Form factors of semileptonic D decays from QCD sum rules

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We give a detailed QCD sum-rule analysis of the semileptonic decays $D^+ \to \bar{K}^0 e^+ \nu_e$ and $D^+ \to \bar{K}^{0*} e^+ \nu_e$. We use the standard method based on expansion in Euclidean distance as well as a new set of sum rules on the light cone. We compare the results with current experimental data and results from quark models. Special attention is paid to the t dependence of the form factors. We find from our sum-rule analysis that for the vector form factors the usual pole model approximation works perfectly well, but is not adequate for the axial-vector form factors.

I. INTRODUCTION

Weak semileptonic decays of charmed mesons have been attracting continuous interest in the last decade due to the relative simplicity of their theoretical description and due to the possibility of extracting mixing angles in the Kobayashi-Maskawa matrix. While the experimental status of $D \to Ke\nu$ decay seems to be settled [1,2], the situation with the decay $D \to K^* e\nu$ remains uncertain [3,4].

Quark models have been generally regarded as giving a successful description of semileptonic decays of heavy quarks and were natural candidates for use in calculating decay rates. These calculations seem to be without problems in decays such as $D \to Ke\nu$ and with polarization in $B \to D^* e\nu$. However, the E691 data [4] for the branching ratio and polarization in the decay $D^+ \to \bar{K}^{*0} e^+ \nu_e$ disagree considerably with quark model predictions [5–7]. By relaxing some of the relations intrinsic to quark models, it turns out to be possible to remove the discrepancy [8,9], but the predictive power of the method then diminishes considerably.

QCD sum rules [10] provide an independent approach for calculating hadronic matrix elements and form factors for systems of both light and heavy quarks. Though sum rules yield generally less detailed results than quark models, their underlying physical assumptions are more directly related to the field theory. In view of an uncertain experimental situation, an independent calculation of decay amplitudes by the sum-rule method seems therefore to be necessary and timely. In this paper we give a detailed analysis of weak semileptonic decays of charmed mesons within the framework of QCD sum rules and put special emphasis on the t dependence of form factors.

The development of the sum-rule technique for three-point functions was initiated by a successful description of the pion electromagnetic form factor at intermediate momentum transfer $t \sim -(0.5-1.5)$ GeV^2 [11,12]. The first calculation for weak decays [13] was done for the value $f_+(0)$ of the form factor $D^+ \to \bar{K}^0 e^+ \nu_e$ in the chiral limit. Later, a similar approach was used for calculations of weak decays of beauty mesons $B \to De\nu$ [14,15], $B \to D^* e\nu$ [14], and $B \to \pi e\nu$ [16]. Until now there has been no detailed analysis of sum rules for the $D \to K^* e\nu$ decay except for an unpublished paper [17] and our Letter [18], reporting first results of the analysis given here.

A new aspect of this paper is the analysis of the sum rules in the physical region for positive values of the momentum squared $t = (p_D - p_M)^2$ transferred to the lepton pair. Although the cut in the t channel starts at $t \sim m_c^2$, and thus, the Euclidean region stretches up to that threshold, existing calculations have avoided this region for the following reasons: first, it has been believed that calculations at $t = 0$ are more reliable since it is deep in the Euclidean region; second, the authors of [15] have claimed the presence of nonintegrable singularities in the double dispersion relation for the three-point function describing the semileptonic decay at $t > 0$. This difficulty has been attributed there to the breakdown of the short-distance expansion. Such an abrupt breakdown would of course also jeopardize the results at $t = 0$. In our paper we show that both arguments are incorrect. The double dispersion relation remains perfectly well defined up to $t = t_{\text{th}} = (m_c + m_s)^2$, and the effect observed in [15] is due to a nonproper treatment of non-Landau-type singularities. This point is discussed in detail in Sec. III.

We also argue that there is no reason to expect better accuracy of sum rules at $t = 0$ than at $t \approx 1$ GeV^2, since it is the distance from the threshold $t_{\text{th}} \approx 2$ GeV^2 that counts. If $|t - t_{\text{th}}|$ is too small, then, indeed, contributions of large distances in the t channel come into the game [19]. The situation is, however, even worse when $|t - t_{\text{th}}|$ becomes too large, since in this case the power corrections which are proportional to powers of $|t - t_{\text{th}}|$ are no longer under control. This difficulty is an artifact of the expansion in local operators and is well known in connection with sum rules for the pion form factor [11,12]. We show in Sec. V that this difficulty is not present if nonlocal condensates are introduced. We compare the results obtained with nonlocal condensates to those obtained in the (local) operator-product expansion and discuss new contributions to the sum rules which are eliminated in the standard procedure by the Borel transformation.

The main results obtained are

$$B(D^0 \to K e \nu) = (2.7 \pm 0.6)\% ,$$

$$B(D^+ \to K^* e \nu) = (4.0 \pm 1.6)\% ,$$

$$\frac{\Gamma(D \to K^*)}{\Gamma(D \to K)} = 0.50 \pm 0.15 ,$$

$$\Gamma_L / \Gamma_T = 0.86 \pm 0.06 ,$$

$$\Gamma_+ / \Gamma_- = 0.09 \pm 0.02 .$$

The t dependence of the form factors corresponding to vector particle exchange in the t channel turns out to be in excellent agreement with the expected dominance of the lowest hadron state, but the axial-vector form factors show a much weaker t dependence than predicted by the pole model, as will be discussed in Sec. IV.

Finally, in Sec. VI we propose a new set of sum rules for the form factors in question, considering a two-point function and making use of existing information on wave functions of K and K^* mesons [20,21] to calculate the relevant meson-to-vacuum transition amplitudes. Although our poor knowledge of wave functions of higher twist limits the present accuracy of this approach, it has the advantage that one needs to extrapolate in one variable only from the Euclidean to the physical region instead of extrapolating in two variables as in the standard procedure. The results agree with those obtained from the three-point function within the expected accuracy.

II. THE STANDARD PROCEDURE

In order to evaluate the form factors for the semileptonic D decays we consider the three-point function

$$T_\mu = i^2 \int d^4x\, d^4y \langle 0 | T\{M(x) J_\mu(0) D(y)\} | 0 \rangle e^{ip_M x - ip_D y} , \quad (2.1)$$

where $J_\mu = \bar{s}\gamma_\mu(1-\gamma_5)c$ is the weak current, $D(y) = \bar{c}(y) i\gamma_5 d(y)$ and $M(x)$ are currents with quantum numbers of the initial (D^+) and final ($\bar{K}^0, \bar{K}^{0*}, \ldots$) mesons, respectively. Specifically we use vector and axial-vector currents as interpolating fields for the K^* and K meson, respectively. The three-point function are expressed through the invariant amplitudes as follows: for the current $M(x)$ interpolating the $\bar{K}^0$ meson we have

$$\Pi_{\mu\nu}(p_D, p_K) = i^2 \int d^4x\, d^4y \langle 0 | T\{\bar{d}(x)\gamma_\nu\gamma_5 s(x) J_\mu(0) D(y)\} | 0 \rangle e^{ip_K x - ip_D y}$$
$$= i g_{\mu\nu} \Pi_0 + i(p_D + p_K)_\mu p_{K\nu} \Pi_+ + i(p_D - p_K)_\mu p_{K\nu} \Pi_- + \cdots , \quad (2.2)$$

and for the current $M(x)$ interpolating the $\bar{K}^*$ meson the expansion reads

$$\Gamma_{\mu\nu}(p_D, p_K) = i^2 \int d^4x\, d^4y \langle 0 | T\{\bar{d}(x)\gamma_\nu s(x) J_\mu(0) D(y)\} | 0 \rangle e^{ip_K x - ip_D y}$$
$$= i g_{\mu\nu} \Gamma_0 - i(p_D + p_K)_\mu p_{D\nu} \Gamma_+ - i(p_D - p_K)_\mu p_{D\nu} \Gamma_- - \epsilon_{\mu\nu\rho\sigma} p_D^\rho p_K^\sigma \Gamma_V + \cdots . \quad (2.3)$$

In the following we evaluate the three-point function (2.1) at negative values of p_D^2 and p_M^2 by the operator-product expansion in QCD on one side, and on the other side by saturating the p_D^2 and p_M^2 channels by hadronic states. Both representations are matched using Borel improvement in p_D^2 and p_M^2 which suppresses higher resonance and continuum contributions as well as the higher-dimensional condensates. The unit operator in the operator-product expansion is given to lowest order in α_s by the triangle diagram of Fig. 1(a). Since we perform a Borel transformation both in p_D^2 and p_M^2, and since we model our higher resonance and continuum by the perturbative contribution above some thresholds (see below), the large integral should be written in the form of a double dispersion integral. The construction of the double spectral function for $t = (p_D - p_M)^2 > 0$ involves some delicate points and is discussed in detail in the next session.

The nonperturbative contributions included in our analysis are shown in Figs. 1(b)–1(j). We have not taken into account the contribution of the gluon condensate which is estimated to be negligible. The terms proportional to the condensate of strange quarks $\langle \bar{s}s \rangle$ and to the corresponding mixed condensate $g\langle \bar{s}\sigma_{\mu\nu}G^{\mu\nu}s \rangle$ become 0 after the Borel transformation in both variables p_D^2 and p_M^2. The complete expressions are collected in Appendix A. Except for the terms proportional to the four-quark condensate they agree with those of Ref. [17]. The latter, however, turn out to be very small numerically.

Second, we saturate the three-point functions (2.2), (2.3) by intermediate hadron states:

$$\Pi_{\mu\nu}(p_D, p_K) = \frac{1}{p_D^2 - m_D^2 + i\epsilon} \frac{1}{p_K^2 - m_K^2 - i\varepsilon} \langle 0 | \bar{d}\gamma_\nu\gamma_5 s | \bar{K}^0 \rangle$$
$$\times \langle \bar{K}^0 | J_\mu | D \rangle \langle D | \bar{c} i\gamma_5 d | 0 \rangle$$
$$+ \text{higher resonances} , \quad (2.4)$$

$$\Gamma_{\mu\nu}(p_D, p_K) = \frac{1}{p_D^2 - m_D^2 + i\epsilon} \frac{1}{p_K^2 - m_K^2 + i\epsilon} \langle 0 | \bar{d}\gamma_\nu s | \bar{K}^*, \lambda \rangle$$
$$\times \langle \bar{K}^*, \lambda | J_\mu | D \rangle \langle D | \bar{c} i\gamma_5 d | 0 \rangle$$
$$+ \text{higher resonances} . \quad (2.5)$$

The standard parametrization for the $D \to K e \nu$ amplitude is

$$\langle K | J_\mu | D \rangle = f_+(t)(p_D + p_K)_\mu + f_-(t)(p_D - p_K)_\mu . \quad (2.6)$$

In the limit of vanishing lepton mass the form factor $f_-(t)$ does not contribute to the decay rate; therefore, we

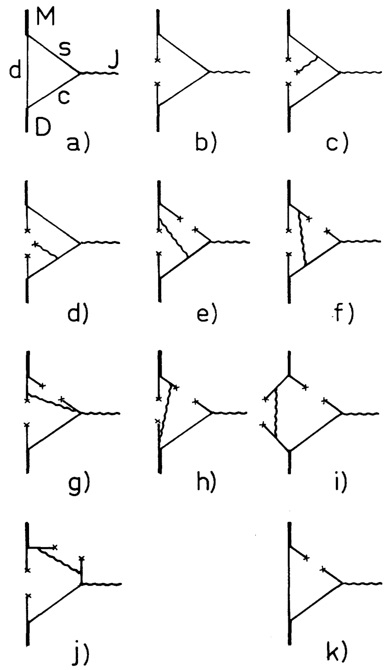

FIG. 1. Graphs for the operator-product expansion of the three-point function. D and M are the mesons in the initial and the final state, J is the weak current. The internal lines represent the c, d, and s quarks, respectively. Lines ending at a cross denote vacuum expectation values. (a) contributes to Γ^{pert}; (b) to $\Gamma^{(3)}$, $\Gamma^{(5)}$, and $\Gamma^{(6)}$; (c) and (d) to $\Gamma^{(5)}$ and $\Gamma^{(6)}$; (e)–(j) to $\Gamma^{(6)}$; (k) vanishes after the Borel transformation, but contributes if a nonlocal condensate (see Sec. V) is introduced.

shall omit it. Note that only the vector part of the weak current J_μ contributes in that case.

For the weak matrix element $D \to K^*$ we use the decomposition adopted in [5]:

$$\langle \bar{K}^*,\lambda|J_\mu|D\rangle = -i(m_D+m_{K^*})A_1(t)\epsilon_\mu^{*(\lambda)} + \frac{iA_2(t)}{m_D+m_{K^*}}(\epsilon^{*(\lambda)}p_D)(p_D+p_K)_\mu + \frac{2V(t)}{m_D+m_{K^*}}\epsilon_\mu{}^{\nu\rho\sigma}\epsilon_\nu^{*(\lambda)}p_{D\rho}p_{K\sigma} , \qquad (2.7)$$

where $t=(p_D-p_K)^2$. The fourth Lorentz invariant $A_3(t)$ multiplying $(\epsilon^{*(\lambda)}p_D)(p_D-p_K)_\mu$ does not contribute in the limit of vanishing lepton mass and is therefore omitted.

The vacuum-to-meson transition amplitudes are written in terms of the corresponding decay constants as

$$\langle 0|\bar{d}i\gamma_5 c|D\rangle = f_D\frac{m_D^2}{m_c} ,$$

$$\langle 0|\bar{d}\gamma_\nu s|\bar{K}^*,\lambda\rangle = \frac{m_{K^*}^2}{g_{K^*}}\epsilon_\nu^{(\lambda)} = f_{K^*}^V m_{K^*}\epsilon_\nu^{(\lambda)} , \qquad (2.8)$$

$$\langle 0|\bar{d}\gamma_\nu\gamma_5 s|\bar{K}\rangle = if_K p_{K\nu} .$$

Finally we perform a Borel transformation in both variables p_D^2, p_M^2 and equate the two representations described above. We assume that the contributions of higher resonances and the continuum in (2.4) and (2.5) are represented by the perturbative contributions above certain thresholds $s_D = p_D^2 \ge s_D^0, s_K = p_K^2 \ge s_K^0$. Thus, the following set of sum rules arises:

$$A_1(t) = \frac{m_c g_{K^*}}{f_D(m_D+m_{K^*})m_D^2 m_{K^*}^2}\exp\left[\frac{m_D^2}{M_D^2}+\frac{m_{K^*}^2}{M_K^2}\right] \times \Gamma_0(M_D^2,M_K^2,t) ,$$

$$A_2(t) = \frac{m_c g_{K^*}(m_D+m_{K^*})}{f_D m_D^2 m_{K^*}^2}\exp\left[\frac{m_D^2}{M_D^2}+\frac{m_{K^*}^2}{M_K^2}\right] \times \Gamma_+(M_D^2,M_K^2,t) , \qquad (2.9)$$

$$V(t) = \frac{m_c g_{K^*}(m_D+m_{K^*})}{2f_D m_D^2 m_{K^*}^2}\exp\left[\frac{m_D^2}{M_D^2}+\frac{m_{K^*}^2}{M_K^2}\right] \times \Gamma_V(M_D^2,M_K^2,t) ,$$

and

$$f_+(t) = \frac{m_c}{f_D f_K m_D^2}\exp\left[\frac{m_D^2}{M_D^2}+\frac{m_K^2}{M_K^2}\right]\Pi_+(M_D^2,M_K^2,t) , \qquad (2.10)$$

where M_D^2 and M_K^2 are the corresponding Borel parameters and

$$\Gamma_i = \Gamma_i^{\text{pert}} + \Gamma_i^{(3)} + \Gamma_i^{(5)} + \Gamma_i^{(6)} + \cdots$$

are the double Borel transformed expressions for the perturbative graph and for the contribution of condensates of dimension 3,5,6, respectively, which are listed in Appendix A.

The perturbative contribution is given by the double dispersion relation

$$\Gamma_i^{\text{pert}} = \int_0^{s_K^0} ds_K \int_0^{s_D^0} ds_D\, \rho_i(s_D,s_K,t)e^{-s_D/M_D^2}e^{-s_K/M_K^2} , \qquad (2.11)$$

where the double spectral densities ρ_i contain contributions of non-Landau-type singularities (cf. Ref. [22], for example) for $t>0$. We outline their origin and structure in the next section.

III. THE t DEPENDENCE OF THE PERTURBATIVE CONTRIBUTION

In order to illustrate the problem, let us consider first the double dispersion relation for the triangle graph of Fig. 1(a) in the case of scalar quarks:

$$T(p_D^2,p_K^2,t) = (4\pi)^2 i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2(p_K+q)^2[(p_D+q)^2-m_c^2]} . \qquad (3.1)$$

The integral in (3.1) is finite and the amplitude vanishes in the limit of infinitely large values of each argument (the other ones kept fixed). Hence the amplitude $T(p_D^2,p_K^2,t)$ can be written in the form of a double dispersion integral without subtractions ($t<m_c^2$):

$$T(p_D^2,p_K^2,t)=\int \frac{\rho(s_D,s_K,t)}{(s_D-p_D^2)(s_K-p_K^2)}ds_D\, ds_K \ . \qquad (3.2)$$

Applying the Cutkosky rules [23] we obtain readily the double spectral function

$$\rho=\frac{1}{\lambda^{1/2}(s_D,s_K,t)}\theta(s_D-s_K^L)\theta(s_K)\ , \qquad (3.3)$$

where $\lambda(s_D,s_K,t)=(s_D+s_K-t)^2-4s_Ds_K$. The quantity s_D^L, the lower limit of the s_D integration, is determined by the condition that all internal particles can be on mass shell (Landau equations [24]), yielding

$$s_D^L=\frac{m_c^2}{m_c^2-t}s_K+m_c^2\ . \qquad (3.4)$$

The double dispersion relation (3.2) with the spectral function (3.3) is shown in Fig. 2 at some values of p_D^2 and p_K^2 below the thresholds. In the region $t>0$ the double dispersion integral (dotted curve) clearly deviates from the Feynman amplitude (3.1) (solid curve), indicating that some contribution to the dispersion integral is missing.

To clarify this discrepancy, we first write (3.1) as a single dispersion relation in p_K^2. A standard calculation yields

$$T(p_D^2,p_K^2,t)=\int_0^\infty \frac{\sigma_K(p_D^2,s_K,t)}{s_K-p_K^2}ds_K\ , \qquad (3.5)$$

where

$$\sigma_K(p_D^2,s_K,t)=\sigma_+(p_D^2,s_K,t)-\sigma_-(p_D^2,s_K,t)\ , \qquad (3.6)$$

$$\sigma_\pm(p_D^2,s_K,t)=\frac{\pi}{\lambda^{1/2}(p_D^2,s_K,t)} \times\ln[\,s_K-p_D^2-t+2m_c^2\pm\lambda^{1/2}(p_D^2,s_K,t)]\ . \qquad (3.7)$$

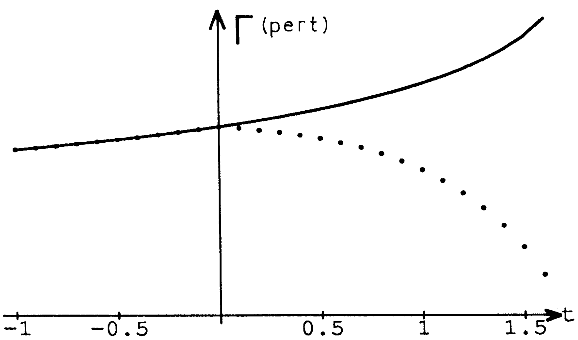

FIG. 2. The scalar triangle graph [Eq. (3.1)] (solid line) and the contribution of the Landau singularity to it (dotted line).

One can easily check that Eq. (3.5) indeed reproduces the exact value of the triangle-diagram equation (3.1).

Next, we discuss $\sigma_K(p_D^2,s_K,t)$ as an analytic function of $s_D=p_D^2$ at fixed s_K and t. For $s_K<(m_c^2-t)^2/t, t>0$, the functions σ_+ and σ_- both have square-root branch points on the physical sheet at $s_D^{(-)}=(\sqrt{s_K}-\sqrt{t}\,)^2$ and $s_D^{(+)}=(\sqrt{s_K}+\sqrt{t}\,)^2$, connected by a cut [dashed lines in Fig. 3(a)]. In addition, the function σ_- has a logarithmic cut starting from $s_D=s_D^L$ [solid line in Fig. 3(a)]. The square-root cuts cancel in the difference $\sigma_K=\sigma_+-\sigma_-$, so that the logarithmic cut of σ_- starting at $s_D=s_D^L$ proves to be the only singularity of σ_K on the physical sheet. Taking the discontinuity of σ_- on this cut, we reproduce Eq. (3.6). A closer inspection shows that the function σ_+ has also a logarithmic cut starting from $s_D=s_D^L$, but it is on the second (unphysical) sheet of the Riemann surface of the square root [dotted line in Fig. 3(a)] and has therefore no influence on the double spectral function. Thus, the situation is effectively the same as for $t<0$, in which case for all values of s_K there are no square-root cuts at all, and only the logarithmic cut in σ_- persists, starting from $s_D=s_D^L$ and giving rise to the discontinuity in Eq. (3.6).

However, for $s_K=(m_c^2-t)^2/t$ it turns out that $s_D^{(+)}=s_D^L$, i.e., the logarithmic and square-root branch points coincide, and for further increasing s_K the logarithmic branch point in σ_+ dives through the square-root cut up onto the physical sheet, whereas the position of the logarithmic branch point of σ_- moves to the second sheet [see Fig. 3(b)]. Thus, on the physical sheet the function σ_+ now has a logarithmic cut from $s_D^{(+)}$ to s_D^L, whereas σ_- has a logarithmic cut from $s_D^{(+)}$ to infinity. Both have also the square-root cuts from $s_D^{(-)}$ to $s_D^{(+)}$. In the difference $\sigma_K=\sigma_+-\sigma_-$ the square-root cuts cancel as before, but the logarithmic cuts add. We thus end up with the double spectral function

$$\rho^0(s_D,s_K,t)=\frac{\theta(s_K)\theta(s_D-s_D^L)}{\lambda^{1/2}(s_D,s_K,t)}+2\theta(t)\theta\left[s_K-\frac{(m_c^2-t)^2}{t}\right] \times\frac{\theta(s_D^L-s_D)\theta(s_D-s_D^{(+)})}{\lambda^{1/2}(s_D,s_K,t)}\ . \qquad (3.8)$$

We have checked that the double dispersion relation (3.2) with the spectral density (3.8) reproduces correctly expression (3.1) for the triangle diagram.

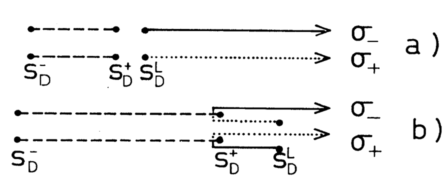

FIG. 3. Singularity structure of the double spectral function of Eqs. (3.6) and (3.7) for (a) $s_K<(m_c^2-t)^2/t$, (b) $s_k>(m_c^2-t)^2/t$. Solid line: logarithmic cut on the physical sheet; dotted line: logarithmic cut on the unphysical sheet; dashed line: square-root branch cut.

The regions of integration are shown in Fig. 4 (for $m_c^2>t>0$). The displayed curves are $s_D=s_D^{(+)}(s_K,t)$ and $s_D=s_D^L(s_K,t)$. Usual Landau-type contributions come from the shaded area I, whereas the second term in (3.8) is nonvanishing in the double-shaded region II.

The structure of singularities in the double dispersion relation remains unchanged for the triangle diagram with internal fermion lines, except that the corresponding double spectral densities may turn out to be more singular at the lower integration limit in s_D. For example, the contribution to the form factor $V(t)$ can be written as

$$\rho^V=\frac{P(s_D,s_K,t)}{\lambda(s_D,s_K,t)}\rho^0(s_D,s_K,t) , \tag{3.9}$$

where ρ^0 is the scalar spectral function (3.8) derived above, and $P(s_D,s_K,t)$ is a polynomial in its arguments vanishing at $s_D=m_c^4/t, s_K=(m_c^2-t)^2/t$ (see Appendix A). A straightforward integration would yield in this case "nonintegrable singularities" [15] due to the additional factor $1/\lambda$, which is singular at $s_D=s_D^{(+)}$. Thus, one should apply the Cauchy theorem accurately, representing $\sigma_K(p_D^2,s_K,t)$ by a contour integral and taking into account the nonvanishing contribution of a small circle around the starting point $s_D^{(+)}$ of the cut [for $s_K>(m_c^2-t)^2/t$]. Then a properly regularized expression emerges, which can be written as

$$\sigma_K(p_K^2,s_K,t)=\int_{s_D^{(+)}}^{\infty}ds_D\left[\frac{1}{s_D-p_D^2}\frac{1}{\lambda(s_D,s_K,t)}P(s_D,s_K,t)\rho^0(s_D,s_K,t)\right.$$
$$\left.-\frac{2}{s_D^{(+)}-p_D^2}\frac{P(s_D^{(+)},s_K,t)}{[(s_D^{(+)}-s_D^{(-)})(s_D-s_D^{(+)})]^{3/2}}\theta\left[s_K-\frac{(m_c^2-t)^2}{t}\right]\theta(t)\right] . \tag{3.10}$$

In the QCD sum-rule analysis the subtraction of the continuum contribution limits the s_K integration to the region $s_K<s_K^0$; hence, for $(m_c^2-t)^2/t>s_K^D$ the non-Landau singularities do not show up.

In the case of the standard dispersion relation in one variable, a nonvanishing contribution coming from a small circle around the branching point signals the presence of kinematical singularities in the amplitude which can be sorted out. For the double dispersion relation this is generally not the case. It is easy to see that the form factors introduced in the previous section are free from any kinematical singularities [22].

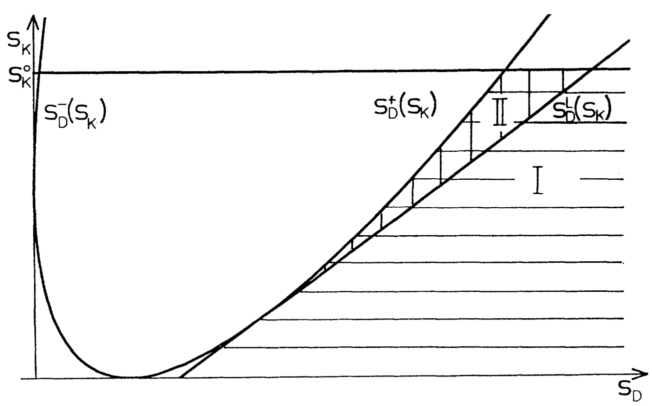

FIG. 4. Regions of integration for the double dispersion integral (3.2). Shaded region (I): Landau singularities, double shaded region (II): non-Landau singularities.

IV. RESULTS FROM THE STANDARD PROCEDURE

The numerical results given below have been obtained using the values of vacuum condensates and quark masses from Ref. [25] (at the normalization point $\mu=1$ GeV):

$$\langle\bar{u}u\rangle=\langle\bar{d}d\rangle=-(230\text{ MeV})^3,\quad \langle\bar{s}s\rangle=0.8\langle\bar{u}u\rangle ,$$
$$g\langle\bar{\psi}\sigma_{\mu\nu}G^{\mu\nu}\psi\rangle=m_0^2\langle\bar{\psi},\psi\rangle,\quad m_0^2=0.8\text{ GeV}^2 , \tag{4.1}$$
$$m_s=160\text{ MeV},\quad m_c=1.3\text{ GeV} .$$

In the Russian literature a somewhat higher value of the quark condensate is preferred: $\langle\bar{u}u\rangle\simeq-(250\text{ MeV})^3$ (at the same normalization point). The effect of this possible change on our results is not large and is discussed below. The contribution of the four-quark condensate turns out to be very small in all cases (see Fig. 5). We have been using factorization for it in the whole range of Borel parameters. All quantities have been renormalization-group improved up to the current value of $\mu^2=M_DM_K$. The values for the continuum thresholds s_D^0 and $s_{K^*}^0$ are taken from QCD sum rules for the corresponding two-point functions

$$s_D^0=6\text{ GeV}^2\quad\text{and}\quad s_{K^*}^0=1.7\text{ GeV}^2 \tag{4.2}$$

(cf. Appendix B). The region of stability in the Borel parameter for the two-point functions is 1–2 GeV^2 for the D and 0.7–1.1 GeV^2 for the K^* meson, and the decay constants extracted from these sum rules are

$$f_D=160\text{ MeV},\quad f_{K^*}^V=210\text{ MeV} . \tag{4.3}$$

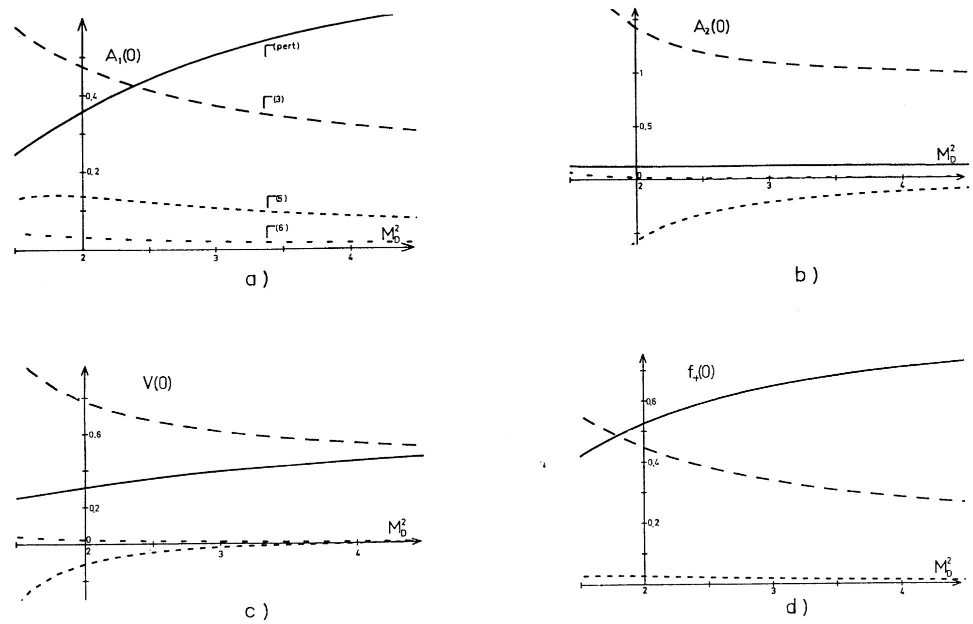

FIG. 5. The various contributions to the operator-product expansion of the form factors as functions of the Borel parameter M_D^2. Solid line: Γ^{pert}; long dashes: $\Gamma^{(3)}$ (quark condensate); short closely spaced dashes: $\Gamma^{(5)}$ (mixed quark-gluon condensate); short widely spaced dashes: $\Gamma^{(6)}$ (four-quark condensate).

For the K meson we take $s_K^0=1.7$ GeV^2 and the experimental value $f_K=160$ MeV.

On evidence of existing calculations [11], we expect the region of stability sum rules for three-point functions to be at values of Borel parameters twice as large as in the corresponding two-point functions. Hence, we evaluate our sum rules in the range $2\le M_D^2 \le 4$ GeV^2 and at a fixed ratio

$$\frac{M_D^2}{M_{K^*}^2}\simeq\frac{m_D^2-m_c^2}{m_{K^*}^2-m_s^2}\simeq 1.8\ , \qquad \frac{M_D^2}{M_K^2}=2\ . \tag{4.4}$$

We have checked that our results depend very weakly on this ratio.

In Fig. 6 we show the values of the form factors A_1, A_2, V, and f_+ at zero-momentum transfer obtained from the sum rules (2.9) and (2.10) as functions of the Borel parameter. The stability is seen to be quite satisfactory in the expected range. The relative contributions of the perturbative graph and of different power corrections are given as functions of M_D^2 in Fig. 5. We see that $A_2(0)$ is mainly determined by the quark condensate, while the perturbative contribution is the largest one for all other form factors. Our final results for the values of form factors at $t=0$ are

$$A_1(0)=0.50\pm0.15,\quad A_2(0)=0.60\pm0.15\ ,$$
$$V(0)=1.1\pm0.25,\quad f_+(0)=0.60^{+0.15}_{-0.10}\ , \tag{4.5}$$
$$\frac{V(0)}{A_1(0)}=2.2\pm0.2,\quad \frac{A_2(0)}{A_1(0)}=1.2\pm0.2\ .$$

The t dependence of form factors for different values of the Borel parameter M_D^2 normalized to unity at $t=0$ is plotted in Fig. 7. The physical t region for the decay $D\to Ke\nu$ extends up to 1.9 GeV. At values $t>1$ GeV^2 the integration region in s_D and s_K [see Eq. (2.11)] shrinks

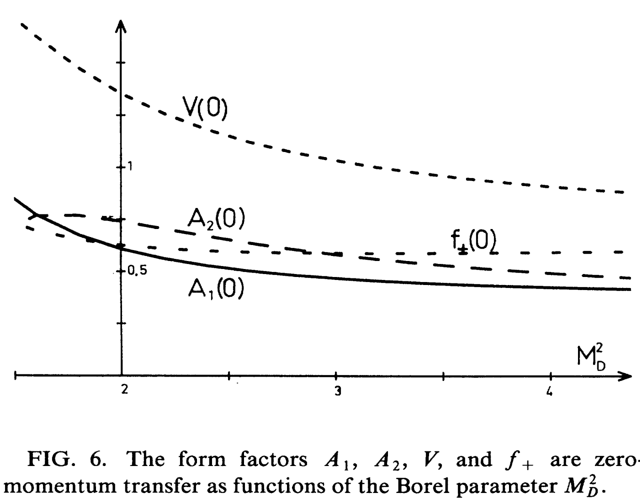

FIG. 6. The form factors A_1, A_2, V, and f_+ are zero-momentum transfer as functions of the Borel parameter M_D^2.

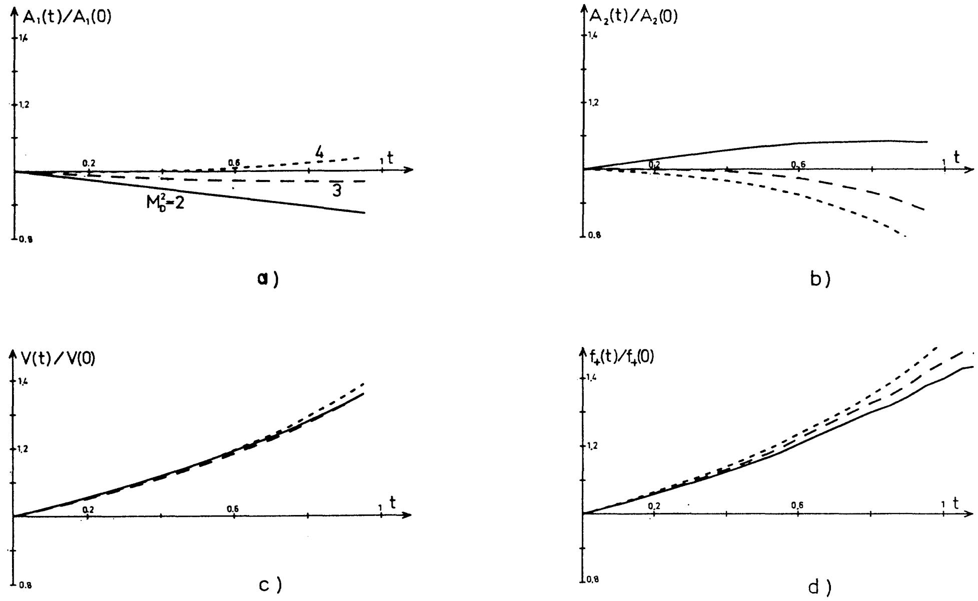

FIG. 7. The t dependence of form factors for various values of M_D^2. Solid line: $M_D^2=2$ GeV^2; long dashes: $M_D^2=3$ GeV^2; short dashes: $M_D^2=4$ GeV^2.

drastically leading to a sharp decrease of the form factor (cf. Fig. 8). This effect is thus simply an artifact of our simple model for the continuum contribution. In addition, for values $t>1$ GeV^2 the distance to the threshold gradually becomes too small and the operator-product expansion breaks down.

The t dependence of $V(t)$ and $f_+(t)$ for $t<1$ GeV^2 can be well approximated by a pole model expression with the pole masses

$$\begin{aligned} m_{\text{pole}}^{V} &= 1.95\pm 0.10 \text{ GeV} , \\ m_{\text{pole}}^{f_+} &= 1.81\pm 0.10 \text{ GeV} . \end{aligned} \tag{4.6}$$

Both $V(t)$ and $f_+(t)$ obtain contributions of the vector current only in the t channel. Our result is thus very compatible with the dominance of a low-lying charmed resonance in the $J^P=1^-$ channel, which is estimated to be at $m_{c\bar{s}1^-}\sim 2.1$ GeV (Ref. [5]), and in the case of $D\to K$ decay is also compatible with the experimental value $m_{\text{pole}}^{f_+}=1.8^{+0.5+0.3}_{-0.2-0.2}$ GeV [1]. In contrast with this pronounced pole-type behavior the other two form factors $A_1(t)$ and $A_2(t)$ show a much weaker t dependence and to our accuracy are nearly constant up to $t=1$ GeV^2. In the case of A_1 this approximate t dependence stems from a mutual cancellation in the sum rule of an increase in the perturbative and a decrease in the quark-condensate contribution. In the case of A_2 the dominant contribution of the quark condensate stays constant in t and the remaining slight t dependence stems from the nondominant contributions of perturbation theory and higher condensates. We conclude that for the axial-vector form factors $A_1(t)$ and $A_2(t)$ the assumption of pole dominance is not adequate.

Finally, we use the form factors to calculate differential decay rates $d\Gamma/dt$ for the decays $D\to K^*e\nu, D\to Ke\nu$, (Fig. 9 and Table I). The decay rates for the decay $D\to\bar{K}^*e\nu$ are evaluated with the t dependence of the form factors as obtained from the sum rules. For the reasons explained above, f_+ is determined by the extrapolation indicated in Fig. 8 in the t range above 1 GeV^2. It should be noted, however, that the influence of this extrapolation on the total rate is quite small. The differential rate of the decay $D\to Ke\nu$ is given by

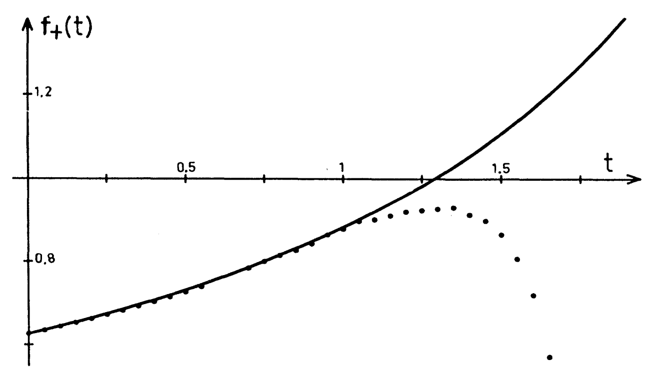

FIG. 8. The t dependence of the form factor $f_+(t)$. Solid line: pole fit with $m_{\text{pole}}=1.81$ GeV; dotted line: result of the sum rule. The falling-off at $t>1$ GeV^2 is an artifact of our simple model of the continuum.

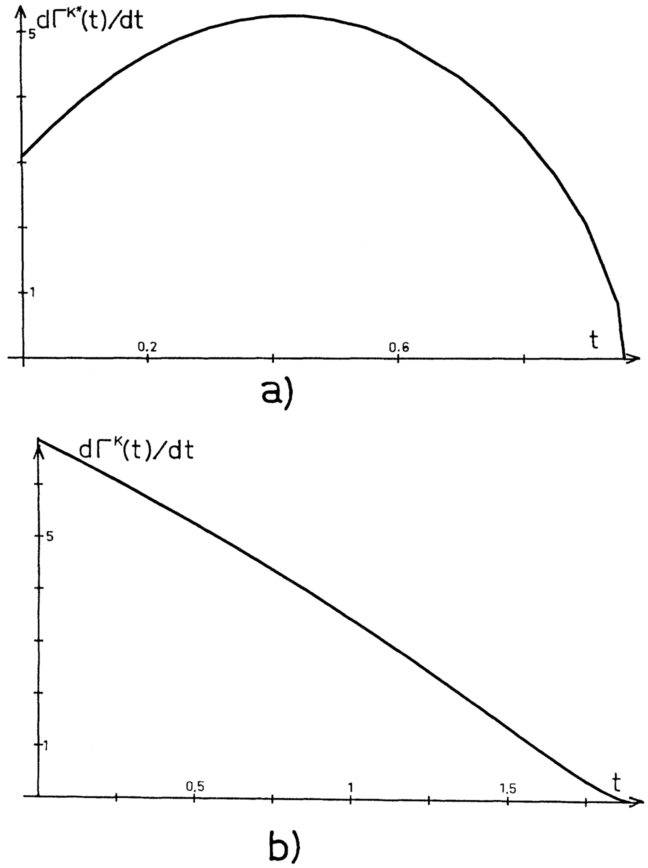

FIG. 9. The decay rates as functions of momentum transfer to the lepton pair in units of 10^{10} sec^{-1} GeV^{-2}: (a) for the decay $D \to K^* e\nu$, (b) for $D \to Ke\nu$.

$$\frac{d\Gamma}{dt} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_D^3}\lambda^{3/2}(m_D^2, m_K^2, t) f_+^2(t) \ .$$

The rates for the decay $D \to \bar{K}^* e\nu$ are written in terms of the helicity amplitudes

$$H_\pm(t) = (m_D + m_{K^*})A_1(t) \mp \frac{\lambda^{1/2}(m_D^2, m_{K^*}^2, t)}{m_D + m_{K^*}} V(t) \ ,$$

$$H_0(t) = \frac{1}{2m_{K^*}\sqrt{t}}\left[(m_D^2 - m_{K^*}^2 - t)(m_D + m_{K^*})A_1(t) - \frac{\lambda(m_D^2, m_{K^*}^2, t)}{m_D + m_{K^*}} A_2(t)\right]$$

so that

$$\frac{d\Gamma_\pm}{dt} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_D^3} t\lambda^{1/2}(m_D^2, m_{K^*}, t)|H_\pm(t)|^2 \ ,$$

$$\frac{d\Gamma_L}{dt} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_D^3} t\lambda^{1/2}(m_D^2, m_{K^*}^2, t)|H_0(t)|^2 \ ,$$

$$\frac{d\Gamma_T}{dt} = \frac{d}{dt}(\Gamma_+ + \Gamma_-), \quad \frac{d\Gamma}{dt} = \frac{d}{dt}(\Gamma_L + \Gamma_T) \ .$$

The comparison to experimental data, to lattice calculations, and to quark models is given in Table II.

The given errors of our results combine the results of extensive calculations modeling intrinsic uncertainties of the sum-rule method and include the possible influence of moderate variations in the above-listed parameters. To check the sensitivity of our results to the continuum model, we have repeated all calculations with the threshold $s_K^0 = 2$ GeV^2 instead of $s_K^0 = 1.7$ GeV^2 with all other parameters fixed. The influence of this change on the form factors turns out to be within 5%. An increase of 30% in the quark condensate [from $(-230 \text{ MeV})^3$ to $(-250 \text{ MeV})^3$ at 1 GeV] yields a $\sim$20% increase in the absolute rates. Taking the ratio of form factors, the effect is reduced to $\leq 12\%$ and is most pronounced for A_2. Changing the ratio of Borel parameters $R = M_D^2/M_K^2$ from $R = 1.8$ to $R = 3$ induces changes in form factors within 5%. The most important source of uncertainty is due to the variation of the Borel parameters $M_D^2 = RM_K^2$ (see Figs. 6 and 7), which amounts to up to 50% in the $D \to K^*$ decay rates and up to 25% in $D \to K$ decay as

TABLE I. Differential decay rates as functions of momentum transfer t in units of 10^{10} sec^{-1} GeV^{-2}. $d\Gamma^{K^*}/dt$ and $d\Gamma^K/dt$: differential decay rates for the decays $D \to K^* e\nu$ and $D \to Ke\nu$, respectively. $d\Gamma_L^{K^*}/dt$, $d\Gamma_T^{K^*}/dt$, $d\Gamma_+^{K^*}/dt$, $d\Gamma_-^{K^*}/dt$: longitudinal, transversal, positive-helicity, and negative-helicity differential decay rates for the decay $D \to K^* e\nu$.

t (GeV^2)	$d\Gamma^{K^*}/dt$	$d\Gamma_L^{K^*}/dt$	$d\Gamma_T^{K^*}/dt$	$d\Gamma_+^{K^*}/dt$	$d\Gamma_-^{K^*}/dt$	$d\Gamma^K/dt$
0	3.09	3.09	0	0	0	6.80
0.20	4.64	2.60	2.04	0.05	1.99	6.15
0.40	5.25	2.15	3.10	0.11	2.99	5.46
0.60	4.89	1.69	3.20	0.20	3.00	4.76
0.80	3.44	1.12	2.32	0.32	2.00	4.03
0.96	0	0	0	0	0	3.21
1.20						2.48
1.40						1.67
1.60						0.87
1.80						0.18
1.89						0

TABLE II. Experimental and theoretical values for decay rates. $\Gamma_{\rm tot}^{+}$: total decay rate for the D^+ decay. $\Gamma_{\rm tot}^{0}$: total decay rate for the D^0 decay. Γ^{K^*}: decay rate for the decay $D^+\to\bar{K}^*e^+\nu_e$. Γ^K: decay rate for the decay $D^0\to\bar{K}e\bar{\nu}_e$. $\Gamma_L,\Gamma_T,\Gamma_+,\Gamma_-$: longitudinal, transversal, positive-helicity, and negative-helicity decay rates, respectively, for the decay $D^+\to\bar{K}^*e^+\nu_e$. The ARGUS Collaboration [3] gives $\Gamma^{K^*}/\Gamma_{\rm tot}^{+}=4.2\pm0.6\pm0.1\%$. A new experimental result of CLEO [33] is $\Gamma^{K^*}/\Gamma^K=0.51\pm0.18\pm0.06$. E691, Ref. [4]; Mark III, Ref. [1]; BWS, Ref. [5]; GS, Ref. [9]; IS, Ref. [7], LMS, Ref. [32].

	Experiment		Sum rules	Quark models			Lattice
	E691	Mark III	This paper	BWS	GS	IS	LMS
$\Gamma^{K^*}/\Gamma_{\rm tot}^{+}$ %	$4.4\pm0.4\pm0.8$	$5.3^{+1.9}_{-1.1}\pm0.6$	4.0 ± 1.6	10.1	10.5	9.4	5.5 ± 2.0
Γ_L/Γ_T	$1.8^{+0.6}_{-0.4}\pm0.3$	$0.5^{+1.0+0.1}_{-0.1-0.2}$	0.86 ± 0.06	0.89	1.20	1.11	1.7 ± 0.6
Γ_+/Γ_-	$0.15^{+0.07}_{-0.05}\pm0.03$		0.09 ± 0.02	0.28	0.18	0.29	
$\Gamma^K/\Gamma_{\rm tot}^{0}$ %	$3.8\pm0.5\pm0.6$	$3.4\pm0.5\pm0.4$	2.7 ± 0.6	3.5			2.1 ± 0.3
Γ^{K^*}/Γ^K	$0.50\pm0.09\pm0.07$	$1.0^{+0.3}_{-0.2}$	0.50 ± 0.15	1.14		1.45	1.1 ± 0.3

M_D^2 varies from 2 to 4 GeV^2. The ratios of rates are much more stable (up to $\sim15\%$) and have the additional advantage of not depending on the decay constants.

V. THE NONLOCAL QUARK CONDENSATE

The nonperturbative contributions to the sum rules (2.9) and (2.10) stay constant for large negative values of t, or even increase. This property is certainly not reasonable, since for asymptotically large momentum transfers the form factors should decrease as $1/t$, and the main contribution to them should be of a completely different origin (from the exchange of a hard gluon [26,27]). It is easy to see that higher-order power corrections to the sum rules would possess an even stronger t dependence and should effectively cancel unreasonably large contributions of the lowest-order condensates.

To model this cancellation and to obtain a semiquantitative estimate for the possible influence of higher-order corrections on our results, we introduce in this section the nonlocal quark condensate

$$\langle\bar{\psi}(x)\exp\left(\int_0^x ig\mathbb{A}_\mu(x)dx_\mu\right)\psi(0)\rangle=\langle\bar{\psi}\psi\rangle f(x^2) \tag{5.1}$$

and investigate its effect on the sum rules (x is the distance in Euclidean space-time). For simplicity, we assume

$$f(x^2)=e^{-x^2/(4\rho)} \tag{5.2}$$

and take $\rho=4/m_0^2$ to be consistent with the short-distance expansion in Euclidean space,

$$\langle\bar{\psi}(x)\psi(0)\rangle=\langle\bar{\psi}\psi\rangle-\tfrac{1}{16}x^2g\langle\bar{\psi}\sigma_{\mu\nu}G_{\mu\nu}\psi\rangle+\cdots\ . \tag{5.3}$$

Nonlocal condensates of type (5.1) are often introduced in the literature in various contexts [28,29]. The physical effect which is taken into account by (5.1) is that quarks in QCD vacuum may actually have a nonvanishing momentum.

The dominant nonperturbative contribution to the sum rules comes from the diagram in Fig. 1(b). Using the nonlocal condensate (5.1), the expression for this graph is modified to

$$\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^\nu\frac{\not{p}_K+\not{k}-m_s}{(p_K+k)^2}\gamma^\mu(1-\gamma_5)\frac{\not{p}_D+\not{k}-m_c}{(p_D+k)^2+m_c^2}\gamma_5\right]\times\langle\bar{\psi}\psi\rangle4\pi^2\rho^2e^{-\rho k^2}\ , \tag{5.4}$$

where we have used Euclidean momenta and γ matrices. In the limit $\rho\to\infty$, Eq. (5.4) reduces to the standard expression for the contribution of the quark condensate.

In order to illustrate the effect, we consider here the form factor $A_2(t)$, which is the most controversial and for which the contribution of the quark condensate plays the most important role. Let us denote by $A_2^{(3)}$ and $A_2^{(3,\mathrm{NL})}$ the contribution from the local condensate and from its nonlocal generalization, respectively. Then a straightforward calculation yields

$$\frac{A_2^{(3,\mathrm{NL})}}{A_2^{(3)}}=\left[1-\frac{2}{\rho M_D^2}\right]\exp\left[-\frac{m_c^2}{M_D^2}\frac{M_D^2+M_K^2}{M_D^2M_K^2\rho-M_D^2-M_K^2}\right]\times\exp\left[\frac{t}{M_D^2M_K^2\rho-M_D^2-M_K^2}\right]\times\theta\left[1-\frac{1}{\rho M_K^2}-\frac{1}{\rho M_D^2}\right]\ . \tag{5.5}$$

This ratio falls off exponentially for large negative t with ρ,M_D^2,M_K^2 fixed. Thus the nonlocal condensate indeed leads to the expected suppression of nonperturbative terms at large negative momentum transfer [$\sim-(2\text{–}3\ \mathrm{GeV})^2$]. For $0\le t\le t_{\max}=(m_D-m_{K^*})^2$ the ratio (5.5) deviates noticeably from unity. However, the major part of the effect is due to the linear term in the Taylor expansion of (5.5) in powers of $1/\rho$ and is taken into account in our sum rules explicitly and consistently as part of the contribution of the mixed condensate. We therefore prefer to stick to the standard procedure.

A particularly interesting effect of nonlocal condensates is the occurrence of terms which are nonanalytic in $1/\rho$ and missing in the standard operator-product expansion. In particular, the contribution of the strange condensate [Fig. 1(k)] is now nonvanishing even after Borel transformation. Denoting the contribution of that diagram by $A_2^{(3,S,\mathrm{NL})}$ we obtain, as before,

$$\frac{A_2^{(3,S,\mathrm{NL})}}{A_2^{(3)}}=\frac{\langle\bar{s}s\rangle}{\langle\bar{\psi}\psi\rangle}M_D^4\frac{\rho M_K^2-1}{(M_D^2+M_K^2)^2}\times\exp\left[-\frac{m_c^2(\rho M_K^2-1)}{M_D^2}\right]\times\exp\left[t\frac{\rho M_K^2-1}{M_D^2+M_K^2}\right]\theta(\rho M_K^2-1)\ . \quad (5.6)$$

This ratio vanishes exponentially for $\rho\to\infty$. It is small but shows a rather strong t dependence, ranging from 0.03 at $t=0$ to 0.15 at $t_{\max}=(m_D-m_{K^*})^2$ (for $M_D^2=2$ GeV^2 and $R=1.8$). As a result the form factor $A_2(t)$, extracted from the sum rules taking into account the nonlocality of the condensate, increases slowly with increasing t, but still the slope remains much smaller than for V or f_+.

We conclude that the effects of nonlocality lead to the necessary damping of power corrections in our sum rules at large negative t, while in the physical region for the decay $0\leq t\leq t_{\max}$ their influence on the rates is negligible and only moderate on the t dependence of the form factors. This indicates the self-consistency of our approach.

VI. SUM RULES ON THE LIGHT CONE

Besides the expansion in distance in Euclidean space-time underlying the standard procedure of QCD sum rules, there exists another expansion in the deviation from the light cone which is more adapted to the kinematics of exclusive processes with large momentum transfer. The factorization theorem allows one to collect contributions of large distances in several hadron wave functions which were studied quite some time ago [20].

In this section we apply this light-cone expansion in a form suggested by studies of exclusive processes in order to obtain a new set of sum rules for the form factors of the weak D decay. Similar sum rules have been suggested recently for B decays in [30].

We consider the two-point correlation function

$$\Pi_\mu=i\int d^4x\, e^{iqx}\langle M(p)|T\{J_\mu(x)D(0)\}|0\rangle\ , \quad (6.1)$$

where $D=\bar{c}i\gamma_5 d$, and $M(p)$ is the K or K^* meson with momentum p. We shall evaluate (6.1) in the region where x^2 is small in Minkowski metric, which implies that $m_c^2-t>1$ GeV^2, but at the same time components of x may be large; i.e., we may have $(px)^2\gg x^2m_M^2$. If one goes to momentum space, the chosen kinematics ensures that the momentum squared $(p+q)^2$ associated with the D meson current is large and negative, while the value of $2qp/(p+q)^2$ is of the order of unity; hence the dependence on this parameter should be calculated exactly.

The leading contribution to the expansion of the T product of currents in (6.1) on the light cone is

$$T\{J_\mu(x)D(0)\}=-\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ikx}}{m_c^2-k^2}\bar{s}(x)\gamma_\mu\times(1-\gamma_5)(m_c-\not{k})d(0)+\cdots\ . \quad (6.2)$$

The relevant matrix elements of the nonlocal operators in (6.2) define wave functions of K (K^*) mesons of leading twist

$$\langle\bar{K}^0(p)|\bar{s}(x)\gamma_\mu\gamma_5 d(0)|0\rangle=-if_Kp_\mu\int_0^1du\, e^{iupx}\varphi_K(u)\ ,$$
$$\langle\bar{K}^{0*}(p)|\bar{s}(x)\gamma_\mu d(0)|0\rangle=\epsilon_\mu^*m_{K^*}f_{K^*}^V\int_0^1du\, e^{iupx}\varphi_V(u)\ , \quad (6.3)$$
$$\langle\bar{K}^{0*}|\bar{s}(x)\sigma_{\mu\nu}d(0)|0\rangle=if_{K^*}^T(\epsilon_\mu^*p_\nu-\epsilon_\nu^*p_\mu)\int_0^1du\, e^{iupx}\varphi_T(u)\ .$$

We use the parametrization of wave functions proposed in [20],

$$\varphi(u)=6u(1-u)\{1+a[(2u-1)^2-\tfrac{1}{5}]+b\tfrac{35}{3}(2u-1)^3\}\ , \quad (6.4)$$

with the following values of parameters (at normalization point $\mu^2\simeq1.5$ GeV^2) [21]:

$$a_K=2,\quad a_V=0.5,\quad a_T=-1.6\ ,\qquad b_K=0.1,\quad b_V=b_T=0.15\ . \quad (6.5)$$

In addition, in the case of $D\to K$ decay, we also take into account the two-particle wave functions of twist 3 [20,31]:

$$\langle\bar{K}^0(p)|\bar{s}(x)i\gamma_5 d(0)|0\rangle=\frac{f_Km_K^2}{m_s+m_d}\int_0^1du\, e^{iupx}\varphi_P(u)\ ,$$
$$\langle\bar{K}^0(p)|\bar{s}(x)\sigma_{\mu\nu}\gamma_5 d(0)|0\rangle=(p_\mu x_\nu-p_\nu x_\mu)\frac{i}{6}\frac{f_Km_K^2}{m_s+m_d}\int_0^1du\, e^{iupx}\varphi_\sigma(u)\ . \quad (6.6)$$

We use the asymptotic wave functions $\varphi_P(u)=1$ and $\varphi_\sigma(u)=6u(1-u)$. In this approximation we do not take into account the contribution of the twist-3 quark-gluon operator (see [31]). The other parameters needed are $f_{K^*}^V=210$ MeV, $f_{K^*}^T=220$ MeV, $f_K=160$ MeV [21].

On the other hand, the contribution of the D meson to the correlation function (6.1) is

$$\frac{f_Dm_D^2}{m_c}\frac{1}{m_D^2-(p+q)^2}\langle M|J_\mu|D^+(p+q)\rangle\ . \quad (6.7)$$

Making the Borel transformation in $(p+q)^2$ and subtracting the continuum contribution above the threshold s_D^0, we obtain the set of sum rules (with $t=q^2$)

$$\frac{f_D m_D^2}{m_c}\begin{bmatrix} f_+(t)\\ A_1(t)\\ V(t)\end{bmatrix} = \exp^{[(m_D^2-m_c^2)/M^2]}\int_0^1 \frac{du}{u}\exp\left[\frac{\bar{u}(t-m_c^2)-\bar{u}um_M^2}{uM^2}\right]\begin{bmatrix}S_1(u)\\ S_2(u)\\ S_3(u)\end{bmatrix}\theta(us_0^D-m_c^2+\bar{u}t-\bar{u}um_M^2)\ , \qquad (6.8)$$

where $m_M^2=m_K^2$ or $m_{K^*}^2$, $\bar{u}=1-u$, and

$$S_1(u)=\frac{f_K}{2}\left[m_c\varphi_K(u)+\frac{m_K^2 u}{m_s+m_d}\varphi_P(u)+\frac{m_K^2}{6(m_s+m_d)}\left[2+\frac{m_c^2+t-u^2m_K^2}{uM^2}\right]\varphi_\sigma(u)\right]\ ,$$

$$S_2(u)=\left[m_{K^*}m_c f_{K^*}^V\varphi_V(u)+\frac{1}{2u}(m_c^2-t+u^2m_{K^*}^2)f_{K^*}^T\varphi_T(u)\right](m_D+m_{K^*})^{-1}\ , \qquad (6.9)$$

$$S_3(u)=\tfrac{1}{2}f_{K^*}^T\varphi_T(u)(m_D+m_{K^*})\ .$$

To the accuracy of the approximation made above it turns out that the form factors A_2 and V coincide, $A_2(t)=V(t)$; thus, their difference is a higher-twist effect. It is easy to see that higher-twist $O(x^2)$ terms are down by powers of $1/(up+q)^2$ under the integral sign, so that after the Borel transformation they behave as $\sim(uM^2)^{-n}/n!$ The average value of u in the integrals (6.8) turns out to be $\langle u\rangle\simeq 0.6$. Thus, one expects the region of stability of the sum rules (6.8) to be at values of the Borel parameter 1/0.6 larger than for the standard two-point correlation function of the D-meson current.

Numerical results for the form factors obtained from (6.8) are plotted in Fig. 10. Figure 10(a) contains the values of the three form factors f_+, A_1, and V at $t=0$, given as a function of the Borel parameter. Their t dependence is shown in Fig. 10(b) for the value $M^2=2.5$ GeV^2, which is in the middle of the expected stability range. The results are qualitatively similar to those in the previous sections. The absolute values of the form factors tend to be 20 –30 % larger, while the effective pole masses for $f_+(t)$ and $V(t)$ are reduced by some 300 MeV. Again we find that the t dependence of A_1 is very weak.

The main advantage of the sum rules (6.8) is that they make use of only one Borel transformation instead of two as in the standard procedure. The principal input is the meson wave function (6.4), which involves new and nontrivial information on the asymptotics of correlation functions in QCD, related to the conformal symmetry [26,27]. The accuracy of (6.8) can be increased considerably by including contributions of wave functions of higher twist [31]. We are going to elaborate on this point in a separate publication.

VII. GENERAL DISCUSSION

We have given a detailed analysis of the semileptonic decays $D\to\bar{K}e\nu$ and $D\to\bar{K}^*e\nu$. The main results are displayed in Tables I–III. Our value for the branching ratio of the $D\to K^*e\nu$ decay agrees with the experimental data of E691 and Mark III as well as of ARGUS [3], which is 4.2±0.6±0.1%. It also agrees with a recent numerical calculation on the lattice [32], but is 2–3 times smaller than predicted by quark models. Our result for the ratio of rates for $D\to\bar{K}^*e\nu$ and $D\to\bar{K}e\nu$ decays is in agreement with the data of E691 and of CLEO [33], but is a factor of 2 smaller than reported by Mark III, the quark models, and the lattice calculations. Our value for the polarization Γ_L/Γ_T is similar to the Mark III result (which has a large error range) and to quark model predictions, but is a factor of 2 below the E691 data.

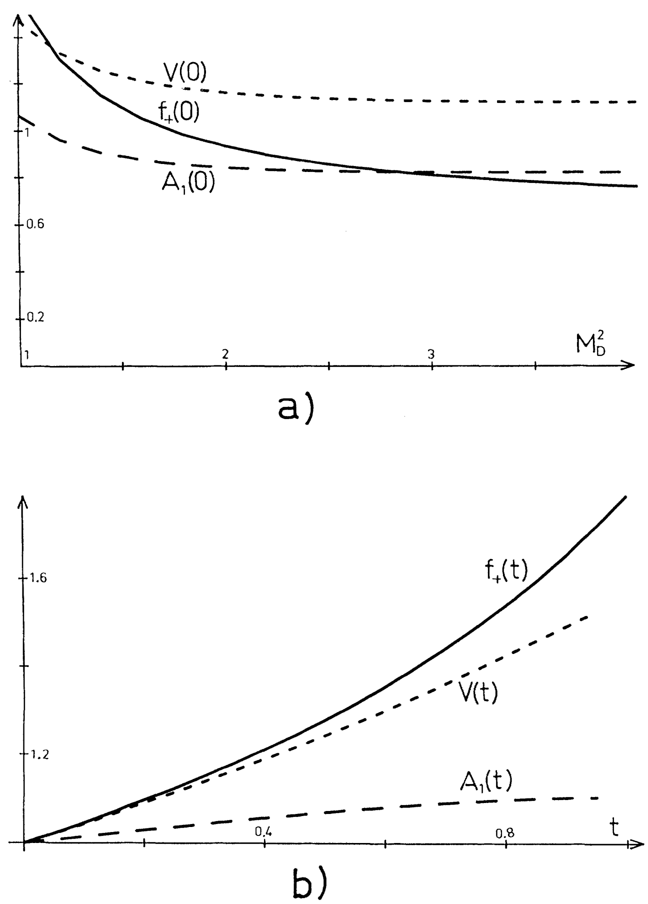

FIG. 10. Results of light-cone sum rules (Sec. VI): (a) the form factors $A_1(0)$, $V(0)$ [$=A_2(0)$ in our approximation], and $f_+(0)$ as functions of the Borel parameter M^2; (b) the t dependence of the form factors at $M^2=2.5$ GeV^2, normalized to unity at $t=0$.

TABLE III. Experimental and theoretical values for the decay form factors at zero-momentum transfer. LC = light cone; AOS, Ref. [17]. The references are the same as in Table II. Dominguez and Paver [35] obtain $f_+(0)=0.75\pm0.05$ using sum rules for two-point functions.

	Experimental	Sum Rules			Quark models			Lattice
		Stand. SR	LC SR	Stand. SR				
	E691	This paper		AOS	BWS	GS	IS	LMS[a]
$f_+(0)$	$0.79\pm0.05\pm0.06$	$0.6^{+0.15}_{-0.10}$	0.8	0.8 ± 0.2	0.76	0.69[b]	0.76[b]	0.58 ± 0.04
$A_1(0)$	$0.46\pm0.05\pm0.05$	0.5 ± 0.15	0.8	0.9 ± 0.2	0.9	0.8	0.8	0.52 ± 0.07
$A_2(0)$	$0.00\pm0.2\pm0.1$	0.6 ± 0.15	1.1	0.8 ± 0.3	1.2	0.6	0.8	0.05 ± 0.35
$V(0)$	$0.9\pm0.3\pm0.1$	1.1 ± 0.25	1.1	1.7 ± 0.6	1.3	1.5	1.1	0.85 ± 0.08

[a]Statistical error probably underestimated [32].
[b]Value recalculated from formulas given in the corresponding paper.

Whereas the latter experimental results lead to a small value (compatible with 0) of the form factor $A_2(0)$, we obtain a ratio $A_2(0)/A_1(0)$ of the order of unity.

Our prediction for the asymmetry Γ_+/Γ_- is considerably smaller than in quark models, and does not contradict the experimental data. Thus the QCD sum-rule predictions are generally in good agreement with data, except for the large polarization of K^*, Γ_L/Γ_T, obtained by E691.

In this paper we have put special emphasis on the t dependence of form factors and have calculated it directly from the sum rules. Our result strongly supports the vector-dominance approximation for the form factors $f_+(t)$ and $V(t)$, while for the axial-vector form factors $A_1(t)$ and $A_2(t)$ we obtain a much weaker t dependence than would be given by dominance of the lowest expected $c\bar{s}$ state in the $J^P=1^+$ channel.

Our sum rules involve several sources of uncertainties which do not allow for an accuracy better than $\sim 20\%$ in form factors and $\sim 50\%$ in decay rates. The two most important sources of error in the standard procedure are the dependence on the Borel parameter and the uncertainty in the value of the quark condensate. Our results for absolute values of form factors at $t=0$ are typically 20–30 % lower than those of an unpublished sum-rule analysis [17]. The difference is partly due to a smaller value of the quark condensate used in our paper, but the main part of the difference is due to our consistent use of the parameters $(s_D^0, s_K^0, M_D^2, \ldots)$ found by using two-point sum rules instead of the usual three-point sum rules. However, the difference from [17] is still within the accuracy of the sum-rule approach.

To obtain semiquantitative estimates of possible effects of higher-power corrections, we have evaluated the sum rules with a nonlocal quark condensate in Sec. V. This sample calculation is interesting in connection with the behavior of form factors at large negative t, where the usual expansion breaks down. A good task for the future is the evaluation of the α_s correction to the triangle diagram in Fig. 1(a), which dominates in the $t\to-\infty$ limit. Since the zeroth-order perturbative contribution to A_2 is particularly small [see Fig. 5(b)], radiative corrections could be important.

The only serious discrepancy between our results and experiment is thus the one in the form factor $A_2(0)$. As mentioned above, the perturbative contribution to this form factor is very small, and the form factor itself is dominated by the term proportional to the chiral-symmetry-breaking quark condensate. It could be reduced by a much larger value of the mixed condensate, which seems, however, to be excluded by other sum rules (cf. [25]) or by an extremely large radiative correction to the three-point function, which had to change sign and had to exceed the zeroth-order contribution by a factor of 4 to 5 in order to be operative. Both possibilities thus seem rather unlikely to us.

Finally, an additional source of error could be in contributions of two-particle states in the hadronic part of the sum rule, (see the figure in Ref. [18]). This type of contribution is generally most difficult to control, although a common belief is that it is not important. Still, even a semiquantitative treatment of such contributions would be desirable in order to judge whether the sum rules indeed exclude a vanishing form factor $A_2(t)$ in the $D\to K^*e\nu$ decay as obtained by the E691 experiment. It should be noted that these corrections are not included in the lattice calculations [32], which yield a small value for $A_2(0)$.

Our results obtained within the standard procedure receive support from an independent set of sum rules on the light cone considered in Sec. VI. This technique is potentially more accurate and reliable since it requires analytical continuation in one variable only, and incorporates new and nontrivial information on asymptotics of three-point functions related to the conformal symmetry. To develop these sum rules to the same (or higher) level of accuracy as the standard ones, one needs to take into account contributions of K- and K^*-meson wave functions of higher twist. We plan to return to this question in a separate publication.

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APPENDIX A: PERTURBATIVE AND NONPERTURBATIVE CONTRIBUTIONS

Generally we take into account the mass of the strange quark, m_s, at most linearly. The perturbative double spectral functions as obtained from Fig. 1(a) by Cutkosky rules (for the corrections of the full double spectral functions, see Sec. III) are

$$\rho_0^C=\frac{3}{8\pi^2\lambda^{1/2}}(m_c s_K+m_s c)+\frac{3}{4\pi^2\lambda^{3/2}}m_c s_K[c(t-m_c^2)+s_K m_c^2]\ ,$$

$$\rho_+^C=\frac{3}{8\pi^2\lambda^{3/2}}[m_c s_K(\Sigma-2c)+m_s(c\Sigma-2s_D s_K)]$$

$$-\frac{3}{4\pi^2\lambda^{5/2}}s_K m_c\{3c^2(2s_K-\Sigma)+2c[\Sigma(s_D-2s_K-t)+2s_K s_D]+s_K[\Sigma(-2s_D+s_K-t)+2s_K s_D]\}\ ,$$

$$\rho_V^C=-\frac{3}{4\pi^2\lambda^{3/2}}[m_s(2s_D s_K-c\Sigma)+m_c s_K(2c-\Sigma)]\ ,$$

where

$$c\equiv s_D-m_c^2\ ,$$

$$\Sigma\equiv s_D+s_K-t\ ,$$

$$\lambda\equiv s_D^2+s_K^2+t^2-2s_D s_K-2s_D t-2s_K t\ .$$

The nonperturbative contributions which survive the Borel transformation in p_D^2 and p_K^2 are

$$\Gamma_0^{(3)}=-\frac{m_c m_s}{cs}-\frac{m_c^2-t}{2cs}\ ,$$

$$\Gamma_+^{(3)}=-\frac{1}{2cs}\ ,$$

$$\Gamma_V^{(3)}=-\frac{1}{cs}\ ,$$

$$\Gamma_0^{(5)}=-\frac{1}{6cs}+\frac{(2m_c^2+3m_c m_s-2t)(m_c^2-t)}{12c^2s^2}+\frac{2m_c^2+3m_c m_s-2t}{12cs^2}+\frac{3m_c^2+9m_c m_s-4t}{12c^2s}+m_c^2\frac{m_c^2+2m_c m_s-t}{4c^3s}\ ,$$

$$\Gamma_+^{(5)}=-\frac{1}{6c^2s}+\frac{m_c^2}{4c^3s}+\frac{2m_c^2-m_c m_s-2t}{12c^2s^2}\ ,$$

$$\Gamma_V^{(5)}=\frac{1}{3c^2s}+\frac{m_c^2}{2c^3s}+\frac{2m_c^2-m_c m_s-2t}{6c^2s^2}\ ,$$

$$\Gamma_-^{(6)_1}=m_c\left[-\frac{1}{9cs^2}-\frac{1}{9c^2s}-\frac{3m_c^2-2t}{36c^3s}+\frac{m_c^2-t}{6c^2s^2}+m_c^2\frac{m_c^2-t}{12c^4s}+\frac{(m_c^2-t)^2}{18c^3s^2}\right]\ ,$$

$$\Gamma_0^{(6)_2}=m_c\left[-\frac{4}{9cs^2}+\frac{4}{9s^2(m_c^2-t)}+\frac{4}{9cs(m_c^2-t)}\right]\ ,$$

$$\Gamma_+^{(6)_1}=m_c\left[-\frac{1}{9c^2s^2}-\frac{2}{9c^3s}+\frac{m_c^2}{12c^4s}+\frac{m_c^2-t}{18c^3s^2}\right]\ ,$$

$$\Gamma_+^{(6)_2}=0\ ,$$

$$\Gamma_V^{(6)_1}=m_c\left[\frac{2}{9c^2s^2}-\frac{2}{9s^3s}+\frac{m_c^2}{6c^4s}+\frac{m_c^2-t}{9c^3s^2}\right]\ ,$$

$$\Gamma_V^{(6)_2}=-\frac{4m_c}{9cs^2(m_c^2-t)}\ ,$$

with

$$c\equiv p_D^2-m_c^2\ ,$$

$$s\equiv p_K^2\ .$$

Coefficients $\Gamma_i^{(6)_1}$ multiply the condensate $\frac{1}{4}g^2\langle\bar{d}\gamma_\mu\lambda^A d\Sigma_{u,d,s}\bar{q}\gamma^\mu\lambda^A q\rangle$, coefficients $\Gamma_i^{(6)_2}$ the condensate $g^2\langle\bar{d}d\rangle\langle\bar{s}s\rangle$.

The perturbative and nonperturbative contributions to Π_+, first given in [17], are

$$\rho_+^C=\frac{3}{8\pi^2\lambda^{3/2}}\{m_c[2c(\Sigma-s_K)+s_K(\Sigma-4s_D)]+m_s(\Sigma c-2s_D s_K)\}$$

$$-\frac{3}{4\pi^2\lambda^{5/2}}m_c[c^2(\Sigma^2-3s_K\Sigma+2s_D s_K)+2cs_K(\Sigma^2-3s_D\Sigma+2s_D s_K)+3s_D s_K^2(2s_D-\Sigma)]\ ,$$

$$\Pi_+^{(3)}=-\frac{1}{2cs},\quad \Pi_+^{(5)}=\frac{1}{6c^2s^2}+\frac{m_c^2}{4c^3s}-\frac{m_c m_s+2t}{12c^2s^2}\ ,$$

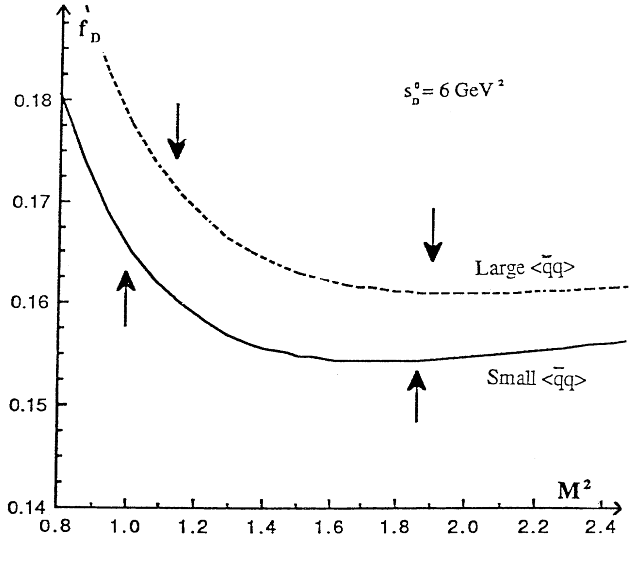

FIG. 11. f_D as calculated from the two-point sum rule for two different values of the quark condensate; contributions taken into account stated in Appendix B. The arrows indicate the limits of the admissible values of the Borel parameter.

with c, Σ, and λ defined as above. The Borel-improved coefficients can easily be obtained by applying the transformations

$$\frac{1}{c^n} \to \frac{1}{(n-1)!}(-1)^n \frac{1}{(M_D^2)^n} e^{-m_c^2/M_D^2} ,$$

$$\frac{1}{s^n} \to \frac{1}{(n-1)!}(-1)^n \frac{1}{(M_K^2)^n} ,$$

$$\int ds' \frac{\rho(s')}{s'-s} \to \frac{1}{M^2} \int ds' \rho(s') e^{-s'/M^2} ,$$

where $M_{K,D}$ are the Borel parameters.

APPENDIX B. THE TWO-POINT FUNCTION FOR THE *D* MESON

For the sake of consistency we use a value of the D-meson decay constant f_D determined from a two-point sum rule with exactly the same ingredients as used for the three-point sum rule, i.e., the bare quark loop and nonperturbative contributions including the quark, the mixed, and the four-quark condensate. In addition to fixing the value of f_D, the two-point sum rule determines the continuum threshold s_D^0 and the range of the Borel parameter M_D^2 to be used in the three-point sum rule, as mentioned in Sec. IV. Using the formulas given in [34], for example, we find the curves $f_D(M_D^2)$ depicted in Fig. 11, which correspond to two different choices of the quark condensate. The working region of M_D^2, which is restricted by the condition that neither the continuum nor the nonperturbative contributions exceed 50% of the value f_D^2, is indicated by the arrows. The curves are given for the continuum threshold $s_D^0 = 6\ \mathrm{GeV}^2$, where the stability of the sum rule is best. We find $f_D = 160 \pm 6$ MeV with a smaller value of the quark condensate, $\langle \bar{q}q \rangle (1\ \mathrm{GeV}) = (-230\ \mathrm{MeV})^3$ within the working region $1.00 \le M_D^2 \le 1.85\ \mathrm{GeV}^2$, and $f_D = 165 \pm 5$ MeV within $1.15 \le M_D^2 \le 1.90\ \mathrm{GeV}^2$ using the larger value $\langle \bar{q}q \rangle$ $(1\ \mathrm{GeV}) = (-250\ \mathrm{MeV})^3$. The values of all other parameters used are given in Sec. IV. We want to point out that these values of f_D must not be considered as a new determination of the D-meson decay constant. They only serve to diminish the influence of contributions not taken into account in our approach such as radiative corrections to the perturbative contribution.

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