Physics Letters B 273 (1991) 316–318
North-Holland

PHYSICS LETTERS B

On vector dominance in the decay $B \to \pi e \bar{\nu}$

Patricia Ball, V.M. Braun [1,2] and H.G. Dosch
Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, W-6900 Heidelberg, FRG

Received 8 August 1991

QCD sum rules strongly support the B^* dominance in the form factor of the semi-leptonic decay $B \to \pi e \nu$ in the whole region of the momentum transfer $0 \leqslant t \leqslant (m_B - m_\pi)^2$.

The semileptonic decay $\bar{B}^0 \to \pi^+ e \bar{\nu}$ can provide important information on the element V_{ub} of the Kobayashi–Maskawa matrix. In order to extract this element from the decay rate the relevant hadronic matrix element of the weak current has to be determined theoretically. Namely, one needs to know the form factor $f_+(t)$ defined as

$$\langle \pi^+(p_\pi) | \bar{u} \gamma_\mu b | \bar{B}^0(p_B) \rangle = (p_\pi + p_B)_\mu f_+(t) + (p_B - p_\pi)_\mu f_-(t) , \quad t = (p_B - p_\pi)^2 .$$

The second form factor $f_-(t)$ does not contribute to the T-matrix element in the limit of vanishing lepton mass and therefore can be neglected.

The value of the form factor $f_+(0)$ at zero momentum transfer has been calculated in the framework of the quark model [1] and QCD sum rules [2–4] [#1]. On the other hand, at the largest possible momentum transfers $t \sim t_{max} = (m_B - m_\pi)^2$ the form factor is generally believed to be dominated by the nearby B^*-pole and independent calculations of the $B^* B \pi$ coupling in refs. [2,4] do not contradict a pole-type formula for $f_+(t)$ in the whole kinematical range. However the accuracy of this approximation is not known. The pole-type behaviour of $f_+(t)$ has been questioned by Isgur and Wise [5], who suggest that the finite extension of the B-meson (and π) sets a scale for the variation of the form factor different from the mass of the B^*-meson. We remind that owing to a very large allowed kinematical range the t-dependence of the form factor in this decay is strong and practically very important.

In this letter we report on a direct calculation of the t-dependence of the form factor $f_+(t)$ for the decay $B \to \pi e \nu$ in the framework of QCD sum rules. Our results turn out to be in remarkable agreement with the pole behaviour

$$f_+(t) = \frac{f_+(0)}{1 - t/m_{B^*}^2} \tag{1}$$

in the wide range $0 < t < 20\ \mathrm{GeV}^2$ and the normalization does not contradict the B^*-dominance relation $f_+(0) = f_{B^*} g_{B^* B \pi} / m_{B^*}$.

In our analysis we start from the three-point function

$$\begin{aligned} T_{\mu\nu}(p_\pi, p_B) &= i^2 \int d^4x\, d^4y \exp(i p_\pi x - i p_B y) \\ &\quad \times \langle 0 | T\{A_\nu(x) V_\mu(0) B^\dagger(y)\} | 0 \rangle \\ &= i g_{\mu\nu} T_0 + i(p_B + p_\pi)_\mu p_{\pi\nu} T_+ + i(p_B - p_\pi)_\mu p_{\pi\nu} T_- \\ &\quad + \ldots , \end{aligned} \tag{2}$$

where $A_\nu = \bar{d} \gamma_\nu \gamma_5 u$, $B = \bar{d} i \gamma_5 b$ are the interpolating fields for π^+ and the B-meson, respectively, and $V_\mu = \bar{u} \gamma_\mu b$ is the weak vector current.

The contribution of the lowest states in the p_π^2 and p_B^2 channels isolates the matrix element of interest:

[1] Alexander von Humboldt Fellow.

[2] On leave of absence from Leningrad Nuclear Physics Institute, SU-188 350 Gatchina, USSR.

[#1] The value given by ref. [2] suffers from a missing factor 12 in the perturbative contribution which caused the author to neglect this contribution at all due to its smallness.

$$T_{\mu\nu}(p_D, p_K) = \frac{1}{p_B^2 - m_B^2 + i\epsilon} \frac{1}{p_\pi^2 - m_\pi^2 + i\epsilon}$$
$$\times \langle 0|\bar{d}\gamma_\nu\gamma_5 u|\pi\rangle \langle\pi|V_\mu|B\rangle \langle B|\bar{b}i\gamma_5 d|0\rangle$$
$$+\text{higher resonances}\,. \qquad (3)$$

In ref. [6] we have examined the analytic structure of the relevant dispersion integrals and have shown that one can apply the QCD sum rule technique in a large part of the kinematically allowed t-range. We evaluate the three-point function (2) by means of the operator product expansion taking into account the lowest order perturbative contribution (triangle diagram), the quark condensate, and the mixed quark–gluon condensate. The resulting expressions are Borel improved in both the variables p_B^2 and p_π^2 and matched with the expansion in hadron states. This leads to the following expression for the form factor $f_+(t)$ of the $\bar{B}^0 \to \pi^+ e\bar{\nu}$ decay:

$$f_+(t) = \frac{m_b}{f_B f_\pi m_B^2} \times \exp\left(\frac{m_B^2}{M_B^2} + \frac{m_\pi^2}{M_\pi^2}\right) T_+(M_B^2, M_\pi^2, t)\,, \qquad (4)$$

where M_B^2 and M_π^2 are the Borel parameters in the p_B^2 and p_π^2 channel, respectively and $T_+(M_B^2, M_\pi^2, t)$ is the Borel transformed expression for the operator product expansion result for the invariant amplitude T_+. The coupling f_B is defined in the usual way as $\langle 0|\bar{b}i\gamma_5 d|\bar{B}^0\rangle = f_B m_B^2/m_b$. The operator coefficients occurring in T_+ are the same as for the decay $D \to Ke\nu$ with the obvious subtitutions $c \to b$ and $s \to u$ [6]. Details of the handling of the double dispersion integral and the derivation of the final sum rule (4) can be found in ref. [6].

As input for the quark masses and condensates we use:

$m_b = 4.7$ GeV (scale independent pole mass),

$\langle\bar{q}q\rangle(1\ \text{GeV}) = -(0.23\ \text{GeV})^3$,

$g\langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}\cdot\frac{1}{2}\lambda q\rangle(1\ \text{GeV}) = m_0^2\langle\bar{q}q\rangle(1\ \text{GeV})$ with $m_0^2 = 0.8\ \text{GeV}^2$.

Higher resonances and the continuum are approximated by the perturbative contribution above the thresholds s_B^0 and s_π^0, respectively. We use the values

$s_B^0 = 35\ \text{GeV}^2$, $s_\pi^0 = 0.75\ \text{GeV}^2$.

These values are adjusted in such a way to get stability in the corresponding two-point functions for f_B and f_π (of course we use the experimental value $f_\pi = 133$ MeV in our sum rules). Following the procedure adopted in our previous paper [6], we use values of the Borel parameters twice as large as the ones following from the corresponding two-point sum rules. We evaluate our sum rule at a fixed value of the ratio of the two Borel parameters $M_B^2/M_\pi^2 \approx 4$. Varying this ratio from 3 to 5 changes our results by at most 3%. The value of f_B in (4) is taken from the two-point sum rule for the B-current without radiative corrections. We take the condensates normalized at 1 GeV. In this way the large logarithms of the quark mass [7,8] in the numerator of (4) cancel those in the constant f_B in the denominator. Cancellations of various uncertainties also occur in this ratio and make the results for the form factor $f_+(t)$ much more stable against a change in the input parameters than those in the two-point sum rule for f_B. Changing the pole mass of the b-quark from 4.6 to 4.8 GeV and allowing the continuum threshold s_B^0 to vary in the wide range 30–50 GeV^2 both changes $f_+(0)$ by at most 10% (cf. fig. 1) and practically does not affect the t-dependence. In fig. 2 we give the t-dependence of f_+ in the range $0 \leqslant t \leqslant 20.5\ \text{GeV}^2$, where the upper limit is set by the requirement of being not very sensitive to the model of the continuum. The displayed curves follow remarkably well a pole fit with a pole mass of 5.20 ± 0.05 GeV, which is only 2% below the observed B*-mass. Applying the same method to the decay $D \to \pi e\nu$ and using $m_c = 1.3$–1.4 GeV, we again find the form factor to be in perfect agreement with

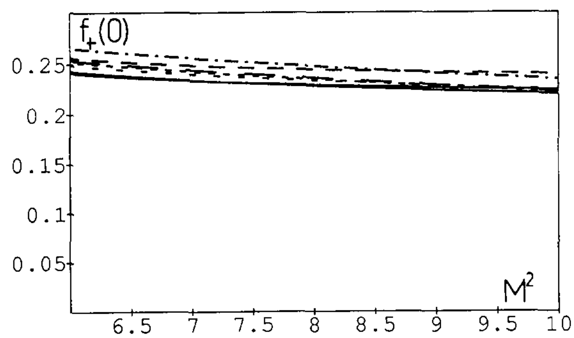

Fig. 1. The form factor f_+ at zero momentum transfer in dependence on the Borel parameter M_B^2. The variety of curves reflects the effect of variations of parameters as explained in the text.

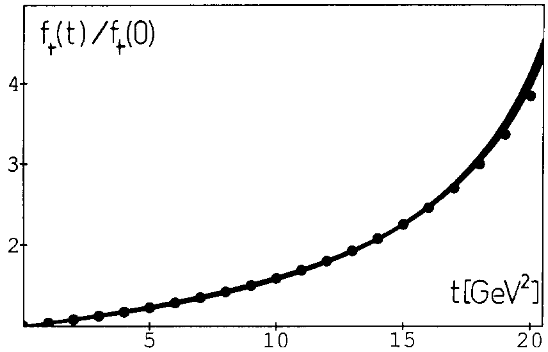

Fig. 2. The t-dependence of the form factor $f_+(t)$. The thickness of the solid curve reflects the effect of variations of the Borel parameter within the interval $6 \leqslant M_B^2 \leqslant 10$ GeV^2. The pole formula fit with $m_{pol} = m_{B^*} = 5.2$ GeV is shown by dots.

a pole fit with pole mass $m_{pol} = 2.00 \pm 0.05$ GeV, which has to be compared with the observed mass of D*, $m_{D^*} = 2.01$ GeV. These results support the observation made in our previous paper [6], that vector form factors, unlike axial vector ones, seem to follow the pole dominance quite well.

Our results strongly suggest the pole dominance approximation over the full kinematical range (note that we just investigate the region away from the pole). The pole-type behaviour and the correct mass scale arise in our approach from a nontrivial interplay of the perturbative contribution and nonperturbative corrections. The perturbative triangle diagram taken alone would yield the cut in the complex t-plane starting at the much smaller mass m_b^2. The contribution of the quark condensate varies much less with t than the perturbative one, and its relative contribution decreases from nearly 60% at $t = 0$ to $\sim 15\%$ at $t = 20$ GeV (the mixed condensate constitutes less than 5%). Thus, the nonperturbative contribution slows down the rise of the form factor and increases the relevant mass scale by $m_{B^*} - m_b \simeq 0.5$ GeV.

Our result is not in contradiction to a bound state picture of hadrons built of *constituent* quarks with masses $m_b^{const} \approx 5$ GeV, $m_q^{const} \approx 0.3$ GeV. In the quark model one would expect the heavy-to-light quark transition form factor $\langle \pi | V_\mu | B \rangle$ considered here to be dominated by the anomalous threshold on the physical sheet at $\sqrt{t} = 5.298$ GeV, which is only slightly below the normal threshold. The situation is drastically different for heavy-to-heavy quark transitions, say $b \to c$, in which case the position of the anomalous threshold may lie considerably lower than the normal threshold, and the vector dominance approximation may be ineffective [9] [#2].

Concerning the absolute value of the form factor, we get

$$f_+(0) = 0.24 \pm 0.025 , \quad (5)$$

yielding a decay rate [#3]

$$\Gamma(\bar{B}^0 \to \pi^+ e \bar{\nu}) = 2\Gamma(B^- \to \pi^0 e \bar{\nu})$$
$$= |V_{ub}|^2 \cdot (4.3 \pm 0.9) \times 10^{12}\ \mathrm{s}^{-1} . \quad (6)$$

These values are somewhat smaller than the quark model prediction [1] which feature has already been observed in semileptonic D-decays, where experiments seem to favour the smaller values obtained by sum rules. The value in (5) is some (20–30)% lower compared also to results [3,4] but this is no contradiction within a conservative estimate of the accuracy of the sum rule approach. The higher value obtained in ref. [3] can well be due to neglecting higher twist effects which are not small according to our estimates. We have carried out the calculation of the form factor $f_+(t)$ using also a more sophisticated modification of sum rules (on the light-cone) and including the nonleading twist-4 effects. The results support the simpler analysis given here. Full details will be presented in a forthcoming publication.

[#2] For the particular case of the vector form factor of the Υ meson considered in ref. [9] one obtains with the value of $m_b^{const} \approx 5$ GeV an anomalous threshold at $\sqrt{t} = 6.13$ GeV considerably below the normal threshold $\sqrt{t} = 10$ GeV.

[#3] On account of the large experimental uncertainty in the lifetime of $\bar{B}^0$ we restrict ourselves to giving the absolute rates only.

References

[1] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 29 (1985) 637.
[2] A.A. Ovchinnikov, Sov. J. Nucl. Phys. 50 (1989) 519.
[3] V.L. Chernyak and I.R. Zhitnitsky, Nucl. Phys. B 345 (1990) 137.
[4] C.A. Dominguez and N. Paver, Z. Phys. C 41 (1988) 217.
[5] N. Isgur and M.B. Wise, Phys. Rev. D 41 (1990) 151.
[6] P. Ball, V.M. Braun and H.G. Dosch, Heidelberg preprint HD-THEP-91-16, submitted to Phys. Rev. D.
[7] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 45 (1987) 292.
[8] H.D. Politzer and M.B. Wise, Phys. Lett. B 206 (1988) 681.
[9] R.L. Jaffe, Phys. Lett. B 245 (1990) 221.