On the Sources of U.S. Stock Market Comovement

Enzo Weber*

March 16, 2010

Nr. 439

* Enzo Weber is Juniorprofessor of Economics at the Department of Economics and Econometrics at the University of Regensburg, 93040 Regensburg, Germany.
Phone: +49-941-943-1952, E-mail: enzo.weber@wiwi.uni-regensburg.de
On the Sources of U.S. Stock Market Comovement

Enzo Weber

Universität Regensburg
D-93040 Regensburg, Germany
enza.weber@wiwi.uni-regensburg.de
phone: +49-941-943-1952 fax: +49-941-943-2735

Abstract

This paper disentangles direct spillovers and common factors as sources of correlations in simultaneous heteroscedastic systems. While these different components are not identifiable by standard means without restrictions, it is shown that they can be pinned down by specifying the variances of the latent idiosyncratic and common shocks as ARCH-type processes. Applying an adapted Kalman filter estimation method to Dow and Nasdaq stock returns, predominant spillovers from the Dow and substantial rising factor exposure are found. While the latter is shown to prevail in the recent global financial crisis, volatility in the dot-com bubble period was driven by Nasdaq shocks.

Keywords: Simultaneous System, Latent Factor, Identification, Spillover, EGARCH
JEL classification: C32, G10

\footnote{This study is partly based on my CRC 649 discussion paper Weber (2007b). I am grateful to Jürgen Wolters and Cordelia Thielitz as well as seminar participants at the ICMA Centre of the University of Reading and the University of Mannheim for their comments. Of course, all remaining errors are my own.}
1 Introduction

Different stocks or portfolios often reveal a high degree of coherence in their fluctuations. For example, from 1971 until 2009, daily Dow Jones Industrial and Nasdaq Composite returns exhibit a correlation of 74%. On a very general basis, such a comovement could result from contemporaneous spillovers between the relevant variables or from common exogenous factors driving all of them alike. Concerning the first alternative, the direction of the transmission effect represents a further refinement. In statistics, the negative statement, "correlation does not imply causation" is widespread; in short, this paper intends to develop a positive answer to the question, to which extent a specific correlation has a truly causal nature, and to which it is based on third-party intervention. From this perspective, the contribution lies in designing statistical methods empirically assessing the classical econometric issue of "instantaneous" causation; see Granger (1988) as one representative from this literature.

In the language of the above financial market example, equal development of different stock indexes can have two reasons: First, observing already realised index movements in one market might influence the decisions and activities of participants in another. "Signalling effects" may be a good label for this case. Besides, liquidity and wealth effects fall into the same category. Evidently, this represents the case of direct causal spillover. Second, certain information is obtained contemporaneously by participants in several markets, for which it is judged equally relevant. Logically, as a classical common factor, this information generates immediate and similar stock price reactions in all segments concerned. It follows that the current study has to unite two types of econometric analyses, one occupied with financial transmission in terms of direct causality, and the other one seeking for the effect of "news" as fundamental factors triggering market responses.

A straightforward way of examining links between financial variables is given by choosing markets with non-overlapping trading time. For example, when the New York Stock Exchange opens, closing prices in Tokyo are already established. Therefore, the direction of propagation can be defined running from the respective daytime to overnight returns. Nonetheless, even here a real direct causal impact is not yet separated from pure incorporation of news arrived overnight, yet already manifested in the daytime trade of a different time zone. In case of parallel trading, naturally given for equity indexes of the same nationality, the issue is additionally complicated by the possibility of bi-directional spillover. Inevitably, the discussion results in a classical econometric identification problem. In this context, consider for illustrational purposes the standard solution to identifying struc-
tural vector autoregressive (SVAR) models: Usually, a Choleski decomposition is applied to the reduced-form residual covariance-matrix, leading to a triangular matrix of recursive contemporaneous effects. Furthermore, the structural innovations have to be assumed uncorrelated. Consequently, two possible sources of contemporaneous correlation, that is half of the causal impacts (the interdependence) and the non-causal connections, are assumed inexistent in this setting.

A strand of recent literature introduced a method that exploits non-constant variances mostly of financial variables to address the simultaneity problem: Following the idea that every shift in the structural variances yields more determining equations from the reduced-form covariance matrix than unknown coefficients, the model structure can be identified “through heteroscedasticity” (see Rigobon 2003). Building upon this logic, different literature contributions proposed specifying the respective variances as ARCH-type processes, either for the reduced-form (Rigobon 2002) or structural residuals (Sentana and Fiorentini 2001, Weber 2007a), thereby virtually providing a continuum of volatility regimes. Further relevant papers include Feenstra (1994) and Lewbel (2008).

Existent methodology typically either assumes that the contemporaneous correlation results exclusively from common factors (i.e., factor models), or that it is to the full extent a product of causal interaction only between the included variables (e.g., SVARs). These are both serious drawbacks: The first variant obviously fails to detect causality between the observed endogenous variables, which might play an important role for indexes as closely connected as Dow and Nasdaq. Concerning the second variant, in presence of neglected exogenous shocks, the estimation is bound to overstate the bilateral linkage. While this problem might in principle be treated by augmenting the model with essential missing variables, much relevant information will be unobservable (see e.g. King et al. 1994) or can hardly be completely covered by time series systems of limited dimension. This stresses the importance of allowing for contemporaneous interaction in the structural innovations. However, as will be shown in the following section, unrestricted time-varying covariances would simply undo the identifiability created by heteroscedasticity.

The current paper contributes to the progress in the research field by including an unobservable common factor into a heteroscedastic system, in addition to unrestricted simultaneous spillovers. Latent factor modelling allows covering general exogenous influences that do not have to be observed or even predisposed. Following the same intuition, Weber (2009) specifies simultaneous systems with constant and dynamic conditional correlations of the structural innovations. While allowing for such correlations is directly linked to the tradition of classical simultaneous systems, the underlying paper might be seen more
in the finance context of factor models. Importantly, the underlying factor setup has the advantage of clear-cut separation of the different fundamental driving forces and allows structural interpretation e.g. in terms of variance decompositions. Moreover, refer as well to Sentana and Fiorentini (2001) and Doz and Renault (2004), who discuss identification of classical factor models in presence of heteroscedasticity; a well-known application is given in King et al. (1994). Estimates of model parameters, factors states and conditional variances are obtained by Quasi Maximum Likelihood (QML) via Kalman filtering. As a further innovation, this paper specifically adapts the EGARCH model to handle unobserved disturbances.

The application to U.S. stock market data shows that mean and volatility spillovers mainly run from Dow to Nasdaq, which are both subject to substantial common factor impacts. This is broadly in line with the findings of Weber (2009). In addition, important developments within the sample period are revealed: While Dow shocks prevailed in the first decades, Nasdaq took over during the dot-com bubble episode. Moreover, the factor influence rises over time, culminating in the recent world financial crisis.

The reader can expect the following: The methodological concept is discussed at length in the next section. Then, section 3 presents empirical results for U.S. equity data. In the end, the summary provides a short overview of outcome, merits and further potentials of the present examination. An appendix contains a mathematical proof.

2 Methodology

In the following paragraphs, a model is constructed that shall finally feature and identify both mutual and common influences among a set of variables. At first, the structural form of the mean equations is presented, followed by a discussion of inherent identification problems in presence of full simultaneity and latent factor exposure. Thereafter, the variances of the idiosyncratic and factor innovations are modelled by EGARCH processes. Finally, Kalman filtering and estimation by Maximum Likelihood are tackled.

2.1 Basic Model and Identification

To begin with, think of the data generating process of the \( n \) variables (e.g., stock returns) \( y_{it} \) being approximated by the system

\[
Ay_t = \varepsilon_t .
\]
The contemporaneous impacts are included in the matrix $A$ with diagonal elements normalised to one, and $\varepsilon_t$ is a $n$-dimensional vector of uncorrelated structural residuals. Of course, this model can easily be adapted to cover vector autoregressive lags or deterministic terms, as considered in the empirical example in section 3.

Representing a simultaneous equation system, (1) as it stands is not identified: In the matrix $A$ with normalised diagonal, $n(n-1)$ simultaneous impacts have to be estimated, whereas the covariance-matrix of $y_t$ delivers only $n(n-1)/2$ covariance equations due to its symmetry. There are two standard solutions: One is to impose a recursive structure on the contemporaneous impacts, thereby restricting $A$ to a triangular matrix. However, this would effectively imply that the research question of uncovering direction and strength of mutual spillovers would have to be answered a priori for some theoretical, but not for empirical reasons. Similar drawbacks apply to the second solution, the use of instrumental variables. Here, exclusion restrictions must be assumed to hold for the instruments.

Heading towards a more appropriate solution, refer to Sentana and Fiorentini (2001), who show that structural latent variable models like (1) can be identified in presence of heteroscedasticity. The concept is as well discussed in Rigobon (2003) and Weber (2009), effectively going back as far as Wright (1928). Since for financial volatility, ARCH-type processes have been widely found to provide appropriate representations, I will follow Weber (2007a) in specifying multivariate EGARCH processes for the structural disturbances.

2.2 Common Factors

Before tackling model setup and estimation in more detail, let us turn our attention to a last problematic point: Conventional approaches to identification through heteroscedasticity are based on the assumption of conditional uncorrelatedness of the structural residuals. However, maintaining this assumption implies that the contemporaneous correlation of the variables in $y_t$ is to be fully taken into account by direct spillovers between the included variables. For example, it seems extremely unlikely that two U.S. indexes like Dow Jones and Nasdaq Composite are not subject to any exogenous common factors, which might at least partly trigger the observed substantial correlations.

Therefore, the present study formally includes a common factor into the simultaneous heteroscedastic system. This allows for time-varying interaction in the structural innovations in addition to the direct spillovers. Consequently, one can discriminate between all possible sources of correlations: two directions of transmission and third-party influences.
Thereby, the inclusion of the common factor allows maintaining mutual uncorrelatedness of the innovations. In particular, identifiability is preserved by representing the dynamic covariance structure in a parsimonious factor setup.

Formalising the preceding argumentation, add the factor $z_t$ (scalar in the bivariate Dow-Nasdaq application) multiplied by the $n \times 1$ vector of loadings $\beta$ to equation (1). As a standard assumption, all $\varepsilon_{jt}$ and $z_t$ are conditionally uncorrelated, including leads and lags. The reduced form results as

$$y_t = A^{-1}(\beta z_t + \varepsilon_t) .$$

(2)

### 2.3 Structural EGARCH

In the following, the heteroscedastic model specification shall be formalised. Besides achieving identification, this serves to increase estimation efficiency and deliver measures of spillovers in the volatility domain (e.g. reflecting information flows, see Ross 1989). First, denote the conditional variances by

$$\text{Var}(\varepsilon_{jt}|\Omega_{t-1}) = h_{jt}, \quad j = 1, \ldots, n, \quad \text{Var}(z_t|\Omega_{t-1}) = h_{zt} ,$$

(3)

where $\Omega_{t-1}$ stands for the whole set of available information at time $t - 1$.

Then, stack the idiosyncratic conditional variances in the vector $H_t = \left( h_{1t} \ldots h_{nt} \right)'$.

At last, obtain the standardised innovations $\tilde{\varepsilon}_{jt} = \varepsilon_{jt}/\sqrt{h_{jt}}, \quad j = 1, \ldots, n$, and $\tilde{z}_t = z_t/\sqrt{h_{zt}}$.

Then, I specify a multivariate EGARCH(1,1)-process for the idiosyncratic variances, which allows for possible volatility spillovers. Following Weber (2007a), write

$$\log H_t = C + G \log H_{t-1} + D(\tilde{\varepsilon}_{t-1} - t\sqrt{2/\pi}) + F\tilde{\varepsilon}_{t-1} ,$$

(4)

where $C$ is a $n$-dimensional vector of constants, $G$, $D$ and $F$ are $n \times n$ coefficient matrices, and $t$ denotes a column vector of $n$ ones. The absolute value operation is to be applied element by element to the $n \times 1$ vector $\tilde{\varepsilon}_{t-1}$. As in Nelson (1991), $\sqrt{2/\pi}$ is subtracted to demean these absolute value terms. Note that the model is invariant in this respect, since any possible difference between $E(\tilde{\varepsilon}_{t-1})$ and $t\sqrt{2/\pi}$, for instance due to deviations from conditional normality, would simply be multiplied by $D$ and merged into $C$. Asymmetric effects are incorporated by including $\tilde{\varepsilon}_{t-1}$ without taking absolute values: Any parameters in $F$ differing from zero indicate that besides the magnitude of a shock its sign contains
valuable information for forecasting the conditional variances. Process orders 1, 1 are
standard in financial econometrics (see Nelson 1992) and will be shown to be appropriate
by ARCH-LM tests.

While (4) has as well been used by Weber (2009) for the innovations in his ”structural
conditional correlation” model, I put up an additional EGARCH equation for the factor
variance:

$$\log h_{zt} = g \log h_{zt-1} + d(|\tilde{z}_{t-1}| - \sqrt{2/\pi}) + f \tilde{z}_{t-1},$$

with $g$, $d$ and $f$ as scalar coefficients. The constant is left out in order to normalise the
unconditional factor variance. Due to the conditional uncorrelatedness of the idiosyncratic
and common factors, covariances need not to be explicitly modelled. Whether spillovers
($A$) and common factor exposure ($\beta$) appropriately pick up the covariation in the data is
an empirical question tested later on.

From (2), the conditional reduced-form covariance-matrix is obtained as

$$\Sigma_t = A^{-1}(\beta h_{zt} \beta' + \begin{pmatrix} h_{1t} & 0 \\ \vdots & \ddots \\ 0 & h_{nt} \end{pmatrix})(A^{-1})'.$$

Since the log-linearised EGARCH equations necessarily deliver positive conditional vari-
ances, the (double) quadratic form (6) conveniently solves the common problem of assuring
the covariance matrix to be positive definite. Importantly, this feature is a consequence
of the structural modelling and does not rely on specific functional forms such as given
e.g. by the BEKK or CCC. Furthermore, two sources of cross-correlation, as represented
by non-zero off-diagonal elements in $\Sigma_t$, become evident: First, the common factor $z_t$
naturally produces a certain degree of comovement though the loadings $\beta$, and second,
changes in a variable can instantaneously spill over according to the coefficients in $A^{-1}$.
The task is to determine the contributions of both effects to the overall correlation as well
as the specific directions of spillover.

2.4 Estimation

For the purpose of estimation, the log-likelihood for a sample of $T$ observations (com-
plemented by an adequate number of pre-sample observations) under the assumption of
conditional normality is constructed as

$$L(\theta) = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log 2\pi + \log |\Sigma_t| + y_t'\Sigma_t^{-1}y_t \right),$$
where the vector $\theta$ stacks all free parameters from $A$, $\beta$, $C$, $G$, $D$, $F$, $g$, $d$ and $f$. That is, maximisation of (7) yields estimates of both the EGARCH parameters and the structural coefficients governing spillovers and common factor exposure. Thereby, I rely on Quasi Maximum Likelihood (see Bollerslev and Wooldridge 1992), using the robust "sandwich" covariance matrix of the parameters adapted to general conditional distributions. Numerical likelihood optimisation is performed using the BHHH algorithm (Berndt et al. 1974).

Obviously, obtaining estimates of $\Sigma_t$ requires knowledge of the non-observable factor states $z_t$ through the EGARCH equation in (4). Approximate Kalman filtering is used to solve this problem. Thereby, the state space model is set up as follows: First, since the factor is serially uncorrelated, the transition equation is trivial in that it simply equals the factor to its innovation. The observation equation is given by the reduced form (2).

Standard Kalman filter recursions predict and update expectation and covariance matrix of the state variables conditional on a given information set; see Lütkepohl (2005) for a textbook treatment of the multivariate case. Specifically, let $\mu_{jt}$ and $\sigma^2_{zt}$, $j = 1, \ldots, n, z$, denote expectation and variance of the $\varepsilon_{jt}$ and $z_t$, conditional on the observable variables in $y$ up to time $t$. Then, $E(z_t|y_t, y_{t-1}, \ldots) = \mu_z$ and $\sigma^2_{zt}$ as the conditional factor variance are directly obtained from Kalman filter updating as usual. The states of the idiosyncratic shocks can be estimated from (2) as $(\mu_{1t}, \ldots, \mu_{nt})' = Ay_t - \beta z_t$. Accordingly, their covariance matrix holding the $\sigma_{zt}$, $j = 1, \ldots, n$, on its main diagonal is given by $\beta \sigma_{zt}$. As a last critical point, note that the EGARCH variances in (4) and (5) depend on the absolute innovations $|\varepsilon_{jt}|$ and $|z_t|$. In the Appendix, the conditional Kalman filter expectation of the $z$ term (and the $\varepsilon$ terms accordingly) is shown to emerge as

$$E(|z_t| |y_t, y_{t-1}, \ldots) = \mu_z (\Phi(\mu_{zt}/\sigma_{zt}) - \Phi(-\mu_{zt}/\sigma_{zt})) + 2\sigma_{zt} \varphi(\mu_{zt}/\sigma_{zt}),$$

with $\varphi$ and $\Phi$ representing the standard normal probability density and cumulative distribution function, respectively. Since (8) contains only quantities that are made available by the recursive filter, we are now well equipped for an application.

---

2Harvey et al. (1992) discuss an equivalent procedure for unobserved component ARCH and GARCH models, using $E_t(z_t^2) = \mu^2_{zt} + \sigma^2_{zt}$. 
3 Blue Chip vs. High Tech

3.1 Data and Empirical Procedure

The empirical part analyses the interaction between two major U.S. stock segments as reflected by the Dow Jones Industrial Average and the Nasdaq Composite, ”blue chip” and ”high tech” in the language of Weber (2009). The sample of daily observations begins on 2/5/1971, where Nasdaq had started, and ends on 12/31/2009; data source is Reuters. Figure 1 presents continuously compounded returns and the well-known picture of the index development. Most eye-catching are the Black Monday in 1987, the extremely volatile period around 2000, where stock prices fell due to the dot-com bubble burst and the general recession, and the recent world financial crisis. The unconditional standard deviations of the returns are 1.09 and 1.26.

![Dow and Nasdaq Indexes](image1)

![Dow and Nasdaq Returns](image2)

Figure 1: Dow Jones and Nasdaq Composite

In preparation for the empirical procedure, the returns$^3$ were filtered by regressing them

$^3$Cointegration could not be established, leading to a model in first differences.
on a constant and four day-of-the-week dummies. Based on the suggestion of the Bayesian information criterion, autoregressive lags were not included. Starting values were determined as follows: The initial factor was obtained as the first principal component and standardised to unit variance. Then, using the respective loadings in \( \beta \), the factor scores were subtracted from the returns. \( A \) was thus initialised as the identity matrix. The EGARCH parameters were then obtained from univariate models for the initial series of the factor and the idiosyncratic residuals. The variance processes were started at the according sample moments. The estimations were carried out in a Gauss programme employing the CML module.

### 3.2 Results and Discussion

Equations (9) display the contemporaneous interactions in the U.S. stock market, based on the parameters from the structural matrix \( A \) and the vector of loadings \( \beta \). The variable names denote daily returns at time \( t \), QML standard errors are in parentheses. The estimates of the spillover coefficients imply that the Dow dominates the mutual transmission. Furthermore, the returns are significantly hit by the common factor. Both outcomes correspond to what economic intuition might have told us in advance.

\[
\begin{align*}
\text{DJIA}_t &= 0.148 \text{NQC}_t + 0.900 \hat{\varepsilon}_t + \hat{\varepsilon}_{1t} \\
\text{NQC}_t &= 0.446 \text{DJIA}_t + 0.591 \hat{\varepsilon}_t + \hat{\varepsilon}_{2t}
\end{align*}
\]

Equation (10) shows the estimated EGARCH processes of the idiosyncratic variances. Again, spillovers (the off-diagonal matrix elements) of Dow shocks to Nasdaq are much stronger than in the reverse direction.\(^5\) In line with many established results for financial data, the variances are quite persistent (as measured by \( G \)) and subject to asymmetrically strong impacts of negative shocks (as measured by \( F \)).

\(^4\) Various further starting values were chosen, including relatively implausible ones. Nothing hinted at a local-maximum-problem. The same applies to the use of different numerical algorithms.

\(^5\) The present results are broadly in line with the structural conditional correlation estimates of Weber (2009), who uses data until 10/31/2007. The current update until the end of 2009 has been checked not to lead to significant changes in the parameter estimates. This should strengthen our confidence in the model that is able to cope with the global financial crisis.
\[
\begin{pmatrix}
\log h^2_{1t} \\
\log h^2_{2t}
\end{pmatrix} = 
\begin{pmatrix}
-0.02 & 0.996 \\
-0.028 & 0.977
\end{pmatrix}
\begin{pmatrix}
\log h^2_{1t-1} \\
\log h^2_{2t-1}
\end{pmatrix} + 
\begin{pmatrix}
0.155 & 0.211 \\
0.221 & 0.024
\end{pmatrix}
\begin{pmatrix}
|\hat{\varepsilon}_{1t-1}| \\
|\hat{\varepsilon}_{2t-1}|
\end{pmatrix}
+ 
\begin{pmatrix}
-0.038 & -0.029 \\
-0.073 & -0.040
\end{pmatrix}
\begin{pmatrix}
\hat{\varepsilon}_{1t-1} \\
\hat{\varepsilon}_{2t-1}
\end{pmatrix}
\]

(10)

The factor variance equation (11) reveals the same properties, i.e. persistence and asymmetry:

\[
\log h_{zt} = 0.975 \log h_{zt-1} + 0.164 |\hat{z}_{t-1}| - 0.086 \hat{z}_{t-1}
\]

(11)

Based on the estimated model (9), (10), (11), the sources of stock market comovement can be assessed. First, the total conditional correlations are calculated from (6). Second, the correlations emerging from common factor exposure only can be obtained from the same covariance matrix by setting \( A = I \). That is, spillovers are neglected, leaving the correlation of the term \( \beta z_t + \epsilon_t \). This corresponds to the structural conditional correlation of Weber (2009), i.e. the fundamental correlation net of spillovers. Both the total and the structural correlation are displayed in Figure 2.

![Figure 2: Total and structural conditional correlations](image)

The average structural correlation (32%) lies near the value of 36% found by Weber (2009). The total correlation reveals troughs in the early eighties and early nineties. The former is evidently due to the surprisingly low factor-based structural correlation at the time. Afterwards however, this structural correlation has been rising considerably, underlining the importance of common information in integrated markets. Further exploring the return comovement, reconsider equation (6), which clarifies that the current model correlations arise from the structural variances of the idiosyncratic and factor shocks. The standard deviations are displayed in Figure 3.
Several interesting observations arise from these figures: The volatile period around 2000 has been caused by idiosyncratic Nasdaq shocks. This is in line with economic intuition, since the growth of the dot-com bubble and its burst are most closely connected to the "high-tech" sector. At the same time, the idiosyncratic Dow volatility reached a minimum, revealing a somewhat unusual pattern\(^6\). Nonetheless, this is a logical consequence of a unique event, the dot-com bubble, dominating stock markets for several years. Beyond that, the common factor has become more and more important. Particularly concerning the recent global financial crisis, the model ascribes the largest part of the turbulences to common factor volatility. Unlike the 2000 bubble, this crisis spread from mortgage and money markets and hit both "blue chip" and "high tech" sectors alike. Logically, the model again performs well in making a plausible choice. The same can be said concerning the Gulf War 1990/91 that shows up in the factor variance.

However, one problem should be acknowledged: The Black Monday 1987 is reflected in the idiosyncratic variances, what appears somehow peculiar in the light of a worldwide stock market crash. The technical explanation of this phenomenon is straightforward: The factor variance has been very low in the 1980s, implying a low Kalman gain (the fraction of the observable variables that the conditional linear projection assigns to the unobservable factor) on 10/19/1987. Logically, the strongly negative returns were not picked up by \( z \), but instead by the \( \varepsilon_t \). Since these shocks drive the EGARCH variances, the spike appears in the idiosyncratic volatilities. Obviously, the otherwise well-performing parsimonious

\(^6\)It might seem that the idiosyncratic Dow volatility has been constant from 2001 until 2005, before it begins to recover. Of course, this would be implausible in a heteroscedastic model. In fact, the conditional variance still varied over time, even if fluctuations appeared on a very low level and are thus hardly detectable in the graphic. The small size of fluctuations simply results from the low variance, i.e. the expected square of the shock. Besides, the elongate curve is to a certain extent induced by the EGARCH model: Since the log variance is not bounded from below, the variance itself can stay near zero for an extended period. Using a standard GARCH model, I found the same curve pattern, but not as pronounced.
model structure is not able to deal with such an extremely short-lasting unique one-time event. Notwithstanding, in a large sample the relevant period comprises only a few days. Neutralising them by use of impulse dummies does not change the estimation outcome, what is reassuring with regard to robustness.

The clear separation of idiosyncratic innovations, spillovers and common factors in (2) is a major strength of the underlying model. Primarily, it facilitates further investigation in terms of variance contributions. The conditional proportions of Dow and Nasdaq return variance that are due to one of the idiosyncratic innovations, respectively, or the common factor are plotted in Figure 4.

Figure 4: Variance contributions of idiosyncratic and factor shocks to DJIA and NDC

The Dow is dominated by own idiosyncratic shocks in the first, and by factor shocks in the second sample half. Additionally, spillovers from Nasdaq gained significant weight during the bubble period. Nasdaq itself was subject to considerable transmission effects from the Dow in the first two decades. Afterwards, the common factor took over, complemented by own shocks predominantly around 2000. In particular, it can be concluded that the quite strong orientation of Nasdaq towards the Dow development has been weakened since the 1990s. This may be interpreted as an emancipation process of Nasdaq, but may above all be seen in the context of the extraordinary phenomenon of the inflating and bursting dot-com bubble.

Naturally, total correlation and return variances may as well be inferred from reduced-
form multivariate volatility models. However, as the preceding discussion shows, in the present context the comovement results from distinct structural market processes. This allows a deeper understanding of the driving forces of financial returns. Furthermore, one might for example conduct correlation forecasts conditioned on special types of shocks, a task that is unfeasible in reduced form.

At last, statistical tests for appropriate specification of the heteroscedasticity are conducted. First, I focus on the return variances. Under the usual null hypothesis, the autocorrelations of the squares of the standardised variables \( y_{jt} / \sqrt{\Sigma_{jjt}} \), with \( \Sigma_{jjt} \) the \( j \)th diagonal element from (6), should equal zero. Indeed, the empirical correlation coefficients are quite small, mostly below 1%, with large p-values of ARCH-LM tests. Only the first Nasdaq autocorrelation reaches statistical significance, but is still limited to 5%. In general, these results support the common literature result that GARCH-type models of orders 1,1 are fairly suitable for financial markets data. Especially for the current model, it might be even more important that the covariance is appropriately modelled given the structural sources of market comovement, the two spillovers and the common factor. Here, the standardised cross-products \( y_{1t} y_{2t} / \Sigma_{12t} \) are clearly free of statistically significant serial correlation, supporting the multivariate structural specification.

4 Concluding Summary

Stock market returns, like those of the Dow Jones and Nasdaq Composite indexes, are often correlated to a substantial degree. This paper aimed at distinguishing the part of a contemporaneous correlation arising from causal spillover between the relevant variables from the one that is due to any third-party influences affecting all of them alike. Logically, an appropriate model has to feature a structural character and must additionally include common factors as sources of model-exogenous impulses. However, such a specification obviously runs into identification problems, which are well known in econometrics from classical simultaneous equation systems.

This study developed a customised adequate solution based on the idea of identification through heteroscedasticity: Both the idiosyncratic innovations of the stock returns as well as their common factor are allowed to exhibit ARCH-type effects, so that the additional information needed for identifying the model structure can be achieved from continually shifting volatility. Parameter estimates likewise factor states and conditional variances are obtained by means of QML Kalman filtering techniques.
The empirical results showed predominant spillovers from the Dow to the Nasdaq compared to the reverse direction. Besides, both stock segments are exposed to significant common factor influence. The latter was moderate in the 1980s, but has been rising since then, dominating in the recent global financial crisis. In contrast, the high volatility in the dot-com bubble period around 2000 was driven by Nasdaq shocks.

This paper contributed to the literature by allowing the researcher to determine common driving forces of different variables while retaining the possibility of mutual contemporaneous interaction between them. Through this methodological innovation it is possible to uncover structural market processes that are normally hidden behind reduced-form correlations. This paves the way to structural interpretations in terms of economics as well as to sophisticated conditioned first and second moment forecasts, both of which are hardly feasible in conventional reduced-form approaches.

Future research might exploit the advance in methodology for finding sources of correlations in further significant applications, respectively for re-examining econometric approaches that traditionally had to rely on non-testable assumptions. Moreover, interest could focus on econometric refinements in terms of theoretical model elaboration, for instance concerning the specification of the factor structure, as well as simplified estimation procedures. At last, complementing the simultaneous factor structure by risk premia from arbitrage pricing as given for example in King et al. (1994) may provide the model with additional economic appeal.

5 Appendix

This appendix proves result (8). First, let us restate \( z_t|\{y_t, y_{t-1}, \ldots\} \sim N(\mu_{zt}, \sigma_{zt}^2) \). In the following, I omit the conditioning information set \( \{y_t, y_{t-1}, \ldots\} \) for simplicity.

Second, define \( w_t := z_t - \mu_{zt} \), \( v_t := w_t^2 \) and \( x_t := w_t/\sigma_{zt} \). Note that this implies \( dw_t = dz_t \), \( dv_t = 2w_t dw_t \) and \( dw_t = \sigma_{zt} dx_t \). Furthermore, \( \varphi \) shall denote the standard normal probability density and \( \Phi \) the according cumulative distribution function.

For the expectation of the absolute value of a continuous random variable, we have

\[
E(|z_t|) = E(z_t|z_t > 0) + E(-z_t|z_t < 0).
\]
The first term results as

\[
E(z_t | z_t > 0) = \frac{1}{\sqrt{2\pi}\sigma_{zt}} \int_0^\infty z_t e^{-\frac{(z_t - \mu_{zt})^2}{2\sigma_{zt}^2}} dz_t = \frac{1}{\sqrt{2\pi}\sigma_{zt}} \int_{-\mu_{zt}}^\infty (w_t + \mu_{zt}) e^{-\frac{w_t^2}{2\sigma_{zt}^2}} dw_t = \frac{1}{\sqrt{2\pi}\sigma_{zt}} \int_{-\mu_{zt}}^\infty w_t e^{-\frac{w_t^2}{2\sigma_{zt}^2}} dw_t + \frac{\mu_{zt}}{\sqrt{2\pi}\sigma_{zt}} \int_{-\mu_{zt}}^\infty e^{-\frac{w_t^2}{2\sigma_{zt}^2}} dw_t
\]

Similarly, one can show for the second term that

\[
E(z_t | z_t < 0) = \sigma_{zt} \varphi(\frac{\mu_{zt}}{\sigma_{zt}}) - \mu_{zt}(1 - \Phi(\frac{-\mu_{zt}}{\sigma_{zt}})).
\]

Similarly, one can show for the second term that

\[
E(z_t | z_t < 0) = \sigma_{zt} \varphi(\frac{\mu_{zt}}{\sigma_{zt}}) - \mu_{zt}(1 - \Phi(\frac{-\mu_{zt}}{\sigma_{zt}})).
\]

In sum, we find

\[
E(|z_t|) = \mu_{zt}(\Phi(\mu_{zt}/\sigma_{zt}) - \Phi(-\mu_{zt}/\sigma_{zt})) + 2\sigma_{zt} \varphi(\mu_{zt}/\sigma_{zt}),
\]

what equals (8). The expectations of \(|\varepsilon_{zt}|, i = 1 \ldots, n,\) can be obtained accordingly.

References


