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QCD sum rules in the effective heavy quark theory

E. Bagan [1,2], Patricia Ball, V.M. Braun [1,3] and H.G. Dosch
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, W-6900 Heidelberg, FRG

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We derive sum rules for the leptonic decay constant of a heavy–light meson in the effective heavy quark theory. We show that the summation of logarithms in the heavy quark mass by the renormalization group technique enhances considerably radiative corrections. Our result for the decay constant in the static limit agrees well with recent lattice calculations. Finite quark mass corrections are estimated.

1. In this paper we give a consistent framework for the construction of QCD sum rules [1] for heavy–light quark systems in the heavy quark limit (HQL), which in that approach has first been discussed by Shuryak [2] and has been further studied by several authors [3,4]. Although for the heavy quark mass m_Q below 10 GeV both the logarithmic and nonlogarithmic contributions are numerically of the same size, for the consistent treatment of the limit $m_Q \to \infty$ it is necessary to sum all corrections of the type $[\alpha_s(m_Q)\ln m_Q]^n$, $\alpha_s(m_Q)[\alpha_s(m_Q)\ln m_Q]^n$, etc. by the renormalization group technique. This "mass factorization" has become one of the most actively discussed topics in the literature, and we find it important to formulate the sum rule approach in such a way that all the scaling laws inherent to the heavy quark expansion (HQE) are automatically fulfilled. This task is interesting for several reasons. First, the sum rules formulated in this way exhibit explicit Isgur–Wise symmetries [5] and so do the physical quantities extracted as their output. Second, quantitative estimates can be made for the finite heavy quark mass corrections. In addition, such a formulation of sum rules facilitates the comparison to the results of lattice calculations [6,7].

A convenient framework for systematically factorizing out the large-mass physics is provided by the effective field theory [5]. The key issue there is the introduction of a separate heavy quark and antiquark field $h_v^{\pm}$ for each four-velocity v in order to implement the velocity superselection rule: the velocity of the heavy quark cannot be changed by the radiation of gluons since it would correspond to infinitely large momentum transfers $\delta p_\mu = m_Q\,\delta v_\mu$. Hence, the part of the lagrangian associated with heavy quarks becomes

$$\mathscr{L}_{\text{heavy}} = \sum_{h=c,b} \int \frac{d^3v}{2v_0} (i\overline{h_v^+} v_\mu D^\mu h_v^+ - i\overline{h_v^-} v_\mu D^\mu h_v^-) \tag{1}$$

from which Feynman rules can be derived for the heavy quark propagator $i(1+\not{v})/(2vk)$ and the heavy quark–gluon vertex $-igv_\mu t^a$. For each composite operator of the full theory we can write an expansion in operators of the effective theory (see refs. [5,8]) [#1]

[1] Alexander von Humboldt Fellow.

[2] On leave of absence from Grup de Física Teòrica, Departament de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra (Barcelona), Spain.

[3] On leave of absence from St. Petersburg Nuclear Physics Institute, SU-188 350 Gatchina, USSR.

[#1] The sign "$\cong$" reminds that the the operator on the LHS of (2) must be sandwiched between hadron states of the full theory, while the operators on the RHS are taken between states of the effective theory, cf. ref. [8].

$$J^{A}_{\mu}(m_Q) \cong \tilde{J}^{A}_{\mu}(m_Q=\infty,\mu)\, C\left(\frac{m_Q}{\mu},\alpha_s(\mu)\right) + \sum_n \frac{\tilde{O}_n(m_Q=\infty,\mu)}{(m_Q)^n}\, C_n\left(\frac{m_Q}{\mu},\alpha_s(\mu)\right), \tag{2}$$

where $J^{A}_{\mu}(m_Q)$ is the axial-vector current. The matrix element of $J^{A}_{\mu}(m_Q)$ between the vacuum and a covariant normalized hadron state defines the physical decay constant f_P:

$$\langle 0|J^{A}_{\mu}|P(p)\rangle = \mathrm{i} f_P p_\mu . \tag{3}$$

Likewise, $\tilde{J}^{A}_{\mu} = \bar{q}\gamma_\mu\gamma_5 h_v$ is the quark current in the effective theory, built of a light antiquark field $\bar{q}$ and a (properly normalized) heavy quark field h_v [5]. Changing to a noncovariant normalization of hadron states [9], we define the decay constant in the static limit by the relevant matrix element of the effective current $\tilde{J}^{A}_{\mu}$:

$$\langle 0|\tilde{J}^{A}_{\mu}|\tilde{P}(v)\rangle = \frac{\mathrm{i}}{\sqrt{2}}\tilde{f}_{\rm stat} v_\mu . \tag{4}$$

The coefficient function $C(m_Q/\mu,\ \alpha_s(\mu))$ in (2) is determined by the matching condition that the effective theory reproduces the results of the full theory at $\mu = m_Q$. One finds [#2]:
$C(1,\alpha_s(m_Q)) = 1 - 2\alpha_s(m_Q)/3\pi$, which in turn implies

$$f_P(m_P) = \frac{1}{\sqrt{m_P}}\left(1 - \frac{2}{3}\frac{\alpha_s(m_P)}{\pi}\right)\tilde{f}_{\rm stat}(\mu=m_P) + O(1/m_Q) . \tag{5}$$

Hereafter we take m_Q to be the scale-invariant pole mass, defined as

$$m_Q(p^2=m_Q^2) = m_{\overline{\rm MS}}(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\tfrac{4}{3} + \ln\frac{\mu^2}{m^2_{\overline{\rm MS}}}\right)\right]. \tag{6}$$

The effective current $\tilde{J}^{A}_{\mu}$ acquires a nontrivial anomalous dimension [10,8,11]:

$$\gamma = \gamma_0\frac{\alpha_s}{4\pi} + \gamma_1\left(\frac{\alpha_s}{4\pi}\right)^2 + \ldots,\quad \gamma_0 = -4\,,\quad \gamma_1 = -\tfrac{254}{9} - \tfrac{56}{27}\pi^2 + \tfrac{20}{9}n_f\,, \tag{7}$$

and hence the effective decay constant scales logarithmically as

$$\tilde{f}_{\rm stat}(m_P) = \left(\frac{\alpha_s(m_P)}{\alpha_s(\mu)}\right)^{\gamma_0/2\beta_0}\left(1 + \frac{\alpha_s(m_P) - \alpha_s(\mu)}{\pi}\Gamma\right)\tilde{f}_{\rm stat}(\mu)\,, \tag{8}$$

where

$$\Gamma = \frac{\gamma_0}{8\beta_0}\left(\frac{\gamma_1}{\gamma_0} - \frac{\beta_1}{\beta_0}\right) \simeq -0.23 \tag{9}$$

and $\beta_0 = 11 - \frac{2}{3}n_f$, $\beta_1 = 102 - \frac{38}{3}n_f$. It is convenient to introduce the renormalization group invariant operator (to two-loop accuracy) and the corresponding scale-invariant decay constant

$$\hat{J} = \tilde{J}(\mu)\alpha_s(\mu)^{-\gamma_0/2\beta_0}\left(1 - \Gamma\frac{\alpha_s(\mu)}{\pi}\right),\quad \hat{f} = \tilde{f}_{\rm stat}(\mu)\alpha_s(\mu)^{-\gamma_0/2\beta_0}\left(1 - \Gamma\frac{\alpha_s(\mu)}{\pi}\right). \tag{10}$$

We remind that in the HQL the Lorentz structure becomes unimportant, it is only parity that counts.

2. We now derive the sum rule for the correlation function of scale-invariant effective currents with negative parity

[#2] This expression disagrees with the corresponding one in ref. [8], where the given answer for f_D corresponds to the case of mesons with positive instead of negative parity.

$$\hat{\Pi}(\omega=vq)=\mathrm{i}\int \mathrm{d}^4x\,\exp(\mathrm{i}qx)\,\langle 0|\tilde{J}(x)\tilde{J}^\dagger(0)|0\rangle\,. \tag{11}$$

to two-loop accuracy. We calculate the correlation function (11) using the technique proposed in ref. [11] in order to evaluate the necessary two-loop integrals. The calculation turns out to be far less tedious than in the full theory. The zeroth order contribution to the correlation function (11) (i.e., the bare quark loop) contains terms $\propto\omega^2\ln\omega/\mu$ which makes the renormalization group analysis of $\hat{\Pi}$ rather cumbersome (cf. footnote on page 411 in ref. [1]). To make the renormalization group improvement simpler we consider the third derivative of $\hat{\Pi}(\omega)$. This is sufficient since later on we shall Borel improve $\hat{\Pi}(\omega)$. The result is

$$\frac{\mathrm{d}^3\hat{\Pi}(\omega)}{\mathrm{d}\omega^3}=-\frac{3}{\omega\pi^2}\,\alpha_s(-2\omega)^{-\gamma_0/\beta_0}\left(1+\frac{\alpha_s(-2\omega)}{\pi}\,(\tfrac{17}{3}+\tfrac{3}{4}\gamma_0+\tfrac{4}{9}\pi^2-2\Gamma)\right)$$
$$-\frac{3}{\omega^4}\,\mathcal{O}_3\left[1+\frac{\alpha_s(-2\omega)}{\pi}\left(2-\frac{\Delta\gamma_1}{8\beta_0}\right)\right]+\frac{15}{4\omega^6}\,\alpha_s(-2\omega)^{(\gamma^{(5)}-2\gamma_0)/2\beta_0}\,\mathcal{O}_5\,, \tag{12}$$

where we have introduced the scale-invariant condensates

$$\mathcal{O}_3=\langle\bar{q}q\rangle(\mu)\,\alpha_s(\mu)^{-\gamma_0^{(3)}/2\beta_0}\left[1-\frac{\alpha_s(\mu)}{4\pi}\,\frac{\gamma_0^{(3)}}{2\beta_0}\left(\frac{\gamma_1^{(3)}}{\gamma_0^{(3)}}-\frac{\beta_1}{\beta_0}\right)\right],$$
$$\mathcal{O}_5=\langle\bar{q}\sigma gGq\rangle(\mu)\,\alpha_s(\mu)^{-\gamma_0^{(5)}/2\beta_0}[1+\mathrm{O}(\alpha_s)]\,. \tag{13}$$

The leading-order anomalous dimensions are $\gamma_0^{(3)}=2\gamma_0=-8$, $\gamma_0^{(5)}=-\frac{4}{3}$, and $\Delta\gamma_1$ is the difference between the two-loop anomalous dimensions of the effective operator $\tilde{J}$ and the quark condensate: $\Delta\gamma_1=2\gamma_1-\gamma_1^{(3)}=\frac{704}{9}-\frac{112}{27}\pi^2$ [11].

Throughout this paper we use the two-loop expression for α_s with $\Lambda_{\overline{\mathrm{MS}}}^{(4)}=200$ MeV, so that $\alpha_s(1\text{ GeV})\simeq 0.34$, $\alpha_s(m_B)\simeq 0.18$. The condensates are taken to be $\langle\bar{q}q\rangle(\mu=1\text{ GeV})=(-240\text{ MeV})^3$, $\langle\bar{q}\sigma gGq\rangle(\mu=1\text{ GeV})=0.8\text{ GeV}^2\times\langle\bar{q}q\rangle(\mu=1\text{ GeV})$. We have not calculated the $\mathrm{O}(\alpha_s)$ correction to the mixed condensate contribution because the latter has little effect on the sum rules. Note that there is no contribution of the gluon condensate in the HQL. We have not shown the contribution of the four-quark condensate, which turns out to be completely negligible.

The large radiative correction to the quark loop in (12) is mainly due (~80%) to one-gluon exchange between heavy and light quarks (in Feynman gauge) and is likely to be the effect of the classical Coulomb interaction [#3]. Putting all the numbers together this correction amounts approximately to $1+7\alpha_s/\pi$ (note that α_s has to be taken at the typical hadron scale of 1 GeV). Since the correction is nearly as large as the leading contribution, one may fear an explosion of the perturbative series. In order to get some intuition and estimate semi-quantitatively the possible size of coulombic effects we have investigated the Coulomb corrections in a nonrelativistic potential model for the heavy–light quark system. We solve numerically the Schrödinger equation for the system of a light and a heavy constituent quark and calculate the decay constant f_P (which is proportional to the wave function at the origin) in three different ways: using the potential $V(r)=\lambda r-4\alpha_s/3r$ we evaluate the full decay constant f_P, the decay constant expanded to first order in α_s, $f_P^{(1)}$, and the constant without coulombic correction, $f_P^{(0)}$.

For a reduced mass of 400 MeV, a linear potential with slope $\lambda=0.2\text{ GeV}^2$ and a Coulomb potential with $\alpha_s=0.3$ the calculation yields $f_P^1/f_P^0=1.34$ and $f_P/f_P^0=1.38$. We notice that the first order Coulomb correction to $(f_P^0)^2$ is large and of similar size as the radiative correction in the correlation function (12) which supports the potential model. Furthermore, the first order contribution yields already a very good approximation to the

[#3] The importance of the coulombic corrections has also been stressed in ref. [12] in the framework of the stochastic vacuum model.

exact result [#4]. Thus this model calculation gives some us confidence that the radiative corrections in (12) are under control, as uncomfortably large as they may seem at first sight.

3. We proceed with the usual QCD sum rule technique [1] and match the operator product expansion in (12) to the dispersion integral over hadron states saturated by the lowest lying level and the continuum. As usual, we model the continuum by the perturbative expression above some threshold to get

$$\Pi = \frac{\hat{f}^2}{2(\Delta m-\omega)-\mathrm{i}\epsilon} + \frac{1}{\pi}\int_{\Delta s}^{\infty} \mathrm{d}\omega'\ \mathrm{Im}\,\Pi^{\mathrm{pert}}(\omega')\frac{1}{\omega'-\omega}, \tag{14}$$

where Δm is the difference between meson and quark mass in the HQL. Applying the Borel improvement, e.g., $(\omega'-\omega)^{-1}\to\Delta M^{-1}\exp(-\omega'/\Delta M)$, we end up with the sum rule

$$\begin{aligned}&\hat{f}^2\exp\left(-\frac{\Delta m}{\Delta M}\right)\\ &=\frac{3}{\pi^2}\alpha_s(2\Delta M)^{-\gamma_0/\beta_0}\left\{2(\Delta M)^3\left(1+\frac{\alpha_s(2\Delta M)}{\pi}\left[\tfrac{17}{3}+\tfrac{4}{9}\pi^2+\tfrac{1}{2}\gamma_0\psi(3)-2\Gamma\right]\right)\right.\\ &\left.-\int_{\Delta s}^{\infty}\mathrm{d}s\,s^2\exp\left(-\frac{s}{\Delta M}\right)\left[1+\frac{\alpha_s(2\Delta M)}{\pi}\left(\tfrac{17}{3}+\tfrac{4}{9}\pi^2-2\Gamma+\tfrac{1}{2}\gamma_0\ln\frac{s}{\Delta M}\right)\right]\right\}\\ &-\left[1+\frac{\alpha_s(2\Delta M)}{\pi}\left(2-\frac{\Delta\gamma_1}{8\beta_0}\right)\right]\mathcal{O}_3+\frac{\mathcal{O}_5}{16(\Delta M)^2}\alpha_s(2\Delta M)^{(\gamma^{(5)}-2\gamma_0)/2\beta_0}\\ &=:\widehat{\mathrm{SR}}(\Delta M,\Delta s)\,.\end{aligned} \tag{15}$$

Here $\psi(3)=\frac{3}{2}-\gamma_E$ is the logarithmic derivative of the gamma function, it comes from the Borel improvement of the running coupling constant [14]. In the continuum contribution we have taken into account the imaginary part of the running coupling $\alpha_s(-2\omega)$; the term $\propto\ln s/\Delta M$ cones from the expansion of $[\alpha_s(2s)/\alpha_s(2\Delta M)]^{-\gamma_0/\beta_0}$ to first order.

In order to estimate the value of Δm we construct another sum rule which is an immediate consequence of (15):

$$\Delta m=(\Delta M)^2\frac{(\mathrm{d}/\mathrm{d}\Delta M)\widehat{\mathrm{SR}}(\Delta M,\Delta s)}{\widehat{\mathrm{SR}}(\Delta M,\Delta s)}\,. \tag{16}$$

In fig. 1a we display the results of that sum rule as a function of the Borel parameter ΔM for different values of the threshold Δs. Apparently values of $\Delta m\approx0.4$ GeV are somewhat favoured, but also $\Delta m\approx0.6$ GeV shows acceptable stability in ΔM. It is tempting to assume that the mass difference m_P-m_Q is not changed much in going from the B-meson to higher masses, so we could insert $\Delta m\simeq m_B-m_b$ to improve our accuracy. However, the two existing analyses of mesons of the ϒ-family by Voloshin [15] and Reinders [16] are contradictory and yield values of the pole mass of the b-quark differing by 200–250 MeV ($m_b=4.8$ and 4.55 GeV with small errors,

[#4] The situation is completely different for a system composed of two heavy quarks: in that case the zeroth order, i.e., the result in the linear potential, is of no significance, since the system is essentially determined by the Coulomb potential and the perturbative expansion made above is senseless. Indeed, it is known that the Coulomb effects in heavy quarkonia should be taken into account exactly [13].

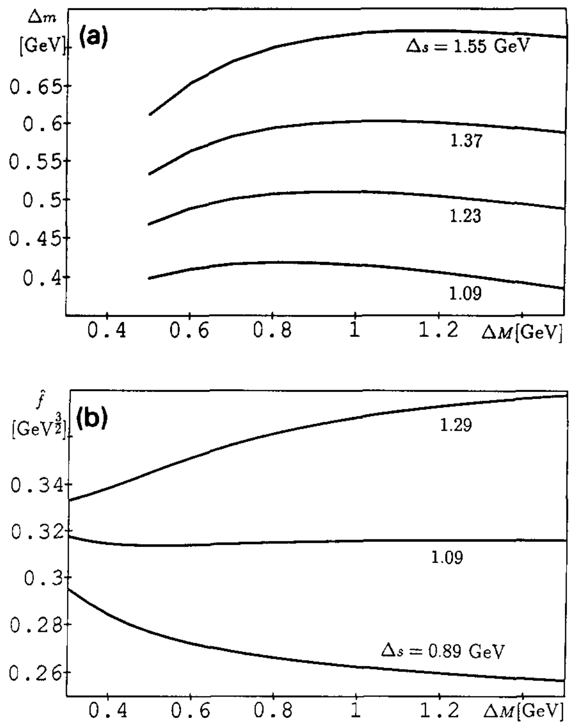

Fig. 1. (a) The sum rule (16) for the mass difference $\Delta m = m_P - m_Q$ as function of the Borel parameter ΔM with continuum thresholds $\Delta s = 1.09$, 1.23, 1.37 and 1.55 GeV, respectively. (b) The sum rule (15) for the scale-invariant decay constant $\hat{f}$ (see (10)) as function of the Borel parameter ΔM for $\Delta m = 0.4$ GeV and $\Delta s = 0.89$, 1.09 and 1.29 GeV, respectively.

respectively), which is just the range of our uncertainty. Thus in the following we use $\Delta m = 0.4$–0.6 GeV as input value for the sum rule for $\hat{f}$, eq. (15). in fig. 1b we show this sum rule for $\Delta m = 0.4$ GeV and different values of the continuum threshold Δs. By requiring maximum stability, we fix Δs to be $\Delta s = 1.09$ GeV which yields $\hat{f} = 0.32$ GeV $^{3/2}$. Applying the same procedure with $\Delta m = 0.6$ GeV, we find $\hat{f} = 0.43$ GeV $^{3/2}$ for $\Delta s = 1.37$ GeV.

From this we get the values of $\tilde{f}_{\mathrm{stat}}$ (5.28 GeV) and f_B^{HQL} by means of eqs. (5) and (8), respectively (in our normalization $f_\pi = 133$ MeV):

$$\Delta m = 0.4\ \mathrm{GeV}: \tilde{f}_{\mathrm{stat}} = 0.47\ \mathrm{GeV}^{3/2}, f_B^{\mathrm{HQL}} = 195\ \mathrm{MeV}\,,$$
$$\Delta m = 0.6\ \mathrm{GeV}: \tilde{f}_{\mathrm{stat}} = 0.64\ \mathrm{GeV}^{3/2}, f_B^{\mathrm{HQL}} = 265\ \mathrm{MeV}\,. \qquad (17)$$

For these values we expect an accuracy of about 10%. Our value for $\tilde{f}_{\mathrm{stat}}$ (5.28 GeV) agrees well with the result of lattice calculations, $\tilde{f}^{\,\mathrm{latt}}_{\mathrm{stat}} = 0.57$ GeV $^{3/2}$, quoted in ref. [6].

4. The values of f_B^{HQL} obtained above from the asymptotic expression do not include power $1/m_Q$ corrections, which we estimate by applying the renormalization group improvement to the sum rule for finite quark masses. To this end we consider the correlation function of two pseudoscalar currents $J_5 = \bar{q}i\gamma_5 Q$:

$$\Pi_5(q^2) = \mathrm{i} \int \mathrm{d}^4x \exp(\mathrm{i}qx)\ \langle 0 | m_{\overline{\mathrm{MS}}} J_5(x) m_{\overline{\mathrm{MS}}} J_5^\dagger(0) | 0 \rangle\ . \qquad (18)$$

The perturbative contribution to (18) is known to two-loop accuracy [17], and we have calculated in addition the $O(\alpha_s)$-correction to the Wilson coefficient of the quark condensate. Retaining the leading contributions in the limit $m_Q \to \infty$ we obtain [#5] with the substitution $q^2 - m_Q^2 \to 2m_Q\omega$:

$$\left(\frac{d}{d\omega}\right)^3 \Pi_5(\omega)=\left[-\frac{3m_Q^2}{\omega\pi^2}\left(1+2\frac{\alpha_s}{\pi}\ln\frac{m_Q}{-2\omega}+\frac{4}{3}\frac{\alpha_s}{\pi}(\tfrac{1}{3}\pi^2+1)\right)-\frac{3m_Q^2}{\omega^4}\langle\bar{q}q\rangle_{(\mu=m_Q)}\left(1+\frac{2}{3}\frac{\alpha_s}{\pi}\right)+...\right]$$
$$\times[1+O(1/m_Q)]\,. \tag{19}$$

This expression should be compared to the correlation function of two effective currents $\tilde{J}$ at the normalization point $\mu=m_Q$, times the coefficient function $C(1, \alpha(m_Q))$ squared. Combining eqs. (5), (10), (12), we obtain

$$C^2(1,\alpha(m_Q))\frac{d^3}{d\omega^3}\,i\int d^4x\,\exp(iqx)\,\langle 0|\tilde{J}(x)\tilde{J}^\dagger(0)|0\rangle_{(\mu=m_Q)}$$
$$=-\frac{3}{\omega\pi^2}\left(\frac{\alpha_s(m_Q)}{\alpha_s(-2\omega)}\right)^{\gamma_0/\beta_0}\left(1+\frac{4}{3}\frac{\alpha_s(-2\omega)}{\pi}(\tfrac{1}{3}\pi^2+1)+\frac{\alpha_s(-2\omega)-\alpha_s(m_Q)}{\pi}(\tfrac{4}{3}-2\Gamma)\right)$$
$$-\frac{3}{\omega^4}\langle\bar{q}q\rangle_{(\mu=-2\omega)}\left(\frac{\alpha_s(m_Q)}{\alpha_s(-2\omega)}\right)^{\gamma_0/\beta_0}\left(1+\frac{2}{3}\frac{\alpha_s(-2\omega)}{\pi}+\frac{\alpha_s(-2\omega)-\alpha_s(m_Q)}{\pi}(\tfrac{4}{3}-2\Gamma)\right). \tag{20}$$

Eqs. (19) and (20) indeed coincide up to the overall normalization factor m_Q^2 to the expected accuracy $O(\alpha_s)$. This gives an independent check of the expression for the coefficient function $C(1, \alpha(m_Q))$ in (5).

Now we are in a position to write the sum rule for the decay constant including both the renormalization group improved contributions of "leading twist" and the finite mass corrections. To this end we make use of the standard technique for factorizing out the leading behaviour of amplitudes, familiar in the studies of hard processes in QCD [18]. We subtract (19) from the third derivative of the correlation function (18) which is available from ref. [19]. The remainder forms a "higher twist" contribution which is suppressed by a power of the heavy quark mass and gives rise after Borel improvement to a finite mass correction to the decay constant. On the other hand, we use the renormalization group improved expression in (20) for the leading twist part. This procedure yields the sum rule

$$f_P^2 m_P^4 \exp[-(m_P^2-m_Q^2)/M^2]=\mathrm{SR}^{\mathrm{h.t.}}(M^2,s_0,m_Q)$$
$$+m_Q^3\alpha_s(m_Q)^{\gamma_0/\beta_0}\left(1-\frac{\alpha_s(m_Q)}{\pi}(\tfrac{4}{3}-2\Gamma)\right)\widehat{\mathrm{SR}}\left(\frac{M^2}{2m_Q},\frac{s_0-m_Q^2}{2m_Q}\right), \tag{21}$$

where $\widehat{\mathrm{SR}}(\Delta M, \Delta s)$ is given by (15), and $\mathrm{SR}^{\mathrm{h.t.}}$ is the result of subtracting the leading twist terms from the full expression given in ref. [19]:

$$\mathrm{SR}^{\mathrm{h.t.}}(M^2,s_0,m_Q)$$
$$=\frac{3}{8\pi^2}\int_{m_Q^2}^{s_0} ds\,\frac{(s-m_Q^2)^3}{s}\exp[-(s-m_Q^2)/M^2]\left\{-1+\frac{4}{3}\frac{\alpha_s}{\pi}\left[\ln\frac{s-m_Q^2}{m_Q^2}\left(\frac{3}{2}-\frac{m_Q^2}{s}\right)-\frac{s}{s-m_Q^2}\right.\right.$$
$$\left.\left.+\frac{m_Q^2}{s-m_Q^2}\ln\frac{s}{m_Q^2}\left(\frac{m_Q^2}{s-m_Q^2}+1+\ln\frac{s}{s-m_Q^2}\right)+2\frac{m_Q^2}{s-m_Q^2}\,\mathrm{Li}\left(\frac{m_Q^2}{s}\right)-\frac{\pi^2}{3}\frac{s}{s-m_Q^2}-\frac{9}{4}\right]\right\}$$
$$+2m_Q^3\frac{\alpha_s}{\pi}\langle\bar{q}q\rangle_{(\mu=m_Q)}\exp(m_Q^2/M^2)\,\mathrm{Ei}\left(-\frac{m_Q^2}{M^2}\right)+\frac{m_Q^2}{12}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle-\frac{m_Q^3}{2M^2}\langle\bar{q}\sigma gGq\rangle_{(\mu=m_Q)}\,, \tag{22}$$

where we neglect contributions of four-quark condensates due to their smallness.

The main result of the summation of leading and next-to-leading logarithms in the heavy quark mass turns

[5] Note that by taking the third derivative one eliminates contributions to the correlation function (18) coming from large internal momenta of the order of the heavy quark mass, which are not present in the correlation function of effective currents and should rather be taken into account separately as contributions of vacuum expectation values of local effective operators.

out to be that the strong coupling constant in the leading twist contribution to (21) must be evaluated at the hadronic scale ~ 1 GeV, rather than at the scale of the quark mass. For definiteness, in the finite mass corrections (22) we have taken α_s at the scale of the Borel parameter M^2 divided by the mass of the heavy quark as in the leading twist terms.

The result for the decay constant of the B-meson are

$$f_B(\Delta m=0.48\ \text{GeV},\ m_b=4.8\ \text{GeV})=195\ \text{MeV}\ (s_0=36\ \text{GeV}^2)\ ,$$
$$f_B(\Delta m=0.68\ \text{GeV},\ m_b=4.6\ \text{GeV})=245\ \text{MeV}\ (s_0=38\ \text{GeV}^2)\ . \tag{23}$$

Our result for f_B in (23) is significantly larger than the value obtained in ref. [19] using the same quark mass $m_b=4.8$ GeV and with the value of α_s taken presumably at the scale of the heavy quark mass. We have shown that α_s must be taken at the hadronic scale ~ 1 GeV. This change of scale results in an increase of f_B, first directly owing to the larger radiative correction, and second because the continuum threshold is pushed to higher values. The increase of s_0 with the rise of radiative corrections is expected, since the Coulomb interaction enhances orbital level splitting. Our value for f_B with $m_b=4.6$ GeV lies within the range of values given in ref. [4].

The difference between the values of f_B given in (17) and (23) is the effect of power $1/m_Q$ corrections. To visualize this explicitly, we have calculated the values of the decay constant from the sum rule (21) at different values of the quark mass under the assumption that the values of $\Delta m=m_P-m_Q$ and $\Delta s=\sqrt{s_0}-m_Q$ stay constant: $\Delta s(\Delta m=0.5\ \text{GeV})=1.23$ GeV, $\Delta s(\Delta m=0.7\ \text{GeV})=1.55$ GeV. The Borel parameter is taken to be $M^2=m_Q\times 1.5$ GeV which is in the expected stability range. In fig. 2 we plot the decay constant, multiplied by the scaling factor

$$\hat{f}(m_P):=\alpha_s(m_P)^{6/25}\left(1+\frac{\alpha_s(m_P)}{\pi}\left(\tfrac{2}{3}-\Gamma\right)\right)\sqrt{m_P}\,f_P \tag{24}$$

as a function of the inverse meson mass. The points show the calculated values (at $\Delta m=0.5$ and 0.7 GeV). The curves present the fit with a quadratic polynomial in $1/m_P$. Actually the contribution of the quadratic term constitutes less than 7% at the scale of the D-meson and the curves are nearly linear:

$$\hat{f}(m_P)=\hat{f}\left(1-\frac{(0.8\text{–}1.1)\ \text{GeV}}{m_P}\right), \tag{25}$$

where the smaller slope corresponds to $\Delta m=0.5$ GeV and the larger one to $\Delta m=0.7$ GeV, respectively. Note that for meson masses around 1.5–2.0 GeV the sum rule becomes insensitive to the input value of Δm.

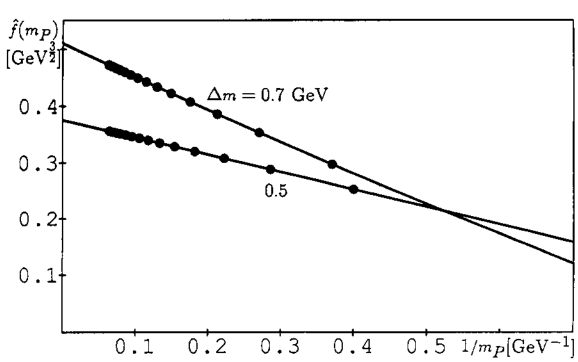

Fig. 2. The scaled decay constant $\hat{f}(m_P)$ [see (24)] as function of the inverse mass of the pseudoscalar meson for $\Delta m=0.5$ and 0.7 GeV, respectively.

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