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ELECTROPRODUCTION OF SOFT PIONS
AT LARGE MOMENTUM TRANSFERS

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We consider pion electroproduction on a proton target close to threshold for Q^2 in the region $1 - 10$ GeV^2. The momentum transfer dependence of the S-wave multipoles at threshold, E_{0+} and L_{0+}, is calculated using light-cone sum rules.

Keywords: threshold electroproduction, chiral symmetry

1. Threshold Pion Production

Pion electroproduction at threshold from a proton target

$$e(l) + p(P) \rightarrow e(l') + \pi^+(k) + n(P')\,,$$
$$e(l) + p(P) \rightarrow e(l') + \pi^0(k) + p(P') \tag{1}$$

can be described in terms of two generalised form factors defined as [1]

$$\langle N(P')\pi(k)|j_\mu^{em}(0)|p(P)\rangle = \tag{2}$$
$$= -\frac{i}{f_\pi}\bar{N}(P')\gamma_5\left\{\left(\gamma_\mu q^2 - q_\mu \slashed{q}\right)\frac{1}{m^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m}G_2^{\pi N}(Q^2)\right\}N(P)\,,$$

which can be related to the S-wave transverse E_{0+} and longitudinal L_{0+} multipoles:

$$E_{0+}^{\pi N} = \frac{\sqrt{4\pi\alpha_{\rm em}}}{8\pi f_\pi}\sqrt{\frac{(2m+m_\pi)^2 + Q^2}{m^3(m+m_\pi)^3}}\left(Q^2 G_1^{\pi N} - \frac{1}{2}mm_\pi G_2^{\pi N}\right),$$

$$L_{0+}^{\pi N} = \frac{\sqrt{4\pi\alpha_{\rm em}}}{8\pi f_\pi}\frac{m|\omega_\gamma^{\rm th}|}{2}\sqrt{\frac{(2m+m_\pi)^2 + Q^2}{m^3(m+m_\pi)^3}}\left(G_2^{\pi N} + \frac{2m_\pi}{m}G_1^{\pi N}\right). \tag{3}$$

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The differential cross section at threshold is given by

$$\frac{d\sigma_{\gamma^*}}{d\Omega_\pi}\Big|_{\text{th}} = \frac{2|\vec{k}_f|W}{W^2 - m^2}\Big[(E_{0+}^{\pi N})^2 + \epsilon\frac{Q^2}{(\omega_\gamma^{\text{th}})^2}(L_{0+}^{\pi N})^2\Big]. \tag{4}$$

Here and below $m = 939$ MeV is the nucleon mass, $W^2 = (k + P')^2$ is the invariant energy, $\vec{k}_f$ and $\omega_\gamma^{\text{th}}$ are the pion three-momentum and the photon energy in the c.m. frame. The generalised form factors in (2) are real functions of the momentum transfer Q^2 at the threshold $W = m + m_\pi$. For generic W the definition in (2) can be extended to specify two of the existing six invariant amplitudes, $G_{1,2}(Q^2) \to G_{1,2}(Q^2, W)$, which become complex functions.

The celebrated low-energy theorem (LET) [2−4] relates the S-wave multipoles or, equivalently, the form factors G_1, G_2 at threshold, to the nucleon electromagnetic and axial form factors for vanishing pion mass $m_\pi = 0$

$$\frac{Q^2}{m^2}G_1^{\pi^0 p} = \frac{g_A}{2}\frac{Q^2}{(Q^2 + 2m^2)}G_M^p\,, \quad G_2^{\pi^0 p} = \frac{2g_A m^2}{(Q^2 + 2m^2)}G_E^p\,, \tag{5}$$

$$\frac{Q^2}{m^2}G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}}\frac{Q^2}{(Q^2 + 2m^2)}G_M^n + \frac{1}{\sqrt{2}}G_A\,, \quad G_2^{\pi^+ n} = \frac{2\sqrt{2}g_A m^2}{(Q^2 + 2m^2)}G_E^n\,.$$

Here the terms in $G_{M,E}$ are due to pion emission off the initial proton state, whereas for charged pion in addition there is a contribution corresponding to the chiral rotation of the electromagnetic current.

The subsequent discussion concentrated mainly on the corrections to (5) due to finite pion mass [5,6]. More recently, the threshold pion production for small Q^2 was reconsidered and the low-energy theorems re-derived in the framework of the chiral perturbation theory (CHPT), see [7] for a review. The new insight gained from CHPT calculations [8] is that the expansion at small Q^2 has to be done with care as the limits $m_\pi \to 0$ and $Q^2 \to 0$ do not commute, in general. The LET predictions seem to be in good agreement with experimental data on pion photoproduction [9], However, it appears [10,7] that the S-wave electroproduction cross section (4) for already $Q^2 \sim 0.1$ GeV^2 cannot be explained without taking into account chiral loops.

For larger momentum transfers the situation is much less studied as the power counting of CHPT cannot be applied. The traditional derivation of LET using PCAC and current algebra does not seem to be affected as long as the emitted pion is 'soft' with respect to the initial and final state nucleons simultaneously. The corresponding condition is, parametrically, $Q^2 \ll \Lambda^3/m_\pi$ (see, e.g. [5]) where Λ is some hadronic scale, and might be satisfied for $Q^2 \sim 1$ GeV^2 or even higher. We are not aware of any dedicated analysis of the threshold production in the $Q^2 \sim 1$ GeV^2 region, however.

It was suggested in Ref. [11] that in the opposite limit of very large momentum transfers the standard pQCD collinear factorisation approach [12,13] becomes applicable and the helicity-conserving $G_1^{\pi N}$ form factor can be calculated for $m_\pi = 0$ in terms of chirally rotated nucleon distribution amplitudes. In practice one expects that the onset of the pQCD regime is postponed to very large momentum transfers because the factorisable contribution involves a small factor $\alpha_s^2(Q)/\pi^2$ and has to win over nonperturbative "soft" contributions that are suppressed by an extra power of Q^2 but do not involve small coefficients.

The purpose of this study is to suggest a realistic QCD-motivated model for the Q^2 dependence of the $G_{1,2}$ form factors alias S-wave multipoles at threshold in the region $Q^2 \sim 1-10$ GeV^2 that can be accessible in current and future experiments in Jefferson Laboratory and elsewhere (HERMES, MAMI).

2. Light-Cone-Sum Rules

In Ref. [14] we have developed a technique to calculate baryon form factors for moderately large Q^2 using light-cone sum rules (LCSR) [15,16]. This approach is attractive because in LCSRs "soft" contributions to the form factors are calculated in terms of the same nucleon distribution amplitudes (DAs) that enter the pQCD calculation and there is no double counting. Thus, the LCSRs provide one with the most direct relation of the hadron form factors and distribution amplitudes that is available at present, with no other nonperturbative parameters.

The same technique can be applied to pion electroproduction. In Ref. [1] the G_1 and G_2 form factors were estimated in the LCSR approach for the range of momentum transfers $Q^2 \sim 5-10$ GeV^2. For this work, we have reanalysed the sum rules derived in [1] taking into account the semi-disconnected pion-nucleon contributions in the intermediate state. We demonstrate that, with this addition, the applicability of the sum rules can be extended to the lower Q^2 region and the LET results in (5) are indeed reproduced at $Q^2 \sim 1$ GeV^2 to the required accuracy $\mathcal{O}(m_\pi)$. The results presented below essentially interpolate between the large-Q^2 limit considered in [1] and the standard LET predictions at low momentum transfers.

Accurate quantitative predictions are difficult for several reasons, e,g, because the nucleon distribution amplitudes are poorly known. In order to minimise the dependence of various parameters in this work we only use the LCSRs to predict certain form factor ratios and then normalise to the electromagnetic nucleon form factors as measured in the experiment, see
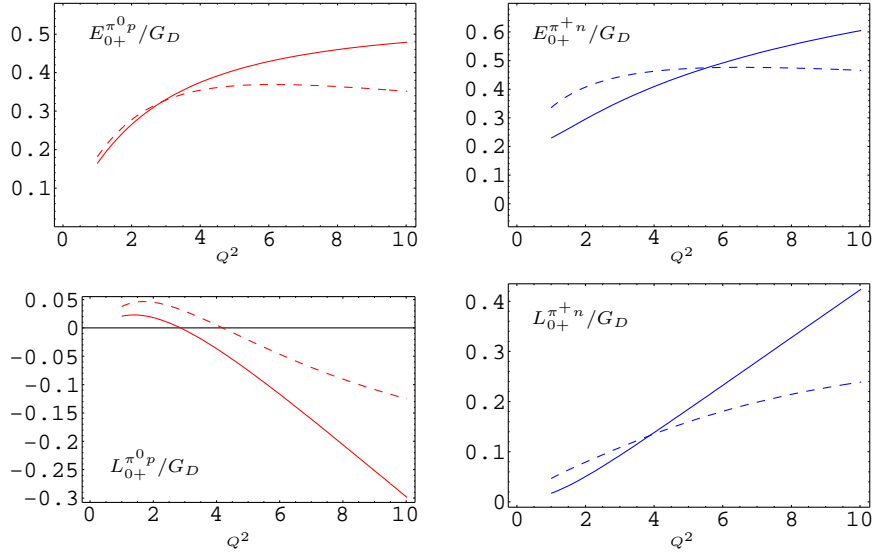
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Fig. 1.   The LCSR-based model (solid curves) for the Q^2 dependence of the electric and longitudinal partial waves at threshold E_{0+} and L_{0+}, (3), in units of GeV^{-1}, normalised to the dipole formula (7).

[17] for the details. In particular we use the parametrisation of the proton magnetic form factor from [18] and for the neutron magnetic form factor from [19]. For the proton electric form factor we use the fit [20,18] to the combined JLab data in the $0.5 < Q^2 < 5.6$ GeV^2 range

$$\mu_p \frac{G_E^p}{G_M^p} = 1 - 0.13(Q^2 - 0.04) \tag{6}$$

and put the neutron electric form factor to zero, which should be good to our accuracy. Note that using (6) for larger values of Q^2 up to 10 GeV^2 is an extrapolation which may be not justified.

The resulting LCSR-based model is shown by the solid curves in Fig. 1, where the four partial waves at threshold that are related to the generalised form factors through the Eq. (3) are plotted as a function of Q^2, normalised to the dipole formula

$$G_D(Q^2) = 1/(1 + Q^2/\mu_0^2)^2 \tag{7}$$

where $\mu_0^2 = 0.71$ GeV^2. This model is used in the numerical analysis presented below. We expect that its accuracy is about 50%. It can be improved in future by the calculation of radiative corrections to the LCSRs, especially

if sufficiently accurate lattice calculations of the moments of nucleon distribution amplitudes become available. To give a rough idea about possible uncertainties, the "pure" LCSR predictions (all form factors and other input taken from the sum rules) are shown by dashed curves for comparison.

3. Moving Away From Threshold

As a simple approximation, we suggest to calculate pion production near threshold in terms of the generalised form factors (2) and taking into account pion emission from the final state which dominates the P-wave contribution in the chiral limit (cf.[11]). In particular, we use the following expression:

$$
\langle N(P')\pi(k)|j_\mu^{em}(0)|p(P)\rangle =
$$
$$
= -\frac{i}{f_\pi}\bar{N}(P')\gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \slashed{q}\right)\frac{1}{m^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^\nu}{2m}G_2^{\pi N}(Q^2) \right\} N(P)
$$
$$
+ \frac{ic_\pi g_A}{2f_\pi[(P'+k)^2)-m^2]}\bar{N}(P')\,\slashed{k}\,\gamma_5(\slashed{P}'+m)\left\{ F_1^p(Q^2)\left(\gamma_\mu - \frac{q_\mu \slashed{q}}{q^2}\right)\right.
$$
$$
\left. + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2^p(Q^2) \right\} N(P) . \tag{8}
$$

Here $F_1^p(Q^2)$ and $F_2^p(Q^2)$ are the Dirac and Pauli electromagnetic form factors of the proton, $c_{\pi^0}=1$ and $c_{\pi^+}=\sqrt{2}$ is the isospin factor, $g_A = 1.267$ and $f_\pi = 93$ MeV.

The separation of the generalised form factor contribution and the final state emission in (8) can be justified in the chiral limit $m_\pi \to 0$ but involves ambiguities in contributions $\sim \mathcal{O}(m_\pi)$. We have chosen not to include the term $\sim \slashed{k}$ in the numerator of the proton propagator in the third line in (8) so that this contribution strictly vanishes at the threshold. In addition, we found it convenient to include the term $\sim q_\mu \slashed{q}/q^2$ in the Lorentz structure that accompanies the F_1 form factor in order to make the amplitude formally gauge invariant. To avoid misunderstanding, note that our expression is not suitable for making a transition to the photoproduction limit $Q^2 = 0$ in which case, e.g. pion radiation from the initial state has to be taken in the same approximation to maintain gauge invariance.

The virtual photon cross section can be written as a sum of terms

$$
d\sigma_{\gamma^*} = \frac{\alpha_{\rm em}}{8\pi}\frac{k_f}{W}\frac{d\Omega_\pi}{W^2 - m^2}|\mathcal{M}_{\gamma^*}|^2 \tag{9}
$$

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with

$$|\mathcal{M}_{\gamma^*}|^2 = M_T + \epsilon\, M_L + \sqrt{2\epsilon(1+\epsilon)}\, M_{LT}\, \cos(\phi_\pi)$$
$$+ \epsilon M_{TT}\, \cos(2\phi_\pi) + \lambda\sqrt{2\epsilon(1-\epsilon)}\, M'_{LT}\, \sin(\phi_\pi)\,; \qquad (10)$$

in the last term λ is the beam helicity.

The complete expressions for the invariant functions are rather cumbersome but are simplified significantly in the chiral limit $m_\pi \to 0$ and assuming $k_f = \mathcal{O}(m_\pi)$. We obtain

$$f_\pi^2 M_T = \frac{4\vec{k}_i^2 Q^2}{m^2}|G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2 - m^2)^2}Q^2 m^2 G_M^2$$
$$+ \cos\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m^2} 4Q^2 G_M \mathrm{Re}\, G_1^{\pi N}\,,$$

$$f_\pi^2 M_L = \vec{k}_i^2 |G_2^{\pi N}|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{(W^2 - m^2)^2}m_N^4 G_E^2$$
$$- \cos\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m^2} 4m^2 G_E \mathrm{Re}\, G_2^{\pi N}\,,$$

$$f_\pi^2 M_{LT} = -\sin\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m^2} Qm\Big[G_M \mathrm{Re}\, G_2^{\pi N} + 4G_E \mathrm{Re}\, G_1^{\pi N}\Big]\,,$$

$$f_\pi^2 M_{TT} = 0\,,$$

$$f_\pi^2 M'_{LT} = -\sin\theta \frac{c_\pi g_A |k_i||k_f|}{W^2 - m^2} Qm\Big[G_M \mathrm{Im}\, G_2^{\pi N} - 4G_E \mathrm{Im}\, G_1^{\pi N}\Big]\,, \quad (11)$$

where $\vec{k}_i$ is the c.m.s. momentum in the initial state. Note that the single spin asymmetry contribution $\sim M'_{LT}$ involves imaginary parts of the generalised form factors that arise because of the final state interaction. In our approximation $M_{TT} = 0$ which is because we do not take into account the D- and higher partial waves. Consequently, the $\sim \cos(2\phi)$ contribution to the cross section is absent.

We find that the integrated cross sections scale like $\sigma_{\gamma^* p \to \pi N} \sim 1/Q^6$, which is in agreement with the structure function measurements in the threshold region by E136 [21]. The S-wave contribution appears to be larger than P-wave up to $W \simeq 1.16$ GeV. The ratio of $\pi^0 p$ and $\pi^+ n$ final states is approximately $1:2$ and almost Q^2-independent.

The comparison of our calculation for the structure function $F_2^p(W, Q^2)$ in the threshold region $W^2 < 1.4$ GeV^2 to the SLAC E136 data [21] at the average value $Q^2 = 7.14$ GeV^2 and $Q^2 = 9.43$ GeV^2 is shown in Fig. 3. The predictions are generally somewhat below these data ($\sim 50\%$), apart from the last data point at $W^2 = 1.4$ GeV^2 which is significantly higher.
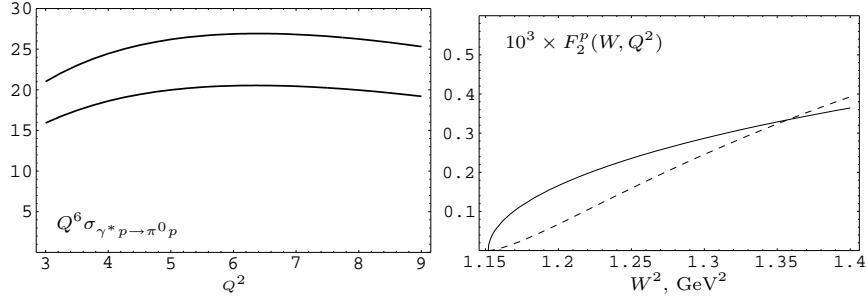
Fig. 2.   *Left panel:* The integrated cross section $Q^6\sigma_{\gamma^*p\to\pi^0p}$ (in units of $\mu b\times\mathrm{GeV}^6$) as a function of Q^2 for $W = 1.11$ GeV (lower curve) and $W = 1.15$ GeV (upper curve). *Right panel:* The S-wave (solid) vs. the P-wave (dashed) contribution to the structure function $F_2^p(W,Q^2)$ as a function of W^2 for $Q^2 = 7.14\,\mathrm{GeV}^2$.
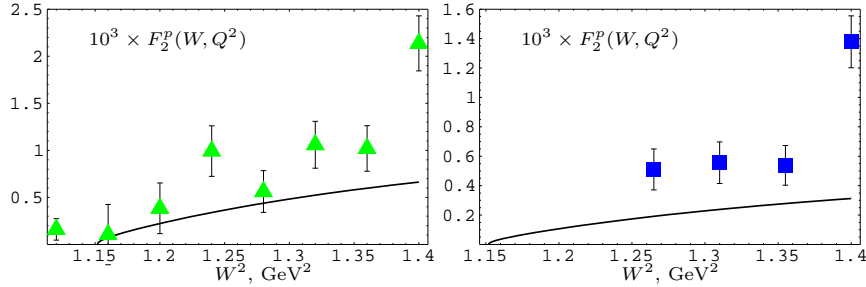

Fig. 3.   The structure function $F_2^p(W,Q^2)$ as a function of W^2 scaled by a factor 10^3 compared to the SLAC E136 data at the average value $Q^2 = 7.14\,\mathrm{GeV}^2$ (left panel) and $Q^2 = 9.43\,\mathrm{GeV}^2$ (right panel).

Note that in our approximation there is no D-wave contribution, and the final state interaction is not included. Both effects can increase the cross section so that we consider the agreement as satisfactory. We believe that the structure function at $W^2 = 1.4$ GeV^2 already contains a considerable D-wave contribution and also one from the tail of the Δ-resonance and thus cannot be compared with our model, at least in its present form.

To avoid misunderstanding we stress that the estimates of the cross sections presented here are not state-of-the-art and are only meant to provide one with the order-of-magnitude estimates of the threshold cross sections that are to our opinion most interesting. These estimates can be improved in many ways, for example taking into account the energy dependence of the generalised form factors generated by the FSI and adding a model for the D-wave contributions. The model can also be tuned to reproduce the

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existing lower Q^2 and/or larger W experimental data.

Acknowledgements

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