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Temperature models for pricing weather derivatives

Quantitative Finance, forthcoming

Frank Schiller* · Gerold Seidler* · Maximilian Wimmer†

*Munich Reinsurance Company
†Department of Finance, University of Regensburg

Maximilian Wimmer
Department of Finance
University of Regensburg

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Maximilian Wimmer
Department of Finance
University of Regensburg

Literature review

Whole zoo of models:

- **Jewson and Penzer (2004)**: Index Modeling
- **Dischel (1998)**: First daily simulation model
- **Cao and Wei (2000)**: AR type process
- **Alaton et al. (2002)**: Sine-shaped seasonality
- **Brody et al. (2002)**: Long autocorrelation in temperature residues
- **Campbell and Diebold (2005)**: Seasonal ARCH
- **Benth and Šaltytė-Benth (2007)**: Standard OU-process with seasonal volatility

Only two contributions compare these models:

- **Oetomo and Stevenson (2005)**
- **Papaziana and Skiadopoulos (2009)**

Maximilian Wimmer
Department of Finance
University of Regensburg

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Department of Finance
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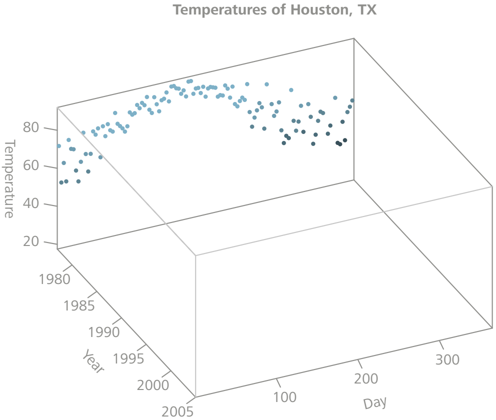
Spline model – Motivation



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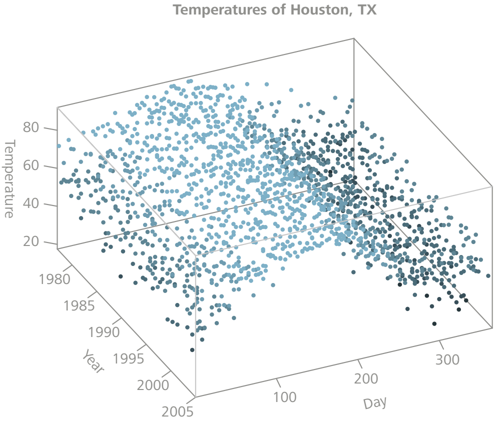

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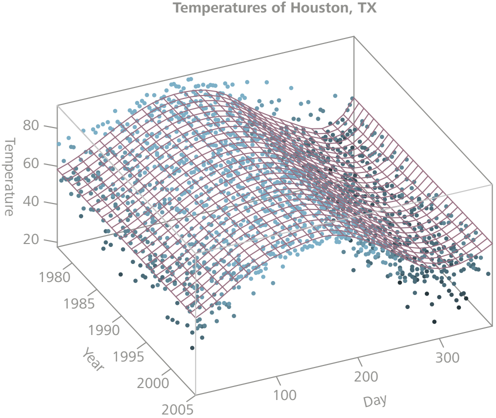

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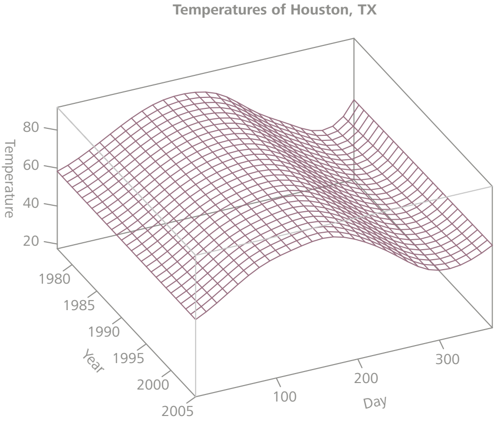

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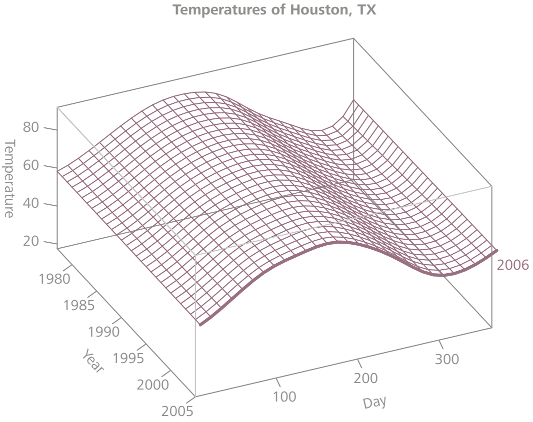

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Spline model – Definition

- Consider the historical temperatures of each year from shortly before the measurement period till the end of the measurement period
- Split the temperatures into a trend and seasonality component in the mean and into a trend and seasonality component in the variance:

$$T_t = \mu_t + \sigma_t R_t,$$

where

$$\mu, \sigma \quad \in \quad \mathbb{S}_{4,K_{\text{Day}}} \otimes \mathbb{S}_{2,K_{\text{Year}}},$$

$$\mathbb{S}_{n,K} = \text{Space of splines of degree } n \text{ with knot sequence } K$$

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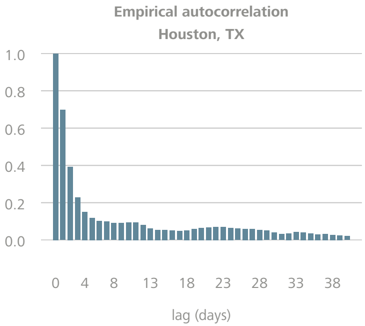
Spline model – Autocorrelation of the residues



- Fast decline of the autocorrelation at the beginning
- **But:** Positive autocorrelations for a long time period

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Spline model – AROMA process

- **Main idea:** Evolution of temperatures is caused by the interaction of different processes with different time scales:

short-term	Changes in the atmosphere
mid-term	Changes of the surface temperature
long-term	Changes of the water temperature

- Modeling the residues with an AROMA process (**Jewson and Caballero, 2003**)

$$R_t = \phi_1 \bar{R}_{m_1,t} + \phi_2 \bar{R}_{m_2,t} + \phi_3 \bar{R}_{m_3,t} + \phi_4 \bar{R}_{m_4,t} + Z_t$$

$$\bar{R}_{m,t} = \frac{1}{m} \sum_{i=1}^{m} R_{t-i}, \quad Z_t \sim N(0, \sigma_\phi^2)$$

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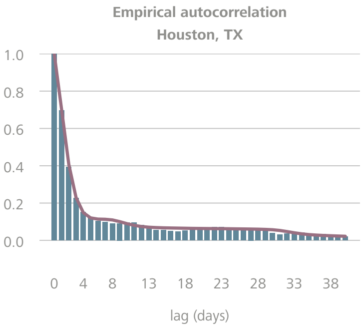
Spline model – Fitting the AROMA process

- For a fixed length the parameters of a AROMA process can be estimated
- Choosing the length so that the empirical autocorrelation is fitted best



$$m_1 = 1, m_2 = 2, m_3 = 8, m_4 = 31$$

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Department of Finance
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Department of Finance
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Results – Backtesting

- Valuation of fictive contracts of the years 1983–2005 using
  - temperature data up to 180 days ahead of the measurement period
  - temperature data up to the middle of the measurement period.
- All models include linear detrending and use temperature data for the last 30 years
- Valuation of 12 typical contracts (6 HDD, 6 CDD) at 35 US locations
- Compare the predicted index values with realized index values
- **Measure:** (mean) relative error and (mean) squared relative error

$$\delta\hat{x} = \frac{\hat{x} - x}{x}, \quad (\delta\hat{x})^2 = \left(\frac{\hat{x} - x}{x}\right)^2$$

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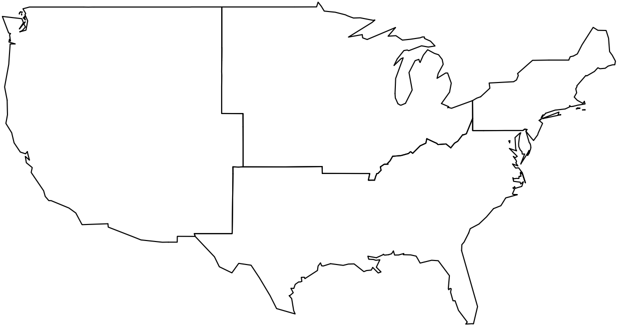
Department of Finance
University of Regensburg

Results – MSRE by geographical regions, 180 days ahead of measurement period



HDD error **CDD error**

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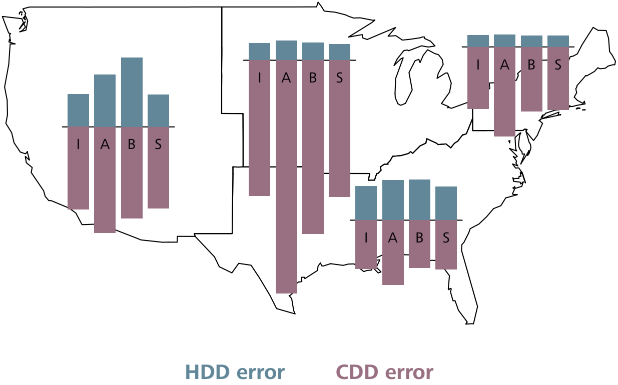

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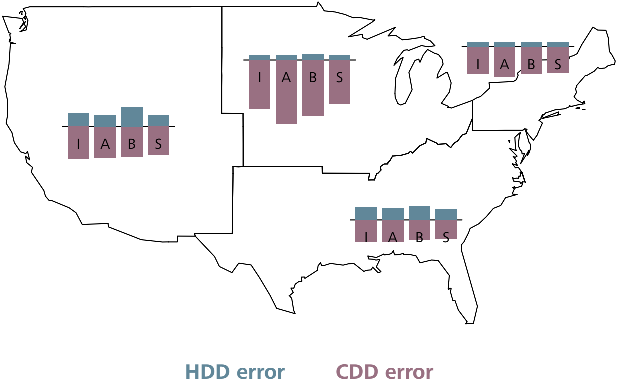
Results – MSRE by geographical regions, middle of measurement period



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Results – Ranking of the models

Mann-Whitney *U* test

- Compare the MSRE of each pair of models
- $H_0 : (\delta\hat{Y})^2_x \geq (\delta\hat{Y})^2_y$ vs. $H_1 : (\delta\hat{Y})^2_x < (\delta\hat{Y})^2_y$
- Significance at 5% level

Evaluated 180 days ahead of the measurement period:

$$\text{Spline model} \prec \text{Index Modeling} \prec \text{Benth model} \prec \text{Alaton model}$$

Evaluated in the middle of the measurement period:

$$\text{Spline model} \prec \text{Alaton model} \prec \text{Index Modeling} \prec \text{Benth model}$$

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Department of Finance
University of Regensburg

Results – Uncertainty

Table: Slope parameters for the relation between the realised standard deviation and the predicted standard deviation.

	Slope	**95% Confidence Interval**
Index Modeling	0.9976	(0.9821, 1.0131)
Alaton model	1.2259	(1.1971, 1.2546)
Benth model	1.0793	(1.0498, 1.1089)
Spline model	1.1556	(1.1387, 1.1726)

All daily simulation models underestimate the uncertainty of the prediction

Maximilian Wimmer
Department of Finance
University of Regensburg

Conclusion

- Models for temperature indices perform better when HDD indices than predicting CDD indices.
- Performance of the models depends on the geographic location of the weather station
- Main advantage of daily simulation models when evaluating contracts during the measurement period
  - Is this still the case when embedding meteorological temperature forecasts into the models?
- Daily simulation models underestimate the uncertainty of the prediction

Universität Regensburg

Dr. Maximilian Wimmer

Department of Finance
93040 Regensburg
Germany

ph: +49 (941) 943 - 2672
fax: +49 (941) 943 - 81 2672
maximilian.wimmer@wiwi.uni-regensburg.de
http://www-finance.uni-regensburg.de