendix A.4). The first equation is:

$$\left(\frac{1}{\alpha} - 1\right) n^{\frac{1}{\alpha}}ma = Rn - \delta Rm, \tag{4.12}$$

which governs the allocation of a given resource pool (i.e. a given unemployment rate) between producing quantities and varieties. The economic intuition behind this condition is more easily seen when rewriting it, yielding:

$$\left(\underbrace{\frac{1}{\alpha}n^{\frac{1}{\alpha}-1}ma}_{\frac{\partial w}{\partial n}} - R\right) = \frac{m}{n}\left(\underbrace{n^{\frac{1}{\alpha}}a}_{\frac{\partial w}{\partial m}} - \delta R\right). \tag{4.13}$$

With a given resource pool, $\frac{m}{n}$ denotes the costs of a marginal variety in terms of quantity loss. Thus, the social planner wants to allocate resources between producing quantity and quality such that the net marginal gain of -for example variety- (the left-hand side) equals the marginal costs of fewer m in utility units. Or (in more familiar microeconomic terms) the reciprocal of the net marginal utility equals the relative price.

Using the two conditions for the dynamic constraint, (4.6) and (4.7) yield the following condition for the optimal choice of θ:

$$B + \theta^{\eta} + \frac{\eta}{\rho + s}\theta = \frac{1-\eta}{\rho + s}\mu_3, \tag{4.14}$$

Plugging in μ_3 from the system of first order conditions eventually gives:

$$\theta^{\eta}\frac{\phi}{1-\eta} + \frac{\phi}{\rho + s}\frac{\eta}{1-\eta}\theta + \frac{1}{\rho + s}m^{-1}R = \frac{1}{\rho + s}\left(\frac{1}{\alpha}n^{\frac{1}{\alpha}-1}a\right). \tag{4.15}$$

This equation corresponds to the free-entry condition in the decentral equilibrium. It governs, as in the decentralised economy, the number of vacancies that will be offered. It states that the social planner should offer vacancies (and as such decrease unemployment) until the marginal costs of decreasing unemployment (the left-hand side) equals the marginal gain (the right-hand side). The marginal gain of lower unemployment is the net marginal utility of either producing more quantity or more variety. The marginal costs consist of the instantaneous costs plus the present value of the stream of expected future search costs.

Since the central planner faces the same structure and restrictions as the decentral economy, the resource constraint is identical in both cases and given by $(1-u)\bar{L} = mn$. This closes the model. Hence, we have -as before- three equations in three endogenous variables, i.e. n, m and θ.

5 Central Planer vs Decentralised Economy

The model we have presented is not analytical solvable in levels. This is true for the decentral as well as for the social optimum. Thus, when comparing these two situations, we have to stick to an indirect strategy, namely relying on pathologic situations and numerical simulations.

In the following we will first of all focus on two polar cases namely one in which the goods market externalities vanish and the other one in which this is true for the labour market externalities. This (simplifying) approach allows to analyse the differences between the decentral and the social planner optimum one by one and we are able to compare these results with the existing literature on the optimality of the search equilibrium and the optimality of product diversity within a framework of monopolistic competition. In a second step we will present the results of some numerical simulations of the model with all externalities at work. This analysis depicts the effects of the externality interaction and contrasts this situation with the one of the polar cases.

Consider an economy in which $\alpha = 1$ and $R = 0$, i.e. entrepreneurs do not face any set-up costs when producing goods (other than searching for the suitable match) and the substitutability between different good varieties is infinite. This implies that entrepreneurs produce a homogenous good and it does not matter whether they choose to produce an established or a new variety. This is the situation which resembles the standard search model of Pissarides (2000). Notice that in this situation the number of firms is indetermined (and irrelevant) as is the case in the standard textbook model. There are no externalities related to goods market imperfections in this economy. How about the efficiency of the unemployment rate? Using (3.5) and (4.15) it is easy to see that labour market tightness is identical in both situation as long as $\eta = \zeta$. This is the well known Hosios (1990) condition. If the bargaining power of workers is equal to the elasticity of the matching function with respect to vacancies, the negative externality of opening up a vacancy (crowding externality) and the positive thick market externality just cancel out. The number of vacancies will be optimal and so will be the equilibrium rate of unemployment.

Let us now turn to the goods market externality. Consider an economy in which there are no labour market externalities. This would be for example the case if $s = 0$. Thus, there are no separations of matched worker-firm pairs and the steady state unemployment rate is zero. The externalities stem from the imperfections in the goods market and hence from the decision of firms whether to produce new or established varieties. This raises the question of how diversity in the decentral case differs from optimal diversity. Since we implicitly assume a full-employment situation, the economy will always be restricted to be on the resource constraint $\bar{L} = mn$ irrespectively of whether the market or the social planner determines the allocation. Assume for the sake of exposition that $\delta = 0$, i.e. no fixed costs have to be sunk when producing established varieties.

Optimum product diversity is given by the solution of (using equation

(4.12) and the resource constraint):

$$\underbrace{\left(\frac{1}{\alpha}-1\right) n^{\frac{1}{\alpha}-1}\frac{\bar{L}}{n}a}_{f(n)} = R$$

whereas product diversity in the decentral case is given by (using (3.3) and the resource restriction):

$$\underbrace{an^{\frac{1-\alpha}{\alpha}}\left(\left(\frac{n}{\bar{L}}\right)^{\alpha-1} - \left(\frac{n}{\bar{L}}+1\right)^{\alpha-1}\right)}_{g(n)} = R. \tag{5.1}$$

Equation (5.1) is not analytically solvable for the optimal number of different products. As such, when comparing optimal product variety in the central and the decentral case, we have to stick to an indirect argument. Using the properties of $f(n)$ and $g(n)$ we can state the following:

Proposition 1 *If $\alpha < 0.5$, equilibrium variety in the decentral case will be smaller than in the social optimum.*

Proof. In the appendix it is shown that for $\alpha < 0.5$, $g'(n) > f'(n) > 0$. Both functions have the same limiting behaviour, thus $g(n) > f(n)$. Hence, equilibrium variety in the decentral case $n^d(= g^{-1}(R))$ is smaller than socially desirable $n^s(= f^{-1}(R))$. ■

Figure B depicts equilibrium variety in the decentral and social planner situation for different degrees of α. Note that $\alpha < 0.5$ is a sufficient condition for the decentral optimum to produce too few varieties. With α exceeding 0.5, however, product variety could be too small, optimal or even too large.

Economically, this result says that if households value variety very much (indicated by the parameter α), the social value of an additional variety producing firm will always exceed the private value plus the negative externality in form of the business stealing effect. Once the social value of variety is small enough ($\alpha > 0.5$) it is ambiguous whether the positive dominates the negative externality.

This result concerning social optimal and equilibrium product diversity in an economy without unemployment complements the results already derived in the literature (see for example Spence (1976), Dixit and Stiglitz (1977) or Whinston and Mankiw (1986)). These papers show that depending on the form of the utility function equilibrium variety may be too small, too large or the effect is ambiguous.

However, the interesting question which, to our best knowledge, has not been analysed in the literature so far is whether there is an interplay between the optimality condition for unemployment and product variety. For example it would be interesting to know whether the Hosios condition for optimal unemployment still holds once we allow for monopolistic competition or whether there exists a trade-off between optimal unemployment and diversity and both goals cannot be achieved simultaneously.

The strength of the presented model lies in the fact that all these questions could be addressed, however since the model is not analytically solvable, we have to rely on simulations to get an understanding of optimal product diversity and unemployment. In the following we present the results of the simulation of four different scenarios. These scenarios differ with respect to the assumed parameter vector.[9] Table 2 gives the different parameters in the analysed scenarios.

Scenarios 1 and 2 assume the Hosios condition to hold (i.e. with competitive labour markets, unemployment would be efficient), but consider different α, hence, these scenarios compare situations in which the heterogeneity of product varieties differs.

Scenarios 3 and 4 assume moderate degrees of heterogeneity, however, they analyse the effects of differences in the bargaining power of workers. This only affects the decentral equilibrium but comparing these two scenarios gives a feeling of how bargaining power affects the labour *and* the goods market and how this relates to the central planner optimum.[10]

	η	ρ	s	ϕ	$\bar{L}$	a	α	δ	ζ
Scenario 1	0.5	0.05	0.15	0.1	1	1	0.95	0	0.5
Scenario 2	0.5	0.05	0.15	0.1	1	1	0.75	0	0.5
Scenario 3	0.5	0.05	0.15	0.1	1	1	0.8	0	0.1
Scenario 4	0.5	0.05	0.15	0.1	1	1	0.8	0	0.9

Table 2: Parameter Vectors for Different Scenarios

[9] The choice of the parameters basically follows the choices in the literature, see e.g. Cahuc and Zylberberg (2005) and the literature given there.

[10] The scenarios we have chosen are only examples for the range of questions which could be addressed within this framework.

Using these parameters, we can calculate equilibrium and optimum quantity, diversity and unemployment. This is done in table 3.[11] These results

	Scenario	n	m	u
Social Planer	1	0.231466	4.10423	5 %
Decentral Equilibrium	1	0.0102244	92.489	5.44 %
Social Planer	2	2.0152	0.47502	4.27 %
Decentral Equilibrium	2	0.592793	1.59267	5.59 %
Social Planer	3	1.26334	0.754588	4.67 %
Decentral Equilibrium	3	0.438468	2.2348	2 %
Social Planer	4	1.26334	0.754588	4.67 %
Decentral Equilibrium	4	0.345983	2.42792	16 %

Table 3: Results for the Different Scenarios

concerning the equilibria in the different scenarios offer some interesting results. Consider scenario 1 in which the Hosios condition holds and product heterogeneity is small ($\alpha = 0.9$). As was already suggested in the previous analysis, the unemployment rate in the decentral case will be close to the optimal rate. However, the allocation of the labour pool between variety and quantity is distorted. The decentral economy produces far too much quantity and too few varieties, although the "love for variety" of households is rather small. The intuition for this is that the private costs of producing quantity are smaller than the social costs. Before a matched entrepreneur-worker pair can produce additional quantities, another entrepreneur must have invested the sunk investment costs R to establish this additional variety. The entrepreneur does not internalise this positive spill-over effect, but the social planner does. The distortion increases as α decreases, since in addition to the wedge between private and social costs of producing quantity, also the wedge between the private and the social value of variety grows larger.

But not only the quantity variety choice will be distorted but also the unemployment rate will be larger than optimal. This is due to the fact that with more firms producing quantities, the profit of a filled job declines which in turn decreases the number of entrepreneurs (=vacancies). Thus the

[11] All results have been computed using Mathematica® 5.1. The Notebooks are available from the author upon request. Note that, as in the whole paper, we do not impose any integer constraints on m or n.

unemployment rate will be too large. Since the Hosios condition holds, the distortion in the labour market is a direct consequence of the goods market imperfection.

Scenarios 3 and 4 demonstrate the effect of an increase in the bargaining power of workers ζ. A change leaves the social planner optimum unchanged, however it has some drastic effects on the goods and labour market situation in the decentral optimum. As expected, the unemployment rate increases. This is due to the fact that with a higher bargaining power, workers capture a larger part of the rent of a filled vacancy. As such, it will ceteris paribus become less attractive to offer vacancies. But not only the labour market is affected by a change in bargaining power, but also the allocation of resources between quantity and variety. With a higher bargaining power, entrepreneurs would rather produce additional quantity than additional variety. This is due to a standard hold-up effect. When producing a new variety, the entrepreneur has to sink some fixed costs R before bargaining takes place. The worker exploits this hands tying by bargaining higher wages. This effect becomes more pronounced as bargaining power increases. Thus, the incentive to produce new variety declines. An interesting point in this context is that employment declines by a smaller amount than the number of varieties. But this implies that (as can be seen from the table) quantity increases.

6 Summary and Conclusion

In this paper we have integrated monopolistic competition into an otherwise standard search model of the labour market. Entrepreneurs not only have the choice of whether to open up a vacancy or not, but also whether to produce additional quantities of an established good or a new variety. With this structure we are able to analyse the labour market effects of imperfect competition without dropping the one firm-one worker assumption of the Pissarides (2000) search model.

Using this model, we were able to derive some comparative static results on the effects of for example a change in bargaining power, a change in search costs or a change in the size of the economy not only on unemployment, but also on the quantity-diversity choice of the economy.

Inherited by its basic building blocks, the model we consider is characterised by a number of (positive and negative) externalities in the goods as well as in the labour market. The question comes up which effect this has on the decentral equilibrium compared to the social planner optimum and whether there is interaction.

To analyse this question we have derived the social planner solution of the model. Due to the fact that both equilibria, the decentral as well as the social planer's are not analytically solvable, the analysis has to rely on special cases and on a number of simulations.

We were able to show that for the two polar cases in which either the goods or the labour market imperfection vanishes, the model generates the same results and conditions concerning optimality as already derived in the literature. Thus, the standard models are nested in our model making it more general and as such better suited to analyse various policy questions. In addition to this, we are able to analyse possible interaction between goods and labour market externalities.

We solve the model numerically for a set of different scenarios, i.e. parameter vectors. The scenarios differ with respect to the strength of the goods and the labour market imperfection. The results of the different simulations reveal some interesting points. As the goods market becomes less competitive (measured by the degree of heterogeneity of product varieties) the optimum and the decentral number of product variety converge whereas we observe divergence for the unemployment rate. An increase in bargaining power of workers has large effects on the difference between the optimal and the decentral unemployment. Its effect on the goods market is rather small. Thus, the different externalities have different "cross market" effects.

Monopolistic competition has become a basic framework for analysing a broad range of economic issues (see for example Brakman and Heijdra (2004)) and is applied in many different fields in economics. However, up to now search unemployment has not been part of the "monopolistic competition revolution" although search frictions are an important feature of many real world labour markets. Thus, the presented model not only yields interesting results concerning the divergence of decentral and central equilibrium, but also offers a basic framework for extending the vast literature which builds on monopolistic competition (endogenous growth theory, new trade theory, new economic geography and so on) by search unemployment. This will make the results from these models richer and presents a fruitful avenue for further research.

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A Appendix

A.1 The Households Maximisation Problem

This is solved using the following Lagrangean:

$$\mathcal{L} = w + \mu_1 \left(I - \sum_{i=1}^{n} p_i x_i - x_0 \right) \tag{A.1}$$

yielding the first order condition:

$$\frac{\partial \mathcal{L}}{\partial x_i} = (w - x_0)^{1/(1-\alpha)} x_i^{\alpha-1} - \mu_1 p_i = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial x_0} = 1 - \mu_1 = 0 \tag{A.3}$$

Deriving the demand function for a variety i:

$$(w - x_0)^{(1-\alpha)} x_i^{\alpha-1} - p_i = 0 \tag{A.4}$$

$$\begin{aligned}
\Leftrightarrow \quad x_i^{\alpha-1} &= \frac{1}{(w - x_0)^{(1-\alpha)}} p_i \qquad\qquad (\text{A.5})\\
\Leftrightarrow \quad x_i &= \left(\frac{1}{(w - x_0)^{1/(1-\alpha)}} p_i \right)^{(\alpha-1)} \\
\Leftrightarrow \quad x_i p_i &= \left(\frac{1}{(w - x_0)^{(1-\alpha)}} \right)^{(\alpha-1)} (p_i)^{\alpha/(\alpha-1)} \\
\Leftrightarrow \quad \sum_1^n x_i p_i &= \left(\frac{1}{(w - x_0)^{(1-\alpha)}} \right)^{(\alpha-1)} \sum_1^n (p_i)^{\alpha/(\alpha-1)}
\end{aligned}$$

Plugging this in gives:

$$x_i = \frac{(p_i)^{1/(\alpha-1)}}{\sum_1^n (p_i)^{\alpha/(\alpha-1)}} \sum_1^n x_i p_i \tag{A.6}$$

Using these first order conditions for optimal consumption, we can derive the minimum expenditures for a bundle of the monopolistic good $(\sum_1^n x_i^{\alpha})^{\frac{1}{\alpha}}$:

$$
\begin{aligned}
x_i &= \frac{(p_i)^{1/(\alpha-1)}}{\sum_1^n (p_i)^{\alpha/(\alpha-1)}} \sum_1^n x_i p_i \qquad \text{(A.7)}\\
\Leftrightarrow\ x_i^\alpha &= \left(\frac{(p_i)^{1/(\alpha-1)}}{\sum_1^n (p_i)^{\alpha/(\alpha-1)}} \sum_1^n x_i p_i \right)^\alpha \\
\Leftrightarrow\ \sum_1^n x_i^\alpha &= \left(\frac{\sum_1^n x_i p_i}{\sum_1^n (p_i)^{\alpha/(\alpha-1)}} \right)^\alpha \sum_1^n (p_i)^{\alpha/(\alpha-1)} \\
\Leftrightarrow\ \left(\sum_1^n x_i^\alpha\right)^{\frac{1}{\alpha}} &= \frac{\sum_1^n x_i p_i}{\sum_1^n (p_i)^{\alpha/(\alpha-1)}} \left(\sum_1^n (p_i)^{\alpha/(\alpha-1)}\right)^{\frac{1}{\alpha}}
\end{aligned}
$$

Thus, minimum expenditures for one bundle are given by:

$$q = \left(\sum_1^n (p_i)^{\alpha/(\alpha-1)}\right)^{(\alpha-1)/\alpha} \qquad \text{(A.8)}$$

With this, we can rewrite the demand function as:

$$x_i = \frac{I - x_0}{q} \left[\frac{p_i}{q}\right]^{1/(\alpha-1)} \Leftrightarrow x_i = \left(\sum_{i=1}^n x_i^\alpha\right)^{\frac{1}{\alpha}} \left[\frac{p_i}{q}\right]^{1/(\alpha-1)} \qquad \text{(A.9)}$$

where we took advantage of the fact that the households spends all of her/his income. The demand function hence reads:

$$p_i = q \left(\frac{x_i}{(\sum_{i=1}^n x_i^\alpha)^{\frac{1}{\alpha}}} \right)^{\alpha-1}, \qquad \text{(A.10)}$$

Using this, we are in a position to determine the price of a firm starting to produce a new variety (in a symmetric situation in which aggregate variables are assumed constant) and a firm which produces additional quantities of a variety:

$$p(a) = \left(\frac{1}{n^{\frac{1}{\alpha}} m}\right)^{\alpha-1} \qquad \text{(A.11)}$$

and (remember that due to the assumed utility structure $q = 1$ must hold)

$$p(ma + a) = p((m+1)a) = \left(\frac{(m+1)}{n^{\frac{1}{\alpha}} m}\right)^{\alpha-1}. \qquad \text{(A.12)}$$

A.2 The Wage Bargain

A.2.1 Existing Varieties

Maximising the Nash product w.r.t. the wage and taking into account that the value of having a job and the value of having a filled vacancy are (linear) functions of ω^E,[12] we get:

$$\frac{\partial \Omega^E}{\partial \omega^E} = \zeta (E^E - S)^{\zeta - 1} J^{E^{1-\zeta}} - (1-\zeta)(E^E - S)^{\zeta} J^{E^{-\zeta}} = 0 \tag{A.13}$$

$$\Leftrightarrow \zeta (E^E - S)^{-1} - (1-\zeta) J^{E^{-1}} = 0 \tag{A.14}$$

$$\Leftrightarrow (1-\zeta)(E^E - S) - \zeta J^E = 0. \tag{A.15}$$

Remember that $J^E = \frac{p_i(x_i+a)a - \omega^E - \delta R}{\rho + s}$ and $E^E = \frac{\omega^E + sS}{\rho + s}$. Plugging this in gives:

$$(1-\zeta)(\frac{\omega^E + sS}{\rho + s} - S) - \zeta \frac{p_i(x_i + a)a - \omega^E - \delta R}{\rho + s} = 0 \tag{A.16}$$

$$\Leftrightarrow (1-\zeta)(\omega^E - \rho S) - \zeta (p_i(x_i + a)a - \omega^E - \delta R) = 0 \tag{A.17}$$

$$\Leftrightarrow \omega^E = \zeta (p_i(x_i + a)a - \delta R) + (1-\zeta)\rho S. \tag{A.18}$$

Thus, workers get a fraction of the monopoly rent plus a fraction of their outside option (i.e. the flow value of being unemployed). This expression can be simplified further (see, Pissarides (2000)). With the bargained wage, the following holds: $E^E - S = \frac{\zeta}{1-\zeta} J^E$. The value of being unemployed is given by (using 2.9): $\rho S = \theta^{1-\eta} \frac{\zeta}{1-\zeta} J^E$. With this the bargained wage is given by:

$$\omega^E = \zeta (p_i(x_i + a)a - \delta R) + (1-\zeta)(\theta^{1-\eta} \frac{\zeta}{1-\zeta} J^E) \tag{A.19}$$

$$\Leftrightarrow \omega^E = \zeta (p_i(x_i + a)a - \delta R) + \zeta \theta \phi \tag{A.20}$$

$$\Leftrightarrow \omega^E = \zeta (p_i(x_i + a)a - \delta R + \theta \phi). \tag{A.21}$$

The wage is a fraction of the rents which accrues to the firm plus a fraction of search costs.

[12] This is due to the implicit assumption that worker-firm-pair bargain the wage separately for every period.

A.2.2 New Varieties

Maximising the Nash product w.r.t. the wage and taking into account that the value of having a job and the value of having a filled vacancy are (linear) functions of ω^I, we get:

$$\frac{\partial \Omega^I}{\partial \omega^I} = \zeta (E^I - S)^{\zeta-1} {J^I}^{1-\zeta} + (1-\zeta)(E^I - S)^{\zeta} {J^I}^{-\zeta} = 0 \tag{A.22}$$

$$\Leftrightarrow \zeta (E^I - S)^{-1} - (1-\zeta) {J^I}^{-1} = 0 \tag{A.23}$$

$$\Leftrightarrow (1-\zeta)(E^I - S) - \zeta J^I = 0. \tag{A.24}$$

Remember that $J^I = \frac{p_i(a)a - \omega^I - R}{1+\rho} + \frac{1-s}{1+\rho} J^E$ and $E^I = \frac{\omega^I + sS}{s+\rho}$. Plugging this in, gives:

$$(1-\zeta)(\frac{\omega^I + sS}{\rho + s} - S) - \zeta \left(\frac{p_i(a)a - \omega^I - R}{1+\rho} + \frac{1-s}{1+\rho} (\rho + s) J^E \right) = 0 \tag{A.25}$$

$$\Leftrightarrow (1-\zeta)(\omega^I - \rho S) - \zeta \left(\left(p_i(a)a - \omega^I - R\right) \frac{\rho + s}{1+\rho} + \frac{1-s}{1+\rho} (\rho + s) J^E \right) = 0 \tag{A.26}$$

$$\Leftrightarrow \omega^I = \frac{\zeta}{1+\rho-\zeta(1-s)} \left((p_i(a)a - R) \frac{\rho + s}{1+\rho} + \frac{1-s}{1+\rho} (\rho + s) J^E \right) + \frac{1-\zeta}{1+\rho-\zeta(1-s)} \rho S. \tag{A.27}$$

With the bargained wage, the following holds $E^I - S = \frac{\zeta}{1-\zeta} J^I$. Thus the value of being unemployed is given by: $\rho S = \theta^{1-\eta} \frac{\zeta}{1-\zeta} J^I$. Using all this, the bargained wage is:

$$\begin{aligned} \omega^I = {} & \frac{\zeta}{1+\rho-\zeta(1-s)} \left((p_i(a)a - R) \frac{\rho + s}{1+\rho} + \frac{1-s}{1+\rho} (\rho + s) J^E \right) \\ & + \frac{1-\zeta}{1+\rho-\zeta(1-s)} \left(\theta^{1-\eta} \frac{\zeta}{1-\zeta} J^I \right) \end{aligned} \tag{A.28}$$

$$\Leftrightarrow \omega^I = \frac{\zeta}{1+\rho-\zeta(1-s)} \left((p_i(a)a - R) \frac{\rho + s}{1+\rho} + \frac{1-s}{1+\rho} (\rho + s) J^E + \theta \phi \right) \tag{A.29}$$

$$\tag{A.30}$$

The wage in the monopoly situation is consists not only a part of the actual monopoly profits and the search costs, but also of the value of having a job when loosing the patent. This is because an agreement today implies the option of getting the value J^E in the next period. The worker is able to get a part of this value. With this, the wage bargained with the new variety producing firm is a function of the wage of a quantity producing firm.

In a Nash equilibrium, in which the following holds $\frac{1-s}{1+\rho}(\rho+s)J^E = \frac{1-s}{1+\rho}\left((1-\zeta)(p_i(x_i+a)a-\delta R)-\zeta\theta\phi\right)$, the bargained wage in the new variety producing firm is given by:

$$\omega^I = \frac{\zeta}{1+\rho-\zeta(1-s)}\left((p_i(a)a-R)\frac{\rho+s}{1+\rho}+\frac{1-s}{1+\rho}(1-\zeta)(p_i(x_i+a)a-\delta R)\right)+\frac{\zeta}{1+\rho}\theta\phi. \tag{A.31}$$

A.3 Equilibrium Determining Equations

A.3.1 The no-arbitrage Relation

Totally differentiating the no-arbitrage relation (3.3) gives:

$$a_1 dm + a_2 dn + a_3 da = a_4 dR - a_5 d\delta, \tag{A.32}$$

where the coefficients are given by:

$a_1 \equiv$	$\frac{1-\alpha}{m}(1-\delta)R + an^{\frac{1-\alpha}{\alpha}}m^{1-\alpha}(1-\alpha)(1+m)^{\alpha-2}$	$a_2 \equiv$	$\frac{1-\alpha}{\alpha}\frac{(1-\delta)R}{n}$
$a_3 \equiv$	$\frac{(1-\delta)R}{a}$	$a_4 \equiv$	$(1-\delta)$
$a_5 \equiv$	R		

A.3.2 The free-entry condition

Totally differentiating the free entry condition (3.5) gives:

$$b_1 dm + b_2 dn + b_3 da = b_4 d\zeta + b_5 d\phi + b_6 dR + b_7 d\delta + b_8 ds + b_9 \frac{s}{\rho} d\rho + b_{10} d\theta, \tag{A.33}$$

where the coefficients are given by:

$b_1 \equiv$	$(1-\alpha)m^{-2}\left(1+\frac{1}{m}\right)^{\alpha-2} n^{\frac{1-\alpha}{\alpha}} a$	$b_2 \equiv$	$\frac{1-\alpha}{\alpha} n^{\frac{1-\alpha}{\alpha}-1}\left(1+\frac{1}{m}\right)^{\alpha-1} a$
$b_3 \equiv$	$n^{\frac{1-\alpha}{\alpha}}\left(1+\frac{1}{m}\right)^{\alpha-1}$	$b_4 \equiv$	$\frac{1}{(1-\zeta^2)}\left((\rho+s)\phi\theta^\eta+\phi\theta\right)$
$b_5 \equiv$	$\frac{\rho+s}{1-\zeta}\theta^\eta+\frac{\zeta}{1-\zeta}\theta$	$b_6 \equiv$	δ
$b_7 \equiv$	R	$b_8 \equiv$	$\frac{\rho}{1-\zeta}\phi\theta^\eta$
$b_9 \equiv$	$\frac{s}{\rho}b_8$	$b_{10} \equiv$	$\frac{\rho+s}{1-\zeta}\phi\eta\theta^{\eta-1}+\frac{\zeta}{1-\zeta}\phi$

A.3.3 The resource constraint

Totally differentiating the resource constraint (3.6) reads:

$$c_1 dm + c_2 dn = -c_3 ds + c_4 d\theta + c_5 d\bar{L}, \tag{A.34}$$

where the coefficients are given by:

$c_1 \equiv$	n	$c_2 \equiv$	m
$c_3 \equiv$	$\frac{s+\theta^{1-\eta}(1-s)}{(s+\theta^{1-\eta})^2)}\bar{L}$	$c_4 \equiv$	$\frac{s}{(s+\theta^{1-\eta})^2}(1-\eta)\theta^{-\eta}$
$c_5 \equiv$	$\left(1-\frac{s}{s+\theta^{1-\eta}}\right)$		

A.3.4 Equilibrium

Stability Before turning to the comparative static analysis, let us first consider the stability of the equilibrium. It turns out that these stability considerations are of major importance for the comparative static results.

Equations (3.3), (3.5) and (3.6) implicitly determine the equilibrium values n^0, m^0 and θ^0. The equations are replicated here for convenience:

$$\underbrace{an^{\frac{1-\alpha}{\alpha}} m^{1-\alpha}(1-(1+m)^{\alpha-1})-(1-\delta)R}_{f^n(m,n)} = 0, \tag{A.35}$$

$$\underbrace{n^{\frac{1-\alpha}{\alpha}}(\frac{1}{m}+1)^{\alpha-1}a-\left(\frac{\rho+s}{1-\zeta}\left(\phi\theta^\eta\right)+\frac{\zeta}{1-\zeta}\phi\theta+\delta R\right)}_{f^\theta(m,n,\theta)} = 0, \tag{A.36}$$

$$\underbrace{\left(1-\frac{s}{s+\theta^{1-\eta}}\right)\bar{L}-mn}_{f^m(m,n,\theta)} = 0. \tag{A.37}$$

Assume such an equilibrium solution exists. The economy starts from this equilibrium and there is some exogenous shock which moves the economy away from the initial equilibrium. The question arises how plausible adjustment paths of the endogenous variables look like once the economy is out of its equilibrium. We consider the following form:

$$\frac{dn}{dt} = h^n(f^n), \tag{A.38}$$
$$\frac{d\theta}{dt} = h^\theta(f^\theta), \tag{A.39}$$
$$\frac{dm}{dt} = h^m(f^m), \tag{A.40}$$

where h^i are some arbitrary sign-preserving functions.

Equation (A.38) says that as long as net profit gain for being the monopolist producing a new variety ($f^n > 0$) there will be new firms which produce additional variety, n increases. Equation (A.39) argues that as long as the gain for opening a vacancy is positive ($f^\theta > 0$), new entrepreneurs offering additional varieties enter the market. Equation (A.40) shows that if resource supply exceeds demand ($f^m > 0$), the number of quantity producing firms increases.

In order to analyse the stability of the adjustment path, we first of all linearise equations (A.38)-(A.40) around some initial equilibrium (m^0, n^0, θ^0). The linearised system reads:

$$\frac{d\bar{n}}{dt} = h^n(0) + h^{n\prime}(0)\left(a_1\bar{m} + a_2\bar{n}\right), \tag{A.41}$$
$$\frac{d\bar{\theta}}{dt} = h^\theta(0) + h^{\theta\prime}(0)\left(b_1\bar{m} + b_2\bar{n} - b_{10}\bar{\theta}\right), \tag{A.42}$$
$$\frac{d\bar{m}}{dt} = h^m(0) + h^{m\prime}(0)\left(-c_1\bar{m} - c_2\bar{n} + c_4\bar{\theta}\right). \tag{A.43}$$

Note that with $h^i(0) = 0$ and $z^i \equiv h^{i\prime}(0) > 0$ the characteristic equation for the above given linearised system is:

$$\begin{vmatrix} z^n a_1 - \lambda & z^n a_2 & 0 \\ z^\theta b_1 & z^\theta b_2 - \lambda & -z^\theta b_{10} \\ -z^m c_1 & -z^m c_2 & z^m c_4 - \lambda \end{vmatrix} = 0,$$

where this is given by:

$$-\lambda^3 + \lambda^2\Delta_1 - \lambda\Delta_2 + z^m z^n z^\theta \Delta_3 = 0, \tag{A.44}$$

	dR	dδ	da	dρ	ds	dζ	dφ	dL
dm	?	?	?	+	+	+	+	-
dn	?	?	?	-	-	-	-	+
dθ	?	?	?	-	?	-	-	+

Table 4: Comparative Static Results Appendix

and the coefficients are given by:

$$\begin{aligned}
\Delta_1 &\equiv (c_4 z^m + a_1 z^n + b_2 z^\theta) > 0, \\
\Delta_2 &\equiv (z^m z^n z^\theta a_1 b_{10} c_2 + a_1 c_4 z^m z^n + b_2 c_4 z^m z^\theta + a_1 b_2 z^n z^\theta) > 0, \\
\Delta_3 &\equiv (a_1 b_2 c_4 + a_2 b_{10} c_1 - a_1 b_{10} c_2 - a_2 b_1 c_4) \gtreqless 0.
\end{aligned}$$

Without doing deeper analysing the roots of the characteristic equation (which is not our primal goal in this analysis) we can state that a necessary condition for the adjustment path to be stable is $\Delta_3 < 0$ (see e.g. Gandolfo (1997)).

Comparative Statics The three equations (A.32)-(A.34) constitute the following equilibrium system:

$$\underbrace{\begin{pmatrix} a_1 & a_2 & 0 \\ b_1 & b_2 & -b_{10} \\ c_1 & c_2 & -c_4 \end{pmatrix}}_{\mathbf{A}} \cdot \begin{pmatrix} dm \\ dn \\ d\theta \end{pmatrix} = \begin{pmatrix} a_4 dR - a_5 d\delta - a_3 da \\ -b_3 da + b_4 d\zeta + b_5 d\phi + b_6 dR + b_7 d\delta + b_8 ds + b_9 \frac{s}{\rho} d\rho \\ -c_3 ds + c_5 d\bar{L} \end{pmatrix}, \tag{A.45}$$

where the determinant of the coefficient matrix is given by $|A| = -a_1 b_2 c_4 - a_2 b_{10} c_1 + a_1 b_{10} c_2 + a_2 b_1 c_4$. By the correspondence principle $|A|$ must be positive for the out of equilibrium adjustment path to be stable. Using this equilibrium system and the information on the sign of the determinant, we can derive the comparative static results which are given in table 1.[13]

[13]Note that the solution of the equilibrium was done with Mathematica 4.2. The notebook with the code and further explanations concerning the determination of the signs of the effects is available on request.

A.4 The Social Planer's Problem

Using the first order condition for the dynamic component of the social planner (controlling the optimal unemployment path), we get derive an expression for (optimal) labour market tightness:

$$-\phi u\bar{L} - \mu_2(1-\eta)\theta^{-\eta}u = 0$$
$$\Leftrightarrow \mu_2 = -\frac{\phi\bar{L}}{1-\eta}\theta^{\eta},$$

from which follows that μ_2 is constant in a steady state. Moreover, we have:

$$-\phi\theta\bar{L} - \mu_2(s+\theta^{1-\eta}) - \mu_3\bar{L} = -\dot{\mu_2} + \rho\mu_2$$
$$\Leftrightarrow \mu_2 = \frac{1}{s+\theta^{1-\eta}+\rho}\left(-\phi\theta\bar{L} - \mu_3\bar{L}\right)$$

Combining these two equations in order to eliminate μ_2, we get:

$$\frac{1}{s+\theta^{1-\eta}+\rho}\left(-\phi\theta\bar{L} - \mu_3\bar{L}\right) = -\frac{\phi\bar{L}}{1-\eta}\theta^{\eta}$$
$$\Leftrightarrow \phi\theta + \mu_3 = \frac{\phi}{1-\eta}\theta^{\eta}(\rho+s) + \frac{\phi}{1-\eta}\theta$$
$$\Leftrightarrow \theta^{\eta} = \frac{\mu_3}{\rho+s}\frac{1-\eta}{\phi} - \frac{\eta}{\rho+s}\theta.$$

Turning to the static constraint, i.e. choosing quantity vs. variety once the amount of filled vacancies is given. The two foc for this problem are given by:

$$\frac{1}{\alpha}n^{\frac{1-\alpha}{\alpha}}ma - R - \mu_3 m = 0$$
$$n^{\frac{1}{\alpha}}a - \delta R - \mu_3 n = 0,$$

where both can be modified to:

$$\frac{1}{\alpha}n^{\frac{1}{\alpha}}ma - Rn = mn\mu_3$$
$$n^{\frac{1}{\alpha}}ma - \delta Rm = mn\mu_3.$$

Combining this yields:

$$\frac{1}{\alpha}n^{\frac{1}{\alpha}}ma - Rn = n^{\frac{1}{\alpha}}ma - \delta Rm$$
$$\Leftrightarrow n\left(\frac{1}{\alpha}n^{\frac{1}{\alpha}-1}ma - R\right) = m\left(n^{\frac{1}{\alpha}}a - \delta R\right) \Leftrightarrow \left(\frac{1}{\alpha}n^{\frac{1}{\alpha}-1}ma - R\right) = \frac{m}{n}\left(n^{\frac{1}{\alpha}}a - \delta R\right)$$

Assumption	Slope	Limiting	Behaviour
$\alpha < 0.5$	$f'(n) > 0$	$\lim_{n\to 0} f(n) = 0$	$\lim_{n\to\infty} f(n) = \infty$
$\alpha > 0.5$	$f'(n) < 0$	$\lim_{n\to 0} f(n) = \infty$	$\lim_{n\to\infty} f(n) = 0$
$0 \leq \alpha \leq 1$	$g'(n) > 0$	$\lim_{n\to 0} g(n) = 0$	$\lim_{n\to\infty} g(n) = \infty$
$\alpha < 0.5$	$g'(n) > f'(n)$		

Table 5: Properties of the equilibrium determining equations

A.5 Social Planer vs Decentral Equilibrium

Deriving the properties of the equilibrium variety determining equations yields:

$$f(n) = \left(\frac{1}{\alpha} - 1\right) n^{\frac{1}{\alpha}-1} \frac{\bar{L}}{n} a$$

$$f'(n) = \frac{1-\alpha}{\alpha} \frac{f(n)}{n} = \frac{1-\alpha}{\alpha} \frac{R}{n}$$

$$g(n) = a n^{\frac{1-\alpha}{\alpha}} \left(\left(\frac{n}{\bar{L}}\right)^{\alpha-1} - \left(\frac{n}{\bar{L}} + 1\right)^{\alpha-1} \right)$$

$$g'(n) = \frac{1-\alpha}{\alpha} \frac{g(n)}{n} + a n^{\frac{1-\alpha}{\alpha}} \left((\alpha - 1) \left(\frac{n}{\bar{L}}\right)^{\alpha-2} \bar{L}^{-1} - (\alpha - 1) \left(\frac{n}{\bar{L}} + 1\right)^{\alpha-2} \bar{L}^{-1} \right) =$$

$$\frac{R}{n} + a n^{\frac{1-\alpha}{\alpha}} (\alpha - 1) \bar{L}^{-1} \left(\left(\frac{n}{\bar{L}}\right)^{\alpha-2} - \left(\frac{n}{\bar{L}} + 1\right)^{\alpha-2} \right) > 0$$

The properties of the two equations are summarized in table A.5.

B Figures

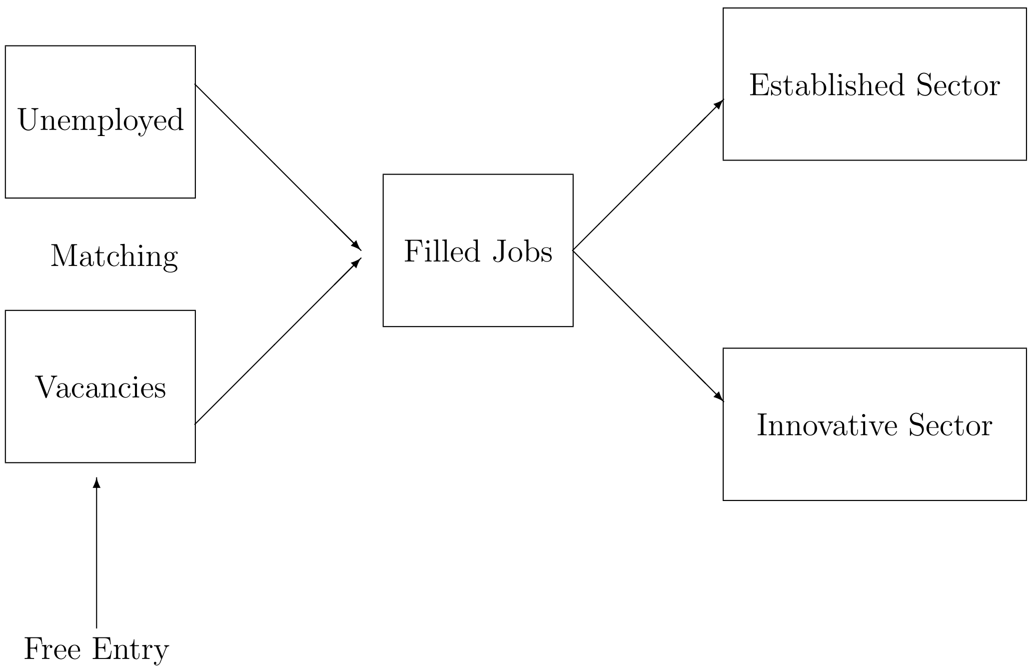

Figure 1: Structure of the Economy

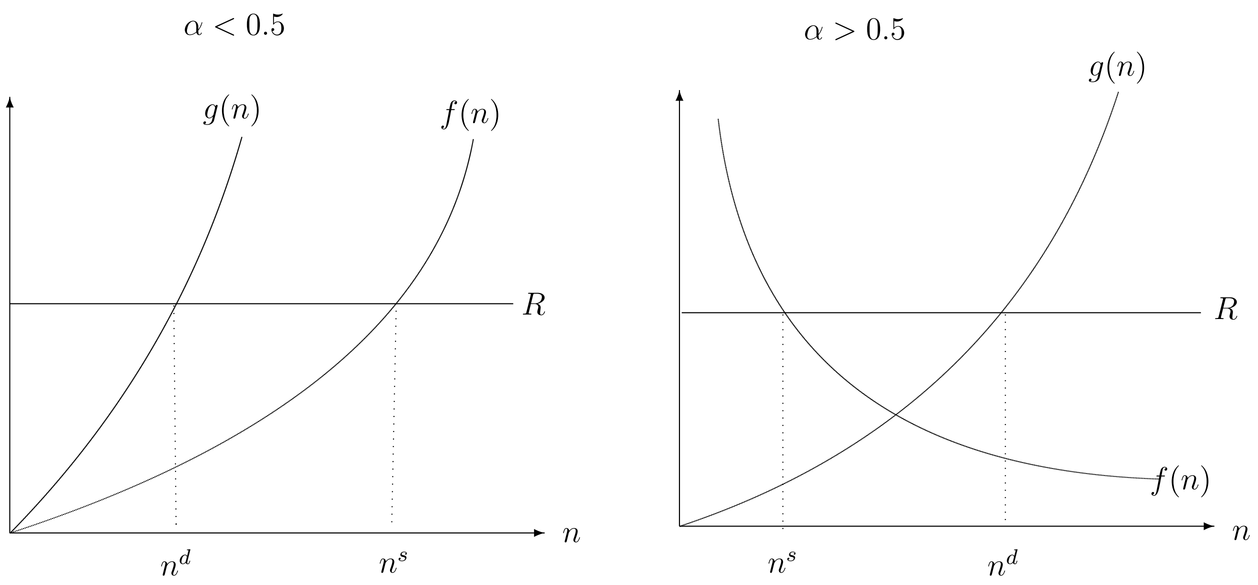

Figure 2: Product Variety in the Decentral and Social Optimum for different α