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LEPTODERMOUS EXPANSION OF FINITE-NUCLEUS INCOMPRESSIBILITY*

R.C. NAYAK[1]

Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Canada H3C 3J7

J.M. PEARSON[2] and M. FARINE[3,4]

Universität München, D-8000 München, Fed. Rep. Germany
and
Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Canada H3C 3J7

P. GLEISSL and M. BRACK

Universität Regensburg, D-8400 Regensburg, Fed. Rep. Germany

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Abstract: We consider the influence of higher-order terms in the leptodermous expansion used to extract the incompressibility K_v of infinite nuclear matter from data on the breathing mode of finite nuclei. The terms we calculate are the curvature term $K_{cv}A^{-2/3}$, the surface-symmetry term $K_{ss}I^2A^{-1/3}$, the quartic volume-symmetry term K_4I^4, and a Coulomb-exchange term. Working within the framework of the scaling model we derive expressions for their coefficients in terms of quantities that are defined for infinite and semi-infinite nuclear matter. We calculate these coefficients for four different Skyrme-type forces, using the extended Thomas-Fermi (ETF) approximation. With the same forces we also calculate the incompressibility $K(A, I)$ for a number of finite nuclei, fit the results to the leptodermous expansion, and thereby extract new results for the same coefficients. The comparison of the two calculations shows that the leptodermous expansion is converging rapidly. Of the new terms, the term K_4I^4 is quite negligible, the curvature term should be included, and we discuss to what extent the other higher-order terms are significant.

1. Introduction

There has recently been a considerable revival of interest in the question of the incompressibility of nuclear matter, prompted in the first instance by the belief that it is of crucial importance for the very occurrence of supernova explosions; for a summary of the current situation, see ref. [1]). Clearly, it is of the greatest importance to see how much information on nuclear-matter incompressibility can be extracted from laboratory nuclear physics. This is a challenge not only to the experimentalist but also to the theoretician, since there has to be an extrapolation from finite nuclei

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[1] Permanent address: Department of Physics, Khallikote College, Berhampur, 760001 India.

[2] Permanent address: Université de Montréal.

[3] Present address: Laboratoire de Spectroscopie Nucléaire, Université de Nantes, 44072 Nantes, France.

[4] Fellow of the A. von Humboldt Foundation.

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to infinite nuclear matter. (Actually, in the astrophysical context this extrapolation is particularly tenuous, since one is dealing with nuclear matter that is much more neutron-rich than is ever the case for stable nuclei.)

Experimental information on the incompressibility of nuclear matter comes from a variety of sources. Thus, Sagawa *et al.* [2]) investigated the sensitivity of the lead isotope shifts (of the r.m.s. charge radius) to K_v, the incompressibility of symmetric infinite nuclear matter, and found that 217 MeV worked much better than 269 MeV. Looking at the details of the charge-density differences between several lead isotopes, Cavedon *et al.* [3]) found their results to be compatible with $K_v = 228$ MeV; $K_v = 209$ MeV would definitely be too low. [A similar study by Co' and Speth [4]) required a K_v of about 345 MeV, but this conclusion was criticized by Bartel *et al.* [5]) on the grounds that pairing had been neglected.]

However, the traditional and most prolific source of information has been the giant isoscalar monopole resonance, the so-called breathing mode. Two different procedures can be used for extracting values of K_v from the measured energies. Firstly, we have the approach of making RPA calculations [6]), or semi-classical approximations thereto [7]), of the breathing mode in several finite nuclei for various effective forces characterized by different values of K_v. The force that has the best agreement with experiment then determines the best value of K_v. A value lying in the range 215 to 230 MeV appears to be quite consistent with the data in this approach [6,7]).

The second, more direct approach [8]), defines a finite-nucleus incompressibility $K(A, I)$ in terms of the corresponding breathing-mode energy, E_{br}, thus

$$K(A, I) = (M/\hbar^2)\langle r^2\rangle E_{br}^2 \tag{1}$$

where $I = (N-Z)/A$ is the neutron-excess parameter, and then makes use of the fact that according to the scaling model of the breathing mode the following leptodermous expansion is possible:

$$K(A, I) = K_v + K_{sf}A^{-1/3} + K_{vs}I^2 + K_{Coul}Z^2A^{-4/3} + \cdots . \tag{2}$$

Fitting eq. (2) to the breathing-mode data then permits in principle a determination not only of K_v, but also of the other parameters, K_{sf}, K_{vs}, etc. At the same time, Blaizot [8]) expresses K_{sf} and K_{vs} in terms of properties of infinite nuclear matter (INM) and semi-infinite nuclear matter (SINM), properties that can be calculated directly for different effective forces.

Treiner *et al.* [9]) have raised the question of higher-order terms, in particular a curvature term, in the leptodermous expansion (2). If all terms of the next order in the small parameters I^2 and $A^{-1/3}$ are included we have

$$K(A, I) = K_v + K_{sf}A^{-1/3} + K_{vs}I^2 + K_{Coul}Z^2A^{-4/3} + K_4I^4 + K_{ss}I^2A^{-1/3} + K_{cv}A^{-2/3} \tag{3}$$

A curvature term $K_{cv}A^{-2/3}$ was in fact included by Sharma *et al.* [10]) in a fit of recent high-precision measurements of breathing-mode energies; they found $K_v \simeq 300$ MeV and $K_{sf} \simeq -750$ MeV. Their numerical fitting procedure using a limited set of finite nuclei was later tested [11]) by using model calculations of $K(A, I)$ for different Skyrme-type forces whose asymptotic values of K_v and K_{sf} are known (see also the results presented in the present paper), confirming thereby the results of ref. [10]).

The main purpose of the present note is to extend the work of ref. [8]) by expressing the new coefficients, K_4, K_{ss} and K_{cv}, in terms of properties of INM and SINM. Thus by calculating scaling-model compressions of both SINM and finite nuclei for different forces we can test the rate of convergence of the expansion (3), and thereby decide which terms should be retained in fitting the breathing-mode data. It must be stressed that all the considerations of this paper suppose the validity of the scaling model, the limitations of which are discussed in refs. [7,11]).

The forces we consider all have the Skyrme form,

$$\begin{aligned} v_{ij} = & \, t_0(1+x_0P_\sigma)\delta(\boldsymbol{r}_{ij}) + t_1(1+x_1P_\sigma)[p_{ij}^2\delta(\boldsymbol{r}_{ij}) + \text{h.a.}]/2\hbar^2 \\ & + t_2(1+x_2P_\sigma)\boldsymbol{p}_{ij}\cdot\delta(\boldsymbol{r}_{ij})\boldsymbol{p}_{ij}/\hbar^2 + \tfrac{1}{6}t_3(1+x_3P_\sigma)\rho^\gamma\delta(\boldsymbol{r}_{ij}) \\ & + (i/\hbar^2)W_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\boldsymbol{p}_{ij}\times\delta(\boldsymbol{r}_{ij})\boldsymbol{p}_{ij}\,. \end{aligned} \tag{4}$$

Specifically, we consider the SkM*, RATP, SkA, and S3 parametrizations, all of which are conveniently summarized by Brack *et al.* [12]) (hereafter referred to as BGH). The energies of all systems, i.e., finite nuclei, INM, and SINM, are calculated in the semi-classical extended Thomas–Fermi (ETF) approximation, without any shell corrections. We use the full fourth-order expansions (in powers of $\hbar$) of the kinetic-energy and spin-current densities, τ_q and $\boldsymbol{J}_q$, respectively, as given by Grammaticos and Voros [13,14]). The energy density $\mathscr{E}(\boldsymbol{r})$, which gives the total energy as

$$E_{\text{ETF}} = \int \mathscr{E}(\boldsymbol{r})\,\mathrm{d}^3\boldsymbol{r}\,, \tag{5}$$

becomes a function of the nucleon densities, ρ_q, and their gradients: see BGH [12]) for more details (we follow them in omitting terms in $\boldsymbol{J}_q^2$).

2. Calculation of the coefficients of the leptodermous expansion

We suppose that the density distributions of neutrons ($q = \text{n}$) and protons ($q = \text{p}$) in finite nuclei (always supposed to be spherical) take the generalized Fermi form

$$\rho_q(r) = \frac{\rho_{cq}}{[1+\exp\{(r-C_q)/a_q\}]^{\gamma_q}}\,. \tag{6}$$

The equilibrium values of all the parameters appearing here are determined by minimization of the total energy. In SINM the distributions are given by

$$\rho_q(z) = \frac{\rho_{cq}}{[1+\exp\{(z-C_q)/a_q\}]^{\gamma_q}}. \tag{7}$$

[The generalization from the ordinary Fermi distribution, $\gamma_n = \gamma_p = 1$, skews the distribution in the surface; this has a significant effect on the equilibrium densities [12,15]), and thus on the calculated incompressibility [7]).]

The scaling model of nuclear compressions supposes that at all points the densities are shifted from their equilibrium values according to

$$\rho_q(r) \to \lambda^3 \rho_q(\lambda r), \tag{8}$$

where the same scaling factor λ is applied to the neutron and proton distributions. The finite-nucleus incompressibility is then given by

$$K(A, I) = \left[\frac{d^2}{d\lambda^2} e(\lambda)\right]_{\lambda=1}$$
$$= 9\left[\rho_c^2 \frac{d^2 e}{d\rho_c^2}\right]_{eq}, \tag{9}$$

where $\rho_c = \rho_{cn} + \rho_{cp}$, e is the energy per nucleon of the system in question, and the right-hand side of the second expression is evaluated at the equilibrium values of ρ_{cn} and ρ_{cp}. Note particularly that the derivative in this latter expression is a total one, so that a variation of density at all points is implied, consistent with the scaling of eq. (8).

We have for the total energy

$$E \equiv eA = e^{\infty}(\rho_c, \delta_c)A + 4\pi R^2 b_0(\rho_c, \delta_c) + 8\pi R b_1(\rho_c, \delta_c) + \cdots, \tag{10}$$

where

$$\delta_c = (\rho_{cn} - \rho_{cp})/\rho_c, \tag{11}$$

$e^{\infty}(\rho_c, \delta_c)$ is the energy per nucleon in INM, while b_0 and b_1 relate to SINM, as described in sect. 5 of BGH [12]). Also, following eq. (5.15) of BGH, we have

$$R = (\rho_0/\rho_c)^{1/3}(r_0 A^{1/3} - c_0 a_0) + O(A^{-1/3}). \tag{12}$$

Here ρ_0 is the equilibrium density of symmetric INM, and $r_0 = (3/4\pi\rho_0)^{1/3}$. Also c_0 and a_0 are determined from symmetric SINM at equilibrium, a_0 being the value of $a_p = a_n$ in eq. (7), and

$$c_0 = \int_{-\infty}^{\infty} dy\{(1+e^y)^{-\gamma} + (1+e^{-y})^{-\gamma} - 1\}. \tag{13}$$

Then

$$e = e^{\infty}(\rho_c, \delta_c) + 4\pi r_0^2(\rho_0/\rho_c)^{2/3} b_0(\rho_c, \delta_c)A^{-1/3}$$
$$+ 8\pi r_0\{(\rho_0/\rho_c)^{1/3} b_1(\rho_c, \delta_c) - c_0 a_0(\rho_0/\rho_c)^{2/3} b_0(\rho_c, \delta_c)\}A^{-2/3}, \tag{14}$$

in which it should be realized that not all the $A^{-1/3}$ dependence has been shown explicitly, since ρ_c is slightly A-dependent. Setting

$$\varepsilon = (\rho_c - \rho_0)/\rho_0 \tag{15}$$

(this is −3 times the ε of the droplet-model literature [16]), we expand (14) in terms of ε and δ_c:

$$\begin{aligned} e^{\infty}(\rho_c, \delta_c) = {} & a_v + \tfrac{1}{18}K_v\varepsilon^2 - \tfrac{1}{162}K'\varepsilon^3 + \tfrac{1}{1944}K''\varepsilon^4 + \cdots \\ & + \delta_c^2\{J + \tfrac{1}{3}L\varepsilon + \tfrac{1}{18}K_{sym}\varepsilon^2 - \tfrac{1}{162}K'_{sym}\varepsilon^3 + \cdots\} \\ & + \delta_c^4\{M + \tfrac{1}{3}U\varepsilon + \tfrac{1}{18}V\varepsilon^2 + \cdots\}, \end{aligned} \tag{16a}$$

$$\begin{aligned} b_0(\rho_c, \delta_c) = {} & \sigma_0 + \tfrac{1}{3}C\varepsilon + \tfrac{1}{18}D\varepsilon^2 - \tfrac{1}{162}G\varepsilon^3 + \cdots \\ & + \delta_c^2\{\tau_0 + \tfrac{1}{3}\Lambda\varepsilon + \tfrac{1}{18}\Gamma\varepsilon^2 + \cdots\}, \end{aligned} \tag{16b}$$

$$b_1(\rho_c, \delta_c) = \mu_0 + \tfrac{1}{3}X\varepsilon + \tfrac{1}{18}Y\varepsilon^2 + \cdots. \tag{16c}$$

Eq. (16a) relates to INM, and all the coefficients are calculated analytically for the different Skyrme forces. The coefficients a_v, J, M, K_v, and L are familiar from the droplet-model literature [16]), with J being the volume-symmetry coefficient, while K_v appears already in eq. (3) as the incompressibility of symmetric INM. Neither a_v nor M are involved in any of the final expressions (21) for the coefficients of eq. (3).

Eqs. (16b) and (16c) both relate to SINM, with

$$\sigma_0 = (1/4\pi r_0^2)a_{sf}, \tag{17a}$$

$$\tau_0 = (9/16\pi r_0^2)J^2/Q, \tag{17b}$$

$$\mu_0 = (1/8\pi r_0)a_{cv} + c_0 a_0 \sigma_0. \tag{17c}$$

Here a_{sf} is the droplet-model surface coefficient, Q the surface-symmetry stiffness coefficient, and a_{cv} the curvature coefficient [16]). These three quantities are defined with respect to the equilibrium configuration of SINM, while all the other coefficients of eqs. (16b) and (16c) represent derivatives of these quantities with respect to the limiting density ρ_c. We describe below in more detail the way in which all the coefficients of eqs. (16b) and (16c) are determined by calculations on SINM. However, we comment here on the presence in (16b) of the C-coefficient, which according to the so-called "$\dot{\sigma} = 0$" theorem [16,17]) should vanish: the point is that this theorem will not be strictly valid in the present calculation since the variations of the density are restricted by the parametrization (7).

To proceed with the formal development, we substitute the expansions (16) into the expression (14) for the energy per nucleon of a finite nucleus. Minimizing with respect to ρ_c (or ε) and δ_c for the given values of I and A determines the equilibrium values of ρ_c and δ_c. For the latter, the usual droplet-model expression [16]),

$$\delta_c = \frac{I}{1 + (9J/4Q)A^{-1/3}} \tag{18}$$

suffices in the present calculation, but for ρ_c we need higher-order terms:

$$\rho_c^{eq}(A, I) = \rho_0 + \frac{3\rho_0}{K_v}[[-LI^2 + (LK_{sym}/K_v + L^2K'/2K_v^2 - U)I^4$$
$$+4\pi r_0^2 A^{-1/3}(2\sigma_0 - C) + A^{-2/3}[8\pi r_0\{\mu_0 - X - c_0 a_0(2\sigma_0 - C)\}$$
$$+16\pi^2 r_0^4(2\sigma_0 - C)\{(-10\sigma_0 - D + 4C)/K_v + K'(2\sigma_0 - C)/2K_v^2\}]$$
$$+4\pi r_0^2 I^2 A^{-1/3}\{2(1 + L/J)\tau_0 - \Lambda - K_{sym}(2\sigma_0 - C)/K_v$$
$$+(10\sigma_0 + D - 4C)L/K_v - K'L(2\sigma_0 - C)/K_v^2\}]] . \tag{19}$$

From eq. (9) we then have, writing $\rho_c^{eq}(A, I) \equiv \bar{\rho}$,

$$K(A, I) = 9\bar{\rho}^2[(d^2e/d\rho_c^2)_{\rho_0} + (\bar{\rho} - \rho_0)(d^3e/d\rho_c^3)_{\rho_0}$$
$$+\tfrac{1}{2}(\bar{\rho} - \rho_0)^2(d^4e/d\rho_c^4)_{\rho_0} + \cdots] . \tag{20}$$

Noting that the scaled compressions (8) always leave δ_c unchanged at its equilibrium value (18), a long and tedious calculation leads to the following expression for all the coefficients of the leptodermous expansion (3):

$$K_{vs} = K_{sym} + L(K'/K_v - 6) , \tag{21a}$$

$$K_4 = V + (K'/K_v - 6)U - 3L^2K'/K_v^2 + 9L^2/K_v$$
$$+(K'_{sym} - K'K_{sym}/K_v)L/K_v + L^2(K'' - K'^2/K_v)/2K_v^2 , \tag{21b}$$

$$K_{sf} = 4\pi r_0^2\{(22 - 2K'/K_v)\sigma_0 + D + (K'/K_v - 10)C\} , \tag{21c}$$

$$K_{ss} = 4\pi r_0^2[\Gamma + (K'/K_v - 10)\Lambda + \{22 - 2K'/K_v + (12L - 2K_{sym} - 2LK'/K_v)/J\}\tau_0$$
$$-(10\sigma_0 - 4C + D)K'L/K_v^2$$
$$+(2\sigma_0 - C)(K_vK'K_{sym} - K_v^2K'_{sym} + LK'^2 - LK_vK''$$
$$+6LK_vK')/K_v^3 + L(44\sigma_0 - 12C + 6D + G)/K_v] , \tag{21d}$$

$$K_{cv} = 8\pi r_0[10\mu_0 - 8X + Y + K'(X - \mu_0)/K_v$$
$$+c_0a_0\{2\sigma_0(K'/K_v - 11) - D + C(10 - K'/K_v)\}]$$
$$+8\pi^2 r_0^4(2\sigma_0 - C)[42C - 12D - 124\sigma_0 - 2G + K'(2D + 8\sigma_0 - 2C)/K_v$$
$$-(2\sigma_0 - C)\{(K'/K_v)^2 - K''/K_v\}]/K_v . \tag{21e}$$

Eqs. (21a) and (21c) are as given by Blaizot [8]).

The coefficients of eqs. (16b) and (16c). All these coefficients are defined with respect to SINM, for which we assume the density profile (7). The surface quantities σ_0 and τ_0 are given by SINM at equilibrium, according to

$$\sigma_0 + (\tau_0 - LC/K_v)\delta_c^2 = \lim_{L\to\infty}\int_{-L}^{L}\{\mathscr{E}^0(z, \delta_c) - e^{\infty}(\rho_c^0, \delta_c)\rho^0(z, \delta_c)\}\, dz . \tag{22}$$

Here the density distribution $\rho^0(z, \delta_c)$ is just the total density given by (7), with ρ_{cn} and ρ_{cp} determined by (11) for the specified value of δ_c, ρ_c taking the equilibrium value ρ_c^0 of the density of INM for this value of δ_c:

$$\rho_c^0 = \rho_0\{1-(3L/K_v)\delta_c^2\}, \tag{23}$$

which is the $A\to\infty$ limit of eq. (19). The quantity $\mathscr{E}^0(z, \delta_c)$ is then the energy density corresponding to this density distribution, for the given force. The calculation has to be performed for at least two values of δ_c to extract σ_0 and τ_0.

In determining the parameters a_q, C_q, and γ_q of the density distribution (7) for which SINM is in equilibrium, it is to be noted that the quantity to be minimized is not the integral of eq. (22) but rather

$$\sigma_\mu(\delta_c) = \lim_{L\to\infty}\int_{-L}^{L}\{\mathscr{E}^0(z,\delta_c)-\mu_n\rho_n^0(z,\delta_c)-\mu_p\rho_p^0(z,\delta_c)\}\,dz\,, \tag{24}$$

where μ_n and μ_p represent the chemical potentials. This is discussed in ref. [18]).

The curvature quantity μ_0 is likewise determined by the equilibrium configuration of SINM, being given as follows (see sect. 5 of BGH [12])). The ETF expressions [13,14]) allow us to write the energy density as

$$\mathscr{E}(\rho) = \mathscr{F}(\rho, (\nabla\rho)^2) + \mathscr{G}(\rho, (\nabla\rho)^2)\nabla^2\rho + \mathscr{H}(\rho, (\nabla\rho)^2)(\nabla^2\rho)^2 \tag{25}$$

where ρ is still a function of z, given by eq. (7), and we are making use of the fact that μ_0 is calculated for the symmetric case, $\rho_n = \rho_p$. Then

$$\mu_0 = \lim_{L\to\infty}\int_{-L}^{L}\left[z\{\mathscr{E}^0(z)-\rho^0(z)a_v\} + \mathscr{G}^0(z)\frac{d\rho^0(z)}{dz} + 2\mathscr{H}^0(z)\frac{d\rho^0(z)}{dz}\frac{d^2\rho^0(z)}{dz^2}\right]dz\,. \tag{26}$$

All the other coefficients in eqs. (16b) and (16c), i.e., all the ones in ε, can then be expressed as derivatives of the leading terms, σ_0, τ_0, and μ_0, with respect to the scaling parameter λ. The scaling calculations then proceed along the usual lines, as described, for example, in ref. [19]). The only problem arises from the fact that while the kinetic-energy density τ scales exactly as λ^5, in the ETF approximation the scaling behaviour is more complicated, although the difference in $K(A, I)$ has been shown to lie in the range 2-5% (see table 4 of ref. [7])). We handled the derivatives arising from eq. (9) analytically (for SINM), using Macsyma.

Coulomb term. So far we have neglected completely the influence of the Coulomb force. If we regard the nucleus as a uniformly charged sphere of radius R, then the classical expression for the Coulomb energy is

$$E_{\text{Coul}}^{\text{dir}} = 3Z^2e^2/5R\,. \tag{27}$$

Neglecting then surface and symmetry effects, we quickly arrive at the Coulomb term of eq. (3), with

$$K_{\text{Coul}} = (3e^2/5r_0)(K'/K_v - 8) \tag{28}$$

as given by Blaizot [8]).

Several corrections to this can be contemplated, the simplest to calculate being the exchange term. Using the Slater approximation for the density of the Coulomb exchange energy,

$$\mathscr{E}^{\text{ex}}_{\text{Coul}} = -\tfrac{3}{4}(3/\pi)^{1/3} e^2 \rho_{\text{p}}^{4/3} , \tag{29}$$

we find, comparing with eq. (27),

$$\begin{aligned} E^{\text{ex}}_{\text{Coul}}/E^{\text{dir}}_{\text{Coul}} &= -5(3/16\pi)^{2/3} Z^{-2/3} \\ &\simeq -0.76 Z^{-2/3} . \end{aligned} \tag{30}$$

The overall contribution to $K(A, I)$ will then be around $-0.3K_{\text{Coul}}$, more or less independently of Z and N. Given the typical values of K_{Coul} shown in table 2, the net correction to $K(A, I)$ will fall in the range 1.4 to 1.8 MeV for any nucleus, according to the force, which is within the limits of present experimental error. We may reasonably expect the other corrections to the usual Coulomb term to be similarly small.

TABLE 1

Parameters of eq. (16)

	SkM*	RATP	SkA	S3
ρ_0 (fm^{-3})	0.1603	0.1598	0.1553	0.1453
a_v (MeV)	−15.77	−16.05	−15.99	−15.85
K_v (MeV)	216.6	239.5	263.1	355.4
K' (MeV)	386.1	349.8	300.1	−101.4
K'' (MeV)	1768.8	1451.9	1014.3	−903.0
J (MeV)	30.03	29.26	32.91	28.16
L (MeV)	45.78	32.40	74.62	9.91
K_{sym} (MeV)	−155.93	−191.22	−78.45	−393.73
K'_{sym} (MeV)	−330.47	−440.69	−174.54	−130.45
M (MeV)	1.9	2.1	2.3	1.7
U (MeV)	3.32	3.94	4.32	2.89
V (MeV)	3.91	5.15	5.98	3.22
c_0	−0.849	−0.876	−0.784	−0.454
a_0 (fm)	0.633	0.637	0.601	0.485
σ_0 ($\text{MeV}\cdot\text{fm}^{-2}$)	1.050	1.125	1.106	1.030
C ($\text{MeV}\cdot\text{fm}^{-2}$)	0.0624	0.186	0.133	−0.067
D ($\text{MeV}\cdot\text{fm}^{-2}$)	−32.94	−35.75	−37.64	−45.40
G ($\text{MeV}\cdot\text{fm}^{-2}$)	78.53	92.93	108.3	195.1
τ_0 ($\text{MeV}\cdot\text{fm}^{-2}$)	3.424	2.527	4.827	1.809
Λ ($\text{MeV}\cdot\text{fm}^{-2}$)	6.474	4.559	8.364	6.055
Γ ($\text{MeV}\cdot\text{fm}^{-2}$)	−55.121	−46.345	−90.217	−15.567
μ_0 ($\text{MeV}\cdot\text{fm}^{-1}$)	−0.1185	−0.1757	−0.1025	0.0943
X ($\text{MeV}\cdot\text{fm}^{-1}$)	3.525	3.723	3.512	2.816
Y ($\text{MeV}\cdot\text{fm}^{-1}$)	25.17	28.68	26.87	20.76

TABLE 2
Coefficients of leptodermous expansion (3) (MeV)

	SkM*	RATP	SkA	S3
K_v	216.6	239.5	263.1	355.4
K_{sf}	−230.9	−260.6	−284.6	−375.4
K_{vs}	−349.0	−338.3	−441.1	−456.0
K_{Coul}	−4.70	−4.94	−5.14	−6.07
K_4	38.3	−5.7	105.9	−19.3
K_{ss}	496.8	312.6	874.6	383.4
K_{cv}	−129.0	−140.7	−142.3	−149.2

Results. All the parameters of eq. (16), along with some other relevant parameters, are listed in table 1 for the four forces that we have considered. The corresponding coefficients of the leptodermous expansion (3) are then given in table 2. One conclusion can be drawn immediately: the term in $K_4 I^4$ will be totally negligible in all possible circumstances.

3. Finite-nucleus incompressibilities

Scaling calculations of $K(A, I)$ for any finite spherical nucleus are almost as straightforward in the Skyrme-ETF approach as in the Skyrme-HF approach, the scale transformation (8) leading in the latter case to simple analytic expressions [see, for example, appendix 2 of ref. [7]), which can be evaluated as in BGH [12])]. The only problem concerns the fact, noted in the previous section, that in the ETF case integrals containing the kinetic-energy density τ have a slightly different scaling behaviour than in the HF method, the differences arising from gradients of the effective masses $M_q^*(\boldsymbol{r})$. The complicated derivatives that thus arose from eq. (9) were calculated numerically in the finite-nucleus case. We checked that the condition

$$\left[\frac{\mathrm{d}}{\mathrm{d}\lambda} E(\lambda)\right]_{\lambda=1} = 0 \tag{31}$$

was well satisfied numerically at the end of the variational calculation, this being a sensitive test of numerical precision. (Actually, eq. (31) is equivalent to the virial theorem.)

Two different sets of finite-nucleus calculations were performed:

(i) $N=Z$ nuclei, Coulomb force switched off. For such nuclei the leptodermous expansion (3) reduces to

$$K(A) = K_v + K_{sf} A^{-1/3} + K_{cv} A^{-2/3} \tag{32}$$

so that a sensitive test of the convergence in powers of $A^{-1/3}$ is possible. We computed the $K(A)$ of 23 such nuclei with $10 \leq A \leq 6000$ for all four of the Skyrme forces

TABLE 3

Values of K_{sf} and K_{cv} (in MeV) extracted from calculations on finite nuclei with $N=Z$ and Coulomb force switched off (see fig. 1). Corresponding SINM values in parentheses (from table 2)

	K_{sf}	K_{cv}
SkM*	−210 (−231)	−90 (−129)
RATP	−249 (−261)	−90 (−141)
SkA	−275 (−284)	−88 (−142)
S3	−360 (−375)	−120 (−149)

discussed in the previous section, and show in fig. 1 the plots of $y=\{K(A)-K_v\}A^{1/3}$ versus $A^{-1/3}$, taking the value of K_v from table 2. The y-axis intercepts of these plots give K_{sf} for the force in question, while the slopes give K_{cv}.

The results thus obtained are shown in table 3, with the quantities in parentheses being the SINM values calculated as described in the preceding section, and given in table 2. The agreement between the two approaches is satisfactory, and indicates that the leptodermous expansion (32) is converging rapidly. In table 4 we show the degree of convergence for the extreme case of $A=20$, comparing the predictions for $K(A=20)$ given by eq. (32) with the values given directly by the finite-nucleus calculations. It will be seen that the discrepancy lies in the range of 1–2%, which is very small indeed.

At the same time, the departure from linearity that can be discerned for $A<20$ shows the influence of terms $O(A^{-1})$ in the leptodermous expansion. Since the net value of these terms is seen to be positive, they can account at least partially for the discrepancies between our two approaches for calculating K_{sf} and K_{cv}. (These discrepancies would have been smaller if we had calculated nuclei with still higher values of A.)

It will be seen that the empirical "law" $K_{sf}\approx -K_v$ found by Treiner *et al.* [9]) is well fulfilled for all four Skyrme forces investigated here. We should emphasize strongly, however, that this "law" appears to hold only within the scaling approxima-

TABLE 4

Comparison of leptodermous expansion (32) with exact finite-nucleus calculation of $K(A)$ (in MeV) for $A=20$

	Eq. (32)	Exact
SkM*	127.1	128.9
RATP	135.7	137.3
SkA	149.9	150.8
S3	206.5	208.6

tion for these forces; in dynamical calculations, which go beyond scaling, K_{sf} tends to have much larger negative values [7,11]).

The negative value for K_{cv} that we find with force S3 (as for the other three forces) is to be contrasted with the positive value found by Treiner *et al.* [9]) with this same force; compensating this is the much more negative value that they find for K_{sf}. The only explanation we can offer for this disagreement is that ref. [9]) does not use the full ETF method. We note that our value of K_{cv} extracted from finite-nucleus calculations is well confirmed by the completely independent SINM calculation described in the preceding section.

Having shown that the leptodermous expansion in powers of $A^{-1/3}$ has converged sufficiently at the curvature term, we ask now whether even the curvature term is necessary. At this point we must realize that in practice it is impossible to draw graphs of the form shown in fig. 1, since one is limited to nuclei with $A \leq 250$, and also one does not know in advance the value of K_v; indeed, this is usually the principal quantity that one wishes to determine. (Another point is that the Coulomb force cannot be switched off in practice, but that is irrelevant for the moment.)

In fig. 2, therefore, we take the same computed data as in fig. 1, limit them to $A \leq 250$, and plot $K(A)$ against $A^{-1/3}$, exactly as one would in analyzing real data. Since no deviation from linearity can be discerned for any of the four forces one might be tempted to drop the curvature term in fitting the leptodermous expansion to experimental data. However, the curvature term still exists, and even if it can be effectively absorbed into the surface term over the limited range of A-values available, the result will be that the extracted value of K_{sf} must become more negative,

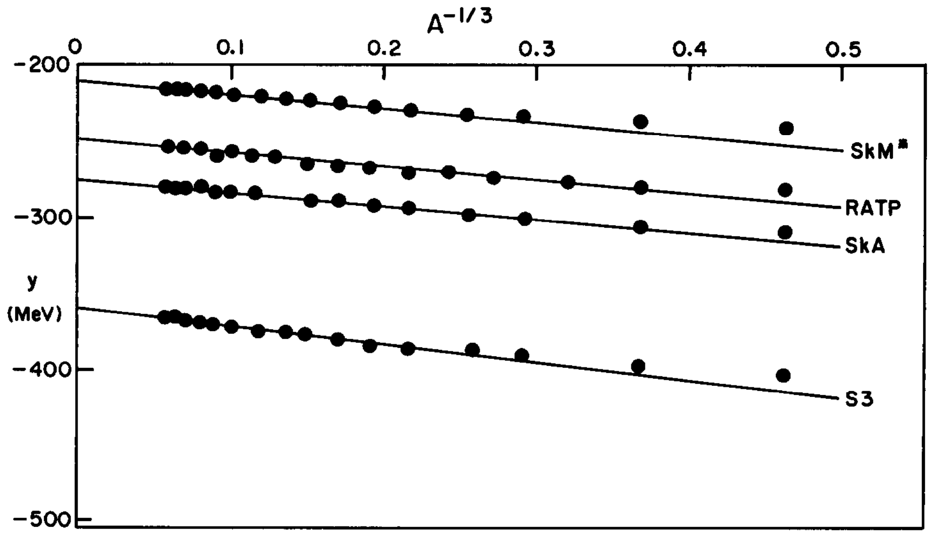

Fig. 1. Plots of $y = \{K(A) - K_v\}A^{1/3}$ versus $A^{-1/3}$ for $N = Z$ nuclei with Coulomb force switched off, calculated with four different Skyrme-type forces.

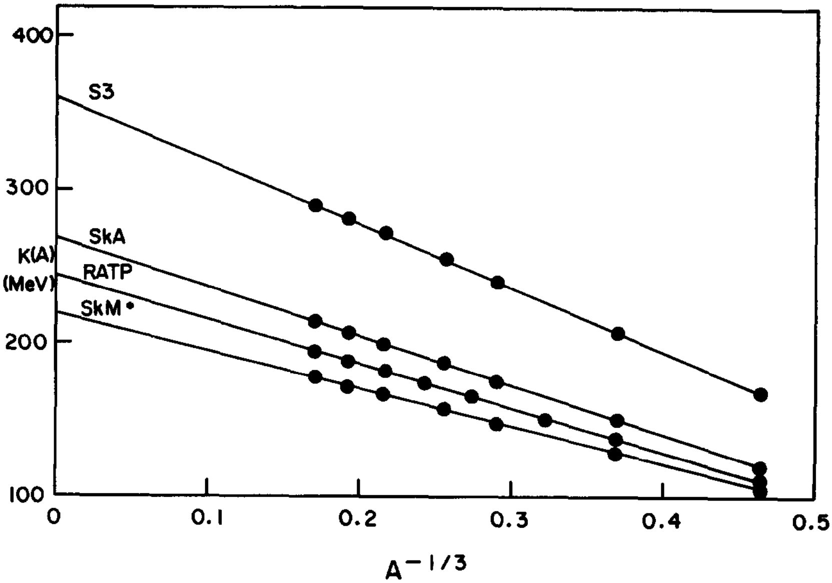

Fig. 2. Plots of $K(A)$ versus $A^{-1/3}$ for same computed data as fig. 1, but with $A \leq 250$.

which in turn will affect the value determined for K_v. The extent to which this happens is seen in table 5, where we summarize the results extracted from the linear fit of fig. 2. For all four forces we see that K_v is overestimated by about 5 MeV (the correct values of table 2, as given by INM, are shown in parentheses). We conclude that in analyzing real data the curvature term should be included, and the parameters determined by a least-squares fit. The curvature coefficient itself will be very badly determined by the data, and the principal effect of its inclusion will be to increase the error bars on K_v.

(ii) Real nuclei. Using each of the four Skyrme forces, we also calculated nine real spherical nuclei, ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{90}Zr, ^{112}Sn, ^{132}Sn, ^{140}Ce, and ^{208}Pb, all with the Coulomb force left switched on. Since the primary object here is to study

TABLE 5

Effective values of K_v and K_{sf} (in MeV) extracted from plot of $K(A)$ versus $A^{-1/3}$ for $A \leq 250$ (see fig. 2). The values of K_v in parentheses are from INM (table 2)

	K_v	K_{sf}
SkM*	220 (217)	−245
RATP	244 (240)	−290
SkA	268 (263)	−320
S3	360 (355)	−410

the possible role of the surface-symmetry term $K_{ss}I^2A^{-1/3}$, we define the quantity

$$z=\{K(A,I)-K_v-K_{sf}A^{-1/3}-K_{cv}A^{-2/3}-K_{Coul}Z^2A^{-4/3}-K_{sym}I^2\} \qquad (33)$$

taking for K_{Coul} and K_{sym} the INM values given in table 2, while for K_{sf} and K_{cv} we use the values derived in fig. 1 and given in table 3. This quantity z is plotted against $I^2A^{-1/3}$ in figs. 3a–d. We see from these values of z that while the surface-symmetry term might be significant for nuclei far from stability, such as ^{132}Sn, for stable nuclei it is probably still safe to neglect it, z being comparable to the errors in $K(A, I)$: see, for example, fig. 2 of ref. [11]). However, it will be advisable to include this term in future analyses of experiments with improved precision.

For this reason we now examine the success that the surface-symmetry term has in fitting our computed data-points of figs. 3a–d. The straight lines in these graphs represent $K_{ss}I^2A^{-1/3}$, with the SINM value of K_{ss} being taken (table 2). We see that the overall trends are well represented; indeed, considering the large cancellations that take place in eq. (33), the agreement is remarkably good, and usually it becomes still better if the Coulomb-exchange term is included, since this lowers all points by between 1.4 and 1.8 MeV (see the preceding section).

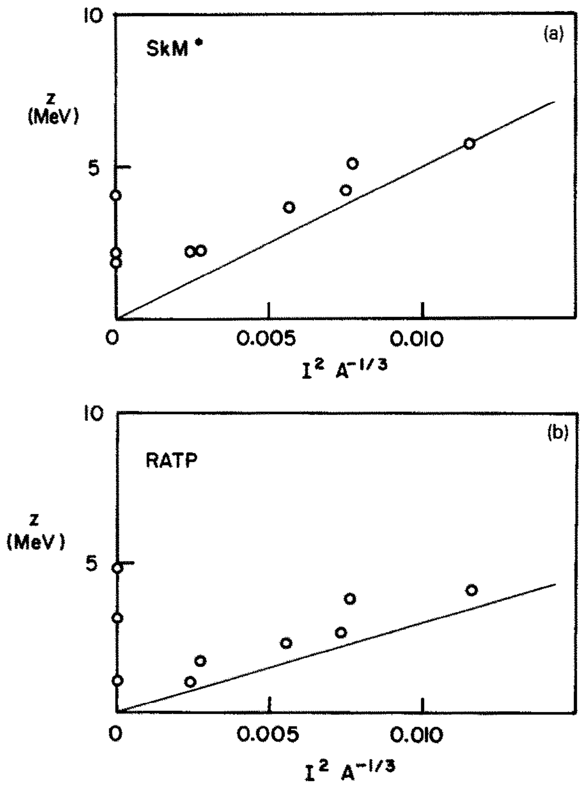

Fig. 3. Plots of z defined in eq. (33) versus $I^2A^{-1/3}$. Solid lines represent $K_{ss}I^2A^{-1/3}$.

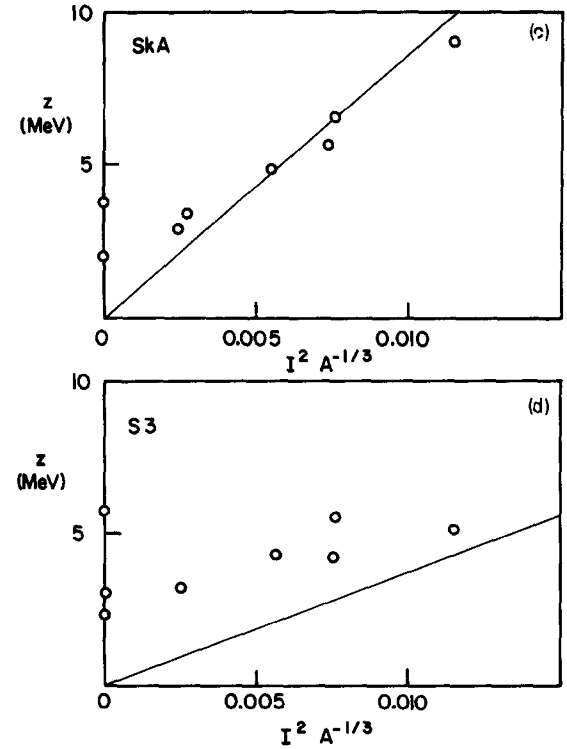

Fig. 3—continued

4. Conclusions

Working within the context of the scaling model, we have considered the inclusion of higher-order terms in the leptodermous expansion of the incompressibility $K(A, I)$ of finite nuclei, as used to extract the incompressibility of infinite nuclear matter, K_v, from experimental data on the breathing mode. Specifically, we have introduced a curvature term $K_{cv}A^{-2/3}$, a surface-symmetry term $K_{ss}I^2A^{-1/3}$, a quartic volume-symmetry term K_4I^4, and a Coulomb-exchange term, and derive expressions for all the coefficients in terms of quantities that can be calculated in INM and SINM, thereby extending the work of ref. [8]). In this way we have been able to calculate the new coefficients for four different Skyrme-type forces, using the ETF approximation.

With the same forces we also calculate $K(A, I)$ for a number of finite nuclei, and fit the results to the leptodermous expansion. The resulting values of the coefficients agree well with those determined from INM and SINM, indicating thereby that the expansion has well converged. We find, in fact, that the term in K_4 is totally negligible, while the surface-symmetry and Coulomb-exchange terms should probably be included in future analyses, should there be any significant improvement in the experimental accuracy. As for the curvature term, the range of A-values over which data are available is too small to permit its experimental determination, but it should still be included in the analysis in order to establish realistic error bars on K_v.

We should emphasize that our conclusions have been established only within the framework of the scaling model. As discussed in refs. [7,11]), the experimental breathing-mode energies of light nuclei cannot be reproduced in the scaling model. In a two-dimensional hydrodynamical approach, which reproduces those data quite well [7]) and which goes beyond scaling, the expansion (3) converges much more slowly and leads to larger negative values of the surface term [11]), as is also the case in the empirical fit of the recent Gronigen data [10,11]).

Nevertheless, the scaling evaluation of the coefficients in (3) gives a very valuable guide to the behaviour of different Skyrme-type forces in predicting breathing-mode energies, particularly for heavy nuclei where scaling reproduces the experimental energies very well, provided the coefficient K_v is small enough, as in the case of force SkM*, for example.

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