A CONSISTENT NONRELATIVISTIC QUARK MODEL
WITH DYNAMICAL PION EXCHANGE

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We investigate the nonrelativistic quark model with a residual pion–quark interaction of the Weinberg–Schwinger type including p-wave and s-wave coupling. The pion–nucleon vertex is renormalised to third order in the pion–quark coupling constant f_π and adjusted to reproduce the empirical pion–nucleon coupling strength, leading to an axial charge $g_A = 1.42$ and a value of $f_\pi = 98.8$ MeV. Including the pion exchange currents dynamically through one-pion loops, we reproduce the nucleon magnetic moments within less than 10% for any quark core radius $0.3 \text{ fm} \leqslant r_q \leqslant 0.7 \text{ fm}$. All these results change by less than 5% if the pion mass is varied from zero to its physical value.

Several attempts have recently been made [1–7] to add pion exchange effects to the nonrelativistic quark model (NRQM). The formal motivation for this is to approximately incorporate chiral symmetry, which in principle can be done also for consitituent quarks [8]. A practical motivation comes from the application of the NRQM to baryon spectroscopy. Indeed, the inclusion of pion exchange as an additional residual interaction between the quarks – beside the one-gluon exchange (OGE) – helps to explain [5,7] the experimentally known smallness of the spin–orbit splittings in the baryon spectrum (especially in the non-strange sector). Since the pion exchange also contributes to the nucleon (N)–delta (Δ) splittings, the quark–gluon coupling constant α_s can hereby be considerably reduced [4,5] (to $\alpha_s \simeq 0.35$) compared to the value ($\simeq 1$) necessary for fitting the ground state N–Δ splitting by the OGE alone [9] which leads to far too strong spin–orbit splittings.

The aim of the present paper is to apply the NRQM with dynamical pion exchange to electromagnetic properties of the nucleons. At the same time we shall bring the model closer to chiral symmetry by including an s-wave type second-order pion–quark coupling. The value of the pion–quark coupling constant (which was an adjustable parameter in refs. [4,5]) is fixed to reproduce the empirical pion–nucleon coupling after a renormalization of the pion–nucleon vertex by pion loop corrections. We shall show that this model is practically equivalent to the cloudy bag model (CBM) [10] and leads to reasonable values of the nucleon magnetic moments and the axial charge g_A rather independently of the quark core radius.

We use the following pion–quark interaction:

$$\mathcal{H}_{\pi q} = \psi^+(r) [(1/2f_\pi)\boldsymbol{\sigma} \cdot i\nabla \boldsymbol{\tau} \cdot \boldsymbol{\phi}(r) - (1/2f_\pi)^2 \boldsymbol{\tau} \cdot \boldsymbol{\phi}(r) \times i\partial_0 \boldsymbol{\phi}(r)] \psi(r) , \tag{1}$$

where $\psi(r)$ is the quark wave function of the NRQM and $\phi(r)$ the quantized pion field; f_π is the coupling constant which will be determined below. The interaction (1) has the usual p-wave part and an s-wave part which is nonlinear in the pion field. Its relativistic equivalent (the "Weinberg lagrangian" [11]) can be derived from a chirally invariant nonlinear coupling of pions to massless quarks by a local chiral rotation of the quark spinors and expansion up to second order in the pion field. (See ref. [12] where this procedure was used to

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derive the "volume coupled" version of the CBM.) In order to determine now the value of f_π, we shall relate it to the pion–nucleon vertex. The general $\pi B_1 B_2$ vertex is $\langle B_2 | \sum_{i=1}^3 \mathcal{H}_{\pi q}(i) | \pi B_1 \rangle$, where $|B_1\rangle$, $|B_2\rangle$ are NRQM three-quark baryon states. This vertex contains a structure form factor $\exp(-q^2/6\alpha_0^2)$ which is just the Fourier transform of the gaussian factor in the quark wave functions. Hereby $\boldsymbol{q}$ is the momentum of the incoming pion and α_0 is the inverse rms radius of the nucleon in the ground state (the "quark core radius" r_q). In the diagrams below, this vertex shall be denoted by an empty blob. Since we are going to include pionic loop corrections to the electromagnetic nucleon vertex below, it is consistent to add such corrections also to the $\pi B_1 B_2$ vertices. This leads, besides renormalizing the coupling strength, to a form factor $F_{\pi B_1 B_2}(q^2)$ shown in the diagrams by a dashed blob. To determine f_π and $F_{\pi B_1 B_2}(q^2)$ consistently, we solve iteratively the following coupled integral equations:

$$\text{(2)}$$

in which $F_{\pi B B'}(k^2)$ appears on the RHS at the dashed vertices under the momentum ($\boldsymbol{k}$) integrals coming from the pion loops. (See ref. [13] for the explicit form of the equations and for calculational details.) In eq. (2), $Z_2^\pi(B)$ is the wave function renormalization factor (see, e.g., ref. [10]) of the baryon B defined by $Z_2^\pi(B) = 1/[1 - (\partial/\partial E)\Sigma_B^\pi(E)|_{E=M_B}]$ in terms of the pionic self energy correction Σ_B^π (see, e.g., refs. [6,10]). We use only one form factor in eq. (2), as well as in Σ_B^π, to avoid double counting of the leading terms of order $(f_\pi)^4$ implicitly included.

In the iterative solution of eq. (2), we restrict the summation over the intermediate states B, B' to the ground state N and Δ only, although reasonable convergence can be obtained [6,13] including higher resonances with the form factors we obtain in this way. The loops in eq. (2) and Σ_B^π are calculated in usual nonrelativistic perturbation theory (using the physical masses of N and Δ and including their recoil in the determinators); summation over all possible time orderings is understood in all diagrams. In each iteration, the form factors are fitted for $q \leqslant q_{max} = 0.8$ GeV^{-1} to a monopole from

$$F_{\pi B_1 B_2}(q^2) = 1/[1 + (q/\Lambda_{\pi B_1 B_2})^2] \,. \tag{3}$$

(The results depend to less than 10% on the precise value of q_{max} up to which the monopole fit is done.) The value of f_π entering at each vertex on the RHS of eq. (2) is determined by equating the LHS of eq. (2) at $q=0$ with the renormalized value f_π^* which is chosen to reproduce the experimentally known πNN coupling constant

$$g_{\pi NN} = 13.45, \quad f_\pi^* = 5M_N/3g_{\pi NN} = 116.2 \text{ MeV} \,. \tag{4}$$

This leads to a cubic equation for the unrenormalized f_π to be solved at each iteration. After convergence we find $f_\pi = 98.8$ MeV when using the standard value $\alpha_0 = 426$ MeV (corresponding to a quark core radius $\langle r^2 \rangle^{1/2} = r_q = 1/\alpha_0 = 0.46$ fm) which ensures a correct spectroscopy [4,5]. Note that this value of f_π differs only by 6% from the empirical value of the pion decay constant $f_\pi = 93.3$ MeV. The corresponding values of $\Lambda_{\pi B_1 B_2}$ in the monopole form factors (3) are

$$\Lambda_{\pi NN} = 1.77 \text{ GeV}, \quad \Lambda_{\pi N\Delta} = 1.54 \text{ GeV}, \quad \Lambda_{\pi\Delta\Delta} = 1.22 \text{ GeV} \,. \tag{5}$$

(Note that if the structure form factor $\exp(-q^2/6\alpha_0^2)$ were included in the monopole fit, one would approximately find 0.90, 0.86 and 0.79 GeV, respectively, for the three cutoffs in eq. (5).) With the above values for f_π and the form factors, we checked the convergence of eq. (2) with respect to inclusion of higher resonances in the intermediate states. Even though these were shown to affect strongly the self energies Σ_B^π in ref. [6], we found here that the $Z_2^\pi(B)$ change much less. The resulting value of f_π varies, in fact, less than 5% and the $Z_2^\pi(B)$ less than 10% upon including intermediate states up to the $N=2$ harmonic oscillator shell, thus justifying our truncation of the model space to N and Δ ground states.
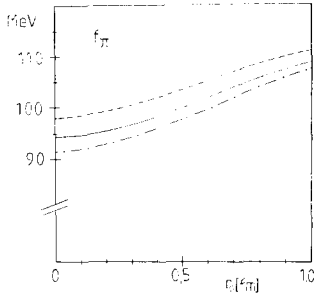
Fig. 1. Coupling constant f_π versus quark core radius r_q. Solid line: including s-wave interaction, $m_\pi = 138$ MeV; dashed line: without s-wave interaction, $m_\pi = 138$ MeV; dash-dotted line: including s-wave interaction, $m_\pi = 0$.

The result for f_π depends also very little on the choice of the quark core radius, i.e. the constant α_0 in the wave functions. In fig. 1 we show the results for f_π versus $r_q = 1/\alpha_0$. Also shown there is the result obtained when the s-wave part of the interaction (1) is omitted. We see thus that its inclusion brings us indeed much closer to the experimental value of f_π which would be that of a chirally symmetric model. Another sign of the closeness of chiral symmetry is the result (shown by a dash-dotted line) obtained in the soft-pion limit, i.e. taking the pion mass m_π to be zero. The behaviour of the curves in fig. 1 can be easily analyzed in a simplified "toy model" in which the pionic form factors (3) are put to unity, recoil and N–Δ mass difference in the intermediate states are ignored and the pion mass is zero. All the loop integrals can then be done analytically and the cubic equation for f_π following from eq. (2) becomes

$$\frac{1}{f_\pi^*} = \frac{1}{f_\pi} \left\{ \frac{4\pi^2 + \frac{45}{8}(\alpha_0/f_\pi)^2}{4\pi^2 + \frac{57}{8}(\alpha_0/f_\pi)^2} \right\} = \frac{1}{f_\pi} Z_1^\pi(N) Z_2^\pi(N) , \tag{6}$$

where $Z_1^\pi(N)$ is the vertex renormalization factor accounting for the diagrams on the RHS of eq. (2) for the πNN vertex at $q = 0$. Eq. (6) becomes linear in the two limits $\alpha_0 = 0$ (infinite radius) and $\alpha_0 = \infty$ (zero radius); the solution for f_π thus has to move between the limits f_π^* and $\frac{45}{57} f_\pi^*$, respectively. The same factors as in eq. (6) also renormalize the axial charge: $g_A = \frac{5}{3} Z_1^\pi(N) \, Z_2^\pi(N)$, leading to $g_A = \frac{5}{3} \cdot \frac{45}{57} = \frac{25}{19} = 1.31$ for zero radius and to the usual (unrenormalized) nonrelativistic value of $\frac{5}{3}$ for an infinite radius. (Without the s-wave pion–quark interaction, one would obtain $\frac{5}{3} \cdot \frac{49}{57} = 1.43$ for zero radius.) In our full calculation corresponding to the solid line in fig. 1, we get $Z_1^\pi(N) = 1.52$, $Z_2^\pi(N) = 0.561$ and thus $g_A = 1.42$ for the standard radius of 0.46 fm. Our results can easily be seen to fulfill the Goldberger–Treiman relation to within less than 10% for quark core radii $r_q \leqslant 0.7$ fm.

It is very interesting that in the CBM [10] with linearized γ_5 coupling, the same toy model calculation ("soft-π" limit, no form factors and degenerate N, Δ without recoil) leads to exactly the same equation (6), replacing 45 by 49 in the numerator, since there is no s-wave quark coupling in this version of the CBM, and replacing α_0^{-2} by $\frac{2}{3} R^2$, where R is the bag radius. This shows that our results are very little dependent on the fact that we use massive quarks and nonrelativistic wavefunctions, once the quark core radius is suitably chosen. Indeed, the quark mass has not entered our model so far; the only parameter besides f_π is the constant α_0. The relativistic quark spinors in the CBM only give rise to a different structure form factor (a Bessel function instead of a gaussian; in fact, in practical calculations in the CBM [10] a gaussian was used, too, for numerical convenience). After integration in the toy model limit, the same algebraical coefficients appear in eq. (6) besides a rescaling of the quark core radius. (A similar analysis can also be made for the nucleon magnetic moments [13].)

We now turn to the discussion of the electromagnetic form factors of the nucleon. To second order in the pion–quark coupling, they are given by the following set of diagrams:

$$(7)$$

which are obtained from the interaction (1) by minimal coupling to the electromagnetic field (see ref. [13] for calculational details). The cross-hatched blob at the vertex on the LHS of eq. (7) signifies the (nonrelativistic) electromagnetic form factors $G^T_{E,M}(q^2)$ ($T=0$ and 1 for isoscalar and isovector case, respectively). On the RHS the renormalized coupling constant f^*_π is used everywhere; the dashed blobs include the selfconsistently obtained form factors (3), (5). The $G^T_E(q^2)$ should be normalized to $G^0_E(0)=G^1_E(0)=e/2$, where e is the unit charge. It is, indeed, straightforward to see that the isoscalar charge, to which only diagrams (a) and (d) in eq. (7) contribute, is exactly $e/2$, i.e. the isoscalar electric current is exactly conserved. (Hereby it is important to include the same number of pion–quark form factors in the corresponding loops of eqs. (2), (7) and in Σ^π_B.) The isovector charge, however, to which also diagrams (b) and (c) contribute (the contributions of (e) and (f) cancel exactly), turns out to deviate by about 3% from the value $e/2$. We see thus that our interaction hamiltonian does not conserve exactly the isovector electric current. The violation is, however, small and we correct it by renormalizing ad hoc the isovector charge to $e/2$. (We should like to emphasize at this point that violation of the isovector charge conservation also occurs in the CBM as used in ref. [10].)

After this renormalization, we obtain the magnetic moments of the nucleons from $G^0_M(0)=(\mu_p+\mu_n)/2$ $G^1_M(0)=(\mu_p-\mu_n)/2$. Since these moments are expressed in units of $e\hbar/2m_q c$, they depend explicitly on the constituents quark mass m_q. In the simple NRQM without poins, one has $\mu_p=e\hbar/2m_q c$ (and $\mu_n=-\frac{2}{3}\mu_p$); the quark mass is therefore chosen to be $m_q=330$ MeV approximately. Here, however, the "quark core term" (diagram (a) in eq. (7)) is reduced by the factor $Z^\pi_2(N)<1$. On the other hand, the diagrams (b), (c) and (d) also contribute positively to μ_p, bringing the total magnetic moment back to approximately the experimental one, similary as in the CBM case [10]. It might be interesting to note that in the *static* one-pion-exchange approximation, we find that the pion exchange currents (sum of Seagull and pole terms) contribute much less to the magnetic moment and depend rather sensitively on the value of the pion mass. For $m_\pi=0$ they give about 13% of the isovector magnetic moment (see also ref. [1]); with $m_\pi=138$ MeV they cancel to about 3%. In contrast to this, the *dynamical* pion exchange through diagrams (b), (c), (d) in eq. (7) gives about 35–45% of the total μ_N (for $r_q\simeq0.5$ fm), rather independently of the pion mass.

Our resulting magnetic moments (in units of the experimental μ_p) are shown in fig. 2 as functions of the quark core radius $r_q=1/\alpha_0$. The solid lines give the results including only ground state N and Δ as intermediate states in the loops in eq. (7); in the dashed lines the contributions of all intermediate states up to the $N=2$
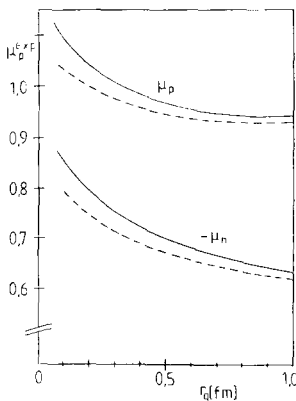

Fig. 2. Proton and neutron magnetic moments (in units of experimental proton moment) versus quark core radius r_q. solid lines: including only the $N=0$ oscillator shell; dashed lines: including $N=0, 1, 2$ oscillator shells for the intermediate states.

harmonic oscillator shell are included. Again a good convergence is found for the summation over intermediate states. The constituents quark mass has been chosen to be the standard value $m_q = 330$ MeV. The results do not depend strongly on the quark core radius; for $r_q \geqslant 0.25$ fm we reproduce the experimental magnetic moments within about 10%. We thus come to the interesting result that the sum of quark core and pionic contributions to the magnetic moments has the right magnitude and depends little on the core radius within reasonable limits, a result which is also found in relativistic chiral quark models [10,14]. Compared to the latter, our model has the advantage that the CM motion is treated correctly and that the convergence of the summation over intermediate states has been ensured without any ad hoc cutoff. (In the CBM calculatations [10] only ground state N and Δ were included; in ref. [14], where pion loops were calculated on the quark level, only intermediate 1s and 1p states were included.)

For the mean squared charge radii we find with $m_q = 330$ MeV and $\alpha_0 = 426$ MeV the following results:

$$\langle r^2 \rangle_E^p = (0.67 \text{ fm})^2, \quad \langle r^2 \rangle_E^n = -(0.30 \text{ fm})^2 . \tag{8}$$

(It should be mentioned that the loops in diagrams (e) and (f) of eq. (7), which contribute to the isovector charge radii, diverge. We have regularized them with a monopole form factor with cutoff $\lambda \simeq 4\pi f_\pi \simeq 1$ GeV (see ref. [8]); their contribution to the radii in eq. (8) then is less than 3%.) If we include the empirical charge form factor of the pion at the $\gamma\pi\pi$-vertices in eq. (7), we obtain the following results:

$$\langle r^2 \rangle_E^p = (0.71 \text{ fm})^2, \quad \langle r^2 \rangle_E^n = -(0.34 \text{ fm})^2 , \tag{9}$$

which are in fair agreement with experiment. (The pionic charge radius can, in prinicple, be obtained from the assumption of vector meson dominance; we feel, however, that we would overstress our model by also including ρ-mesons.)

In summary, we can say that we obtain very reasonable electromagnetic properties of the nucleons in our NRQM using the pion–quark interaction (1) and including pion loops. Note that we could fine-tune our results by changing the quark mass m_q and/or choosing an appropriate oscillator parameter α_0 (i.e. quark core radius). We want, however, to keep the spectroscopy of the baryon resonances in mind, which is the particular strength of the NRQM. We therefore have demonstrated that we can obtain reasonable results with values of m_q and α_0 that are compatible with baryon spectroscopy [4,5].

As already noted in ref. [6], the dynamical pion exchange alone does not yield the full experimental N–Δ splitting of 293 MeV, if the pion–quark coupling constant is adjusted to the empirical pion–nucleon coupling constant as we have done it here. In fact, with the present results of eqs. (2)–(5) we find a pionic contribution to this splitting (including only N and Δ intermediate states and the standard parameters $m_q = 330$ MeV, $\alpha_0 = 426$ MeV) of $\text{Re}(\Sigma_\Delta^\pi) - \Sigma_N^\pi = 220$ MeV. This splitting is smaller than the empirical one; it would be further reduced by higher intermediate states [6]. In order to obtain the correct N–Δ splitting, one still has to include a weak residual OGE interaction; in first-order perturbation we would need a quark–gluon coupling constant $\alpha_s \simeq 0.25$–0.35 to account for the full splitting. Note that this is in line with the value $\alpha_s \simeq 0.28$ obtained in ref. [8]. This residual OGE interaction will, of course, also affect the electromagnetic form factors of the nucleon (see, e.g., ref. [15]) and thus modify our above results. A new calculation of the baryon spectrum, taking pion *and* gluon exchange into account and aiming at a simultaneous fit of electromagnetic properties of the nucleons and the baryonic excitation spectrum, is in progress [16].

In conclusion, it appears possible to formulate an approximately chiral NRQM with a residual interaction dominated by dynamical pion exchange, yielding satisfactory electromagnetic properties of the nucleons and applicable to baryon spectroscopy due to a correct treatment of the centre of mass. The parameters of the model are: (1) the pion–quark coupling constant f_π which can be taken to be the empirical pion decay constant; (2) the constituent quark mass m_q of the order of (but not necessarily equal to!) 330 MeV to give the correct nucleon magnetic moments; (3) the harmonic oscillator constant $\alpha_0 = \sqrt{\hbar\omega m_q}$ of the order 400–450 MeV, corresponding to an oscillator frequency $\hbar\omega = 500$–550 MeV, to give the correct gross structure of the baryon spectrum; and (4) an OGE coupling constant α_s of the order of 0.25–0.35 to obtain a reasonable fine structure of the spectrum.

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