Rethinking risk capital allocation in a RORAC framework

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Motivation

Bank

Two lines of business

Retail lending

Structured finance

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Individual EC Retail lending

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Diversification

Total EC

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Two lines of business

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Total EC

Capital allocation

Retail lending

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Diversification

EC Retail lending

EC Structured finance

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Motivation

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Total EC

Capital allocation

Total RORAC = \frac{Total Profit}{Total EC}

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\[ \Delta RORAC_{RL} = \frac{\Delta \text{Profit}_{RL}}{\Delta EC_{RL}} \]

\[ \Delta RORAC_{SF} = \frac{\Delta \text{Profit}_{SF}}{\Delta EC_{SF}} \]

\[ \text{Total RORAC} = \text{Total Profit} \frac{1}{\text{Total EC}} \]

Expand business

Reduce business

\[ \Delta RORAC_{RL} > \text{Total RORAC} \]

\[ \Delta RORAC_{SF} < \text{Total RORAC} \]
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Total EC

Capital allocation

Total RORAC = \frac{Total \ Profit}{Total \ EC}

\Delta RORAC_{RL} = \frac{\Delta Profit_{RL}}{\Delta EC_{RL}}

\Delta RORAC_{RL} > Total \ RORAC

Expand business

\Delta RORAC_{SF} = \frac{\Delta Profit_{SF}}{\Delta EC_{SF}}

\Delta RORAC_{SF} < Total \ RORAC

Reduce business
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Total EC

Capital allocation

Total RORAC = \frac{\text{Total Profit}}{\text{Total EC}}

\Delta \text{RORAC}_{RL} = \frac{\Delta \text{Profit}_{RL}}{\Delta \text{EC}_{RL}}

\Delta \text{RORAC}_{RL} > \text{Total RORAC}

Expand business

\Delta \text{RORAC}_{SF} = \frac{\Delta \text{Profit}_{SF}}{\Delta \text{EC}_{SF}}

\Delta \text{RORAC}_{SF} < \text{Total RORAC}

Reduce business

\Delta \text{Profit}_{RL} = \Delta \text{RORAC}_{RL} \cdot \Delta \text{EC}_{RL}

\Delta \text{Profit}_{SF} = \Delta \text{RORAC}_{SF} \cdot \Delta \text{EC}_{SF}

max?
Outline

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1 Literature review

2 Notation and preliminaries

3 Example for failing capital allocation

4 Second-order approach

5 Conclusion
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Literature review

- Mathematical finance context
  - Denault (2001): Axiomatic approach in a game theory setting
  - Kalkbrener (2005): Axiomatic system
  - Tasche (2004): Suitability for performance measurement, gradient allocation principle
  - Buch and Dorfleitner (2008): Coherence of gradient allocation principle

- Insurance-linked perspective
  - Dhaene et al. (2003): Coherent risk measures not optimal
  - Furman and Zitikis (2008): Weighted allocation

- Financial economics viewpoint
  - Merton and Perold (1993): Incremental allocation

Does gradient allocation lead a firm to its optimal RORAC?
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Model and notation

\( F \) Firm consisting of \( n \) individual segments

\( u_k \) Number of contracts written by segment \( k \), \( u = (u_1, \ldots, u_n) \)

\( Y_k(u_k) \) Profit of segment \( k \), \( Y_k(u_k) = M_k(u_k) + X_k(u_k) \)

\( M_k(u_k) \) Expected profit of segment \( k \), \( M(u) = \sum_{k=1}^{n} M_k(u_k) \)

\( X_k(u_k) \) Profit fluctuation of segment \( k \), \( X(u) = \sum_{k=1}^{n} X_k(u_k) \), \( X(u) \) is linear WRT \( u \)

\( \rho \) Convex risk measure

\( \rho_X \) Risk function \( \rho_X : \mathbb{R}^n \rightarrow \mathbb{R} \), \( \rho_X : u \mapsto \rho(X(u)) \)

Information asymmetry

- Headquarters can evaluate the risk \( \rho(\cdot) \), but has only knowledge of the current expected profit \( M(u) \)

- Divisions can estimate the expected profit \( M_k(u_k + \epsilon_k) \), but not the overall risk \( \rho(\cdot) \)
Preliminaries—RORAC and marginal RORAC

**Definition**
The function \( r_{M,\rho_X} : U \rightarrow \mathbb{R} \) defined as

\[
r_{M,\rho_X} : u \mapsto \frac{M(u)}{\rho_X(u) - M(u)}
\]

is called **return function associated with** \( m, X, \) and \( \rho \)

**Definition**
Given per-unit risk contribution \( a_k \), such that \( \sum_k a_k(u)u_k = \rho_X(u) \), one can define the **marginal RORAC** by

\[
\frac{M'_k(u_k)}{a_k(u) - M'_k(u_k)}
\]
Preliminaries—Suitability for performance measurement

**Definition**
An allocation $a_1, \ldots, a_n$ is called **suitable for performance measurement** if there holds:

1. For all portfolios $u \in U$ and for all differentiable profit functions $M : U \to \mathbb{R}$ with $\rho_x(u) \neq 0$ and $k \in \mathbb{N}$ the inequality
   \[
   \frac{M'_k(u_k)}{a_k(u_k) - M'_k(u_k)} > r_{M,\rho_x}(u)
   \]
   implies that there is an $\epsilon > 0$ such that for all $\tau \in (0, \epsilon)$ we have
   \[
   r_{M,\rho_x}(u) < r_{M,\rho_x}(u + \tau e_k)
   \]

2. “The other way around”

**Theorem**
The gradient or Euler allocation, i.e.,
\[
a_k(u) = \frac{\partial \rho_x(u)}{\partial u_k},
\]
*is the only allocation that is suitable for performance measurement*
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Example for failing capital allocation

General setup:

Two divisions

\[ M_1(u_1) = \log(u_1 + \frac{1}{2}) \]
\[ M_2(u_2) = \log(u_2 + \frac{1}{2}) \]

\[ X_{1,2} \sim N(0, 1) \]

\[ \text{corr}(X_1, X_2) = 0.5 \]

\[ \rho = 99.97\%-\text{VaR} \]
Example for failing capital allocation

Let $u_1^{(1)} = 1.5$, $u_2^{(1)} = 1.7$

Then,

$$r_{M, X}(u^{(1)}) = 18.451\%$$

Marginal RORAC analysis leads to

$$\frac{M_1'(u_1^{(1)})}{a_1(u^{(1)}) - M_1'(u_1^{(1)})} = 20.775\%$$

and

$$\frac{M_2'(u_2^{(1)})}{a_2(u^{(1)}) - M_2'(u_2^{(1)})} = 17.647\%$$
Example for failing capital allocation

How far to go?
Approximate additional profit by

$$M_k(u_k + \epsilon_k) - M_k(u_k)$$

instead of $$\epsilon_k M'_k(u_k)$$ and replace

$$\frac{\epsilon_k M'_k(u_k)}{\epsilon_k a_k(u) - \epsilon_k M'_k(u_k)} > r_{M,\rho_X}(u)$$

by

$$\frac{M_k(u_k + \epsilon_k) - M_k(u_k)}{\epsilon_k a_k(u) - (M_k(u_k + \epsilon_k) - M_k(u_k))} > r_{M,\rho_X}(u)$$
Example for failing capital allocation

Let \( u_1^{(2)} = 1.85, \ u_2^{(2)} = 1.55 \)

Then,

\[
M_1'(u_1^{(2)}) \over a_1(u^{(2)}) - M_1'(u_1^{(2)}) \] = 16.190\%

Marginal RORAC analysis leads to

and

\[
M_2'(u_2^{(2)}) \over a_2(u^{(2)}) - M_2'(u_2^{(2)}) \] = 20.397\%
Example for failing capital allocation

Let
\[ u_1^{(2)} = 1.85, \quad u_2^{(2)} = 1.55 \]

Then,
\[ r_{M,\rho_X}(u^{(1)}) = 18.410\% \]

Marginal RORAC analysis leads to
\[
\frac{M'_1(u_1^{(2)})}{a_1(u^{(2)}) - M'_1(u_1^{(2)})} = 16.190\%
\]

and
\[
\frac{M'_2(u_2^{(2)})}{a_2(u^{(2)}) - M'_2(u_2^{(2)})} = 20.397\%
\]
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A second-order approach

**Theorem**

Assume that

- $H(u) = \left[ \frac{\partial^2 \rho_X(u)}{\partial u_i \partial u_j} \right]$ is the Hessian of $\rho_X(u)$
- $\|H(u)\|$ is bounded on a convex set $U \subseteq \mathbb{R}_n^{\geq 0}$
- $\Lambda \geq \max_{u \in U} \lambda_{\text{max}}(H(u))$ is an upper bound for the largest eigenvalue of $H(u)$
- $u \in U$, $u + \epsilon \in U$, $M(u) > 0$, $r_{M,\rho_X}(u) > 0$
- For all $k \in N$ there holds

$$\frac{M_k(u_k + \epsilon_k) - M_k(u_k)}{(\epsilon_k a_k(u) + \frac{1}{2} \epsilon_k^2 \Lambda) - (M_k(u_k + \epsilon_k) - M_k(u_k))} \geq r_{M,\rho_X}(u),$$

with strict inequality given for at least one $k \in N$

Then there also holds

$$r_{M,\rho_X}(u + \epsilon) > r_{M,\rho_X}(u)$$
Example (continued)
Example (continued)
Example (continued)
Example (continued)

\[
\begin{align*}
&1 \quad 1.50 \\
&2 \quad 1.55 \\
&3 \quad 1.60 \\
&4 \quad 1.65 \\
&5 \quad 1.70 \\
&6 \quad 1.75 \\
&7 \quad 1.80
\end{align*}
\]

\[u^{\text{opt}}\]
Example (continued)
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Conclusion

- (Slightly) extended setting as in Tasche (2004): Concave expected profit function
- Here: Only strictly stationary profit process considered
- The implementation of a naïve gradient capital allocation in firms can be suboptimal if division managers are allowed to venture into all business whose marginal RORAC exceeds the firm’s RORAC
- If the marginal RORAC requirements are refined by adding a risk correction term that takes into account the interdependencies of the risks of different lines of business, it can be guaranteed that the optimal RORAC will be achieved eventually (under the assumption of a strictly stationary profit process)

Financial crisis check

- Higher requirements on the yields of signed contracts
- Less piling of tons of CDO tranches
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