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## An Ontology of Intensional Entities

Take first-order predicate logic with identity and description and add a dyadic predicate  $P$  and three monadic categorial predicates  $Z^1, Z^0, Z^{<0>}$ .  $xPy$  says that state-of-affairs  $x$  is *intensional part* of state-of-affairs  $y$  (for example, *that George is an animal* is intensional part of *that George is a human being*).  $Z^1, Z^0, Z^{<0>}$  express being a state-of-affairs, an individual, a property (of individuals) respectively; they are mutually exclusive;  $Z^0$  is assumed to be non-empty.

For  $P$  we assume the formal characteristics of a complete atomistic Boolean algebra (as far as these are expressible in a first-order language). All Boolean functions and constants can then be defined by the itself definable operator  $cxA[x]$ : *the conjunction of all states-of-affairs that are A*:

$$\iota x[Z^1(x) \wedge \forall y (Z^1(y) \wedge A[y] \supset yPx) \wedge \forall z (Z^1(z) \wedge \forall y (Z^1(y) \wedge A[y] \supset yPz) \supset xPz)].$$

For example:  $\text{dis}(x,y) := cz(zPx \wedge zPy)$  (*the [state-of-affairs which is the] disjunction of  $x$  and  $y$* ),  $\text{con}(x,y) := cz(zPx \vee zPy)$  (*the conjunction of  $x$  and  $y$* ),  $\underline{\text{t}} := cz(z \neq z)$  (*the tautological state-of-affairs*),  $\underline{\text{c}} := czZ^1(z)$  (*the contradictory state-of-affairs*). Moreover, modal functions can be defined as well:  $\text{nec}(x) := cz(x \neq \underline{\text{t}} \wedge z = \underline{\text{c}})$ , for example, is the modal function expressed by the sentential operator of analytical necessity.

By introducing a special constant  $\underline{w}$  (*the real world*) we can define truth/actuality for states-of-affairs by:  $\bar{T}(x) := xP\underline{w}$ .  $\underline{w}$  is a possible world, which is a maximal-consistent state-of-affairs; being a maximal-consistent state-of-affairs is in its turn readily definable:

$$\text{MC}(x) := Z^1(x) \wedge x \neq \underline{\text{c}} \wedge \forall y (Z^1(y) \wedge \neg yPx \supset \text{con}(x,y) = \underline{\text{c}}).$$

Thus, truth-laws become provable for the various Boolean functions, and for the modal functions as well; for example, for all states-of-affairs  $x,y$ :

$$T(\text{con}(x,y)) \equiv T(x) \wedge T(y), T(\text{nec}(x)) \equiv \forall y (\text{MC}(y) \supset xPy).$$

We now introduce two more expressions:  $(y,x)$  (*the satiation of  $y$  by  $x$* ),  $\lambda o\pi[o]$  (*the  $\pi$ -extract*), which are characterized as follows:

$\forall y \forall x Z^1((y,x))$ ,  $Z^{<0>}(\lambda o\pi[o])$  ( $o, o', \dots$  are called "extraction-variables"; they occur only in connection with  $\lambda$ ; note that the  $\lambda$ -operator operates on terms, not on predicates);

$$\forall y \forall x (\neg Z^{<0>}(y) \vee \neg Z^0(x) \supset (y,x) = \underline{\text{c}}), \forall x (\neg Z^1(\pi[x]) \supset (\lambda o\pi[o], x) = \underline{\text{c}})$$

( $x$  not in  $\pi[o]$ );

$$\forall x (Z^1(\pi[x]) \wedge Z^0(x) \supset (\lambda o\pi[o], x) = \pi[x]) \quad (x \text{ not in } \pi[o]).$$

For properties we have the following identity-principle:

$$\forall x \forall y (Z^{<0>}(x) \wedge Z^{<0>}(y) \wedge \forall z (Z^0(z) \supset (x,z)=(y,z)) \supset x=y).$$

And for states of affairs:  $\forall x \forall y (xPy \wedge yPx \supset x=y)$ .

What can be done with this theory? The theory is a small part of a much larger intensional ontology which has infinitely many categorial predicates, while the presently considered one has only three. Yet its expressive power is surprisingly great:

$f(x) := T((f,x))$  ( $:= (f,x)Pw$ ); this is the definition of "x exemplifies f", "f is satisfied by x".

$EZ^{<0>}(y) := Z^{<0>}(y) \wedge \forall x (Z^0(x) \supset (y,x)=\underline{1} \vee (y,x)=\underline{c})$ ; this is the definition of "y is an essential property". We can prove:

$$\forall f \forall g (EZ^{<0>}(f) \wedge EZ^{<0>}(g) \wedge \forall z (Z^0(z) \supset (f(z) \equiv g(z)) \supset f=g);$$

$$\exists f (EZ^{<0>}(f) \wedge \forall z (f(z) \equiv A[z])),$$

provided  $\forall z (A[z] \supset Z^0(z))$  [consider  $\lambda ocy(-A[o] \wedge y=\underline{c})$ ].

This shows that essential properties can serve as *sets of individuals*.

The following functions are the equivalents of the quantifiers on all individuals:  $cy\exists z (Z^0(z) \wedge yP(f,z))$  (of the all-quantifier),  $cy\forall z (Z^0(z) \supset yP(f,z))$  (of the existential-quantifier). They can be shown to satisfy mirror-images of the theorems of predicate logic.

Boolean and modal functions for properties are easily definable.

For example:  $con^{<0>}(f,g) := \lambda o con((f,o), (g,o))$  (*the property of being f and g*),  $nec^{<0>}(f) := \lambda o nec((f,o))$  (*the property of being necessarily f*).

The next step is to introduce more than two-placed satiation-expressions, and more than one-place extract-expressions:  $(x, z_1, \dots, z_n)$ ,  $\lambda o_1 \dots o_n \pi[o_1, \dots, o_n]$ . This means that besides properties n-place relations (n greater 1) between individuals are drawn into consideration. By this move the theory is strengthened to such an extent that the semantics of modal (first-order) predicate logic becomes expressible (the expressions of the formal language are being treated as abstract *individuals*) without making use of sets (as primitive entities). This kind of intensional semantics in intensional ontology has a considerably different appearance from orthodox modal semantics that is developed within the extensional framework of sets and possible worlds (which, in the eyes of extensionalism, are special individuals).

It seems that the intensional ontology of which a fragment has been sketched in this paper (and which may be considered as a synthesis of ontological ideas derived from Frege and Wittgenstein) can be a real competitor to set-theory. It is no less precise than the latter, and it is closer to our ontological intuitions, albeit these are nowadays somewhat obscured by the long habituation to set-theory and (needlessly) intimidated by extensionalistic polemics (especially Quine's); but (non-nominalistic philosophical tradition up to the 19th century is intensionalistic throughout.

A final remark: For comprehensive details concerning all the points addressed in this paper the interested reader is referred to the book and the article named at the end.

*Literature*

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