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Rethinking risk capital allocation in a RORAC framework

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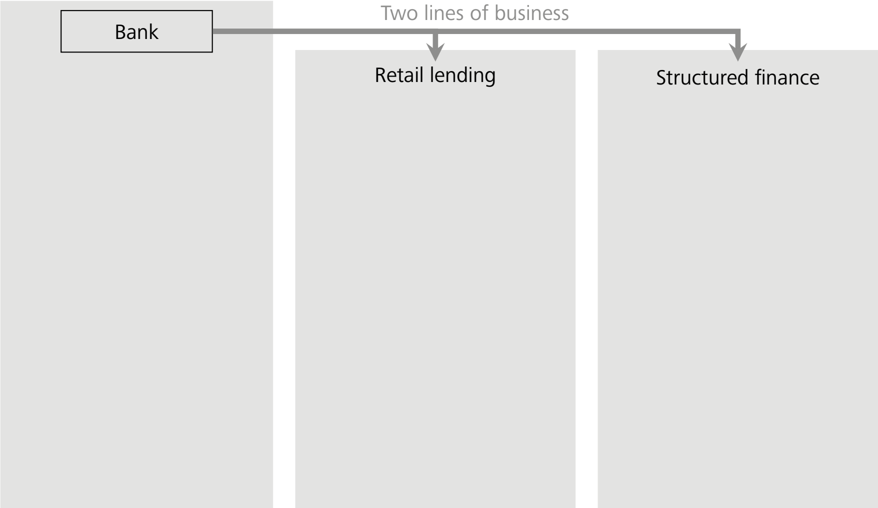
Motivation



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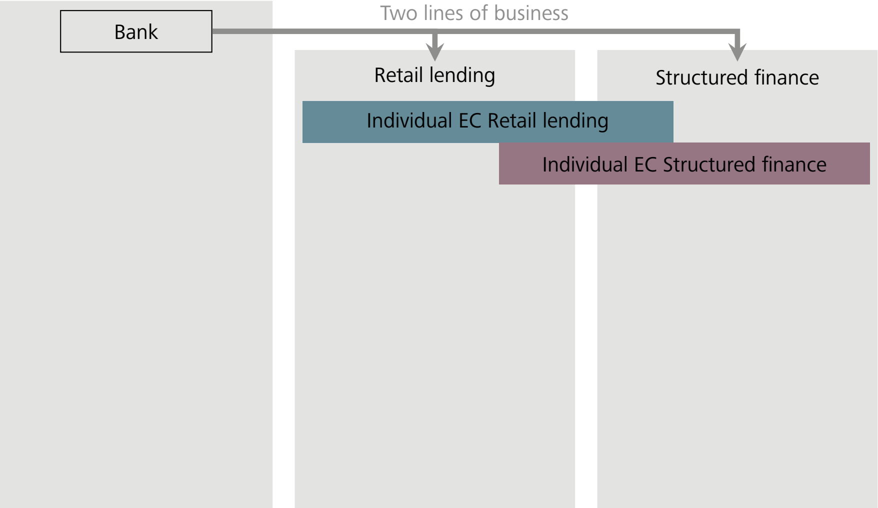

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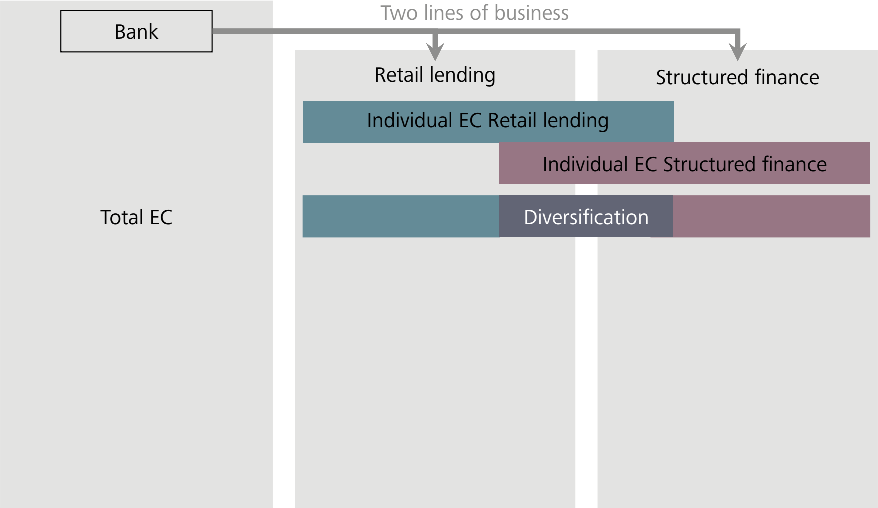

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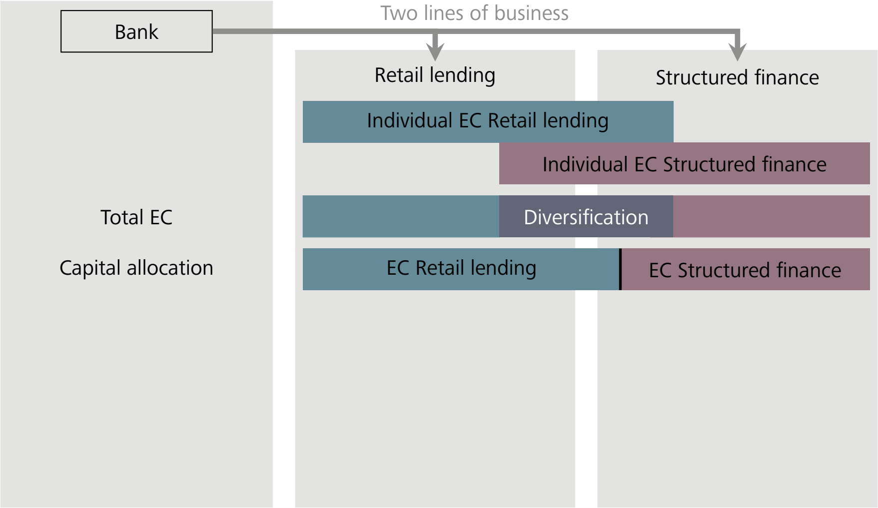

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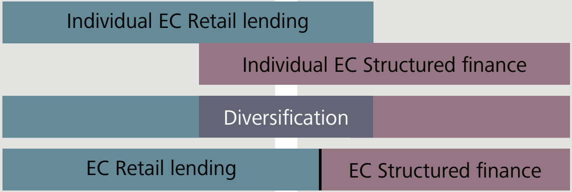
Motivation

Bank

Two lines of business

Retail lending | Structured finance



Total EC

Capital allocation

$$\text{Total RORAC} = \frac{\text{Total Profit}}{\text{Total EC}}$$

$$\Delta \text{RORAC}_{RL} = \frac{\Delta \text{Profit}_{RL}}{\Delta \text{EC}_{RL}}$$

$$\Delta \text{RORAC}_{SF} = \frac{\Delta \text{Profit}_{SF}}{\Delta \text{EC}_{SF}}$$

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Motivation

Bank

Two lines of business

Retail lending

Structured finance

Individual EC Retail lending

Individual EC Structured finance

Total EC

Diversification

Capital allocation

EC Retail lending

EC Structured finance

$$\text{Total RORAC} = \frac{\text{Total Profit}}{\text{Total EC}}$$

$$\Delta \text{RORAC}_{RL} = \frac{\Delta \text{Profit}_{RL}}{\Delta \text{EC}_{RL}}$$

$$\Delta \text{RORAC}_{RL} > \text{Total RORAC}$$

Expand business

$$\Delta \text{RORAC}_{SF} = \frac{\Delta \text{Profit}_{SF}}{\Delta \text{EC}_{SF}}$$

$$\Delta \text{RORAC}_{SF} < \text{Total RORAC}$$

Reduce business

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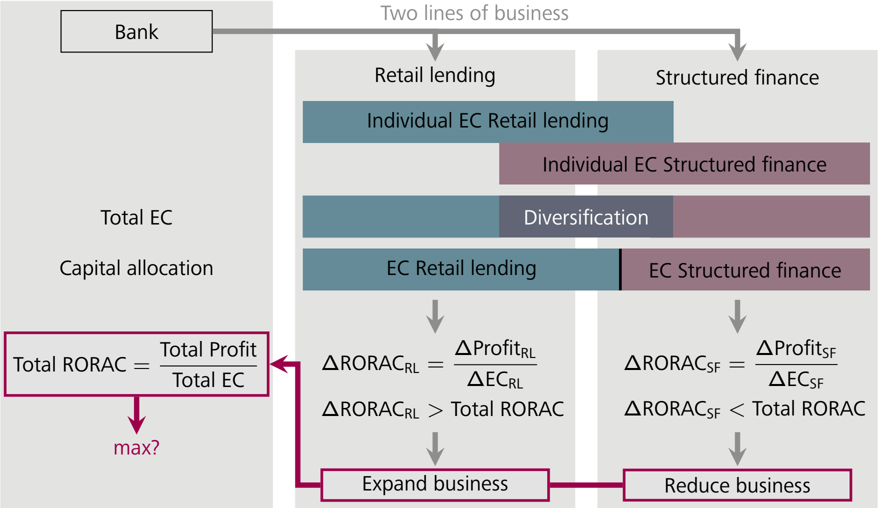

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Outline

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Literature review

- Mathematical finance context
  - **Denault (2001)**: Axiomatic approach in a game theory setting
  - **Kalkbrener (2005)**: Axiomatic system
  - **Tasche (2004)**: Suitability for performance measurement, gradient allocation principle
  - **Buch and Dorfleitner (2008)**: Coherence of gradient allocation principle
- Insurance-linked perspective
  - **Dhaene et al. (2003)**: Coherent risk measures not optimal
  - **Furman and Zitikis (2008)**: Weighted allocation
- Financial economics viewpoint
  - **Merton and Perold (1993)**: Incremental allocation
  - **Stoughton and Zechner (2007)**: Economic optimization problem

Does gradient allocation lead a firm to its optimal RORAC?

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Model and notation

F	Firm consisting of n individual segments
u_k	Number of contracts written by segment k, $u = (u_1, \dots, u_n)$
$Y_k(u_k)$	Profit of segment k, $Y_k(u_k) = M_k(u_k) + X_k(u_k)$
$M_k(u_k)$	Expected profit of segment k, $M(u) = \sum_{k=1}^{n} M_k(u_k)$
$X_k(u_k)$	Profit fluctuation of segment k, $X(u) = \sum_{k=1}^{n} X_k(u_k)$, $X(u)$ is linear WRT u
ρ	Convex risk measure
ρ_X	Risk function $\rho_X : \mathbb{R}^n \to \mathbb{R}$, $\rho_X : u \mapsto \rho(X(u))$

Information asymmetry

- Headquarters can evaluate the risk $\rho(\cdot)$, but has only knowledge of the current expected profit $M(u)$
- Divisions can estimate the expected profit $M_k(u_k + \epsilon_k)$, but not the overall risk $\rho(\cdot)$

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Preliminaries—RORAC and marginal RORAC

Definition

The function $r_{M,\rho_X} : U \to \mathbb{R}$ defined as

$$r_{M,\rho_X} : u \mapsto \frac{M(u)}{\rho_X(u) - M(u)}$$

is called **return function associated with m, X, and ρ**

Definition

Given per-unit risk contribution a_k, such that $\sum_k a_k(u) u_k = \rho_X(u)$, one can define the **marginal RORAC** by

$$\frac{M'_k(u_k)}{a_k(u) - M'_k(u_k)}$$

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Preliminaries—Suitability for performance measurement

Definition

An allocation $a_1, \ldots, a_n$ is called **suitable for performance measurement** if there holds:

1. For all portfolios $u \in U$ and for all differentiable profit functions $M : U \to \mathbb{R}$ with $\rho_X(u) \neq 0$ and $k \in N$ the inequality

$$\frac{M_k'(u_k)}{a_k(u) - M_k'(u_k)} > r_{M,\rho_X}(u)$$

   implies that there is an $\epsilon > 0$ such that for all $\tau \in (0, \epsilon)$ we have

$$r_{M,\rho_X}(u) < r_{M,\rho_X}(u + \tau e_k)$$

2. "The other way around"

Theorem

The gradient or Euler allocation, i.e.,

$$a_k(u) = \frac{\partial \rho_X(u)}{\partial u_k},$$

is the only allocation that is suitable for performance measurement

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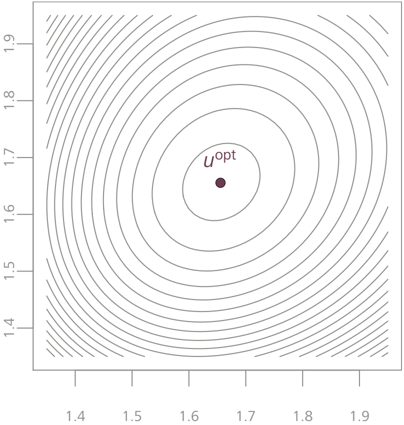
Example for failing capital allocation



General setup:

Two divisions

$$M_1(u_1) = \log(u_1 + \tfrac{1}{2})$$

$$M_2(u_2) = \log(u_2 + \tfrac{1}{2})$$

$$X_{1,2} \sim \mathrm{N}(0,1)$$

$$\mathrm{corr}(X_1, X_2) = 0.5$$

$$\rho = 99.97\%\text{-VaR}$$

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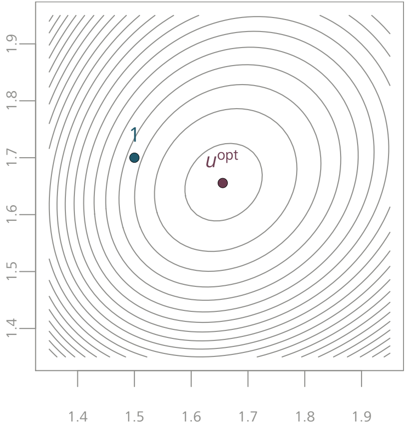
Example for failing capital allocation



Let

$$u_1^{(1)} = 1.5,\ u_2^{(1)} = 1.7$$

Then,

$$r_{M,\rho_X}(u^{(1)}) = 18.451\%$$

Marginal RORAC analysis leads to

$$\frac{M_1'(u_1^{(1)})}{a_1(u^{(1)}) - M_1'(u_1^{(1)})} = 20.775\%$$

and

$$\frac{M_2'(u_2^{(1)})}{a_2(u^{(1)}) - M_2'(u_2^{(1)})} = 17.647\%$$

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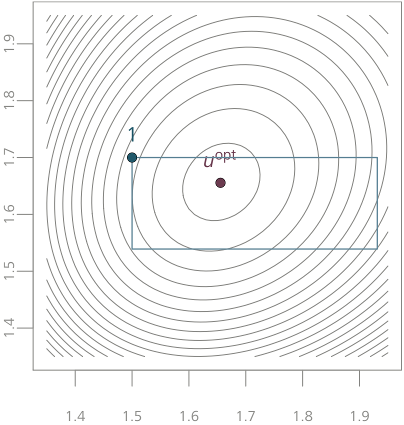
Example for failing capital allocation



How far to go?
Approximate additional profit by

$$M_k(u_k + \epsilon_k) - M_k(u_k)$$

instead of $\epsilon_k M_k'(u_k)$ and replace

$$\frac{\epsilon_k M_k'(u_k)}{\epsilon_k a_k(u) - \epsilon_k M_k'(u_k)} > r_{M,\rho_X}(u)$$

by

$$\frac{M_k(u_k + \epsilon_k) - M_k(u_k)}{\epsilon_k a_k(u) - (M_k(u_k + \epsilon_k) - M_k(u_k))} > r_{M,\rho_X}(u)$$

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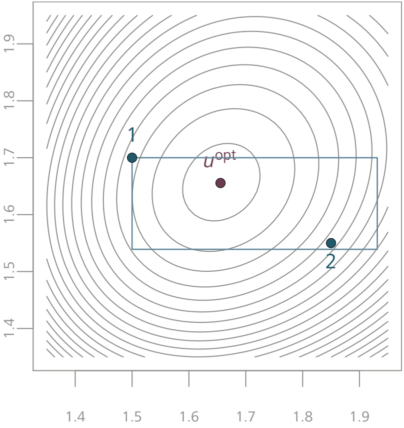
Example for failing capital allocation



Let

$$u_1^{(2)} = 1.85,\ u_2^{(2)} = 1.55$$

Then,

$$r_{M,\rho_X}(u^{(1)}) = 18.410\%$$

Marginal RORAC analysis leads to

$$\frac{M_1'(u_1^{(2)})}{a_1(u^{(2)}) - M_1'(u_1^{(2)})} = 16.190\%$$

and

$$\frac{M_2'(u_2^{(2)})}{a_2(u^{(2)}) - M_2'(u_2^{(2)})} = 20.397\%$$

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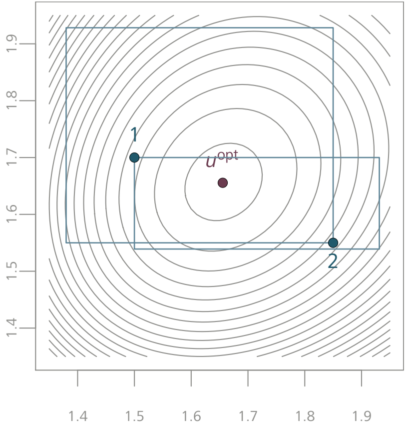
Example for failing capital allocation



Let

$$u_1^{(2)} = 1.85,\ u_2^{(2)} = 1.55$$

Then,

$$r_{M,\rho_X}(u^{(1)}) = 18.410\%$$

Marginal RORAC analysis leads to

$$\frac{M_1'(u_1^{(2)})}{a_1(u^{(2)}) - M_1'(u_1^{(2)})} = 16.190\%$$

and

$$\frac{M_2'(u_2^{(2)})}{a_2(u^{(2)}) - M_2'(u_2^{(2)})} = 20.397\%$$

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A second-order approach

Theorem

Assume that

- $H(u) = \left[\frac{\partial^2 \rho_X(u)}{\partial u_i \partial u_j}\right]$ *is the Hessian of* $\rho_X(u)$
- $\|H(u)\|$ *is bounded on a convex set* $U \subseteq \mathbb{R}^n_{\geq 0}$
- $\Lambda \geq \max_{u \in U} \lambda_{\max}(H(u))$ *is an upper bound for the largest eigenvalue of* $H(u)$
- $u \in U$*,* $u + \epsilon \in U$*,* $M(u) > 0, r_{M,\rho_X}(u) > 0$
- *For all* $k \in N$ *there holds*

$$\frac{M_k(u_k + \epsilon_k) - M_k(u_k)}{(\epsilon_k a_k(u) + \frac{1}{2}\epsilon_k^2 \Lambda) - (M_k(u_k + \epsilon_k) - M_k(u_k))} \geq r_{M,\rho_X}(u),$$

with strict inequality given for at least one $k \in N$

Then there also holds

$$r_{M,\rho_X}(u + \epsilon) > r_{M,\rho_X}(u)$$

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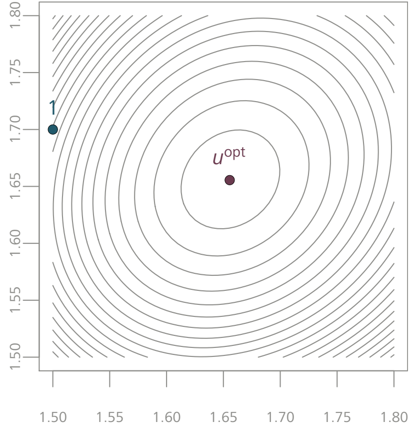
Example (continued)



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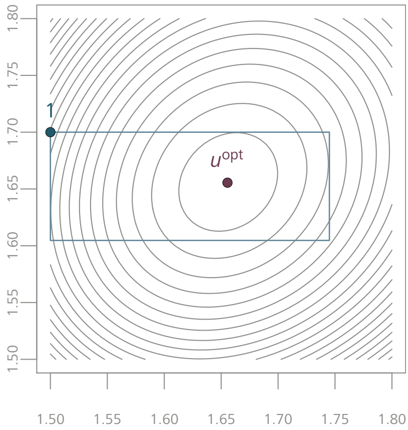
Example (continued)



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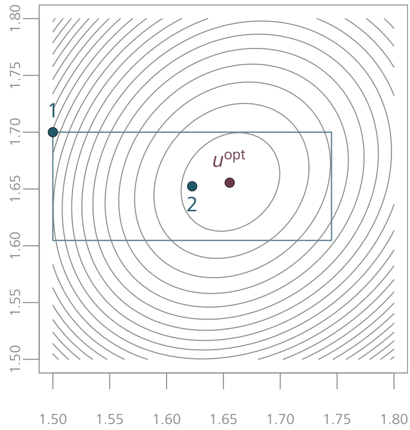
Example (continued)



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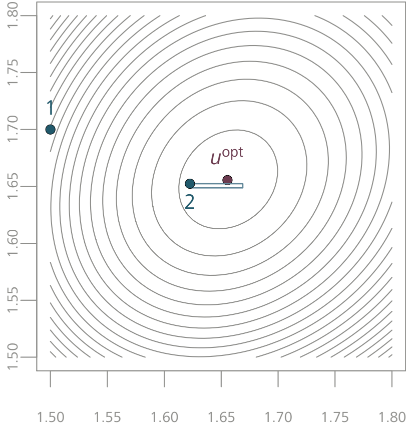

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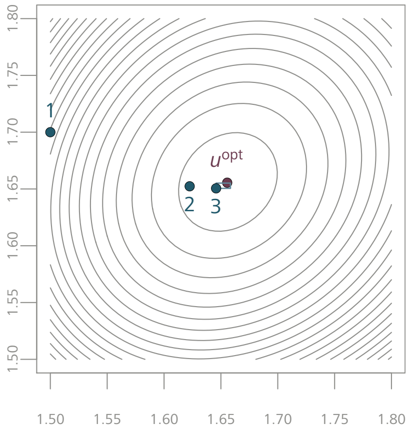
Example (continued)



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Conclusion

- (Slightly) extended setting as in **Tasche (2004)**: Concave expected profit function
- Here: Only strictly stationary profit process considered
- The implementation of a naïve gradient capital allocation in firms can be suboptimal if division managers are allowed to venture into all business whose marginal RORAC exceeds the firm's RORAC
- If the marginal RORAC requirements are refined by adding a risk correction term that takes into account the interdependencies of the risks of different lines of business, it can be guaranteed that the optimal RORAC will be achieved eventually (under the assumption of a strictly stationary profit process)

Financial crisis check

- Higher requirements on the yields of signed contracts
- Less piling of tons of CDO tranches

✓

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