Temperature models for pricing weather derivatives

Quantitative Finance, forthcoming

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Agenda

1 Literature review

2 Spline model

3 Results

4 Conclusion
Literature review

Whole zoo of models:

- **Jewson and Penzer (2004):** Index Modeling
- **Dischel (1998):** First daily simulation model
- **Cao and Wei (2000):** AR type process
- **Alaton et al. (2002):** Sine-shaped seasonality
- **Brody et al. (2002):** Long autocorrelation in temperature residues
- **Campbell and Diebold (2005):** Seasonal ARCH
- **Benth and Šaltytė-Benth (2007):** Standard OU-process with seasonal volatility

Only two contributions compare these models:

- **Oetomo and Stevenson (2005)
- **Papaziana and Skiadopoulos (2009)**
Literature review

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Spline model – Motivation

Temperatures of Houston, TX

- Day: 100, 200, 300
- Temperature: 20, 40, 60, 80
Spline model – Motivation

Temperatures of Houston, TX

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Spline model – Motivation

Temperatures of Houston, TX
Spline model – Motivation

Temperatures of Houston, TX

Year

1980
1985
1990
1995
2000
2005

Day

100
200
300

Temperature

20
40
60
80
Spline model – Motivation

Temperatures of Houston, TX

Temperature

Year

Day
Spline model – Definition

- Consider the historical temperatures of each year from shortly before the measurement period till the end of the measurement period
- Split the temperatures into a trend and seasonality component in the mean and into a trend and seasonality component in the variance:

\[ T_t = \mu_t + \sigma_t R_t, \]

where

\[ \mu, \sigma \in S_{4, K_{\text{Day}}} \otimes S_{2, K_{\text{Year}}}, \]

\[ S_{n, K} = \text{Space of splines of degree } n \text{ with knot sequence } K \]
Spline model – Autocorrelation of the residues

- Fast decline of the autocorrelation at the beginning
- **But:** Positive autocorrelations for a long time period
Spline model – AROMA process

**Main idea:** Evolution of temperatures is caused by the interaction of different processes with different time scales:

- **short-term** Changes in the atmosphere
- **mid-term** Changes of the surface temperature
- **long-term** Changes of the water temperature

**Modeling the residues with an AROMA process** (Jewson and Caballero, 2003)

\[ R_t = \phi_1 \bar{R}_{m,1,t} + \phi_2 \bar{R}_{m,2,t} + \phi_3 \bar{R}_{m,3,t} + \phi_4 \bar{R}_{m,4,t} + Z_t \]

\[ \bar{R}_{m,t} = \frac{1}{m} \sum_{i=1}^{m} R_{t-i}, \quad Z_t \sim N(0, \sigma^2) \]
Spline model – Fitting the AROMA process

- For a fixed length the parameters of a AROMA process can be estimated
- Choosing the length so that the empirical autocorrelation is fitted best

$m_1 = 1, m_2 = 2, m_3 = 8, m_4 = 31$
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Results – Backtesting

- Valuation of fictive contracts of the years 1983–2005 using
  - temperature data up to 180 days ahead of the measurement period
  - temperature data up to the middle of the measurement period.
- All models include linear detrending and use temperature data for the last 30 years
- Valuation of 12 typical contracts (6 HDD, 6 CDD) at 35 US locations
- Compare the predicted index values with realized index values
- Measure: (mean) relative error and (mean) squared relative error

\[ \delta \hat{x} = \frac{\hat{x} - x}{x}, \quad (\delta \hat{x})^2 = \left( \frac{\hat{x} - x}{x} \right)^2 \]
Results – MSRE by geographical regions, 180 days ahead of measurement period

HDD error  CDD error
Results – MSRE by geographical regions, 180 days ahead of measurement period
Results – MSRE by geographical regions, middle of measurement period

HDD error

CDD error
Results – Ranking of the models

Mann-Whitney \( U \) test

- Compare the MSRE of each pair of models
- \( H_0 : (\delta \hat{Y})_x^2 \geq (\delta \hat{Y})_y^2 \) vs. \( H_1 : (\delta \hat{Y})_x^2 < (\delta \hat{Y})_y^2 \)
- Significance at 5% level

Evaluated 180 days ahead of the measurement period:

Spline model \( \preceq \) Index Modeling \( \preceq \) Benth model \( \preceq \) Alaton model

Evaluated in the middle of the measurement period:

Spline model \( \preceq \) Alaton model \( \preceq \) Index Modeling \( \preceq \) Benth model
Results – Uncertainty

Table: Slope parameters for the relation between the realised standard deviation and the predicted standard deviation.

<table>
<thead>
<tr>
<th>Model</th>
<th>Slope</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index Modeling</td>
<td>0.9976</td>
<td>(0.9821, 1.0131)</td>
</tr>
<tr>
<td>Alaton model</td>
<td>1.2259</td>
<td>(1.1971, 1.2546)</td>
</tr>
<tr>
<td>Benth model</td>
<td>1.0793</td>
<td>(1.0498, 1.1089)</td>
</tr>
<tr>
<td>Spline model</td>
<td>1.1556</td>
<td>(1.1387, 1.1726)</td>
</tr>
</tbody>
</table>

All daily simulation models underestimate the uncertainty of the prediction.
Conclusion

- Models for temperature indices perform better when HDD indices than predicting CDD indices.
- Performance of the models depends on the geographic location of the weather station.
- Main advantage of daily simulation models when evaluating contracts during the measurement period:
  - Is this still the case when embedding meteorological temperature forecasts into the models?
- Daily simulation models underestimate the uncertainty of the prediction.