ON THE MAGICITY OF THE NEUTRON NUMBER N = 228

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I. INTRODUCTION

The possible magicity of the neutron number N = 228 corresponding to the closure of the almost degenerate 1k 17/2 and 2h 11/2 subshells, has been early recognized (Refs. 1-3). For such a neutron filling, one has generally found a rather important gap (larger than 2 MeV) near the Fermi level. Hartree-Fock calculations of N = 228 nuclei, performed with a phenomenological effective force have further shown that this gap was unchanged upon varying the proton number from 114 to 138 (Refs. 1 and 4). For these nuclei, the proton numbers producing beta-stability are to be found around 126. As a result of our calculations the latter is neither a magic number nor close to one. However the corresponding proton shell correction energy is found to be almost vanishing. Therefore the fission stability of such nuclei is heavily dependent upon the magicity of the neutron number N = 228.

Clearly, it is of great importance to study how this magicity depends upon the particular choice of the phenomenological effective interaction in use. It is the aim of this paper to contribute to such a study. For this purpose we will study Hartree-Fock solutions calculated with a variety of effective forces of the Skyrme type (Refs. 5-7). Even though such a work is restricted to a specific force parametrization, we find it useful to perform a thorough investigation of this problem within a well-defined framework. This is all the more true that we are backed up by a careful study of the static properties of experimentally known magic nuclei (Ref. 8). Since these properties are reasonably well reproduced with this type of effective force, we found it justified to extend their use to non-magic nuclei. Such an extrapolation has been shown already to be highly successful for both stable well-deformed nuclei (Ref. 9) and nuclei far from the beta stability line (Ref. 10). In our opinion, these successes may generate some confidence in the predictive power of the whole approach, in spite of some well-known bold simplifications which are involved, like the absence of a tensor force, the linear character of the density dependence, and the rough mocking-up of finite range effects.

To go beyond qualitative studies of spherical magicity, one should assess precise values for the fission halflives. Even if the effective force were unambiguously known, in the present state of art in theoretical nuclear physics such halflives would not be free from uncertainties due to the necessity of further approximations and numerical short cuts.

A complete study of the stability of the considered nuclei must include the alpha, beta and electron-capture decay properties (not to mention the particle emission stability which is granted in this case). As a consequence the variation of these decay properties when changing the effective force needs also to be investigated.

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2. THE MAGICITY OF THE NEUTRON NUMBER N = 228

2.1 The Effective Forces

In our selfconsistent calculations, the Skyrme effective interaction has been used. It is a two-body density-dependent force defined with usual notation as:

$$v(\vec{r}_{12}) = t_0(1 + x_0P_\sigma)\ \delta(\vec{r}_{12}) + \frac{1}{2}\ t_1[\delta(\vec{r}_{12})\vec{k}^2 + \vec{k}^{\dagger 2}\ \delta(\vec{r}_{12})] + t_2\ \vec{k}^{\dagger}\cdot\delta(\vec{r}_{12})\ \vec{k}$$

$$+ iW(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{k}^{\dagger} \times \delta(\vec{r}_{12})\ \vec{k} + \frac{1}{6}\ t_3(1 + P_\sigma)\ \delta(\vec{r}_{12})\ \rho[(\vec{r}_1 + \vec{r}_2)/2] \quad (1)$$

and

$$\vec{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/2i \quad . \quad (2)$$

Such a force is determined by six parameters. In Table 1 the five sets of parameters which we have used are given. They have previously been shown (Ref. 8) to reproduce fairly well experimental binding energies and r.m.s. radii of the charge distribution of magic and

TABLE 1 Parameters of the Skyrme Interactions in Use. The interactions have been ordered according to the decreasing value of the parameter t_3.

	t_0 (MeV-fm^3)	t_1 (MeV-fm^5)	t_2 (MeV-fm^5)	t_3 (MeV-fm^6)	x_0	W (MeV-fm^5)
SVI	-1101.81	271.67	-138.33	17000.0	0.583	115.0
SIII	-1128.75	395.0	- 95.0	14000.0	0.45	120.0
SII*	-1169.9	586.6	- 27.1	9331.1	0.23	130.0
SIV	-1205.6	765.0	35.0	5000.0	0.05	150.0
SV	-1248.29	970.56	107.22	0.0	-0.17	150.0

semi-magic nuclei from ^{16}O to ^{208}Pb. They all yield thus correct saturation properties of nuclear matter as seen in Table 2. They differ however by the mechanism they imply to achieve

TABLE 2 Nuclear Matter Properties of the Interactions in Use. Binding energy per particle E/A, Fermi momentum k_F, incompressibility coefficient K, effective mass ratio m^*/m and symmetry coefficient ε_1 (linear term of the expansion of E/A in powers of $[(N-Z)/A]^2$) in nuclear matter.

	E/A (MeV)	k_F (fm^{-1})	K (MeV)	m^*/m	ε_1 (MeV)
SVI	- 15.77	1.29	364	0.95	26.9
SIII	- 15.87	1.29	356	0.76	28.2
SII*	- 16.00	1.30	342	0.58	29.4
SIV	- 15.98	1.31	325	0.47	31.2
SV	- 16.06	1.32	306	0.38	32.7

this saturation, i.e. by the relative importance in that respect, of velocity-dependent and density-dependent terms. This variation can be characterized by the effective mass in nuclear matter. The ratio of the latter to the nucleonic mass ranges in our case (see Table 2) from 0.38 (no density dependence) to 0.95 (almost no velocity dependence). Even though the four fundamental liquid drop quantities (volume energy, symmetry energy, surface energy, and saturation density) are too poorly determined to provide precise constraints in an actual optimization of the force, they may be viewed however as imposing definite values to four out of the five parameters of the central part of the force; the fifth one is then determined by the effective mass. The determination of the spin-orbit parameter W is made by fitting some spin-orbit splitting energies or the level orderings in heavy magic nuclei. Such a procedure does not yield a very definite answer but rather a range of reasonable values. This is exemplified on Figs. 1 and 2, and in Table 3 in the case of the SIII force. Whereas the energy

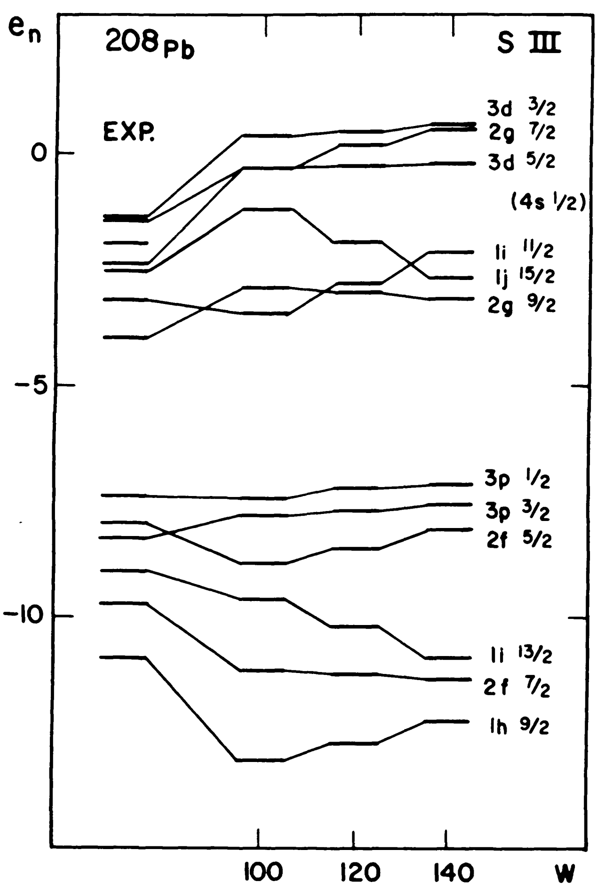

Fig. 1. Variation of neutron single-particle energies (in MeV) with respect to the spin-orbit parameter W (in MeV-fm^5) in the vicinity of the Fermi level for the ^{208}Pb nucleus. The SIII force has been used. Experimental levels (Ref. 11) are given for comparison.

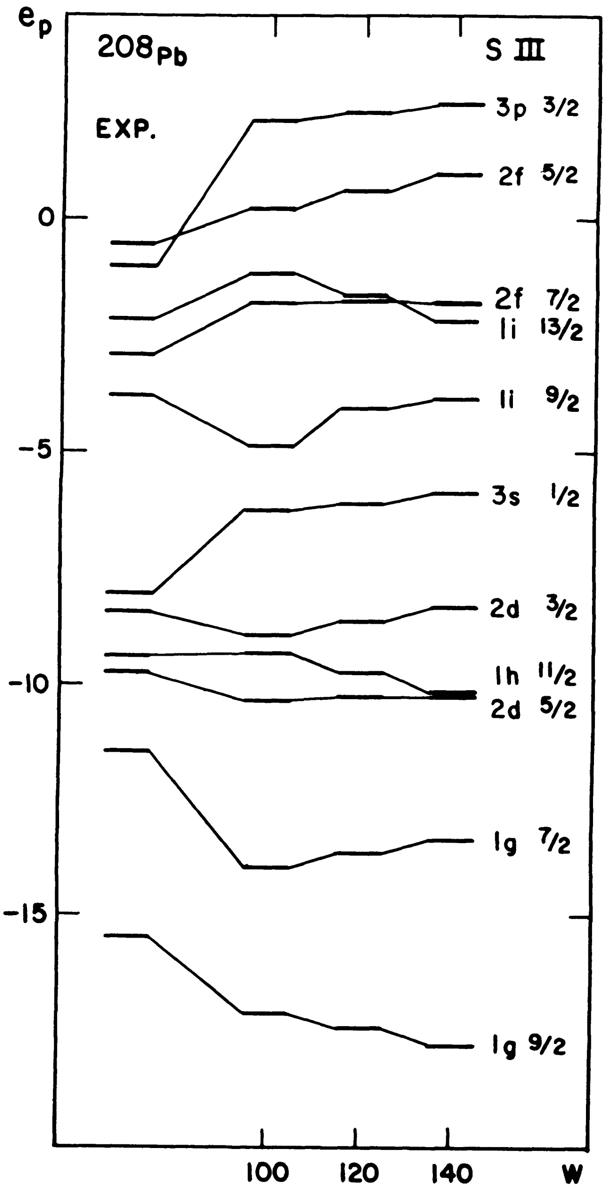

Fig. 2. Same as Fig. 1 for protons.

TABLE 3 Spin-orbit Splitting of the 1p Levels as a Function of the W Parameter. The energy differences ΔE^n_{1p} (ΔE^n_{1p}) between the two neutron (proton) 1p levels have been calculated for the ^{16}O nucleus and with the SIII central force. The experimental values (labelled with asterisks) are extracted from Ref. 11.

W(MeV-fm^5)		100	120	140
ΔE^n_{1p} (MeV)	6.16$^{(*)}$	4.50	5.38	6.25
ΔE^p_{1p} (MeV)	6.33$^{(*)}$	4.43	5.30	6.17

differences of the 1p levels in ^{16}O would be better reproduced with W = 140 MeV-fm^5, the obtention of the correct level orderings in ^{208}Pb imposes a smaller value. As a result it seems realistic to assess an error bar of about ± 20% on the spin-orbit parameter.

Since the number Z = 126 does not correspond in our calculations to a shell closure we have included pairing correlations for the protons only, through an Hartree-Fock + BCS approximation in the constant pairing matrix element limit. The value of the latter is given by

$$G(\text{MeV}) = 12.5/(11 + A) \quad , \tag{3}$$

where A is the total number of nucleons.

2.2 Results

We have performed selfconsistent (Hartree-Fock + BCS) calculations of the spherically symmetric solution of the nucleus N = 228, Z = 126. Exact numerical solutions of the system of (one-dimensional) differential equations have been obtained according to the methods detailed in Refs. 6 and 8. The resulting single-particle spectra near the Fermi level have been plotted on Figs. 3 and 4 for all the considered effective forces. Upon increasing the effective mass m^*, one observes the well-known increase of the single-particle level density. As a consequence, upon increasing m^* one produces a diminishment of the N = 228 neutron gap energy. However the latter has never been found smaller than 2.7 MeV. In view of the previously discussed uncertainty on the spin-orbit force parameter W, we have also studied the variation of the single-particle energies corresponding to its change by about ± 20%. As exemplified in Figs. 5 and 7, a decrease of W results in a slight reduction of the N = 228 energy gap, due to the squeezing of the 2h levels and the raising of the 1k 17/2 level. Nevertheless, the energy gap always exceeds 2.2 MeV.

Upon varying the forces and the spin-orbit parameter as specified above, the global pattern for proton spectra remains quite unchanged (see Figs. 4, 7, 8). The proton number Z = 126 is found within a major shell (between Z = 120 and Z = 138) including the 3p and the 1i 11/2 levels. This filling however corresponds to the beginning of that shell yielding, as already pointed out, to an approximately vanishing shell effect energy.

3. FISSION DECAY PROPERTIES

3.1 Method of Calculation

One may view the evaluation of fission halflives as a three-step calculation which may be schematically summarized as: i) evaluation of a potential energy curve (or surface) within a constrained Hartree-Fock framework, ii) evaluation of relevant adiabatic inertial parameters within the adiabatic limit of the time-dependent Hartree-Fock approximation, iii) evaluation of the halflife within the WKB approximation for a given path in the possibly multi-dimensional space of retained collective variables. In view of the number of particles

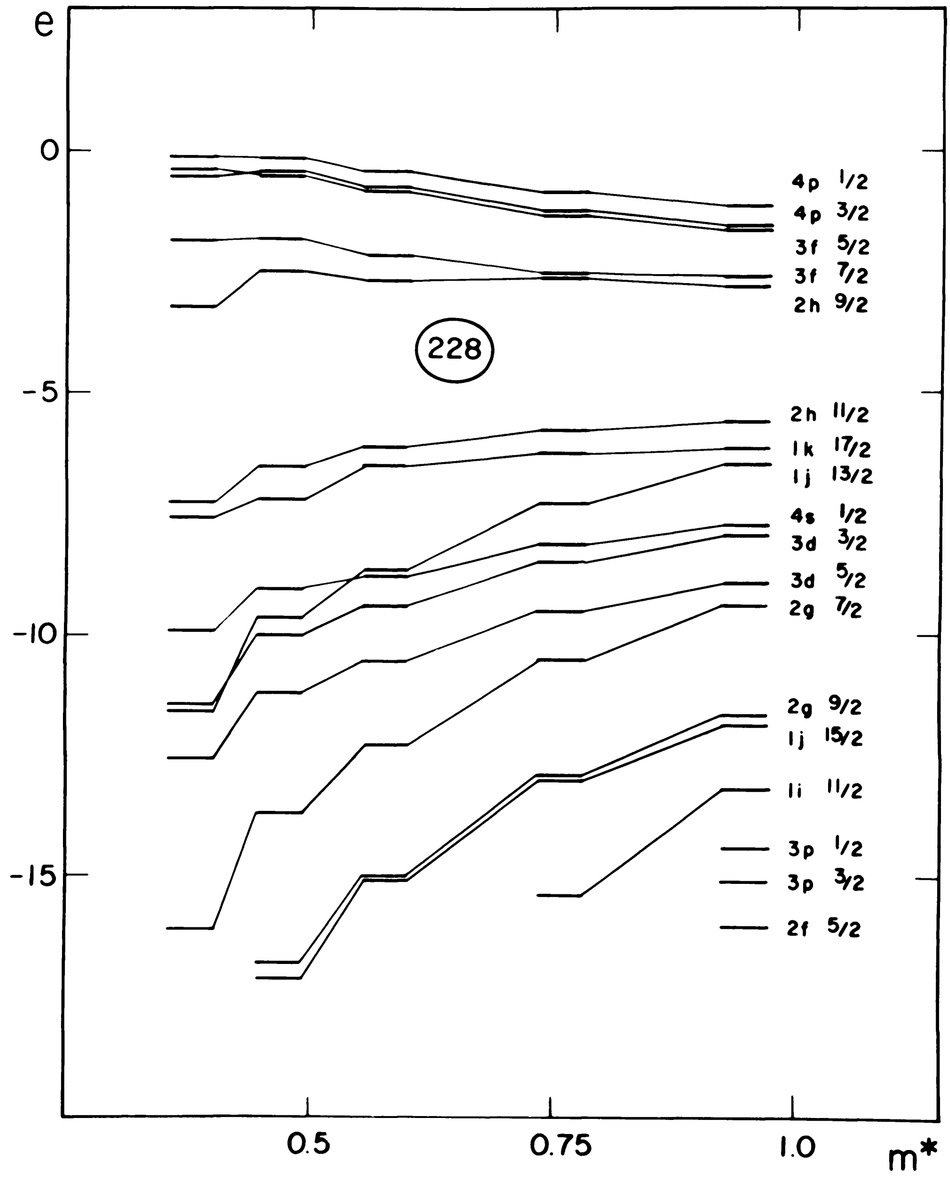

Fig. 3. Neutron single-particle energies (in MeV) as functions of the nuclear matter effective mass (in units of the nucleonic mass) in the vicinity of the Fermi level for the N = 228, Z = 126 nucleus.

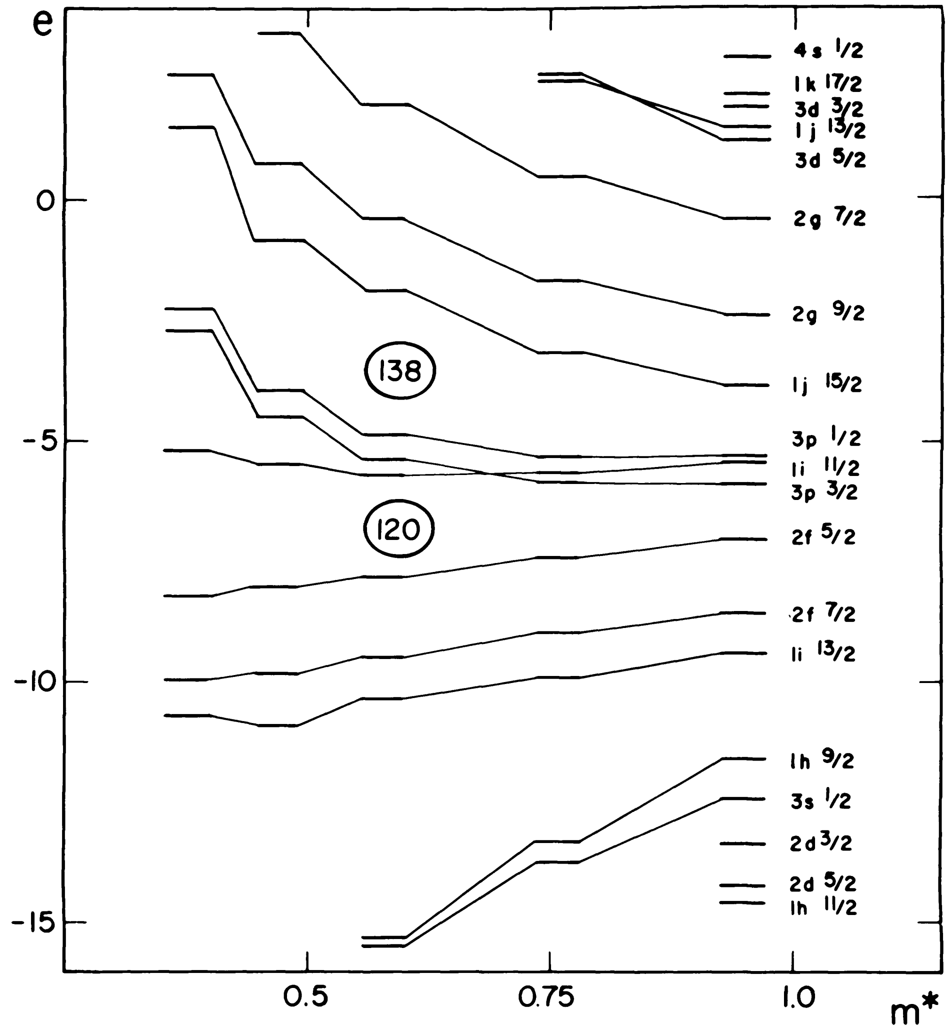

Fig. 4. Same as Fig. 3 for protons.

involved in a super-heavy nucleus, the careful completion of step i) represents a tremendous numerical effort (see Ref. 4). Actual calculations of step ii) if *a priori* possible (according to the method of Ref. 12) have not yet been attempted. One needs definitely good substitudes for that.

3.2 Fission Barriers

For the calculation of fission barriers, we have used the expectation value method (Refs. 13-14) as an approximation to constrained Hartree-Fock calculations. This method involves the following steps:

i) From a Hartree-Fock calculation of the spherically symmetric solution for each given nucleus and each given force, one gets in $\vec{r}$ representation the (local) mean field and the effective mass. These quantities are then fitted by simple analytical functions (e.g. a Woods-Saxon potential for the central part of the average potential).

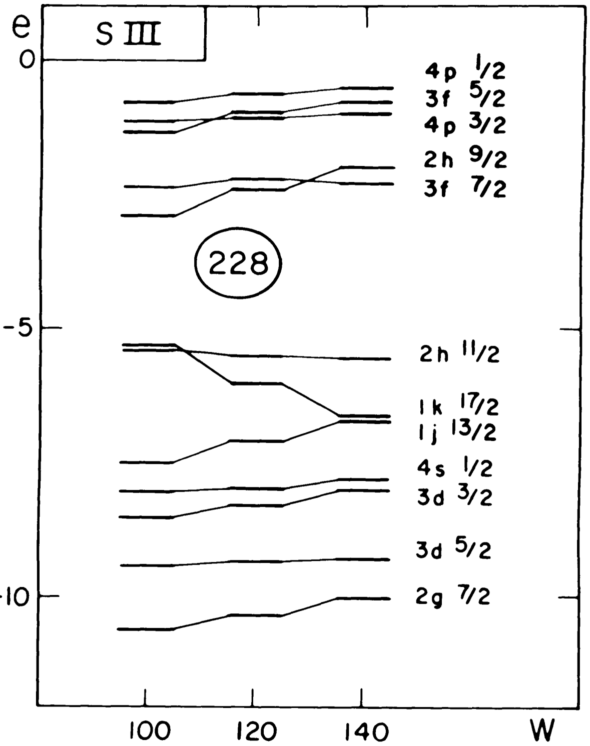

Fig. 5. Same as Fig. 1 for the N = 228, Z = 126 nucleus.

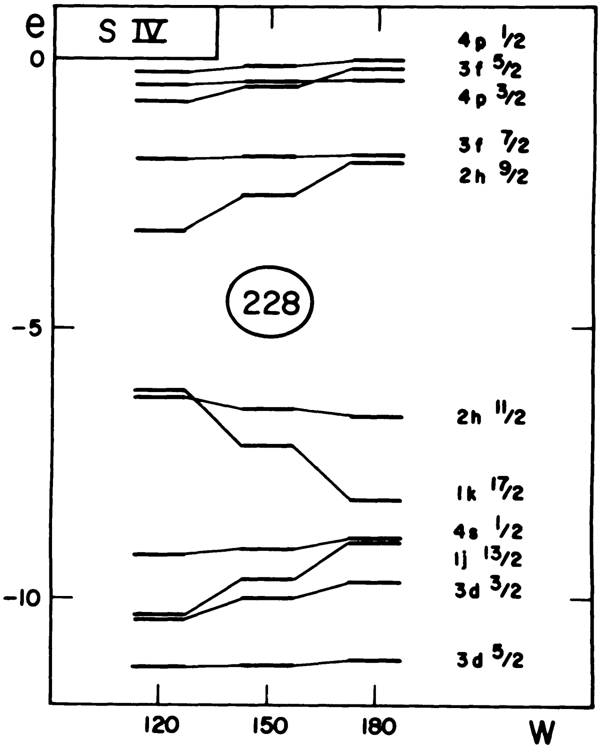

Fig. 6. Same as Fig. 1 for the N = 228, Z = 126 nucleus and for the SIV force.

ii) The various parts of the approximated Hartree-Fock hamiltonian are consistently deformed using a prescription proposed in Ref. 15.

iii) The one-body Schrödinger equation corresponding to this deformable Hartree-Fock hamiltonian is solved upon projecting its solutions on a truncated(†) harmonic oscillator basis (allowing axially symmetric deformation). The independent-particle solution so obtained is then used to evaluate the expectation value of the effective hamiltonian. Upon varying the deformation of the basis one generates a deformation energy curve. The total energy is dependent for each deformation upon the other basis parameter (which is a size parameter like $\omega_0 = (\omega_\perp{}^2\omega_z)^{1/3}$ with usual notation). However, the analytical dependence of the total energy E with respect to ω_0 makes it easy to determine the value of this parameter by minimization of E.

iiii) Finally pairing effects are included within the BCS approximation with a constant pairing matrix element the value of which is fixed by the uniform gap method of Ref. 15. The resulting variation of this matrix element with deformation is found to be negligible.

Our results for the fission barriers are shown on Figs. 9-11. Their accuracy is evaluated by comparing with exact Hartree-Fock calculations in a few cases (see Figs. 9 and 11). In the case of the SIII force the height of the first fission barrier B_I is correctly approximated, but the expectation value method fails in giving the correct sign of the energy difference E_{is} between the ground and the isomeric states. These results together with other numerical

†The actual size of the basis corresponds to 13 major oscillator shells.

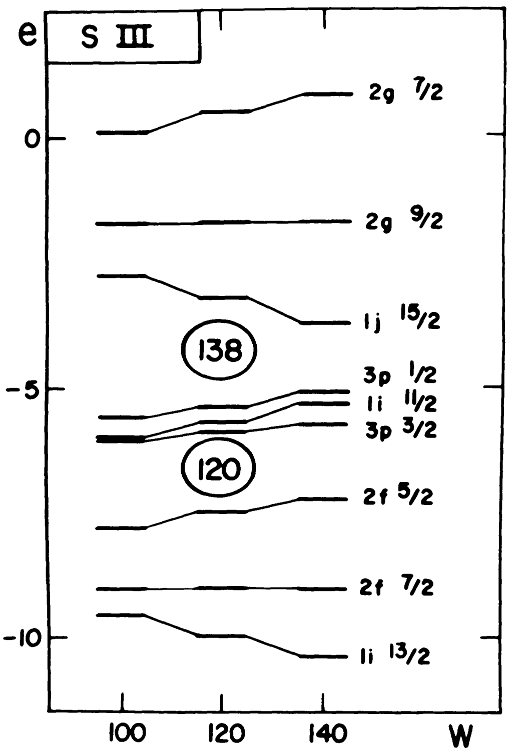

Fig. 7. Same as Fig. 1 for the N = 228, Z = 126 nucleus and for protons.

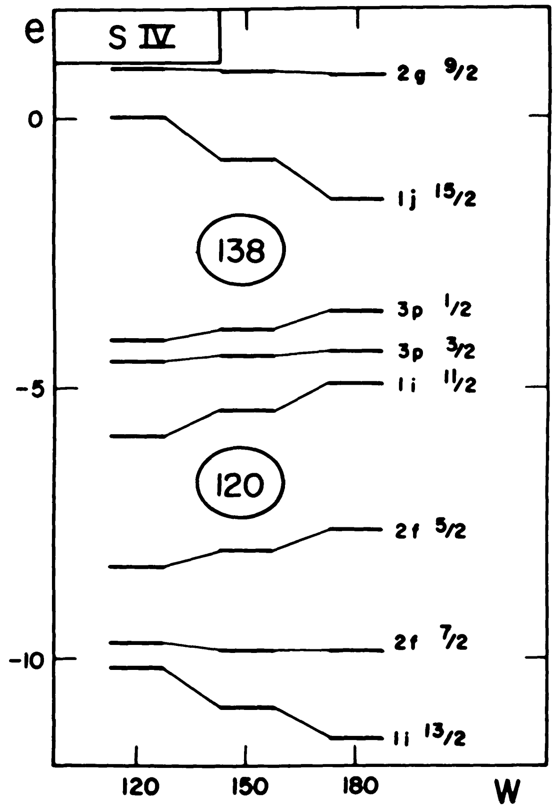

Fig. 8. Same as Fig. 1 for the N = 228, Z = 126 nucleus, for protons and for the SIV force.

evidence (see Ref. 14) lead in this case to an estimation of a 2 MeV error bar on relative energies. The approximation seems less good in the case of the SIV force, where B_I and E_{is} are overestimated by about 5 MeV and 0.5 MeV, respectively.

These deficiencies are likely to be due to a wrong assumption upon the deformation of the Hartree-Fock hamiltonian. This poor accuracy prevents our making use of the approximated results as such, in precise assessments of the fission halflives. These results however are particularly useful to sketch systematical trends of fission barriers characterized by variations much larger than the errors associated with the method. As seen on Fig. 10 in the case of the SIII central force, upon varying W from 100 to 140 MeV-fm^5, B_I increases from ~ 5 MeV to ~ 17 MeV. A similar variation is observed in Fig. 11, in the case of the SIV central force. This demonstrates the dramatic influence of the poorly determined value of the spin-orbit parameter, upon the fission decay properties of the studied nucleus.

From our (exact) Hartree-Fock calculations, the values of B_I are 9 MeV and 15 MeV with the SIII and the SIV forces. This dependence of the fission barrier heights with respect to the force (or equivalently in our case with the effective mass) is indeed rather large. However it is our opinion (see the general discussion of Ref. 8 and the study of the force dependence of the single-particle level density of deformed nuclei, in Ref. 9) that the SIII set of parameters ($m^* \sim 0.75$) is to be preferred to the SIV one ($m^* \sim 0.5$). It appears therefore that the phenomenological freedom still allowed in the choice of the parametrization of the central part of the force plays here a lesser role than the one associated with the spin-orbit part.

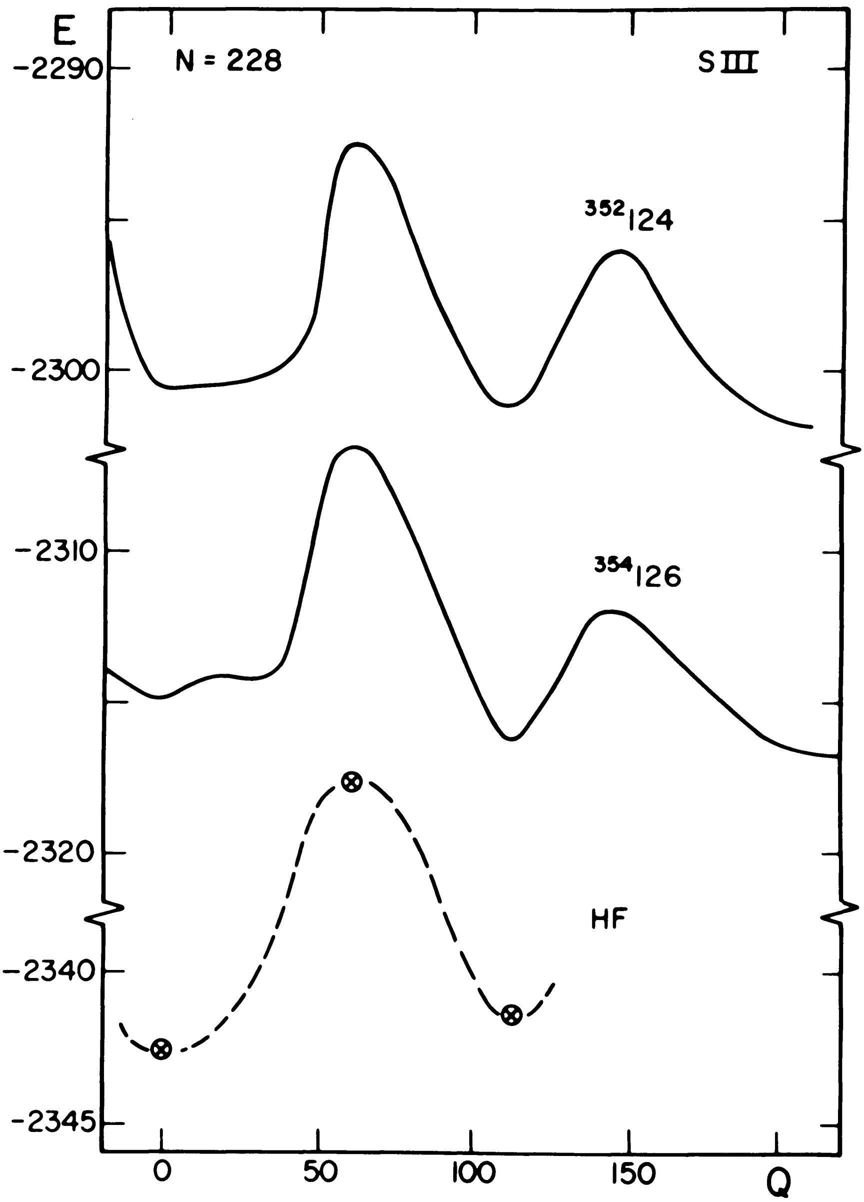

Fig. 9. Fission barriers of the N = 228, Z = 124 and 126 nuclei approximated within the expectation value method. The total energies E are given (in MeV) as functions of the quadrupole moments Q (in barn) of the mass distribution. In the lower part of the figure the results of Hartree-Fock calculations for the two equilibrium points and the top of the first barrier are given in the A = 354 case, for comparison with their approximations. The Skyrme SIII force has been used.

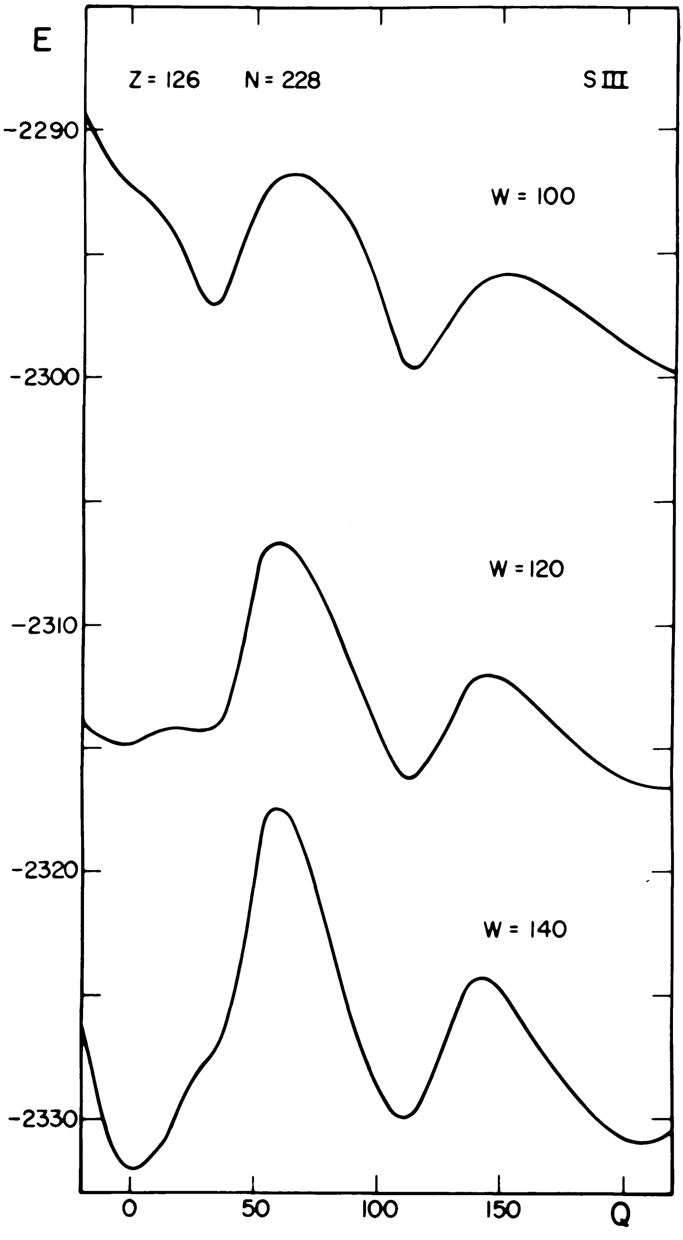

Fig. 10. Same as Fig. 9 but for three different values of the spin-orbit parameter W (in MeV-fm^5). For the central part of the force, the parameters are those of the SIII force. Consequently the W = 120 curve corresponds to the standard SIII force.

It is worth discussing now a particular aspect of our calculations. To preserve the simplicity of the Hartree-Fock equations resulting from the use of a Skyrme effective interaction, we have made a local approximation of the Slater type for the exchange contribution of the Coulomb interaction (Refs. 16-17):

$$E^{exch}_{coul} = - \frac{3e^2}{4} \left(\frac{3}{\pi}\right)^{1/3} \int [\rho_p(\vec{r})]^{4/3} \, d^3\vec{r} \quad , \qquad (4)$$

where $\rho_p(\vec{r})$ is the proton density. This approximation has been shown to be quite satisfactory for ground state solutions (Ref. 18). Now when evaluating fission barrier heights, one would like to know whether or not such an approximation reproduces correctly the deformation dependence of the exact Coulomb exchange energy. The answer seems negative. It has been recently shown (Ref. 19) for Slater determinants built on pure harmonic oscillator states that the ratio of direct to exchange Coulomb energies remains fairly constant upon deforming the harmonic oscillator. The proportionality factor is indeed very close to the nuclear matter ratio (i.e.: $- 5[3/16\pi Z]^{2/3}$, Z being the proton number). Therefore, whereas the exchange energy approximated by Eq. (4) is almost constant with deformation, the exact one is an increasing function of elongation. If this result extends to the Slater determinants considered in this study (which are not built from pure oscillator states), this would have the consequence of raising significantly the fission barriers(†) as shown on Fig. 11.

3.3 Adiabatic Masses

In view of the uncertainties in the determination of fission barriers, a rough estimate of adiabatic masses is quite sufficient. That is why we will use in what follows the ansatz proposed by Fiset and Nix (Ref. 20), with the parametrization of Ref. 21. In terms of a purely prolate ellipsoidal elongation characterized by a ratio q of the major to the minor axis, the adiabatic mass $M(q)$ defined by

$$E_{coll\ kin} = \frac{1}{2} M(q) \, \dot{q}^2 \qquad (5)$$

is written

$$M(q) = \frac{m r_0^2 A^{5/3}}{16 \, q^{2/3}} \left\{ 1 + 16 \, \frac{17}{15} \exp\left[- \frac{32}{17} (q^{2/3} - 1) \right] \right\} \quad , \qquad (6)$$

where r_0 is the size liquid-drop parameter equal to 1.175 fm (Ref. 21). Besides, the quadrupole moment Q of the mass distribution is related to the parameter q by

$$Q = (2 r_0^2 A^{5/3}/5)(q^{4/3} - q^{-2/3}) \quad . \qquad (7)$$

3.4 Fission Halflives

We have roughly estimated the fission barriers of the nucleus N = 228, Z = 126, in the following way:

i) In cases (SIII central force with W values different from 120 MeV-fm^5) where the Hartree-Fock results for the first fission barrier were not known, we have used the findings of the expectation value method for B_I and the Hartree-Fock result with the SIII force for E_{is}.

ii) For the second fission barrier heights we have used the findings of the expectation value method concerning the energy difference between the second saddle point and the fission shape isomer.

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†This remark can also be applied to the fission barrier calculations for the nucleus N = 184, Z = 114, presented in Ref. 4.

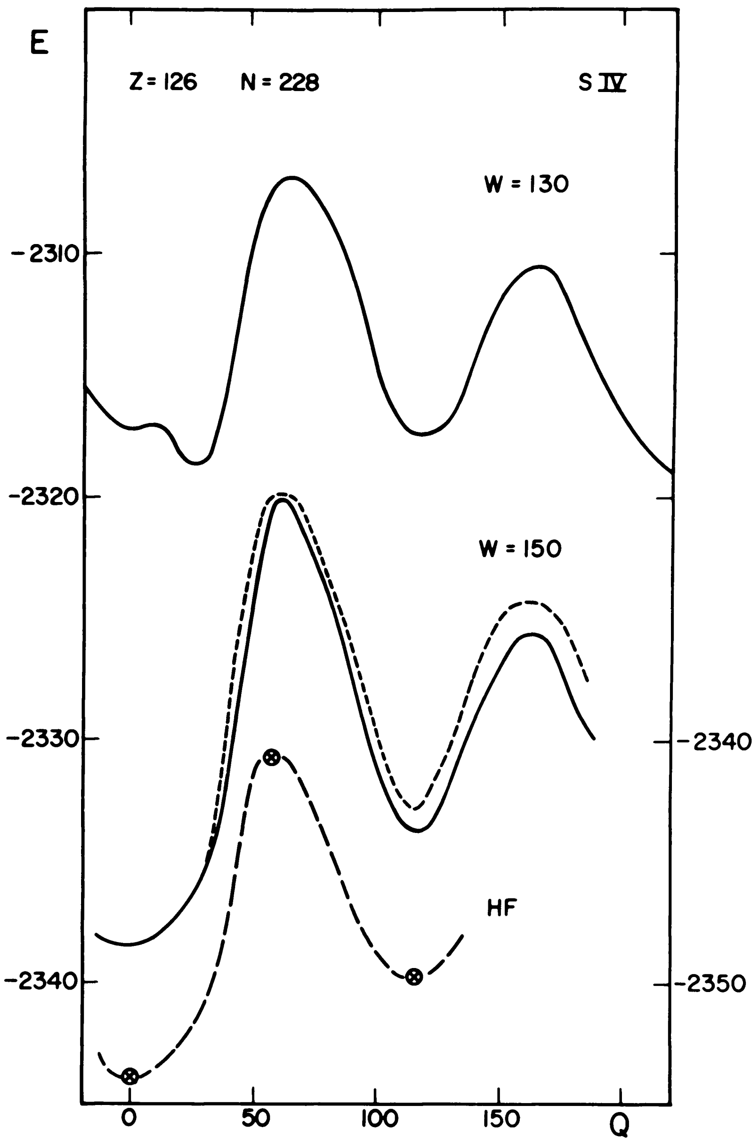

Fig. 11. Same as Fig. 9 but for two different values of the spin-orbit parameter W (in MeV-fm^5) and for the SIV central part of the force. The W = 150 curve corresponds to the standard SIV force. The latter curve is compared with the fission barrier obtained when assuming a more realistic deformation dependence of the Coulomb exchange energy (small dash curve). The Hartree-Fock calculations (HF curve) have been performed for the standard SIV force. The energy scale (in MeV) of the l.h.s. of the figure corresponds to the two upper curves, whereas the one of the r.h.s. corresponds to the lower curve.

iii) Each resulting curve has been approximated by two parabolas whose parameters are given in Table 4. It is necessary to emphasize the very tentative character of the fission barriers so determined.

TABLE 4 Rough Estimates of Fission-decay Halflives for the Nucleus N = 228, Z = 126. For each force the fission barriers have been roughly estimated (see text). Each barrier (labelled i) is characterized by two quadrupole moments $Q_1{}^i$ and $Q_2{}^i$ of the mass distribution and by one energy H^i. The former quantities are the abscissa values corresponding in the parabolic potential to the penetration energy. The latter one is the height of the top of the barrier, relative to the penetration energy. All fission barriers have been estimated from calculations including the Coulomb exchange energy according to the approximation (4), except for the one labelled SIII (Coul. Exch.), where the ratio of direct to exchange Coulomb energies has been kept constant throughout the whole curve. The two last results correspond to calculations using the central part of the SIII force with various values for the spin-orbit coefficient W.

Force	$Q_1{}^1$ (barn)	$Q_2{}^1$ (barn)	H^1 (MeV)	$Q_1{}^2$ (barn)	$Q_2{}^2$ (barn)	H^2 (MeV)	τ (year)
SIV	12	121	12.8	106	218	12.0	10^{39}
SIII	17	110	8.6	106	192	5.0	10^{15}
SIII (Coul. exch.)	17	117	9.0	107	213	6.4	10^{22}
SIII (W = 140 MeV-fm^5)	8	108	14.5	105	200	7.5	10^{32}
SIII (W = 100 MeV-fm^5)	25	110	4.9	110	181	4.1	10^{2}

From these deformation energy curves and the adiabatic mass parameters (6) one deduces the fission halflives within the WKB approximation as (see, e.g., Ref. 21):

$$\tau(\text{year}) = 10^{-28.04}\left(1 + \exp\left\{\int_{q_1}^{q_2} 2\left[\frac{2M(q)}{\hbar^2}\,V(q)\right]^{1/2} dq\right\}\right) , \tag{8}$$

where the frequency of assaults on the barrier is assumed to be 1 MeV/$\hbar$ and V(q) is the potential energy relative to the penetration energy. The latter has been determined under the assumption that the actual ground state lies 0.5 MeV above the minimum of the deformation energy curve, to take roughly into account zero-point motion energy effects in the elongation mode.

Resulting fission halflives are reported in Table 4. From both SIII and SIV forces calculations, we may conclude that the N = 228, Z = 126 is stable against fission decay. However upon changing slightly the spin-orbit parameter W, one changes dramatically the fission halflives. As a matter of fact, with an approximately 20% smaller value of W, the previous conclusion about the fission stability is no longer valid. It is also worth noticing in Table 4 that the approximation (4) for the Coulomb exchange energy may be responsible for a lowering of the fission halflife by seven orders of magnitude.

4. ALPHA, BETA, ELECTRON-CAPTURE DECAY PROPERTIES

4.1 Alpha-decay Properties

The energy release corresponding to the emission of an alpha-particle by a nucleus having N neutrons and Z protons, is given by

$$Q_\alpha(N,Z) = B(2,2) - B(N,Z) + B(N-2, Z-2) \quad , \tag{9}$$

where B stands for the (positive) binding energy. The experimental value--28.3 MeV--of B(2,2) has been used (Ref. 22). The binding energy difference B(N - Z) - B(N - 2, Z - 2) has been estimated as $-2(e_n + e_p)$ (where e_n and e_p are the energies of the last neutron and proton occupied levels in the (N,Z) nucleus). This approximation is known to be satisfactory (Refs. 1 and 8) and is of course good enough in view of the uncertainties related to the effective force.

The alpha-decay halflives are deduced from the Q_α values according to the prescription of Viola and Seaborg (Ref. 23) resulting from a fit in heavy nuclei, namely

$$\log_{10} \tau(\mathrm{sec}) = (2.11329\, Z - 48.9879)/[Q_\alpha(\mathrm{MeV})]^{1/2} - 0.39004\, Z - 16.9543 \quad . \tag{10}$$

TABLE 5 Alpha-decay Properties of the Nucleus N = 228, Z = 126. For each force, the Q_α values and the estimated alpha-decay halflives τ, are reported. The last two columns correspond to the central part of the SIII force with different values of the spin-orbit parameter W (in MeV-fm^5).

Force	SV	SIV	SII*	SIII	SVI	SIII (W = 100)	SIII (W = 140)
Q_α(MeV)	3.4	4.3	4.7	5.5	6.3	5.8	6.6
τ (year)	10^{44}	10^{31}	10^{26}	10^{19}	10^{13}	10^{16}	10^{11}

As seen in Table 5, the considered nucleus seems almost stable against alpha-decay with all the forces in use. The uncertainty about the spin-orbit parameter generates a variation of halflives much smaller in this case than in the fission-decay case. One may notice however that Hartree-Fock calculations near magic nuclei are known to slightly underestimate experimental Q_α values (see Ref. 4). This implies that our alpha-decay halflives are probably overestimated. A reasonable guess of the error on Q (in the case of the studied semi-magic nucleus) is 0.5 MeV. For a Q_α value of approximately 5 MeV this leads to an overestimation of τ by about five orders of magnitude (according to Eq. (10)).

4.2. Beta and Electron-capture Decay Properties

The energy release corresponding to the capture of an electron by a nucleus having N neutrons and Z protons, is given by

$$Q_{ec}(N,Z) = B(N+1, Z-1) - B(N,Z) - \Delta E \quad , \tag{11}$$

where the quantity $\Delta E = 0.78$ MeV and is defined as

$$\Delta E = (M_{neutron} - M_{proton} - M_{electron})\, c^2 \quad . \tag{12}$$

As an approximation for the binding energy difference (neglecting in particular pairing effects for protons) one writes

$$B(N+1, Z-1) - B(N,Z) \simeq e_p^{(N+1,Z)} - e_n^{(N+1,Z)} \quad , \tag{13}$$

where $e_p^{(N+1,Z)}$ ($e_n^{(N+1,Z)}$) is the energy of the last occupied proton (neutron) single-particle state in the nucleus (N + 1,Z). Now, in the vicinity of the Fermi surface one may crudely estimate the dependence of e_p and e_n with the neutron number as

$$e_p^{(N+1,Z)} \simeq e_p^{(N,Z)} - \delta e_p \quad ,$$
$$e_n^{(N+1,Z)} \simeq e_n^{(N,Z)} - \delta e_n \quad . \tag{14}$$

For the SIII force one finds roughly (see Fig. 2 of Ref. 4):

$$\delta e_p \simeq 0.15 \text{ MeV}$$
$$\delta e_n \simeq 0.05 \text{ MeV} \quad . \tag{15}$$

Neglecting then a possible force-dependence of these figures, one gets for the Q_{ec} values (all energies in MeV):

$$Q_{ec}(N,Z) \simeq e_p^{(N,Z)} - e_n^{(N,Z)} - 0.88 \quad . \tag{16}$$

TABLE 6 Beta-decay Properties of the Nucleus N = 228, Z = 126. For each force, the Q_β values and the estimated beta-decay halflives τ (if finite), are reported. The last two columns correspond to the central part of the SIII force with different values of the spin-orbit parameter W (in MeV-fm^5). In the last row, the Q_{ec} values are also given.

Force	SV	SIV	SII*	SIII	SVI	SIII (W = 100)	SIII (W = 140)
Q_β(MeV)	-1.9	-0.8	-0.2	0.1	0.1	0.6	-0.4
τ (day)	-	-	-	200	200	4	-
Q_{ec}(MeV)	-2.9	-3.9	-3.9	-3.9	-3.5	-3.7	-3.7

As seen in Table 6, the nucleus N = 228, Z = 126 is found stable against electron capture for all forces in use.

The energy release corresponding to the emission of an electron by a nucleus having N neutrons and Z protons is given by

$$Q\ (N,Z) = B(N-1,Z+1) - B(N,Z) + \Delta E \quad . \tag{17}$$

As in the electron-capture case one makes the following approximations:

$$B(N-1,\ Z+1) - B(N,Z) \simeq e_n^{(N,Z+1)} - e_p^{(N,Z+1)} \tag{18}$$

and

$$e_n^{(N,Z+1)} \simeq e_n^{(N,Z)} - \delta e_n'$$
$$e_p^{(N,Z+1)} \simeq e_p^{(N,Z)} + \delta e_p' \quad . \tag{19}$$

Furthermore for the SIII force one gets roughly (see Figs. 3 and 4 of Ref. 4):

$$\delta e_n' \simeq \delta e_p' \simeq 0.3 \text{ MeV} \quad . \tag{20}$$

This leads to the following crude estimate for Q_β (all energies in MeV):

$$Q_\beta(N,Z) \simeq e_n^{(N,Z)} - e_p^{(N,Z)} + 0.18 \quad . \tag{21}$$

From these Q_β values (if positive), one deduces beta-decay halflives according to the average method described in Ref. 20, namely for our spherical singly magic even nucleus (Q_β being given in MeV)

$$\tau(\text{day}) = 6.69/[(Q_\beta + 0.511)^6 - (0.511)^6] \quad . \tag{22}$$

The results listed in Table 6 show that for all forces but two the nucleus N = 228, Z = 126 seems stable against beta-decay. With the SIII and SIV forces the positive Q_β values are pretty small. Upon slightly increasing the spin-orbit parameter W from its standard value in the SIII force, one yields beta-stability. As a result it appears that the nucleus N = 228, Z = 128 should be stable by beta emission and electron capture (one sees from the proton spectra and Eqs. (19) and (20) that the Q_β and Q_{ec} values should be modified by ~ - 1.2 and ~ 1.2 MeV, respectively). The alpha-decay properties on the other hand should be almost unchanged by the addition of two protons, whereas the first fission barrier should be slightly lowered (due to a small increase of the spherical shell correction).

5. CONCLUSIONS

The present study is rather of an exploratory nature. Granted the Hartree-Fock (+ BCS) framework and the use of an effective force, a lot of different approximations have been made. Some of them (related to alpha- and beta-decay) could be released at a minor cost, some others (related to fission-decay) are almost necessary to keep the problem in a tractable form. It is our opinion however that, in spite of these approximations, our work allows us to draw worthwhile conclusions about two important issues, as we will specify now.

Our primary aim was to study the magicity of the neutron number N = 228 and its effect on the fission-decay stability of corresponding superheavy elements. We have found that this neutron magicity could be responsible alone for the fission stability since the proton numbers producing beta-stability for these N = 228 nuclei do not correspond in our case to a closed shell filling. During the last two years, a number of similar investigations within self-consistent or not frameworks have been published (Refs. 24-29). They were all more or less intended as a search of a magic number around Z = 126. We would like to point out that such a proton magicity may be not necessary; it could be sufficient that the spherical proton shell-effect energy almost vanishes. However the spherical neutron shell-effect energy does not do all in building up the fission-decay stability. For instance, one may notice the fair similarity of our single-particle spectra with the ones of Ref. 21, resulting though in a complete disagreement for deformation energy curves. It could be argued that as in Ref. 30 (where the second fission barrier of ^{240}Pu calculated with the SIII force was found too high) that our deformed solution has followed a wrong collective path. Conversely, one may question the quality of the extrapolation to nuclear regions far from experimentally known elements of both the liquid drop and the mean field parameters in use in Ref. 21. These questions remain (among others!) unanswered at the moment.

We have studied, within a specific phenomenological parametrization of the effective interaction, the influence of various force parameters upon the stability properties of nuclei with N = 228. We have seen that due to a proper consideration of bulk properties of nuclei, there were only two really free parameters: one related to the saturation mechanism (velocity dependence vs density dependence), the other being the spin-orbit strength. Upon varying either one, one generates equally important variations of the stability properties. However one has more stringent arguments to assess precise values for the former than one has for the latter. The spin-orbit strength must thus be considered as the most critical parameter.

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DISCUSSION

S.G. Nilsson: I notice that we agree on a common worry about the spin-orbit force. Have you applied the same analysis to the Z=114 island?

P. Quentin: The point is that we shall look at it but we have not yet done it. I have a suspicion that it would be the same story, namely that the uncertainty upon the spin-orbit parameter would be the major source of theoretical error bars.

C.Y. Wong: With regard to the different density dependence of the Skyrme interaction used by Kohler, is the incompressibility of his Ska force lower than the incompressibility of Skyrme forces?

P. Quentin: Yes it exists a strong correlation between the exponent of the density dependence of the central part of the force and the incompressibility modulus in nuclear matter. Consequently it is no surprise that instead of 300 and up for standard Skyrme forces, the Skyrme Ska force leads to about 200 for this modulus.

W.M. Howard: If you believe that element 126 is found in nature, you must not only have element 126 stable, but also have a chain of stable isotopes from N=184 to N=228 stable against neutron capture. Since the stability of any Z=114 or Z=126 is due to strong neutron shell closures at N=184 and N=228, the region between these closures are likely to be highly unstable.

P. Quentin: The calculations I have presented are indeed rather lengthy (particularly as far as fission decay properties are concerned even though they are in the present stage only rough approximations). I appreciate the importance of the point you are making, but I am afraid that the investigation you think of could exceed the computational possibilities at the moment.

F. Petrovitch: In regard to Howard's question I mention that Schramm at the University of Chicago has looked at the r-process in light of a reasonable shell correction at Z=126, N=228. The production of Z=126, N=228 by the r-process seems a marginal possibility.

P. Quentin: No comment.