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Abstract

In this short review, we introduce the mathematical framework of the principle of the fermionic projector and set up a variational principle in discrete space-time. The connection to the continuum theory is outlined. Recent results and open problems are discussed.

The principle of the fermionic projector [1] provides a new model of space-time together with the mathematical framework for the formulation of physical theories. It was proposed to formulate physics in this framework based on a particular variational principle. In this short review article we explain a few basic ideas of the approach and report on recent results and open problems.

It is generally believed that the concept of a space-time continuum (like Minkowski space or a Lorentzian manifold) should be modified for distances as small as the Planck length. We here assume that space-time is discrete on the Planck scale. Our notion of "discrete space-time" differs from other discrete approaches (like for example lattice gauge theories or quantum foam models) in that we do not assume any structures or relations between the space-time points (like for example the nearest-neighbor relation on a space-time lattice). Instead, we set up a variational principle for an ensemble of quantum mechanical wave functions. The idea is that for mimimizers of our variational principle, these wave functions should induce relations between the discrete space-time points, which, in a suitable limit, should go over to the topological and causal structure of a Lorentzian manifold. More specifically, in this limit the wave functions should group to a configuration of Dirac seas.

For clarity, we first introduce the mathematical framework (Section 1) and discuss it afterwards, working out the underlying physical principles (Section 2). Then we outline the connection to the continuum theory (Sections 3 and 4). We conclude with a statement of results and an outlook (Sections 5 and 6).

1 A Variational Principle in Discrete Space-Time

We let $(H, <.|.>)$ be a complex inner product space of signature (N, N). Thus $<.|.>$ is linear in its second and antilinear in its first argument, and it is symmetric,

$$\overline{<\Psi \mid \Phi>} \;=\; <\Phi \mid \Psi> \qquad\qquad \text{for all } \Psi, \Phi \in H\,,$$

and non-degenerate,

$$<\Psi \mid \Phi> \;=\; 0 \quad \text{for all } \Phi \in H \quad \Longrightarrow \quad \Psi \;=\; 0\,.$$

In contrast to a scalar product, $<.|.>$ is *not* positive. Instead, we can choose an orthogonal basis $(e_i)_{i=1,\ldots,2N}$ of H such that the inner product $<e_i \,|\, e_i>$ equals $+1$ if $i = 1, \ldots, N$ and equals -1 if $i = N+1, \ldots, 2N$.

A *projector* A in H is defined just as in Hilbert spaces as a linear operator which is idempotent and self-adjoint,

$$A^2 = A \qquad \text{and} \qquad <A\Psi \,|\, \Phi> = <\Psi \,|\, A\Phi> \quad \text{for all } \Psi, \Phi \in H \, .$$

Let M be a finite set. To every point $x \in M$ we associate a projector E_x. We assume that these projectors are orthogonal and complete in the sense that

$$E_x \, E_y \;=\; \delta_{xy} \, E_x \qquad \text{and} \qquad \sum_{x \in M} E_x \;=\; \mathbb{1} \, . \tag{1}$$

Furthermore, we assume that the images $E_x(H) \subset H$ of these projectors are non-degenerate subspaces of H, which all have the same signature (n, n). We refer to (n, n) as the *spin dimension*. The points $x \in M$ are called *discrete space-time points*, and the corresponding projectors E_x are the *space-time projectors*. The structure $(H, <.|.>, (E_x)_{x \in M})$ is called *discrete space-time*.

We introduce one more projector P on H, the so-called *fermionic projector*, which has the additional property that its image $P(H)$ is a *negative definite* subspace of H. We refer to the rank of P as the *number of particles* $f := \dim P(H)$.

A space-time projector E_x can be used to project vectors of H to the subspace $E_x(H) \subset H$. Using a more graphic notion, we also refer to this projection as the *localization* at the space-time point x. For example, using the completeness of the space-time projectors (1), we readily see that

$$f \;=\; \mathrm{Tr}\, P \;=\; \sum_{x \in M} \mathrm{Tr}(E_x P) \, . \tag{2}$$

The expression $\mathrm{Tr}(E_x P)$ can be understood as the localization of the trace at the space-time point x, and summing over all space-time points gives the total trace. When forming more complicated composite expressions, it is convenient to use the short notations

$$P(x,y) \;=\; E_x \, P \, E_y \qquad \text{and} \qquad \Psi(x) \;=\; E_x \, \Psi \, . \tag{3}$$

The operator $P(x,y)$ maps $E_y(H) \subset H$ to $E_x(H)$, and it is often useful to regard it as a mapping only between these subspaces,

$$P(x,y) \;:\; E_y(H) \;\rightarrow\; E_x(H) \, .$$

Using (1), we can write the vector $P\Psi$ as follows,

$$(P\Psi)(x) \;=\; E_x \, P\Psi \;=\; \sum_{y \in M} E_x \, P \, E_y \, \Psi \;=\; \sum_{y \in M} (E_x \, P \, E_y) \, (E_y \, \Psi) \, ,$$

and thus

$$(P\Psi)(x) \;=\; \sum_{y \in M} P(x,y) \, \Psi(y) \, . \tag{4}$$

This relation resembles the representation of an operator with an integral kernel. Therefore, we call $P(x,y)$ the *discrete kernel* of the fermionic projector.

We can now set up our variational principle. We define the *closed chain* A_{xy} by

$$A_{xy} \;=\; P(x,y)\,P(y,x) \;=\; E_x\,P\,E_y\,P\,E_x\;; \qquad (5)$$

it maps $E_x(H)$ to itself. Let $\lambda_1, \ldots, \lambda_{2n}$ be the zeros of the characteristic polynomial of A_{xy}, counted with multiplicities. We define the *spectral weight* $|A_{xy}|$ by

$$|A_{xy}| \;=\; \sum_{j=1}^{2n} |\lambda_j|\;.$$

Similarly, one can take the spectral weight of powers of A_{xy}, and by summing over the space-time points we get positive numbers depending only on the form of the fermionic projector relative to the space-time projectors. Our variational principle is to

$$\text{minimize} \qquad \sum_{x,y \in M} |A^2_{xy}| \qquad (6)$$

by considering variations of the fermionic projector which satisfy the constraint

$$\sum_{x,y \in M} |A_{xy}|^2 = \text{const}\;. \qquad (7)$$

In the variation we also keep the number of particles f as well as discrete space-time fixed. Using the method of Lagrangian multipliers, for every minimizer P there is a real parameter μ such that P is a stationary point of the *action*

$$\mathcal{S}_\mu[P] \;=\; \sum_{x,y \in M} \mathcal{L}_\mu[A_{xy}] \qquad (8)$$

with the *Lagrangian*

$$\mathcal{L}_\mu[A] \;=\; |A^2| - \mu\,|A|^2\;. \qquad (9)$$

This variational principle was first introduced in [1]. In [2] it is analyzed mathematically, and it is shown in particular that minimizers exist:

Theorem 1.1 *The variational principle (6, 7) attains its minimum.*

2 Discussion, the Underlying Physical Principles

We come to the physical discussion. Obviously, our mathematical framework does not refer to an underlying space-time continuum, and our variational principle is set up intrinsically in discrete space-time. In other words, our approach is *background free*. Furthermore, the following physical principles are respected, in a sense we briefly explain.

- The **Pauli Exclusion Principle**: We interpret the vectors in the image of P as the quantum mechanical states of the particles of our system. Thus, choosing a basis $\Psi_1, \ldots, \Psi_f \in P(H)$, the Ψ_i can be thought of as the wave functions of the occupied states of the system. Every vector $\Psi \in H$ either lies in the image of P or it does not. Via these two conditions, the fermionic projector encodes for every state Ψ the occupation numbers 1 and 0, respectively, but it is impossible to describe higher occupation numbers. More technically, we can form the anti-symmetric many-particle wave function

$$\Psi \;=\; \Psi_1 \wedge \cdots \wedge \Psi_f \,.$$

Due to the anti-symmetrization, this definition of Ψ is (up to a normalization constant) independent of the choice of the basis $\Psi_1, \ldots, \Psi_f$. In this way, we can associate to every fermionic projector a fermionic many-particle wave function which obeys the Pauli Exclusion Principle. For a detailed discussion we refer to [1, §3.2].

- A **local gauge principle**: Exactly as in Hilbert spaces, a linear operator U in H is called *unitary* if

$$<\!U\Psi \,|\, U\Phi\!> \;=\; <\!\Psi \,|\, \Phi\!> \qquad \text{for all } \Psi, \Phi \in H.$$

It is a simple observation that a joint unitary transformation of all projectors,

$$E_x \;\to\; U E_x U^{-1} \,, \qquad P \;\to\; U P U^{-1} \qquad \text{with } U \text{ unitary} \tag{10}$$

keeps our action (6) as well as the constraint (7) unchanged, because

$$\begin{aligned} P(x,y) \;&\to\; U\, P(x,y)\, U^{-1} \,, \qquad A_{xy} \;\to\; U A_{xy} U^{-1} \\ \det(A_{xy} - \lambda \mathbb{1}) \;&\to\; \det\big(U (A_{xy} - \lambda \mathbb{1})\, U^{-1}\big) \;=\; \det(A_{xy} - \lambda \mathbb{1}) \,, \end{aligned}$$

and so the λ_j stay the same. Such unitary transformations can be used to vary the fermionic projector. However, since we want to keep discrete space-time fixed, we are only allowed to consider unitary transformations which do not change the space-time projectors,

$$E_x \;=\; U E_x U^{-1} \qquad \text{for all } x \in M \,. \tag{11}$$

Then (10) reduces to the transformation of the fermionic projector

$$P \;\to\; U P U^{-1} \,. \tag{12}$$

The conditions (11) mean that U maps every subspace $E_x(H)$ into itself. Hence U splits into a direct sum of unitary transformations

$$U(x) \;:=\; U E_x \;:\; E_x(H) \;\to\; E_x(H) \,, \tag{13}$$

which act "locally" on the subspaces associated to the individual space-time points.

Unitary transformations of the form (11, 12) can be identified with local gauge transformations. Namely, using the notation (3), such a unitary transformation U acts on a vector $\Psi \in H$ as

$$\Psi(x) \;\longrightarrow\; U(x)\, \Psi(x) \,.$$

This formula coincides with the well-known transformation law of wave functions under local gauge transformations (for more details see [1, §1.5 and §3.1]). We refer to the group of all unitary transformations of the form (11, 12) as the *gauge group*. The above argument shows that our variational principle is *gauge invariant*. Localizing the gauge transformations according to (13), we obtain at any space-time point x the so-called *local gauge group*. The local gauge group is the group of isometries of $E_x(H)$ and can thus be identified with the group $U(n,n)$. Note that in our setting the local gauge group cannot be chosen arbitrarily, but it is completely determined by the spin dimension.

- The **equivalence principle**: At first sight it might seem impossible to speak of the equivalence principle without having the usual space-time continuum. What we mean is the following more general notion. The equivalence principle can be expressed by the invariance of the physical equations under general coordinate transformations. In our setting, it makes no sense to speak of coordinate transformations nor of the diffeomorphism group because we have no topology on the space-time points. But instead, we can take the largest group which can act on the space-time points: the group of all permutations of M. Our variational principle is obviously *invariant under permutations* of M because permuting the space-time points merely corresponds to reordering the summands in (6, 7). Since on a Lorentzian manifold, every diffeomorphism is bijective and can thus be regarded as a permutation of the space-time points, the invariance of our variational principle under permutations can be considered as a generalization of the equivalence principle.

An immediate objection to the last paragraph is that the symmetry under permutations of the space-time points is not compatible with the topological and causal structure of a Lorentzian manifold, and this leads us to the discussion of the physical principles which are *not* taken into account in our framework. Our definitions involve **no locality** and **no causality**. We do not consider these principles as being fundamental. Instead, our concept is that the causal structure is induced on the space-time points by the minimizer P of our variational principle. In particular, minimizers should spontaneously break the above permutation symmetry to a smaller symmetry group, which, in a certain limiting case describing the vacuum, should reduce to Poincaré invariance. Explaining in detail how this is supposed to work goes beyond the scope of this short article (for a first step in the mathematical analysis of spontaneous symmetry breaking see [3]). In order to tell the reader right away what we have in mind, we shall first simply assume the causal structure of Minkowski space and consider our action in the setting of relativistic quantum mechanics (Section 3). This naive procedure will *not* work, but it will nevertheless illustrate our variational principle and reveal a basic difficulty. In Section 4 we will then outline the connection to the continuum theory as worked out in [1].

3 Naive Correspondence to a Continuum Theory

Let us see what happens if we try to get a connection between the framework of Section 1 and relativistic quantum mechanics in the simplest possible way. To this end, we just replace M by the space-time continuum $\mathbb{R}^4$ and the sums over M by space-time integrals. For a vector $\Psi \in H$, the corresponding $\Psi(x) \in E_x(H)$ as defined by (3) should be a 4-component Dirac wave function, and the scalar product $<\Psi(x) \,|\, \Phi(x)>$ on $E_x(H)$ should correspond to the usual Lorentz invariant scalar product on Dirac spinors $\overline{\Psi}\Phi$ with $\overline{\Psi} = \Psi^\dagger \gamma^0$ the adjoint spinor. Since this last scalar product is indefinite of signature $(2,2)$, we are led to choosing $n = 2$, so that the spin dimension is $(2,2)$.

In view of (4), the discrete kernel should in the continuum go over to the integral kernel of an operator P on the Dirac wave functions,

$$(P\Psi)(x) \;=\; \int_M P(x,y)\, \Psi(y)\, d^4y \,.$$

The image of P should be spanned by the occupied fermionic states. We take Dirac's concept literally that in the vacuum all negative-energy states are occupied by fermions forming the so-called *Dirac sea*. This leads us to describe the vacuum by the integral over the lower mass shell

$$P(x,y) \;=\; \int \frac{d^4k}{(2\pi)^4}\,(\not{k}+m)\,\delta(k^2-m^2)\,\Theta(-k^0)\,e^{-ik(x-y)} \tag{14}$$

(we consider for simplicity only one Dirac sea of mass m; the factor $(\not{k}+m)$ is needed in order to satisfy the Dirac equation $(i\not{\partial}_x - m)\,P(x,y)=0$).

We now consider our action for the above fermionic projector. Since we do not want to compute the Fourier integral (14) in detail, we simply choose x and y for which the integrals in (14) exist (for details see below) and see what we get using only the *Lorentz symmetry* of P. We can clearly write $P(x,y)$ as

$$P(x,y) \;=\; \alpha\,(y-x)_j\gamma^j + \beta\,\mathbb{1}$$

with two complex parameters α and β. Taking the complex conjugate of (14), we see that

$$P(y,x) \;=\; \overline{\alpha}\,(y-x)_j\gamma^j + \overline{\beta}\,\mathbb{1}\,.$$

As a consequence,

$$A_{xy} \;=\; P(x,y)\,P(y,x) \;=\; a\,(y-x)_j\gamma^j + b\,\mathbb{1} \tag{15}$$

with real parameters a and b given by

$$a \;=\; \alpha\overline{\beta} + \beta\overline{\alpha}\,, \qquad\qquad b \;=\; |\alpha|^2\,(y-x)^2 + |\beta|^2\,. \tag{16}$$

Using the formula $(A_{xy} - b\mathbb{1})^2 \;=\; a^2\,(y-x)^2$, one can easily compute the zeros of the characteristic polynomial of A_{xy},

$$\lambda_1 \;=\; \lambda_2 \;=\; b + \sqrt{a^2\,(y-x)^2}\,, \qquad\qquad \lambda_3 \;=\; \lambda_4 \;=\; b - \sqrt{a^2\,(y-x)^2}\,.$$

If the vector $(y-x)$ is spacelike, we conclude from the inequality $(y-x)^2<0$ that the argument of the above square root is negative. As a consequence, the λ_j appear in *complex conjugate pairs*,

$$\overline{\lambda_1} \;=\; \lambda_3\,, \qquad\qquad \overline{\lambda_2} \;=\; \lambda_4\,.$$

Furthermore, the λ_j all have the same absolute value $|\lambda_j| =: |\lambda|$, and thus the action (6) reduces to

$$\mathcal{S}_\mu[A] \;=\; |\lambda|^2\,(4-16\,\mu)\,.$$

This simplifies further if we choose the Lagrangian multiplier equal to $\frac{1}{4}$, because then the action vanishes identically. If conversely $(y-x)$ is timelike, the λ_i are all real. Using (16), one easily verifies that they are all *positive* and thus $\mathcal{S}_{\frac{1}{4}}[A] = (\lambda_1-\lambda_3)^2$. We conclude that

$$\mathcal{S}_{\frac{1}{4}}[A_{xy}] \;=\; \left\{ \begin{array}{cl} 4a^2\,(y-x)^2 & \text{if } (y-x) \text{ is timelike} \\ 0 & \text{if } (y-x) \text{ is spacelike}\,. \end{array} \right. \tag{17}$$

This consideration gives a simple *connection to causality*: In the two cases where $(y-x)$ is timelike or spacelike, the spectral properties of the matrix A_{xy} are completely different (namely, the λ_j are real or appear in complex conjugate pairs, respectively), and this leads to a completely different form of the action (17). More specifically, if the λ_j are non-real, this property is (by continuity) preserved under small perturbations of A_{xy}. Thinking of a dynamical situation, this suggests that perturbations of $P(x,y)$ for spacelike $(y-x)$ should not effect the action or, in other words, that events at points x and y with spacelike separation should not be related to each other by our variational principle. We remark that choosing $\mu = \frac{1}{4}$ is justified by considering the Euler-Lagrange equations corresponding to our variational principle, and this also makes the connection to causality clearer (see [1, §3.5 and §5]).

Apart from the oversimplifications and many special assumptions, the main flaw of this section is that the Fourier integral (14) does not exist for all x and y. More precisely, $P(x,y)$ is a well-defined distribution, which is even a smooth function if $(y-x)^2 \neq 0$. But *on the light cone* $(y-x)^2 = 0$, this distribution is *singular* (for more details see [1, §2.5]). Thus on the light cone, the pointwise product in (15) is ill-defined and our above arguments fail. The resolution of this problem will be outlined in the next section.

4 The Continuum Limit

We now return to the discrete setting of Section 1 and shall explain how to get a rigorous connection to the continuum theory. One approach is to study the minimizers in discrete space-time and to try to recover structures known from the continuum. For example, in view of the spectral properties of A_{xy} in Minkowski space as discussed in the previous section, it is tempting to introduce in discrete space-time the following notion (this definition is indeed symmetric in x and y, see [1, §3.5]).

Def. 4.1 *Two discrete space-time points $x, y \in M$ are called* **timelike** *separated if the zeros λ_j of the characteristic polynomial of A_{xy} are all real and not all equal. They are said to be* **spacelike** *separated if the λ_j are all non-real and have the same absolute value.*

The conjecture is that if the number of space-time points and the number of particles both tend to infinity at a certain relative rate, the above "discrete causal structure" should go over to the causal structure of a Lorentzian manifold. Proving this conjecture under suitable assumptions is certainly a challenge. But since we have a precise mathematical framework in discrete space-time, this seems an interesting research program.

Unfortunately, so far not much work has been done on the discrete models, and at present almost nothing is known about the minimizers in discrete space-time. For this reason, there seems no better method at the moment than to impose that the fermionic projector of the *vacuum* is obtained from a Dirac sea configuration by a suitable regularization process on the Planck scale [1, Chapter 4]. Since we do not know how the physical fermionic projector looks like on the Planck scale, we use the *method of variable regularization* and consider a large class of regularizations [1, §4.1].

When introducing the fermionic projector of the vacuum, we clearly put in the causal structure of Minkowski space as well as the free Dirac equation ad hoc. What makes the method interesting is that we then introduce a *general interaction* by inserting a general (possibly nonlocal) perturbation operator into the Dirac equation. Using methods of hyperbolic PDEs, one can describe the fermionic projector with interaction in detail [1, §2.5]. It turns out that the regularization of the fermionic projector with interaction is completely determined by the regularization of the vacuum (see [1, §4.5 and Appendix D]). Due to the regularization, the singularities of the fermionic projector have disappeared, and one can consider the *Euler-Lagrange equations* corresponding to our variational principle (see [1, §4.5 and Appendix F]). Analyzing the dependence on the regularization in detail, we can perform an expansion in powers of the Planck length. This gives differential equations involving Dirac and gauge fields, which involve a small number of so-called *regularization parameters*, which depend on the regularization and which we treat as free parameters (see [1, §4.5 and Appendix E]). This procedure for analyzing the Euler-Lagrange equations in the continuum is called *continuum limit*. We point out that only the singular behavior of $P(x,y)$ on the light cone enters the continuum limit, and this gives causality.

5 Obtained Results

In [1, Chapters 6-8] the continuum limit is analyzed in spin dimension $(16, 16)$ for a fermionic projector of the vacuum, which is the direct sum of seven identical massive sectors and one massless left-handed sector, each of which is composed of three Dirac seas. Considering general chiral and (pseudo)scalar potentials, we find that the sectors spontaneously form pairs, which are referred to as *blocks*. The resulting *effective interaction* can be described by chiral potentials corresponding to the effective gauge group

$$SU(2) \times SU(3) \times U(1)^3 .$$

This model has striking similarity to the standard model if the block containing the left-handed sector is identified with the leptons and the three other blocks with the quarks. Namely, the effective gauge fields have the following properties.

- The $SU(3)$ corresponds to an unbroken gauge symmetry. The $SU(3)$ gauge fields couple to the quarks exactly as the strong gauge fields in the standard model.
- The $SU(2)$ potentials are left-handed and couple to the leptons and quarks exactly as the weak gauge potentials in the standard model. Similar to the CKM mixing in the standard model, the off-diagonal components of these potentials must involve a non-trivial mixing of the generations. The $SU(2)$ gauge symmetry is spontaneously broken.
- The $U(1)$ of electrodynamics can be identified with an Abelian subgroup of the effective gauge group.

The effective gauge group is larger than the gauge group of the standard model, but this is not inconsistent because a more detailed analysis of our variational principle should give further constraints for the Abelian gauge potentials. Moreover, there are the following differences to the standard model, which we derive mathematically without working out their physical implications.

- The $SU(2)$ gauge field tensor F must be simple in the sense that $F = \Lambda\, s$ for a real 2-form Λ and an $su(2)$-valued function s.
- In the lepton block, the off-diagonal $SU(2)$ gauge potentials are associated with a new type of potential, called nil potential, which couples to the right-handed component.

6 Outlook

The results so far are very encouraging. But of course, many questions remain open. Since the mathematical framework is very recent, numerous open problems seem easily accessible. I see the following directions for future work:

- *finite-dimensional systems:* Except for very simple examples [2, Section 3], nothing is known about the minimizers of our variational principle in discrete space-time. In view of the physical picture described in Section 3, it seems most interesting to begin with spin dimension $(2,2)$ in the so-called *critical case* $\mu = \frac{1}{4}$. It seems promising to tackle such problems numerically. Analytically, it seems possible to construct minimizers with additional symmetry properties (see [3] for the general framework). It would be extremely useful to have a method for analyzing the minimizers in the limit of a large number of space-time points and many particles.

- *stability of the vacuum:* The goal is to show that Dirac sea configurations are stable minima of our variational principle. One approach is to try to take a suitable limit of finite-dimensional systems. Another method is to start from the continuum (see [1, §5.6]) and to analyze if there are suitable regularizations of Dirac sea configurations which are stable minima.

- *analysis of the continuum limit:* It is a major task to analyze the continuum limit in more detail. The next steps are the derivation of the field equations and the analysis of the spontaneous symmetry breaking of the chiral gauge bosons. Furthermore, except for [4, Appendix B], no calculations for gravitational fields have been made so far.

- *field quantization:* As explained in [1, §3.6], our conception is that the field quantization effects should be a consequence of a "discreteness" of the interaction described by our variational principle. This effect could be studied and made precise for finite-dimensional systems.

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