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Valleys in Minkowski space and instanton-induced cross sections

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We generalize the valley method for the calculation of instanton-induced cross sections, and reformulate it directly in the Minkowski space using the Keldysh diagram technique. The summation over all finite states is made implicit without our approach, and the fields of outgoing W-bosons are effectively taken into account by a classical field with specific analytic properties, which we call the "alien" field.

1. Introduction

High-energy behaviour of instanton-induced cross sections in gauge theories is attracting a growing interest fuelled by an intriguing possibility of the observation of baryon number violation (BNV) at SSC energies. Since the pioneering work of 't Hooft [1] it is known that the BNV is induced by the tunneling process between two adjacent gauge vacua separated by a potential barrier with the height $E_0 = 8\pi^2 m_W/g_W^2$ ($\sim$ 14 TeV) called the "sphaleron energy". Hence the probability of BNV is strongly damped by the huge Gamov factor $\exp(-16\pi^2/g_W^2) \sim 10^{-160}$ (which is often called in this case "the 't Hooft suppression factor"). Owing to such a strong suppression the instanton induced BNV has been considered being of pure academical interest for nearly 15 years.

The present interest to this problem has been triggered by the observation [2] that the 't Hooft suppression factor can be compensated in high-energy collisions by a large probability of emission of accompanying gauge W-bosons. The key point

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is that the amplitude of the emission of each additional W in the presence of a strong nonperturbative instanton field is enhanced by the large factor $1/g$. Hence the BNV cross section of the production of N secondary W's is rather written as $\sigma_{\mathrm{BNV}}^{N} \sim (1/g^2)^N \mathrm{e}^{16\pi^2/g^2}$ times the phase volume. The 't Hooft factor can in principle be compensated if the energy is large enough to allow for the production of the large number of W's, namely $N \sim 16\pi^2/g^2 \sim E_0/m_{\mathrm{W}}$.

First calculations of the BNV cross section [2–4] have indeed yielded an exponential growth with the energy associated with the production of a very large number of accompanying W-bosons. The total BNV cross section is written to the exponential accuracy in terms of the function $F(\epsilon)$ depending on the dimensionless parameter $\epsilon = E/E_0$:

$$\sigma_{\mathrm{BNV}} = \mathrm{const} \times \exp\left[-\frac{16\pi^2}{g^2} F(\epsilon)\right]. \tag{1.1}$$

The first few terms of the expansion of $F(\epsilon)$ in powers of ϵ are known:

$$F(\epsilon) = 1 - \tfrac{1}{2}\left(\tfrac{3}{2}\epsilon\right)^{4/3} + \tfrac{3}{8}\epsilon^2 + \mathrm{O}(\epsilon^{8/3}). \tag{1.2}$$

We see that at $\epsilon \sim 1$ the 't Hooft suppression factor can indeed be eaten up.

It was understood [5] that the rapid growth of the cross sections of multiparticle production is related to large-order behaviour of the perturbative theory and can be reproduced by a direct evaluation of the tree-level perturbative graphs [6]. The main open question is whether the cross section will grow further to energies of the order of the sphaleron energy, or will be stopped by some mechanism. For perturbative calculations one could doubt the relevance of the tree-level approximation. In the case of instantons, there is a variety of corrections, e.g. coming from multi-instantons, which may blow up at energies where naive estimates would violate the unitarity bound [17,18]. In addition, it is not clear whether all the energy of the colliding particles can be transferred to a single degree of freedom corresponding to the transition along the topological coordinate through the barrier. The corresponding overlap integrals for wave functions may be small [7].

The "soft–soft" corrections to Ringwald's approximation which stem from the interaction of the produced particles in the final state (see fig. 1a) are relatively well understood at the moment. An existing technique [19] is in principle efficient for their evaluation to arbitrary order, but more remarkable is the valley method [10] which allows one to calculate the whole sequence of soft–soft corrections semiclassically, evaluating the classical action on the distorted in a specific way instanton–anti-instanton "valley" configuration. The starting point here has been the remark by Zakharov [8] who noticed that as far as the final-state interaction is concerned, an optical theorem enables one to relate the calculation of the BNV cross section to the calculation of the imaginary part of the forward scattering

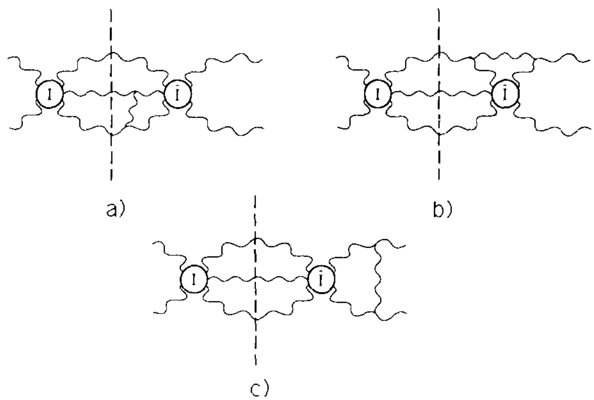

Fig. 1. (a) Soft–soft and (c) hard–hard corrections to Ringwald's approximation.

amplitude in the instanton–anti-instanton background. As in the well-known case of deep inelastic scattering one can start then from the euclidean space, and the problem can be reformulated in terms of the instanton–anti-instanton $(I\bar{I})$ interaction U_{int} which has been discussed for a long time in connection with modelling the QCD vacuum (see, e.g., ref. [9]). It is known that the relevant $(I\bar{I})$ configuration can be defined as a valley trajectory in the functional space [10]. A direct method exists to calculate the amplitudes in the valley background (at least in principle), and a simple conformal ansatz has been proposed [11] which well approximates the numerical solution [12]. The first corrections to the leading behaviour given in (1.2) have been calculated first using this technique [13] and the result has been confirmed using the various methods in refs. [14–16].

As is well known, however, in a general situation the physical cross sections cannot be obtained by analytical continuation of a suitable amplitude from euclidean space, which issue is related simply to the fact that the amplitude contains various imaginary parts related to different physical processes. In our case, going over from the euclidean to the Minkowski region and trying to find the instanton-induced cross section as an imaginary part of the $I\bar{I}$ amplitude, we should pick up the contribution of the $\langle 2 \mid I \mid N \rangle \langle N \mid \bar{I} \mid 2 \rangle$ discontinuity and not of

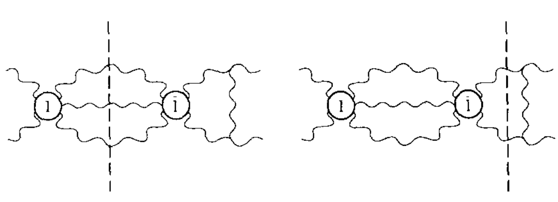

Fig. 2. The discontinuities of the instanton–anti-instanton contribution to the forward scattering amplitude, corresponding to subprocesses (a) with, and (b) without baryon number violation.

the $\langle 2 | N \langle\langle | I\bar{I} | 2 \rangle$ discontinuity, see fig. 2, whereas the total imaginary part obtained from euclidean calculations contains both of them. This problem shows up when including "hard–hard" corrections (fig. 1c) which are induced by initial-state interactions of ingoing high-energy quanta. The form of (1.1) suggests that a kind of semiclassical procedure may be developed for their treatment. The aim of this paper is to find out a language in which such a quasiclassical treatment would become possible. In particular, we reformulate the valley approach of ref. [10] in Minkowski space, so that it becomes applicable for the semiclassical expansion around certain classical fields in the particular discontinuities of the Feynman amplitudes.

We find it suitable to use the same operator technique as in calculations in perturbation theory of higher-twist effects in inclusive particle production in e^+e^- annihilation [20]. It turns to be convenient to incorporate ideas from nonequilibrium statistical physics and the Keldysh diagram technique [21–23] in particular. In this approach it is possible to trace which fields stand in the amplitude to the right of the cut (labeled as $(+)$ fields) and which ones appear to be to the left (so-called $(-)$ fields). This doubling of species of fields in the functional integral allows one to calculate a particular discontinuity in the amplitude in the operator language. The matrix elements of field operators are given by (perturbative) diagrams with internal lines of different type: the $(++)$ Green functions are the ordinary Feynman propagators with singularities of the type $-p^2 - i\epsilon$, the $(--)$ propagators possess complex conjugated singularities $-p^2 + i\epsilon$, while the $(-+)$ propagators equal $2\pi\delta(p^2)\theta(p_0)$. (The corresponding singularities in the coordinate space are $-x^2 + i\epsilon$, $-x^2 - i\epsilon$, and $-x^2 + i\epsilon x_0$, respectively). In this paper we generalize this technique to the presence of classical external fields, restricting ourselves to a particular $\bar{I}I$ contribution as an example.

We argue that classical fields should have the same structure of singularities in Minkowski space as quantum fields: the singularities of the instanton field to the right of the cut have the form $\rho^2 - x^2 + i\epsilon$; the anti-instanton to the left of the cut yields $\rho^2 - x^2 - i\epsilon$. The summation over all intermediate states in the calculation of the cross section can be done implicitly and actually induces a common boundary condition in both the $(+)$ and $(-)$ functional integrals (standing for the direct and the final amplitudes, respectively). Owing to this boundary condition the anti-instanton field of the type $\rho^2 - x^2 - i\epsilon x_0$ participates in the "alien" righthand side functional integral over the $(+)$ fields. The counterpart "alien" instanton configuration in the $(-)$ functional integral has the conjugate singularities of the type $\rho^2 - x^2 + i\epsilon x_0$. The structure of singularities is a direct consequence of the standard vacuum boundary conditions (no ingoing waves) at time $t \to -\infty$ in both the $(+)$ and $(-)$ functional integrals, while at $t \to +\infty$ the outgoing waves exist and correspond to real particles produced in the intermediate state (at the cut). The summation over intermediate states implies that the boundary conditions for the $(+)$ and $(-)$ functional integrals at $t \to +\infty$ should coincide. Thus, we arrive at a pair of the "native" and the "alien" instantons (the $I\bar{I}$ pair, to be precise) for which we take the valley configuration with the measure obtained by analytical continuation from euclidean space. We expand around the two-component valley field in the double ((+) and (−)) functional integral with the following analytical structure:

$$A^{\mathrm{v}}_{+} = \left(\rho_2^2 - x^2 + i\epsilon\right)^{-1} + \left(\rho_1^2 - (x-R)^2 - i\epsilon(x-R)_0\right)^{-1},$$

$$A^{\mathrm{v}}_{-} = \left(\rho_2^2 - x^2 + i\epsilon x_0\right)^{-1} + \left(\rho_1^2 - (x-R)^2 - i\epsilon\right)^{-1}. \tag{1.3}$$

Here R is the $I\bar{I}$ separation. Of course, eq. (1.3) is a schematical one – we have displayed the structure of singularities only and omitted all the color and spin factors. It is easy to see that both the fields have indeed no ingoing waves at $t \to -\infty$, and coincide at large positive times.

Thus, we calculate the forward scattering amplitude at the background of the conformal valley configuration of the above type. As long as we do not take into account the initial-state interactions, there is only one discontinuity (one real W cannot decay into several ones), and this way we reproduce the result (1.2) of the euclidean calculation at the background of the valley field,

$$A^{\mathrm{v}}_{\mathrm{Eucl}} = \left(\rho_2^2 + x^2\right)^{-1} + \left(\rho_1^2 + (x-R)^2\right)^{-1}. \tag{1.4}$$

(Again, only the structure of poles is shown.) We discuss in detail the matching of the valley method with the direct calculation of the cross section of the emission of large numbers of W's at the one-instanton background [19].

2. The double functional integral

We are going to rewrite the BNV cross section in the form of a functional integral with the doubling of species of the fields instead of the usual treatment as a product of two functional integrals [19]. The effect of this procedure will be that the summation over all intermediate states (the partial cross sections $2 \to N$) is made implicit and is replaced by imposing specific boundary conditions on the fields. We first remind the derivation of the usual diagram technique [22] starting from this functional integral. Then a quasiclassical expansion around certain classical field configurations in this functional integral is formulated and we show that any classical field, put "by hands" in a particular, say (−) sector of the functional integral, induces its "mirror reflection" in the opposite (+) sector, and the structure of singularities of both of them is fixed by the boundary conditions. In what follows we shall refer to such an induced classical field as to the "alien" field. Finally, we discuss the relation of our technique to the quasiclassical expansion in ref. [19].

The cross section under consideration is

$$\sum_N \langle 2 | \tilde{T} \exp(iH_I T) | N \rangle \langle N | T \exp(-iH_I T) | 2 \rangle, \tag{2.1}$$

where $T = t_f - t_i$, $t_f \to +\infty$, $t_i \to -\infty$, $|2\rangle$ is the initial two-boson state, H_I is the interaction hamiltonian, and the sum goes over all the intermediate states with a nonvanishing baryon number. Note that the time ordering in the conjugate amplitude is reversed. Applying the LSZ reduction formula, one obtains in the usual way

$$\begin{aligned} &A(p, k; \lambda, \sigma) \\ &\quad = e_\mu^\lambda(p) e_\nu^\sigma(k) p^2 k^2 \Big\{ \Big\langle 0 | \tilde{T}\big\{ A_\mu(p) A_\nu(k)\ \mathrm{e}^{iH_I T} \big\} | N \Big\rangle \\ &\quad \times \Big\langle N | T\Big\{ \big\{ A_\alpha(-p) A_\beta(-k)\ \mathrm{e}^{-iH_I T} \big\} | 0 \Big\rangle \Big\} p^2 k^2 e_\alpha^\lambda(-p) e_\beta^\alpha(-k), \end{aligned} \tag{2.2}$$

where $e^\lambda(p)$ are the polarization vectors of the W-bosons, and $A(p)$ are the Fourier transforms of the fields,

$$A_\mu(p) = \int \mathrm{d}x\ \mathrm{e}^{ipx} A_\mu(x), \tag{2.3}$$

etc. Note that we have not displayed the gauge indices of the W-bosons.

The relevant direct and final amplitudes are represented by the corresponding functional integrals and we can write

$$\begin{aligned} &\Big\langle 0 | \tilde{T}\big\{ A_\mu(p) A_\nu(k)\ \mathrm{e}^{iH_I T} \big\} | N \Big\rangle \Big\langle N | T\big\{ A_\alpha(-p) A_\beta(-k)\ \mathrm{e}^{-iH_I T} \big\} | 0 \Big\rangle \\ &\quad = \int DA^- DA^+\ \mathrm{e}^{-iS^- + iS^+} A_\mu^-(p) A_\nu^-(k) A_\alpha^+(-p) A_\beta^+(-k), \end{aligned} \tag{2.4}$$

where we have introduced the labels $(-)$ and $(+)$ to distinguish the fields from direct and final functional integrals, respectively. Integrations over the Higgs fields and the fermions are suppressed for brevity. We are going to treat the expression in (2.4) as *one* functional integral but with a doubled number of fields and must specify to this end the corresponding boundary conditions. As usual, the vacuum initial state implies that no ingoing particle waves exist at $t_i \to -\infty$. Hence the $(+)$ fields should have only negative frequencies at $t \to -\infty$, while the $(-)$ fields contain in this limit only fields with positive frequencies (the difference being due to an opposite sign in front of the $(-)$ action in eqs. (2.2) and (2.4)). The only nontrivial point are the boundary conditions at large positive times. The summation over all intermediate states implies that the $(+)$ and $(-)$ fields should coincide at $t = t_{\mathrm{f}} \to \infty$. More accurately, the unity operator which we have written down as the sum over all the Fock states $|N\rangle\langle N|$ can equivalently be rewritten as the sum over all intermediate states in the so-called "coordinate representation" $|A(x)\rangle\langle A(x)|$ where $|A(x)\rangle$ are the eigenvectors of the field operator $A(x)$ with the eigenvalues equal to the classical field $A(x)$ in close analogy with ordinary quantum mechanics. It is obvious that in the integrand of (2.4) one should take $A^{-}(\boldsymbol{x}, t_{\mathrm{f}}) = A^{+}(\boldsymbol{x}, t_{\mathrm{f}}) = A(\boldsymbol{x})$. In other words, the boundary conditions at $t = +\infty$ are such that the $(+)$ and $(-)$ fields coincide.

Let us remind the derivation of the Keldysh diagram technique [22], starting from the functional integral in eq. (2.4). Let us consider for simplicity the propagator of the Higgs boson * and introduce to this end two scalar sources J_{-} and J_{+}:

$$Z(J_{-}, J_{+}) = \int D\phi_{-} D\phi_{+} \exp\Big\{ -iS_{-} + iS_{+} + i\int \mathrm{d}x\big[-\phi_{-}(x)J_{-}(x) + \phi_{+}(x)J_{+}(x)\big]\Big\}, \tag{2.5}$$

where

$$S_{-} = \int \mathrm{d}x\ \phi_{-}(x)(-\Box + i\epsilon)\phi_{-}(x),$$

$$S_{+} = \int \mathrm{d}x\ \phi_{+}(x)(-\Box - i\epsilon)\phi_{+}(x). \tag{2.6}$$

Thus we obtain the generating functional for the Green functions. This functional integral is gaussian and hence it is calculated by shifting the integration variables. One should remember, however, that $(+)$ and $(-)$ variables are not completely independent since they should coincide at $= t_{\mathrm{f}}$. Therefore, the shift of the $(+)$ variables induces the shift in $(-)$ variables and vice versa. The shift has the form

$$\delta_{-}(x) \to \phi_{-}(x) + \tilde{\phi}_{-}(x),$$

$$\phi_{+}(x) \to \phi_{+}(x) + \tilde{\phi}_{+}(x), \tag{2.7}$$

* Here and below we put the mass of the Higgs field to zero which is sufficient for our purposes.

with

$$\tilde{\phi}_-(x) = \int dz \frac{-i}{4\pi^2\left(-(x-z)^2 - i\epsilon\right)} J_-(z)$$

$$+ \int dz \frac{i}{4\pi^2\left(-(x-z)^2 - i\epsilon(x-z)_0\right)} J_+(z), \tag{2.8}$$

$$\tilde{\phi}_+(x) = \int dz \frac{i}{4\pi^2\left(-(x-z)^2 + i\epsilon\right)} J_+(z)$$

$$+ \int dz \frac{-i}{4\pi^2\left(-(x-z)^2 - i\epsilon(x-z)_0\right)} J_-(z). \tag{2.9}$$

The shifts $\tilde{\phi}_\pm$ satisfy the equations $\Box\tilde{\phi}_\pm = J_\pm$ and the first terms in the r.h.s. of eqs. (2.8) and (2.9) are simply the integrals of the corresponding Green functions $(\Box \pm i\epsilon)^{-1}$ with the "same sign" sources and with a correct behaviour at $t \to -\infty$. In turn, the additional second terms are the solutions of the corresponding homogeneous equations, added in order to ensure the proper (coinciding) boundary conditions at plus infinity. After this shift, the functional integration is easily performed, and by differentiating with respect to the sources we obtain the following set of free propagators of the scalar field:

$$\phi_+(x)\phi_+(z) = -i\langle x|\left(-p^2 - i\epsilon\right)^{-1}|z\rangle = \frac{1}{4\pi^2\left(-(x-z)^2 + i\epsilon\right)},$$

$$\phi_-(x)\phi_-(z) = i\langle x|\left(-p^2 + i\epsilon\right)^{-1}|z\rangle = \frac{1}{4\pi^2\left(-(x-z)^2 - i\epsilon\right)},$$

$$\phi_-(x)\phi_+(z) = \left\langle x|2\pi\delta(p^2)\theta(p_0)|z\right\rangle = \frac{1}{4\pi^2\left(-(x-z)^2 - i\epsilon(x-z)_0\right)},$$

$$\phi_+(x)\phi_-(z) = \left\langle x|2\pi\delta(p^2)\theta(-p_0)|z\right\rangle = \frac{1}{4\pi^2\left(-(x-z)^2 + i\epsilon(x-z)_0\right)}, \tag{2.10}$$

where we have used the Schwinger notations for the Green functions,

$$\langle x|F(p)|z\rangle = \int \frac{dp}{(2\pi)^4} e^{ip(x-z)} F(p). \tag{2.11}$$

Obviously, the propagators of the gauge bosons and the fermions have the same structure of singularities.

We are going to describe a procedure for calculating the instanton-induced BNL cross section by some quasiclassical evaluation of the functional integral in (2.4). As usual, saying "instanton-induced", we have in mind that the shift of integration variables is made

$$A_\mu^- \to A_\mu^{\bar{I}-} + B_\mu^- ,$$

$$A_\mu^+ \to A_\mu^{I+} + B_\mu^+ , \tag{2.12}$$

where $A_\mu^{\bar{I}-}$ and A_μ^{I+} are the instanton field in direct, and the anti-instanton field in the final amplitude, respectively, rotated to Minkowski space *

$$A_\mu^{I+} = \frac{i\rho_2^2}{g} \frac{\sigma_\mu \bar{x} - x_\mu}{(-x^2 + i\epsilon)(\rho_2^2 - x^2 + i\epsilon)}, \tag{2.13}$$

$$A_\mu^{\bar{I}-} = \frac{i\rho_1^2}{g} u \frac{\bar{\sigma}_\mu (x-R) - (x-R)_\mu}{\left(-(x-R)^2 - i\epsilon\right)\left(\rho_1^2 - (x-R)^2 - i\epsilon\right)} \bar{u}, \tag{2.14}$$

where u is the unitary matrix of the $I\bar{I}$ relative orientation. The remaining "quantum" fields B_μ describe the creation of particles in the presence of the classical instanton fields A_μ **.

To have a quasiclassical expansion we would like to deal with quantum fields B of the order of unity at the background of large classical fields $\sim 1/g$. However, as discussed in detail in ref. [19], the situation is not that simple. In our language the problem is to satisfy the boundary condition $A_\mu^{\bar{I}-} + B_\mu^- = A_\mu^{I+} + B_\mu^+$ at $t = t_f$. We easily see that the quantum fields B_+ defined in eq. (2.12) appear to be of the order of $1/g$ at least at $t \to t_f$. In the technique of ref. [19] the constraint $A^-(\boldsymbol{x}, t = t_f) = A^+(\boldsymbol{x}, t = t_f)$ is removed at the price of an additional integration over creation and annihilation operators of the coherent states, see below. Thus the so-called "$\Re$-term" comes into the game which is given by the product of the classical and quantum fields at $t = t_f$, times $1/g$.

Another possibility, which we suggest in this paper, is to require that the boundary conditions at $t = t_f$ are satisfied for both the classical and the quantum

* We use the notations $\sigma^\mu = (1, \boldsymbol{\sigma})$, $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$, and $x = x_\mu \sigma^\mu$, $\bar{x} = x_\mu \bar{\sigma}^\mu$. The standard relations to 't Hooft symbols are $\sigma_\mu \bar{\sigma}_\nu = g_{\mu\nu} - i\eta^a_{\mu\nu}\tau^a$, $\bar{\sigma}_\mu \sigma_\nu = g_{\mu\nu} - i\eta^a_{\mu\nu}\tau^a$.

** This statement may appear to be confusing since in the LSZ formalism the amplitudes for particle production (to leading order) are obtained by amputation of the classical instanton fields A_μ [2]. However, this prescription is actually a technical trick and enters because we use the LSZ formalism to relate $2 \to N$ amplitudes to vacuum–vacuum transitions (Green functions) in the presence of external classical sources. If we start instead directly from the functional integral representation of the $2 \to N$ sector of the S-matrix, then, obviously, outgoing waves corresponding to emission of secondary particles are described by quantum excitations B_μ above the classical background fields A_μ.

fields separately. To assure this property, we make an additional shift of integration variables, and extract "classical" pieces from the quantum fields B_μ^- and B_μ^+. Thus, instead of eq. (2.12) we write

$$A_\mu^- \to A_\mu^{\bar{I}-} - \tilde{A}_\mu^- + B_\mu^- ,$$

$$A_\mu^+ \to A_\mu^{I+} + \tilde{A}_\mu^+ + B_\mu^+ , \tag{2.15}$$

and require that $A^{\bar{I}-}(x,\ t=t_f) + \tilde{A}^-(x,\ t=t_f) = A^{I+}(x,\ t=t_f) + \tilde{A}^+(x,\ t=t_f)$, and $B_\mu^-(x,\ t=t_f) = B_\mu^+(x,\ t=t_f)$. From the above discussion of the bare propagators it is clear that the simplest choice of the "alien" fields $\tilde{A}^-$ and $\tilde{A}^+$ in order to satisfy these constraints is to take them in the form of the anti-instanton and the instanton field configurations in eq. (2.13) and (2.14), respectively, but change the prescription to go around the singularities to

$$\tilde{A}_\mu^+ = \tilde{A}_\mu^{\bar{I}+} = A_\mu^{\bar{I}-}(i\epsilon - i\epsilon(x_0 - R_0)),$$

$$\tilde{A}_\mu^- = \tilde{A}_\mu^{I-} = A_\mu^{I+}(i\epsilon \to i\epsilon x_0). \tag{2.16}$$

However, as the word "quantum" was not accurate in describing the field $B_\mu^\pm$ in eq. (2.12) owing to large contributions at $t = t_f$, now the word "classical" becomes ambiguous with regard to the field $A^I + \tilde{A}^{\bar{I}}$, since the latter no longer satisfies the equations of motion. We have eliminated the $\mathfrak{R}$-term at the boundary $t = t_f$, but only at the cost of having instead linear terms in the action inside both the $(+)$ and $(-)$ sectors. Fortunately, this second type of linear terms is known for a long time in connection with the problem of $I\bar{I}$ interactions, and can be treated by the valley method of ref. [10]. The idea is that choosing the alien field configuration in the form of a slightly modified instanton field we can minimize this linear term in some sense.

Let us trace in detail the connection of our approach to the technique of ref. [19] on the simplest example of Ringwald's correction to the BNV cross section. To this accuracy the simplest ansatz in eq. (2.16) is sufficient. Using the formalism of coherent states we can write the cross section of interest as [19]

$$\sum_b \langle a^* | e^{iH_1 T} | b \rangle \langle b^* | e^{-iH_1 T} | a \rangle$$

$$= \int DA_i^-(x) DA_f^-(x) DA^-(x) Db^*(k) Db(k) DA_i^+(x) DA_f^+(x) DA^+(x)$$

$$\times \langle a^* | A_i^- \rangle e^{-iS(A^-)} \langle A_f^- | b \rangle \langle b^* | A_f^+ \rangle e^{iS(A^+)}$$

$$\times \langle A_f^+ | a \rangle \exp\left[-\int dk\, b^*(k) b(k) \right], \tag{2.17}$$

where

$$\langle A(\boldsymbol{x}) \mid a^{\lambda}\rangle = \text{const} \times \exp\Big[-\tfrac{1}{2}\int \mathrm{d}\boldsymbol{k}\, a^{\lambda}(\boldsymbol{k}) a^{\lambda}(-\boldsymbol{k})$$

$$-\tfrac{1}{2}\int \mathrm{d}\boldsymbol{k}\ \omega_k e_{\mu}^{\lambda}(\boldsymbol{k}) A_{\mu}(\boldsymbol{k}) e_{\nu}^{\lambda}(-\boldsymbol{k}) A_{\nu}(-\boldsymbol{k})$$

$$+\int \mathrm{d}\boldsymbol{k} \sqrt{2\omega_k}\, a^{\lambda}(\boldsymbol{k}) e_{\mu}^{\lambda}(\boldsymbol{k}) A_{\mu}(-\boldsymbol{k})\Big],$$

$$\langle a^{*\lambda} \mid A(\boldsymbol{x})\rangle = \text{const} \times \exp\Big[-\tfrac{1}{2}\int \mathrm{d}\boldsymbol{k}\, a^{*\lambda}(\boldsymbol{k}) a^{*\lambda}(-\boldsymbol{k})$$

$$-\tfrac{1}{2}\int \mathrm{d}\boldsymbol{k}\ \omega_k e_{\mu}^{\lambda}(\boldsymbol{k}) A_{\mu}(\boldsymbol{k}) e_{\nu}^{\lambda}(-\boldsymbol{k}) A_{\nu}(-\boldsymbol{k})$$

$$+\int \mathrm{d}\boldsymbol{k} \sqrt{2\omega_k}\, a^{*\lambda}(\boldsymbol{k}) e_{\mu}^{\lambda}(\boldsymbol{k}) A_{\mu}(-\boldsymbol{k})\Big] \tag{2.18}$$

are the wave functions of the coherent states in the coordinate representation. (Here $A(\boldsymbol{k}) = \int \mathrm{d}^3x\ \mathrm{e}^{ikx} A(\boldsymbol{x})$). The cross section in eq. (2.4) is obtained by differentiating (2.18) four times with respect to $a(\boldsymbol{p})$, $a(\boldsymbol{k})$, $a^*(-\boldsymbol{p})$, and $a^*(-\boldsymbol{k})$ [see ref. [19] for details].

It is worth noting that if we integrate explicitly over the intermediate coherent states, then the boundary condition for the functional integral in (2.4) at $t = t_{\mathrm{f}}$ is readily reproduced, since

$$\int Db^*(\boldsymbol{k}, \lambda) Db(\boldsymbol{k}, \lambda)\ \exp\Big[-\int \mathrm{d}\boldsymbol{k}\, b^*(\boldsymbol{k}) b(\boldsymbol{k})\Big] \langle A_{\mathrm{f}}^{-} \mid b, \lambda\rangle \langle b^*, \lambda \mid A_{\mathrm{f}}^{+}\rangle$$

$$= \prod_x \delta(A_{\mathrm{f}}^{-}(\boldsymbol{x}) - A_{\mathrm{f}}^{+}(\boldsymbol{x})). \tag{2.19}$$

However, instead of summation over intermediate states, we shall now follow ref. [19] and first take the functional integrals over the $(+)$ and $(-)$ fields in the semiclassical approximation, expanding them around instanton fields in eqs. (2.13) and (2.14). Making the appropriate shift of integration variables (2.12) we obtain to lowest order in g^2 [19],

$$e^{-\frac{2}{g^2}S_0}\int \mathrm{d}R\ \mathrm{d}\mu_I\ \mathrm{d}\mu_{\bar{I}} \int Db^*(\boldsymbol{k}) Db(\boldsymbol{k})\ \exp\Big\{iER_0 - \int \mathrm{d}\boldsymbol{k}\, b^*(\boldsymbol{k}) b(\boldsymbol{k})$$

$$+\frac{1}{g}\int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2\omega_k}} \big[\mathrm{e}^{i\omega_k R_0 - i\boldsymbol{k}\cdot\boldsymbol{R}}\ b^*(\boldsymbol{k}, \lambda) e_{\mu}^{\lambda}(\boldsymbol{k}) \Re_{\mu}(\boldsymbol{k})$$

$$+ b(\boldsymbol{k}, \lambda) e_{\mu}^{\lambda}(\boldsymbol{k}) \Re_{\mu}^{*}(\boldsymbol{k})\big]\Big\}, \tag{2.20}$$

where $d\mu_I$, $d\mu_{\bar{I}}$ denote integrations over the collective coordinates and the "residue" $\mathfrak{R}_\mu(\boldsymbol{k})$ is determined by the asymptotics of the (anti)instanton field at large t:

$$A_\mu^I(\boldsymbol{x}, t) = \frac{1}{g}\int \frac{d\boldsymbol{k}}{(2\pi)^3 2\omega_k} e^{i\omega_k t + i\boldsymbol{k}\cdot\boldsymbol{x}} \mathfrak{R}_\mu(\boldsymbol{k}),$$

$$A_\mu^{\bar{I}}(\boldsymbol{x}, t) = \frac{1}{g}\int \frac{d\boldsymbol{k}}{(2\pi)^3 2\omega_k} e^{-i\omega_k(t-R_0) + i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{R})} \mathfrak{R}_\mu^*(\boldsymbol{k}),$$

$$\mathfrak{R}_\mu(\boldsymbol{k}) = 2\pi^2\rho_2^2\left(\sigma_\mu \bar{k} - k_\mu\right),$$

$$\mathfrak{R}_\mu^*(\boldsymbol{k}) = 2\pi^2\rho_1^2 u\left(\bar{\sigma}_\mu k - k_\mu\right)\bar{u}, \qquad k_\mu = (\omega_k, \boldsymbol{k}). \tag{2.21}$$

Performing the gaussian integration over b, b^* in eq. (2.20) we end up with the familiar dipole–dipole $I\bar{I}$ interaction induced by the emission of W-bosons,

$$\exp\left\{iER_0 - \frac{2S_0}{g^2} + \frac{1}{g^2}\int \frac{d\boldsymbol{k}}{(2\pi)^3 2\omega_k} e^{i\omega_k R_0 - i\boldsymbol{k}\cdot\boldsymbol{R}} e_\mu^\lambda(\boldsymbol{k})\mathfrak{R}_\mu(\boldsymbol{k}) e_\nu^\lambda(\boldsymbol{k})\mathfrak{R}_\nu^*(\boldsymbol{k})\right\}$$

$$= \exp\left\{iER_0 - \frac{2S_0}{g^2} + \frac{32\pi^2}{g^2}\frac{\rho_1^2\rho_2^2}{R^6}\left(4(uR)^2 - R^2\right)\right\}. \tag{2.22}$$

Let us demonstrate now that the same expression follows from the above formalism (to the lowest order in ρ^2/R^2). Within our framework the $I\bar{I}$ interaction is defined as the defect of the classical action,

$$\exp\{-iS^- + iS^+\} = \exp\left\{\frac{i}{2}\int dx\, G_{\mu\nu}^- G_{\mu\nu}^- - \frac{i}{2}\int dx\, G_{\mu\nu}^+ G_{\mu\nu}^+\right\}$$

$$= \exp\left\{-\frac{2S_0}{g^2} + \frac{i}{g^2}U_{\text{int}}\right\}, \tag{2.23}$$

where to leading semiclassical accuracy we should insert the classical fields in the form of a sum of a "native" A_μ and an "alien" $\tilde{A}_\mu$ instantons. At first glance the action in eq. (2.23) in the limit of large $I\bar{I}$ separation equals $4S_0/g^2$ as the sum of the contributions of the "native" and "alien" fields in each (+) and (−) sector. However, while the action at the "native" instanton equals S_0/g^2, the action at the "alien" instanton is zero, since all singularities in x_0 lie at one side of the contour of integration. Hence we end up with an instanton action taken twice, plus the interaction "native"–"alien", as anticipated. If we now restrict ourselves to first order in the parameter ρ^2/R^2 (which is related to the energy as $\rho^2/R^2 \sim$

$(E/E_{\text{sph}})^{2/3})$, then the "alien" field is small, of the order of ρ^2/R^2, compared to the "native" one. (Of course, both are $\sim 1/g$). Hence we can pick up the terms of the first order only in the "alien" field and write, e.g.

$$i\int \mathrm{d}x\, G_{\mu\nu}^{\bar{I}I-} G_{\mu\nu}^{\bar{I}I-} = i\int \mathrm{d}x \left(D_\mu^{I-} \tilde{A}_\nu^{\bar{I}-} - D_\nu^{I-} \tilde{A}_\mu^{\bar{I}-} \right) G_{\mu\nu}^{I-}, \tag{2.24}$$

where the covariant derivative and the strength tensor contain the "native" field only. Integrating by parts and taking into account that the instanton field satisfies the equations of motion, we end up with the surface terms only, which read

$$i\int \mathrm{d}\boldsymbol{x}\, G_{\mu\nu}^{\bar{I}I-} G_{\mu\nu}^{\bar{I}I-} = i\int \mathrm{d}x\, \tilde{A}_i^{\bar{I}-}(\boldsymbol{x}, t_\mathrm{f}) G_{0i}^{I-}(\boldsymbol{x}, t_\mathrm{f}). \tag{2.25}$$

The Fourier transform of the "alien" field is written similarly to eq. (2.22), but contains negative frequencies at $t \to \infty$ corresponding to outgoing waves of W-bosons:

$$\bar{A}_\mu^{\bar{I}-}(\boldsymbol{x}, t) = \frac{1}{g}\int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3 2\omega_k} \mathrm{e}^{+i\omega_k(t-R_0) - i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{R})} \mathfrak{R}_\mu^*(\boldsymbol{k}). \tag{2.26}$$

Hence, the exponent $\mathrm{e}^{i\omega t_\mathrm{f}}$ from the "alien" field (2.26) cancels the rapidly oscillating factor $\mathrm{e}^{-i\omega t_\mathrm{f}}$ in the surface integral in eq. (2.25) coming from the "native" field in eq. (2.21). All the dependence on t_f disappears, and after a little algebra we recover the expression (2.22). More accurately, we obtain in this way only one half of the interaction; the other half originates from the term in (2.23) with the $(+)$ fields.

Technically, the same answer arises because performing the gaussian integration in eq. (2.20) we actually make the same shift of integration variables,

$$b_\mu^*,(\boldsymbol{k}) \to b_\mu^*(\boldsymbol{k}) + \frac{1}{g}\frac{1}{(2\pi)^{3/2}\sqrt{2\omega_k}} \mathfrak{R}_\mu^*(\boldsymbol{k}),$$

$$b_\mu(\boldsymbol{k}) \to b_\mu(\boldsymbol{k}) + \frac{1}{g}\frac{1}{(2\pi)^{3/2}\sqrt{2\omega_k}} \mathrm{e}^{i\omega_k R_0 - i\boldsymbol{k}\cdot\boldsymbol{R}} \mathfrak{R}_\mu(\boldsymbol{k}). \tag{2.27}$$

Precisely this shift ensures the proper boundary conditions at $t = t_\mathrm{f}$ since the shifts in (2.27) coincide with the Fourier components of the "alien" fields $\tilde{A}$. It can be said that the second shift (2.15) is done to eliminate the linear terms in (2.27). In a general situation (not to lowest order in ρ^2/R^2), this shift eliminates all the "surface" linear terms $\sim b^*(\boldsymbol{k})\mathfrak{R}(\boldsymbol{k})$, but at the cost of producing instead "volume" linear terms $\sim D_\mu G_{\mu\nu}$, which are not vanishing because the sum of "native" and "alien" instantons does not satisfy the equations of motion. The linear terms of this type are, however, familiar from the analysis of $I\bar{I}$ interaction and are treated in a systematic way by the valley method.

3. The valley field configurations

Our objective is to calculate the functional integral (2.4) in the vicinity of the $I\bar{I}$ configuration which is not trivial due to the following reasons. First, the $I\bar{I}$ configuration is not a solution of classical field equations and hence the unpleasant linear term (in the quantum deviations) survives in the action. Second, at large $I\bar{I}$ separations there are quasizero modes in the functional space (at infinite separations they are converted into the double set of instanton zero modes) and the corresponding integrations in the functional space are non-gaussian. Third, one should properly define the $I\bar{I}$ field, since the instantons are distorted by the interaction. These difficulties are cured by the valley method which we recall in this section in its standard form in euclidean space. A detailed discussion is given in refs. [10,11,24].

To illustrate the idea, let us consider the simplest case of quantum mechanics with the double-well potential,

$$Z = N^{-1}\int \mathscr{D}\phi \, \exp\left[-\frac{1}{g^2}\int \mathrm{d}t\left(\tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\phi^2(1-\phi)^2\right)\right] \tag{3.1}$$

and calculate the contribution to the vacuum energy coming from the $I\bar{I}$ pair. At infinitely large separations the $I\bar{I}$ configuration is well defined. It obeys the classical field equations and possesses two zero modes corresponding to an independent translation of each instanton. The zero modes can be rediagonalized in such a way that one of them describes a trivial total translation of the $I\bar{I}$ configuration as a whole, whereas the other zero mode corresponds to the separation of instantons. When the separation is not infinite but large as compared to the instanton size (which is 1 in our toy example), then the mode related to the $I\bar{I}$ separation becomes a quasizero mode: the action varies much more slowly along this direction in functional space as compared to the orthogonal directions corresponding to normal modes. Pictorially, this landscape in the functional space looks like a valley corresponding to the quasizero mode surrounded by a steep canyon of usual (gaussian) modes; see fig. 3.

We want to evaluate the contribution to the functional integral (3.1) coming from the vicinity of this valley and shall do this in two steps: first we perform all the gaussian integrations in the directions orthogonal to the valley and then perform an exact (nongaussian) integration along the valley. The valley can rigorously be defined as a trajectory $\phi_{\mathrm{v}}(t, \alpha)$ in the functional space described by some parameter α and corresponding to the minimum of the action in an

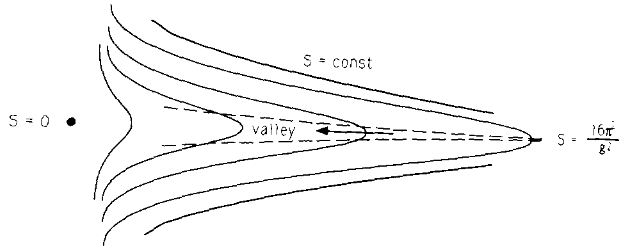

Fig. 3. The topography of the functional space near the valley. The solid lines show surfaces of constant action. The arrow denotes the direction of the negative mode, and the point $S = 0$ corresponds to the perturbation theory. The region between the two dashed curves shows the "bottom" of the valley, where the deviation from the negative mode is $O(1/\xi^2)$. All the valley trajectories lying in this region are acceptable and correspond to different choices of the weight function.

orthogonal slice in the vicinity of the valley. Thus, we should find the minimum of the action with the constraint

$$(\phi - \phi_v, \omega(\alpha)\ \mathrm{d}\phi_v/\mathrm{d}\alpha) = 0, \tag{3.2}$$

where (f, g) denotes the usual scalar product of functions $\int \mathrm{d}t\, f(t)g(t)$, and $\omega(t, \alpha)$ is a suitable weight function. (Of course, ϕ_v depends also on an additional parameter T describing the total translation of the $I\bar{I}$ configuration but we shall ignore this trivial dependence at the moment). Applying the standard technique of Lagrange multipliers we arrive at the following constrained classical equation (the "valley equation" [10]):

$$\left.\frac{\delta S}{\delta\phi(t)}\right|_{\phi=\phi_v} = \chi(\alpha)\omega(t, \alpha)\frac{\mathrm{d}\phi_v(t, \alpha)}{\mathrm{d}\alpha}, \tag{3.3}$$

where $\chi(\alpha)$ is the Lagrange multiplier which takes into account the possibility of a reparametrisation of the valley parameter. The boundary conditions are that $\phi_v(t, \alpha)$ approaches the infinitely separated instanton and anti-instanton pair at $\alpha \to \infty$:

$$\phi_v(t, \alpha) \to \phi_I(t + \alpha) - \phi_{\bar{I}}(t - \alpha). \tag{3.4}$$

It is easy to see that the action increases monotonically along the valley since

$$\frac{\mathrm{d}S(\phi_v)}{\mathrm{d}\alpha} = \chi(\alpha)\left(\frac{\mathrm{d}\phi_v}{\mathrm{d}\alpha}, \omega(\alpha)\frac{\mathrm{d}\phi_v}{\mathrm{d}\alpha}\right) \tag{3.5}$$

and the increase only stops at the solution of classical equation $\delta S/\delta\phi = 0$ where $\chi = 0$. Therefore, the valley (3.3) actually interpolates the two classical solutions: the infinitely separated $I\bar{I}$ configuration and the perturbative vacuum.

Now we are in a position to integrate in the region of functional space close to the valley. To this end we need to replace the integration over ϕ with the integration over the collective coordinate α of the "classical" valley field ϕ_v and integrations over "quantum" deviations $\phi - \phi_v$ in directions orthogonal to the valley. Using the standard Faddeev–Popov trick we insert

$$1 = -\int \mathrm{d}\alpha\ \delta(\phi - \phi_v,\ \omega\phi'_v)\left[(\phi'_v,\ \omega\phi'_v) - (\phi - \phi_v,\ (\omega\phi'_v)')\right], \tag{3.6}$$

where $\phi'_v = \mathrm{d}\phi_v/\mathrm{d}\alpha$, make the shift $\phi \to \phi + \phi_v$ and expand the action in powers of the deviation from the valley:

$$S(\phi + \phi_v) = S(\phi_v) + (\phi,\ \delta S/\delta\phi_v) + \tfrac{1}{2}(\phi,\ \Box_v\phi) + \mathrm{O}(\phi^3), \tag{3.7}$$

where $\Box_v = -\partial^2 + 1 - 6\phi_v + 6\phi_v^2$ is the operator of the second derivative of the action. Now comes the central point: the linear term in this expansion vanishes due to the $\phi(\phi,\ \phi'_v)$ factor in the integrand and thanks to the valley equation. Hence, to the semiclassical accuracy we are left with the following functional integral:

$$N^{-1}\int \mathrm{d}\alpha(\phi'_v,\ \omega\phi'_v)\ \mathrm{e}^{-S(\phi_v)/g^2}\int \mathscr{D}\phi\ \delta(\phi,\ \omega\phi'_v)\ \mathrm{e}^{-(\phi,\ \Box_v\phi)/2g^2}. \tag{3.8}$$

All the effects proportional to $1/g^2$ originate from the classical action only. The quantum corrections come from the terms of $\mathrm{O}(\phi^3)$ in the expansion of the action in (3.7) and from the collective coordinate jacobian in eq. (3.6).

The simplest form for the valley field is given by the sum of kinks,

$$\phi_v(t,\ \alpha) = \tfrac{1}{2}\tanh\left(\frac{t+\alpha}{2}\right) - \tfrac{1}{2}\tanh\left(\frac{t-\alpha}{2}\right), \tag{3.9}$$

which obeys the valley equation (3.3) with the weight function *

$$\omega(t,\ \alpha) = \tfrac{1}{4}\sinh\alpha\ \mathrm{e}^{\alpha}(\cosh t\cosh\alpha + 1)^{-1}. \tag{3.10}$$

The corresponding Lagrange multiplier equals

$$\chi = 12/\xi^2, \qquad \xi = 2\cosh\alpha, \tag{3.11}$$

The main point in the above consideration is the absence of the linear term, and it does not depend on the particular choice of the weight function $\omega(t,\ \alpha)$. Thus, at

* Actually it is the product $\omega(t,\ \alpha)\chi(\alpha)$ which is determined from the valley equation. We tacitly imply that the measure $\omega(t,\ \alpha)$ is of the order of unity at $\alpha \to \infty$. Then it turns out that $\chi \sim 1/\xi^2$. The saddle point value of ξ in the evaluation of the vacuum energy is large: $\xi^2 \sim 1/g^2$ [27], so that effectively $\chi \sim g^2$.

first glance any weight function and hence any valley field (which starts from infinitely separated $I\bar{I}$ configuration) are appropriate. In fact the situation is more delicate since a bad choice of the valley can lead to an artificial enhancement of quantum corrections. Namely, one should take the valley configuration which streams from the infinitely separated $I\bar{I}$ field following the direction of the negative quasizero mode of the operator $\Box_{\mathrm{v}}$ to $O(1/\xi^2)$ accuracy,

$$\phi'_{\mathrm{v}} = \phi_- + O(1/\xi^2),$$

$$\Box\phi_- = -\lambda\phi_-, \qquad \lambda = O(1/\xi^2). \tag{3.12}$$

If this condition is not satisfied, then the quantum corrections will be enhanced by a large factor ξ^2. Indeed, let us evaluate the quantum correction coming from the collective coordinate jacobian. To this end we need to calculate the projection on the field $(\omega\phi')'$ of the constrained propagator,

$$G(t_1, t_2) = \int \mathscr{D}\phi\, \phi(t_1)\phi(t_2)\delta(\phi, \omega\phi'_{\mathrm{v}})\, \mathrm{e}^{-(\phi, \Box\phi)/2g^2}$$

$$= g^2\left\langle t_2\left|\frac{1}{\Box}\right|t_1\right\rangle - g^2\frac{\left\langle t_2\left|\frac{1}{\Box}\right|\omega\phi'_{\mathrm{v}}\right\rangle\left\langle\omega\phi'_{\mathrm{v}}\left|\frac{1}{\Box}\right|t_1\right\rangle}{\left\langle\omega\phi'_{\mathrm{v}}\left|\frac{1}{\Box}\right|\omega\phi'_{\mathrm{v}}\right\rangle}, \tag{3.13}$$

where we have used the standard Schwinger notations. Using the valley equation (3.3) we obtain

$$(\omega\phi')' = \frac{1}{\chi}\Box\phi' - \frac{\chi'}{\chi}\omega\phi'. \tag{3.14}$$

Owing to the constraint in the definition of the propagator in (3.13) the second term in eq. (3.14) does not contribute and we are left with

$$g^2\int \mathrm{d}t_2\, G(t_1, t_2)(\omega\phi')'(t_2) = \frac{g^2}{\chi}\left(\phi'_{\mathrm{v}}(t_1) - \frac{(\phi'_{\mathrm{v}}, \omega\phi'_{\mathrm{v}})\left\langle\omega\phi'_{\mathrm{v}}\left|\frac{1}{\Box}\right|t_1\right\rangle}{\left\langle\omega\phi'_{\mathrm{v}}\left|\frac{1}{\Box}\right|\omega\phi'_{\mathrm{v}}\right\rangle}\right)$$

$$\sim \frac{g^2}{\chi}\left(\phi'_{\mathrm{v}}(t_1) - \phi_-(t_1)\frac{(\phi'_{\mathrm{v}}, \omega\phi'_{\mathrm{v}})}{(\phi_-, \omega\phi'_{\mathrm{v}})}\right), \tag{3.15}$$

where in the last line we have retained the contribution of the negative quasizero mode only. Recall that $1/\chi$ is large, of the order of ξ^2, see eq. (3.10). Thus unless

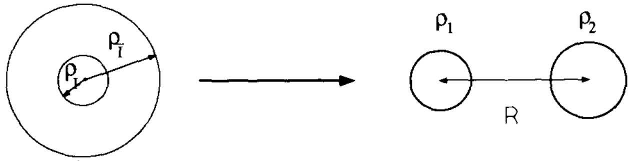

Fig. 4. The conformal transformation converts an instanton–anti-instanton pair with coinciding centers and with parametrically different radii $\rho_I = \rho/\xi$, $\rho_{\bar{I}} = \rho\xi$, $\xi \gg 1$ to an instanton–anti-instanton pair at a large separation R, and with $\rho_1 \sim \rho_2 \ll R$.

the two terms in parenthesis cancel to $O(1/\xi^2)$ accuracy (which is equivalent to the restriction in eq. (3.12)), the resulting quantum correction is enhanced parametrically by a factor ξ^2. In principle we can work with this Green function as well, but the correct answer, say to the vacuum energy, to the given order in g^2 will only be restored in the sum of the infinite series of "quantum" corrections. It is easy to check that the particular valley field in (3.9) indeed satisfies the condition in (3.12).

In order to find a suitable valley trajectory in the gauge theory it is convenient to use the conformal invariance of QCD at the tree level. This symmetry enables one to build the whole family of $I\bar{I}$ configurations with arbitrary centres starting from a single spherically symmetric $I\bar{I}$ configuration with coinciding centres by means of appropriate conformal transformations [26], see fig. 4. It is known that for this spherical ansatz (and for coinciding gauge orientations) QCD is equivalent at the tree level to the ordinary double-well quantum mechanics. If we take

$$A_\mu(x) = -\frac{i}{g}\left[\sigma_\mu \bar{x} - x_\mu\right] x^{-2} \phi_{\mathrm{v}}(t, \alpha), \tag{3.16}$$

where $t = \ln(x^2/\rho^2)$, then the QCD action coincides up to a normalization factor to the simple quantum mechanical expression in (3.1). Taking the quantum-mechanical valley field in form (3.9) we come to the corresponding gauge field

$$A_\mu^{\mathrm{v}} = -\frac{i}{g}\left[\sigma_\mu \bar{x} - x_\mu\right]\left\{\frac{\rho^2/\xi}{x^2\left[x^2+\rho^2/\xi\right]} + \frac{1}{x^2+\xi\rho^2}\right\}, \tag{3.17}$$

which is simply the sum of the instanton field in the regular gauge and the anti-instanton field in the singular gauge and obeys the valley equation

$$\left.\frac{\delta S}{\delta A_\mu(x)}\right|_{A=A^{\mathrm{v}}} = \chi(\xi)\omega(x, \xi)\frac{\mathrm{d}}{\mathrm{d}\xi}A_\mu^{\mathrm{v}}, \tag{3.18}$$

with the weightfunction (3.10) taken at $t = \ln x^2/\rho^2$. Here ρ^2 is the common scale of I and $\bar{I}$. Note that in the case of QCD it is convenient to take $\xi = 2\cosh\alpha$ as the valley parameter. For $\xi \to \infty$ we have almost non-interacting I and $\bar{I}$ fields and at $\xi = 1$ we arrive at a pure gauge field (with zero topological number) equivalent to the perturbative vacuum. The action at the valley (3.17) takes the form [12,25]

$$S^{\mathrm{v}} = \frac{48\pi^2}{g^2}\left[\frac{6\xi^2 - 14}{(\xi - 1/\xi)^2} - \frac{17}{3} + \ln(\xi)\left(\frac{(5/\xi - \xi)(\xi + 1/\xi)^2}{(\xi - 1/\xi)^3} + 1\right)\right]$$

$$= \frac{16\pi^2}{g^2}\left(1 - \frac{6}{\xi^2} + \mathrm{O}(\ln(\xi)/\xi^4)\right). \tag{3.19}$$

As it should be, the classical action does not depend on the common scale of I and $\bar{I}$. This dependence reveals itself on the quantum level only and comes from the argument of g^2: the bare coupling constant changes to $g^2(\rho)$ after taking into account the UV divergent part of the determinant det $\square^{\mathrm{v}}$ [1]. Note that if we had started from a valley which does not satisfy the criterion in eq. (3.12) (e.g. from the simple $I\bar{I}$ ansatz), then the quantum correction discussed above would contribute on equal grounds as the $\mathrm{O}(1/\xi^2)$ term in the action, and would make the final answer independent on the choice of the valley field.

Performing the conformal transformation (shift and inversion) we obtain the general form of the $I\bar{I}$ valley configuration with arbitrary separations and scales: *

$$A_\mu^{\mathrm{v}} = \left(A_\mu^{\bar{I}}\right)^{\Omega_1} + \left(A_\mu^{I}\right)^{\Omega_2},$$

$$(A_\mu)^{\Omega} = \Omega A_\mu \Omega^\dagger + \frac{i}{g}\Omega\partial_\mu\Omega^\dagger, \tag{3.20}$$

where A^I and $A^{\bar{I}}$ are the instanton positioned at the point $x_1 = R$ and having the size ρ_1, and the anti-instanton of the size ρ_2^2 standing at $x_2 = 0$, respectively

$$A_\mu^{\bar{I}} = -\frac{i\rho_1^2}{g}\frac{R\left(\bar{\sigma}_\mu(x-R) - (x-R)_\mu\bar{R}\right)}{R^2(x-R)^2\left(\rho_1^2 + (x-R)^2\right)}, \tag{3.21}$$

$$A_\mu^{I} = -\frac{i\rho_2^2}{g}\frac{\sigma_\mu\bar{x} - x_\mu}{x^2\left(\rho_2^2 + x^2\right)}. \tag{3.22}$$

Here Ω_1 and Ω_2 are the matrices of the gauge rotation which reduce to unity matrices at infinity:

$$\Omega_1 = \frac{R(\bar{x} - \bar{b})(x-R)\bar{R}}{R^2\sqrt{(x-b)^2(x-R)^2}}, \tag{3.23}$$

* Actually in what follows we shall need the valley (3.17) with the parallel (= maximum attractive) orientation only. Of course, in this case the action coincides with (3.19).

$$\Omega_2 = \frac{(x-a)\bar{x}}{\sqrt{(x-a)^2 x^2}}, \tag{3.24}$$

and additional singular points arising from the gauge rotations are *

$$a = -R\frac{\rho_2/\xi}{\rho_1 - \rho_2/\xi} \tag{3.25}$$

$$b = R\frac{\rho_2}{\rho_2 - \rho_1/\xi}. \tag{3.26}$$

The conformal parameter ξ in these variables takes the form

$$\xi = \frac{R^2+\rho_1^2+\rho_2^2}{2\rho_1\rho_2} + \sqrt{\frac{\left(R^2+\rho_1^2+\rho_2^2\right)^2}{4\rho_1^2\rho_2^2} - 1} \simeq \frac{R^2}{\rho_1\rho_2}\left\{1 + \mathrm{O}\left(\frac{\rho_1\rho_2}{R^2}\right)\right\} \tag{3.27}$$

and the weight function (3.10) becomes [24]

$$\omega(x, \xi) = \frac{24(\xi-\xi^{-1})}{\left(\left((x-R)^2+\rho_1^2\right)^2/\rho_1^2 + \left(\rho_2^2+x^2\right)^2/\rho_2^2\right)^2}. \tag{3.28}$$

Note that it is positive definite everywhere.

Thus, the conformal valley is given by a sum of somewhat modified instanton and anti-instanton fields in the singular gauge with the positions of the singularities slightly different (at small ξ) from the instanton centres. It is worth noting that the two valleys of the type (3.20) with different sets of x's and ρ's are connected by the conformal transformation (i.e. by the inversion back to the spherical form, translation, possibly dilatation, and again inversion).

Substituting (3.27) into eq. (3.19) we readily obtain

$$S^{\mathrm{v}} = \frac{16\pi^2}{g^2}\left(1 - \frac{6\rho_1^2\rho_2^2}{R^4} + \frac{12\rho_1^2\rho_2^2(\rho_1^2+\rho_2^2)}{R^6} + \mathrm{O}\left(\frac{\rho^8}{R^8}\right)\right). \tag{3.29}$$

Note that the $\mathrm{O}(1/\xi^2)$ term in the action determines *two* terms in the expansion of the action in the inverse powers of the $I\bar{I}$ separation R. The second of them readily gives the third term $\sim\varepsilon^2$ of the expansion of the function $F(\epsilon)$ in eq. (1.2) in powers of ϵ [13]. The result of the valley calculation has been confirmed by the

* One of the additional singular points could be removed by an appropriate gauge rotation (see ref. [24]) but we conform here to the above form which is symmetric with respect to interchanging of I and $\bar{I}$.

direct computation of the relevant "quantum" corrections at the one-instanton background as described in sect. 2.

4. Valleys in Minkowski space

In this section we suggest the valley configuration for the double functional integral (2.4) corresponding to the euclidean valley (3.20) and satisfying the boundary conditions specified above: no ingoing waves at $t = -\infty$ in both the $(+)$ and $(-)$ integrals, and coinciding outgoing waves at $t \to +\infty$. As discussed in sect. 2 the relevant configuration consists of pairs of "native" and "alien" instantons in each sector. Since the euclidean valley (3.20) is given by the sum of I and $\bar{I}$ with the gauge rotations (3.23), (3.24) needed to satisfy the valley equation, we apply the same gauge rotations to the simple $I\bar{I}$ ansatz described in sect. 2 to obtain the Keldysh-type valley solution in the form

$$A_\mu^{\mathrm{v}-} = \left(A_\mu^{\bar{I}-}\right)^{\Omega_1} + \left(\tilde{A}_\mu^{I-}\right)^{\Omega_2}, \tag{4.1}$$

$$A_\mu^{\mathrm{v}+} = \left(A_\mu^{I+}\right)^{\Omega_2} + \left(\tilde{A}_\mu^{\bar{I}+}\right)^{\Omega_1}, \tag{4.2}$$

where the A's are the "native" and the $\tilde{A}$'s are the "alien" instantons,

$$A_\mu^{\bar{I}-} = \frac{i\rho_1^2}{g} \frac{R\left(\bar{\sigma}_\mu(x-R) - (x-R)_\mu\right)\bar{R}}{R^2\left(-(x-R)^2 - i\epsilon\right)\left(\rho_1^2 - (x-R)^2 - i\epsilon\right)}, \tag{4.3}$$

$$A_\mu^{I+} = \frac{i\rho_2^2}{g} \frac{\sigma_\mu \bar{x} - x_\mu}{\left(-x^2 + i\epsilon\right)\left(\rho_2^2 - x^2 + i\epsilon\right)}, \tag{4.4}$$

$$\tilde{A}_\mu^{\bar{I}+} = \frac{i\rho_1^2}{g} \frac{R\left(\bar{\sigma}_\mu(x-R) - (x-R)_\mu\right)\bar{R}}{R^2\left(-(x-R)^2 - i\epsilon(x-R)_0\right)\left(\rho_1^2 - (x-R)^2 - i\epsilon(x-R)_0\right)}, \tag{4.5}$$

$$\tilde{A}_\mu^{I-} = \frac{i\rho_2^2}{g} \frac{\sigma_\mu \bar{x} - x_\mu}{\left(-x^2 + i\epsilon x_0\right)\left(\rho_2^2 - x^2 + i\epsilon x_0\right)}, \tag{4.6}$$

and Ω_1 and Ω_2 are the gauge matrices given in eqs. (3.23), (3.24) but with the structure of the singularities determined by those of the corresponding field under rotation. Namely, the singularities in $(x-a)^2$ are encircled similarly as the ones in x^2, while the singularities in $(x-b)^2$ are the same as in $(x-R)^2$. It is easy to see that the valley solutions in eqs. (4.1) and (4.2) indeed satisfy the proper boundary conditions. We now proceed to the calculation of the double functional integral in the vicinity of our valley configuration.

Similar as in the case of the euclidean valley in eq. (3.18) the linear term $D_\mu^- G_{\mu\nu}^-$ vanishes thanks to the $\delta(A^- - A_{\rm cl}^-,\ D_\mu^- G_{\mu\nu}^-)$ factor inserted as a Faddeev–Popov constraint. Of course, the linear term built of $(+)$ fields also disappears. Hence, to evaluate the functional integral in (2.4) to semiclassical accuray we should insert the classical valley fields $A_\mu^{\rm v-}$ and $A_\mu^{\rm v+}$ (and take into account the relevant determinants at the valley field background). The quantum corrections now become well defined and are of the order of g^2 compared to the leading term *. The action at the Keldysh-type valley configuration in eqs. (4.1), (4.2) can be evaluated by a straightforward calculation. After some algebra we get

$$
\begin{aligned}
&\frac{i}{2}\int \mathrm{d}x\, G_{\mu\nu}^{v-}G_{\mu\nu}^{v-} - \frac{i}{2}\int \mathrm{d}x\, G_{\mu\nu}^{v+}G_{\mu\nu}^{v+} \\
&= \frac{i}{2}\int \mathrm{d}x \left\{ \frac{\rho_1^2}{\left(\rho_1^2-(x-R)^2-i\epsilon\right)^2}\left(\frac{\rho_1}{\rho_1^2-(x-R)^2-i\epsilon}-\frac{\rho_2/\xi}{\rho_2^2-x^2+i\epsilon x_0}\right)^2\right. \\
&\qquad \left. + \frac{\rho_2^2}{\left(\rho_2^2-x^2+i\epsilon x_0\right)^2}\left(\frac{\rho_2}{\rho_2^2-x^2+i\epsilon x_0}-\frac{\rho_1/\xi}{\rho_1^2-(x-R)^2-i\epsilon}\right)^2\right\} \\
&\quad - \frac{i}{2}\int \mathrm{d}x \left\{ \frac{\rho_1^2}{\left(\rho_1^2-(x-R)^2-i\epsilon(x-R)_0\right)^2}\right. \\
&\qquad \times\left(\frac{\rho_1}{\rho_1^2-(x-R)^2-i\epsilon(x-R)_0}-\frac{\rho_2/\xi}{\rho_2^2-x^2+i\epsilon}\right)^2 \\
&\qquad \left. + \frac{\rho_2^2}{\left(\rho_2^2-x^2+i\epsilon\right)^2}\left(\frac{\rho^2}{\rho_2^2-x^2+i\epsilon}-\frac{\rho_1/\xi}{\rho_1^2-(x-R)^2-i\epsilon(x-R)_0}\right)^2\right\} \\
&= -S_{\rm Eucl}^{\rm v}\left(R^2 \to -R^2+i\epsilon R_0\right).
\end{aligned}
\tag{4.7}
$$

Thus we reobtain the answer (3.19) for the euclidean valley with the substitution $R_{\rm Eucl}^2 \to -R_{\rm Mink}^2 + i\epsilon R_0$ which corresponds to taking the imaginary part with respect to $(p+k)^2$ after the Fourier transformation.

* As pointed out by Mueller [28], the quantum corrections $\sim g^2$ can become large when multiplied by a large factor E^2. Corrections of this type are due to the interaction of the colliding W's in the initial state.

To evaluate the functional integral in eq. (2.4) we need to calculate the Fourier transforms of the valley fields $A^{\mathrm{v}}(p)$. This step requires some care. The problem is that this Fourier transform involves terms $\sim p_\mu(pR)p^{-4}$ which produce double-pole contributions after the rotation to Minkowski space and have no interpretation in terms of emitted particles. Note that the problem is indeed related to the Minkowski metric: in euclidean space $p_\mu p_\nu \sim p^2$ while in Minkowski space the components of a light-like vector p can be large. This difficulty is actually not a surprise, since the amplitudes on mass-shell have an interpretation in terms of physical particles in physical gauges only. Let us go over to the temporal gauge $A_0 = 0$ in which a consistent hamiltonian formalism exists for the Yang–Mills theory. The transition to this gauge is performed by the gauge matrix

$$U(x) = [x,\, x+\infty e]^{\mathrm{v}} = P\,\exp\left\{-ig\int_0^\infty \mathrm{d}\lambda\; e_\mu A^{\mathrm{v}}_\mu(x+\lambda e)\right\},$$

$$U^\dagger(x) = [\infty e + x,\, x]^{\mathrm{v}}, \tag{4.8}$$

where $e = (1,0,\,0,\,0)$ is the unity vector in the time direction. The valley field in eqs. (4.1)–(4.6) tacitly implies an arbitrary covariant gauge. Hence we should identify the production of physical particles not with the fields A_μ but rather with the gauge-rotated field

$$A^{\mathrm{v},(t)}_\mu(x) = [\infty e + x,\, x]^{\mathrm{v}}\left(\frac{i}{g}\partial_\mu + A^{\mathrm{v}}_\mu\right)[x,\, x+\infty e]^{\mathrm{v}}. \tag{4.9}$$

The question which particular field A^{v}_μ or $A^{\mathrm{v},(t)}_\mu$ enters the LSZ formula (2.2) makes no difference in euclidean space. Indeed, going over to the mass-shell $k^2 \to 0$, $p^2 \to 0$ we pick up asymptotics of the field $x^2 \to \infty$ in the coordinate space. Since the valley field decreases as $1/x^3$ the gauge factor in (4.9) turns to unity. Hence the residue of the physical amplitude on the mass-shell is gauge independent. The situation in Minkowski space is more complicated. It is easy to see that the relevant asymptotics $p^2 \to 0$, $(pR) =$ fixed, corresponds in coordinate space to the "Bjorken limit": both $x^2 \to \infty$, and $(xR) \to \infty$, but with the fixed ratio $(xR)/x^2 \sim (pR)$ of the order of unity. In this region the valley field decreases not fast enough (as $(xR)/x^4 \sim 1/x^2$ only), and the gauge factor in eq. (4.9) is nontrivial. An explicit calculation yields, see appendix A,

$$[x,\, x+\infty e]^{\mathrm{v}}_- = \frac{R\bar{x}}{2(Rx)}\sqrt{\frac{(x-a)^2 - i\epsilon(x-a)_0}{(x-b)^2 + i\epsilon}\,\frac{(x-R)^2 + i\epsilon}{x^2 - i\epsilon x_0}}$$

$$+\frac{x\bar{R}}{2(Rx)}\sqrt{\frac{(x-b)^2 + i\epsilon}{(x-a)^2 - i\epsilon(x-a)_0}\,\frac{x^2 - i\epsilon x_0}{(x-R)^2 + i\epsilon}} + \mathrm{O}(1/x^2),$$

$$[x, x+\infty e]_{+}^{v} = \frac{R\bar{x}}{2(Rx)}\sqrt{\frac{(x-a)^2 - i\epsilon}{(x-b)^2 + i\epsilon(x-b)_0}\,\frac{(x-R)^2 + i\epsilon(x-R)_0}{x^2 - i\epsilon}}$$
$$+\frac{x\bar{R}}{2(Rx)}\sqrt{\frac{(x-b)^2 + i\epsilon(x-b)_0}{(x-a)^2 - i\epsilon}\,\frac{x^2 - i\epsilon}{(x-R)^2 + i\epsilon(x-R)_0}}$$
$$+\mathrm{O}(1/x^2), \tag{4.10}$$

The structure of the singularities is the same as in eqs. (4.1)–(4.6).

Inserting (4.10) into eq. (4.9) one obtains after a little algebra the expression for the valley field in the temporal gauge in the limit $x^2 \sim (xR) \to \infty$,

$$A_{\mu}^{\mathrm{v}-,(t)}(x) = \frac{i}{g}\frac{R\bar{x}\sigma_{\mu}\bar{R}x\bar{R}\rho_1^2(1-\rho_2/\rho_1\xi)}{2(Rx)R^2\left((x-R)^2 + i\epsilon\right)^2} + \frac{i}{g}\frac{x\bar{\sigma}_{\mu}R\bar{x}\rho_2^2(1-\rho_1/\rho_2\xi)}{2(Rx)\left(x^2 - i\epsilon x_0\right)^2},$$

$$A_{\mu}^{\mathrm{v}+,(t)}(x) = \frac{i}{g}\frac{R\bar{x}\sigma_{\mu}\bar{R}x\bar{R}\rho_1^2(1-\rho_2/\rho_1\xi)}{2(Rx)R^2\left((x-R)^2 + i\epsilon(x-R)_0\right)^2} + \frac{i}{g}\frac{x\bar{\sigma}_{\mu}R\bar{x}\rho_2^2(1-\rho_1/\rho_2\xi)}{2(Rx)(x^2 - i\epsilon)^2} \tag{4.11}$$

We see that the additional gauge singularities have disappeared. This cancellation of gauge singularities is actually quite general and does not depend on the particular choice of the valley (see appendix A). The final result for the Fourier transform of the valley field is

$$A_{\mu}^{\mathrm{v}+(t)}(-p) = \frac{2\pi^2\rho_2^2}{g(p^2 - i\epsilon)}\left(1 - \frac{\rho_1}{\rho_2\xi}\right)\frac{p\bar{\sigma}_{\mu}R\bar{p}}{2(pR)} + \text{regular terms},$$

$$A_{\mu}^{\mathrm{v}-(t)}(p) = \frac{2\pi^2\rho_1^2}{g(p^2 + i\epsilon)}\left(1 - \frac{\rho_2}{\rho_1\xi}\right)\frac{R}{\sqrt{R^2}}\,\frac{\bar{p}\sigma_{\mu}\bar{R}_p}{2(\cancel{pR})}\,\frac{\bar{R}}{\sqrt{R^2}}\mathrm{e}^{ipR} + \dots, \tag{4.12}$$

and the residues in front of the poles coincide to the required accuracy with the residues of the pure instanton and the pure anti-instanton fields (in the singular gauge) in eqs. (2.21).

Now we are in a position to write down the final answer for the BNV cross section in the leading semiclassical approximation. Combining eqs. (4.7) and (4.12) we obtain the contribution of the vicinity of the valley (4.1), (4.2) to the functional integral (2.4) in the following form:

$$\sigma_{\mathrm{BNV}} = \int\frac{\mathrm{d}\rho_1}{\rho_1^5}\frac{\mathrm{d}\rho_2}{\rho_2^5}\int\mathrm{d}R\;\mathrm{d}\mu_{I,\bar{I}}\;\mathrm{e}^{iER_0 - S(\xi)/g^2 - S_{\mathrm{H}}}, \tag{4.13}$$

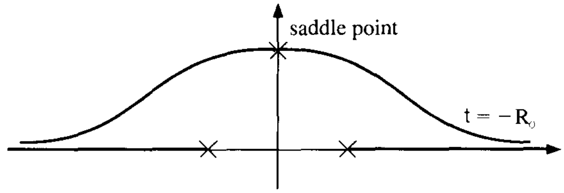

Fig. 5. The integration contour in R_0 in eq. (4.13).

where

$$\xi = \frac{\rho_1^2 + \rho_2^2 - R^2 + i\epsilon R_0}{2\rho_1\rho_2} + \sqrt{\frac{\left(\rho_1^2 + \rho_2^2 - R^2 + i\epsilon R_0\right)^2}{4\rho_1^2\rho_2^2} - 1}$$

$$= \frac{-R^2 + i\epsilon R_0}{\rho_1\rho_2}\left(1 + O(\rho^2/R^2)\right). \tag{4.14}$$

Here $d\mu_{I,\bar{I}}$ is the measure in the space of a double set of instanton and anti-instanton collective coordinates proportional to the product of gauge, fermion, Higgs, and ghost determinants at the valley background. At large $I\bar{I}$ separations it factorizes into a product of two one-instanton measures $d\mu_I d\mu_{\bar{I}}$ up to $O(1/\xi^2)$ corrections.

We have added here the contribution of the Higgs fields S_H (not displayed in eq. (2.4)) which deserves some discussion. At large $I\bar{I}$ separations it equals the sum of the classical actions of the Higgs components of the instanton and anti-instanton fields plus the leading $1/R^2$ Coulomb-type correction which is obtained similarly to the dipole–dipole term in the gauge interaction potential:

$$S_H = \pi^2 v^2\left(\rho_1^2 + \rho_2^2\right) + 2\pi^2 v^2 \rho_1^2\rho_2^2 R^{-2} + O\left(\rho^4/R^4\right). \tag{4.15}$$

Here v is the vacuum expectation value of the Higgs field (we tacitly assume the standard model with one Higgs doublet). When the $I\bar{I}$ separation is not large, one should solve the valley equation for the Higgs and gauge fields simultaneously *.

The contour of integration over R_0 in eq. (4.13) is going "in the Minkowski region" along the real axis, see fig. 5. However, the integral can be calculated by the steepest descent method with the saddle point lying "in the euclidean region" on the imaginary axis. At small energies this saddle point is fixed by the first

* Since the Higgs component of the valley should approach the perturbative vacuum $\phi = v$ together with the gauge field, one can bear in mind a simple model for the Higgs interaction $S_H + 4\pi^2 v^2(\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2/\xi)$ which has the correct behaviour in both the limits $R \to \infty$ and $R \to 0$, $\rho_1 \to \rho_2$.

dipole–dipole interaction term in $S(\xi)$ [see the expansion in eq. (3.29)]. It is easy to find the saddle points in the integrals over R and ρ_1, ρ_2 [8]:

$$-R_0 = i\tau_* = \frac{4i}{m_W}(3\epsilon)^{1/3},$$

$$\rho_1 = \rho_2 = \rho_* = \frac{4}{m_W}\left(\tfrac{3}{8}\epsilon^4\right)^{1/6},$$

$$\rho_*^2/\tau_*^2 = \left(\epsilon^2/24\right)^{1/3}. \tag{4.16}$$

The saddle point in the integration over d^3R is $\boldsymbol{R} = 0$ and the saddle point in the integration over orientations corresponds to the most attractive orientation $u = 1$. (We have dropped the orientation dependence in sect. 3 having in mind that the gaussian integral over orientations is already performed. This dependence is shown explicitly for the dipole–dipole term (which determines the position of the saddle point) in eq. (2.22)). After substituting the saddle point values of ρ and R into the expansion of $S(\xi)$ in eq. (3.29) we reobtain the first three terms of the expansion of $F(\epsilon)$ in powers of ϵ. (Additional contributions from the Coulomb $I\bar{I}$ interaction induced by the Higgs exchanges (4.15) and the correction due to the mass of the W-boson are implied, see refs. [13,15,16].)

It should be emphasized that in our Minkowski-space calculation we have reproduced precisely the imaginary part arising from the analytical continuation of the corresponding contribution to the forward W-boson scattering amplitude given by the euclidean valley in (3.20). This has happened since at the semiclassical level the relevant diagrams possess only one imaginary part, namely the one corresponding to the BNV process $\langle 2 | \bar{I} | N \rangle \langle N | I | 2 \rangle$. Indeed, the dangerous discontinuities of the type $\langle 2 | N \rangle \langle N | I\bar{I} | 2 \rangle$ or $\langle 2 | I\bar{I} | N \rangle \langle N | 2 \rangle$ all come from initial-state interactions. As demonstrated in ref. [28], these quantum corrections are of the order of $\epsilon^{10/3}$ so that starting from this term they should be taken into account in the function $F(\epsilon)$ in eq. (1.2). To handle these corrections within our approach we should find the high-energy behaviour of the propagator in the non-euclidean external field (4.1), (4.2).

5. Concluding remarks

We have proposed a certain modification of the valley approach for the calculation of the instanton-induced cross sections which has the advantage of being formulated in Minkowski space. Namely, we work out a systematic technique to do the semiclassical expansion around certain classical field trajectories in the double functional integral. From the viewpoint of the standard technique [19] for the expansion around pure instanton fields, what we actually propose is to make one more shift of variables in the double functional integral, such that large $1/g$ fields of the produced W-bosons are simulated by a classical field with particular analytic properties which we call the "alien" field. By our procedure we automatically pick up the proper cut of the forward scattering amplitude in fig. 1, corresponding to processes with baryon number violation. This separation becomes important to $\epsilon^{10/3}$ accuracy, from which the "hard–hard" quantum corrections come into the game [16].

The contribution to the function $F(\epsilon)$ (1.2) of the order of $\sim \epsilon^{8/3}$ still can be evaluated by the analytical continuation of the euclidean functional integral. It is known [30] that the sum of soft–soft corrections depends to this accuracy on the choice of constraints in the gauge propagator at the one-instanton background and this dependence should be remedied by soft–hard corrections. Similarly, in the valley approach the semiclassical answer depends to this accuracy on the particular choice of the valley. This ambiguity can most easily be demonstrated by a direct calculation, choosing a different quantum mechanical valley field instead of the simplest kink–antikink configuration in (3.9), and retaining the conformal ansatz.

Namely, we remind of an old result [10], that the field

$$\phi_{\tilde{v}}(t,\alpha)=\tfrac{1}{2}m\left[\tanh\left(\tfrac{1}{2}m(t+\alpha)\right)-\tanh\left(\tfrac{1}{2}m(t-\alpha)\right)\right]+\mathrm{O}(1/\xi^4),$$

$$m=1-6/\xi^2+\mathrm{O}(1/\xi^4),\qquad \xi=2\cosh m\alpha \tag{5.1}$$

provides an approximate solution of the valley equation with the weight function $\omega(t,\alpha)=1$:

$$\frac{\mathrm{d}\phi_{\tilde{v}}}{\mathrm{d}\alpha}=-L(\phi)\frac{\xi^2}{12}+\mathrm{O}(1/\xi^2). \tag{5.2}$$

We evaluate the action on the QCD valley corresponding to (5.1). A simple calculation yields

$$S^{\tilde{v}}=\frac{16\pi^2}{g^2}\left(1-\frac{6}{\xi^2}-\frac{42}{\xi^4}+\dots\right). \tag{5.3}$$

(Note that the accuracy of (5.1) is enough to get the third term.) The $\mathrm{O}(1/\xi^4)$ contribution to the action of the modified valley in (5.3) clearly deviates from the corresponding term in (3.19), as expected. One can easily convince oneself that the saddle point values of collective coordinate are left intact by this modification of the valley, and hence the corresponding contribution to $F(\epsilon)$ is easily calculated by the insertion of the value of ξ_* corresponding to (4.16). It is worth while to remember that variations of the valley $>\mathrm{O}(1/\xi^2)$ are forbidden, since it must coincide to $\sim \mathrm{O}(1/\xi^2)$ accuracy to the negative mode, see sect. 3. Thus, quite generally, the action S^{v} depends on the choice of the valley to the accuracy $\sim 1/\xi^4$, and yields different contributions to the function $F(\epsilon)$ (1.2) to the order of $\sim \epsilon^{8/3}$.

Since all the freedom in the valley equation (3.3) is due actually to a variety of possibilities to insert the unit factor (3.6) in the functional integral, it is clear that the complete answer cannot depend on the choice of the valley. If the integration near the valley is done rigorously, i.e. if all "quantum" corrections to the semiclassical result are taken into account, then the answer must become unambiguous. In the particular case of the $O(\epsilon^{8/3})$ contribution this means that the difference must be compensated by the corresponding change of the soft-hard corrections or by the change of the Higgs component of the valley, which we do not consider in this paper. An interesting question is whether the valley field can be chosen in such a way that all soft-hard corrections vanish to the required $O(1/\xi^4)$ accuracy (in the exponent). The finding of such an "improved" valley would mean that it is possible to treat soft–hard corrections semiclassically, which is not trivial. This question is under study and we plan to discuss it in a separate publication.

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Appendix A. Gauge phase factors at the background of the valley

The straight-line ordered phase factor

$$[x+\lambda e,\ x+\infty e]^{\mathrm{v}} = P\exp\left\{ig\int_{\infty}^{\lambda}\mathrm{d}t\ \mathrm{e}_{\mu}A_{\mu}^{\mathrm{v}}(x+te)\right\} \tag{A.1}$$

is determined as the solution of the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}[x_{\lambda},\ x+\infty e]^{\mathrm{v}} = ige_{\mu}A_{\mu}^{\mathrm{v}}(x_{\lambda})[x_{\lambda},\ x+\infty e]^{\mathrm{v}} \tag{A.2}$$

(where $x_{\lambda}=x+\lambda e$) with the boundary condition

$$[x_{\lambda},\ x+\infty e]^{\mathrm{v}} \xrightarrow{\lambda\to\infty} 1. \tag{A.3}$$

The structure of singularities of the gauge factor (A.1) follows that of the background field. Thus it is sufficient to work out the formulas for one particular case, say, for the $I\bar{I}$ valley with the Feynman prescription of encircling the singularities. Going over to the pair of "native" and "alien" fields one only needs to change the prescriptions to those in eqs. (4.1)–(4.6). Therefore, in what follows we drop all the necessary $i\epsilon$ terms which are easily recovered in the final expressions.

We remind that $e = (1, 0, 0, 0)$ is the unity vector in the time direction. Unfortunately, we have not succeeded in obtaining an analytical solution of (A.2) at the background of an $I\bar{I}$ valley configuration with an arbitrary $I\bar{I}$ separation R. However, to the semiclassical accuracy it is sufficient to evaluate the gauge factors at the saddle-point configuration in eq. (4.16) with the saddle-point value of R_* collinear to the time axis. In this case the solution of eq. (A.2) is easily found to be

$$[x_\lambda, x + \infty e]^{\mathrm{v}} = \frac{1}{2}\left(1 + \frac{R\bar{x} - (Rx)}{(Rx)\sqrt{\kappa}}\right)\sqrt{F_\lambda} + \frac{1}{2}\left(1 - \frac{R\bar{x} - (Rx)}{(Rx)\sqrt{\kappa}}\right)\frac{1}{\sqrt{F_\lambda}}, \tag{A.4}$$

where

$$F_\lambda = \left(\frac{(Rx_\lambda) + (Rx)\sqrt{\kappa_2}}{(Rx_\lambda) - (Rx)\sqrt{\kappa_2}}\right)^{\sqrt{\kappa/\kappa_2}} \left(\frac{(Rx_\lambda - R) - (Rx)\sqrt{\kappa_1}}{(Rx_\lambda - R) + (Rx)\sqrt{\kappa_1}}\right)^{\sqrt{\kappa/\kappa_1}}$$
$$\times \left(\frac{(Rx_\lambda - a) - (Rx)\sqrt{\kappa}}{(Rx_\lambda - a) + (Rx)\sqrt{\kappa}}\right)\left(\frac{(Rx_\lambda - b) + (Rx)\sqrt{\kappa}}{(Rx_\lambda - b) - (Rx)\sqrt{\kappa}}\right), \tag{A.5}$$

and

$$\kappa = 1 - \frac{R^2 x^2}{(Rx)^2},$$
$$\kappa_{1(2)} = 1 - \frac{R^2(x^2 - \rho^2_{1(2)})}{(Rx)^2}. \tag{A.6}$$

By a staightforward calculation one obtains

$$\frac{\mathrm{d}F_\lambda}{\mathrm{d}\lambda} = 2(Rx)\sqrt{\kappa}\left(-\frac{1}{x_\lambda^2 - \rho_2^2} + \frac{1}{(x_\lambda - R)^2 - \rho_1^2} + \frac{1}{(x_\lambda - a)^2} - \frac{1}{(x_\lambda - b)^2}\right)F_\lambda, \tag{A.7}$$

from which we get

$$ige_\mu A'_\mu(x_\lambda) = \frac{1}{2}\frac{R\bar{x} - (Rx)}{(Rx)\sqrt{\kappa}}\frac{1}{F_\lambda}\frac{\mathrm{d}F_\lambda}{\mathrm{d}\lambda}. \tag{A.8}$$

Using eqs. (A.7) and (A.8) it is easy to check that the gauge factor in (A.4) satisfies eq. (A.2).

As mentioned in the main text, in order to find the amplitude for the emission of real W's with virtuality $p^2 \to 0$, one should pick up the contribution of the region in the coordinate space corresponding to the "Bjorken limit" $x^2 \to \infty$, $(xR) \to \infty$, $(xR)/x^2 \sim 1$. In this limit the function F_λ simplifies to

$$F_\lambda = \frac{(x_\lambda - R)^2 (x_\lambda - a)^2}{(x_\lambda - b)^2 x_\lambda^2}, \tag{A.9}$$

and the expression for the gauge factor in (A.4) reduces to (4.10) where the prescriptions to go around the singularities are shown explicitly. The expression for the valley field in the temporal gauge is easily obtained using the representation in (4.9). The only nontrivial point is that simplification of the expression in (A.5) to the one in (A.9) can only be done after the differentiation with respect to x_μ, since e.g. $e^2x^2 \ll (ex)^2$, but $2e^2x_\mu \sim 2(ex)e_\mu$. The answer is given in eq. (4.11).

It is instructive to demonstrate that the observed cancellation of additional gauge singularities is a general effect which does not depend on the particular expression for the valley field in eqs. (3.20)–(3.24). To this end we consider the generalized expression for the valley in (3.16) with an arbitrary profile function ϕ (going to unity in both limits $x^2 \to 0$ and $x^2 \to \infty$ to ensure zero topological charge of the valley field):

$$A_\mu^{\mathrm{v}}(x) = -\frac{i}{g}\left[\sigma_\mu \bar{x} - x_\mu\right] x^{-2} \phi\left(\ln\left(\frac{-x^2}{\rho^2}\right)\right). \tag{A.10}$$

Making the shift $x \to x - x_0$ and the inversion with respect to the point a_μ: $(x-a)_\mu \to r^2/(x-a)^2 (x-a)_\mu$ we obtain the generalized valley field in the form

$$A_\mu^{\mathrm{v}}(x) = -\frac{i}{g}\left\{\frac{(x-a)\bar{\sigma}_\mu (x-b)(\bar{b}-\bar{a})}{(x-a)^2(x-b)^2} - \frac{(x-b)_\mu}{(x-b)^2} + \frac{(x-a)_\mu}{(x-a)^2}\right\}\phi\left(\ln\left(-\frac{(x-b)^2 c^2}{(x-a)^2 \rho^2}\right)\right), \tag{A.11}$$

where $c_\mu = (x_0 - a)_\mu$, $b_\mu = a_\mu + r^2 c_\mu / c^2$, $c^2 = r^4/(b-a)^2$. For completeness we give the necessary formulas relating the parameters x_0, a, r^2 of the conformal transformation to the final parameters $x_1 = R$, $x_2 = 0$, ρ_1, ρ_2 of the $I\bar{I}$ configuration,

$$0 = a + c\frac{r^2}{-c^2 + \rho^2 \xi},$$

$$R = a + c\frac{r^2}{-c^2 + \rho^2/\xi},$$

$$\rho_1 = \frac{r^2\rho}{\sqrt{\xi}\,(-c^2+\rho^2/\xi)},$$

$$\rho_2 = \frac{r^2\rho\sqrt{\xi}}{-c^2+\rho^2\xi}. \tag{A.12}$$

Note that we use the minkowskian notations here in contrast to sect. 3.

It can be demonstrated that the expression for the gauge factor in (A.4) holds true with the substitution of (A.5) by

$$F_\lambda = \exp\left\{2(Rx)\sqrt{\kappa}\int_\infty^\lambda \mathrm{d}t\left(\frac{1}{(x_t-a)^2}-\frac{1}{(x_t-b)^2}\right)\phi\left(\ln\left(-\frac{(x_t-b)^2c^2}{(x_t-a)^2\rho^2}\right)\right)\right\} \tag{A.13}$$

which simplifies in the required limit $x^2\to\infty$, $(xR)\to\infty$, $(xR)/x^2\sim 1$ to

$$F_\lambda = \exp\left\{\int_{\ln(-c^2/\rho^2)}^{\ln(-(x_\lambda-b)^2c^2/(x_\lambda-a)^2\rho^2)}\mathrm{d}\tau\;\phi(\tau)\right\}. \tag{A.14}$$

Proceeding in the same way as above one obtains the expression for the valley field in the temporal gauge in the form

$$igA_\mu^{\mathrm{v},(t)}(x) = \frac{R\bar{x}\sigma_\mu\bar{R}x\bar{R}}{8(Rx)^3}\left\{F_0-1+2\frac{(x,\,b-a)}{(x-b)^2}\phi F_0\right\}$$

$$+\frac{R^2x\bar{\sigma}_\mu R\bar{x}}{8(R\bar{x})^3}\left\{\frac{1}{F_0}-1-2\frac{(x,\,b-a)}{(x-a)^2}\phi\frac{1}{F_0}\right\} \tag{A.15}$$

where the argument of the function ϕ is the same as in (A.11), and $F_0 = F_{\lambda=0}$. It is seen now that for $x\to a$ and $x\to b$ the upper limit of the integration in (A.14) tends to $+\infty$, $-\infty$, respectively. Since $\phi(\tau)\to 1$ in both limits, we obtain

$$F \xrightarrow{x\to a} \frac{1}{(x-a)^2},$$

$$F \xrightarrow{x\to b} (x-b)^2. \tag{A.16}$$

From (A.16) it easily follows that both expressions in the braces in (A.15) are nonsingular for $x\to a$ and $x\to b$. It can also be shown that the Fourier transform of (A.14) is free of the unphysical poles $\sim 1/p^4$.

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