

$$-\lim_{\eta \rightarrow 0} \operatorname{Re} \left[\frac{\beta}{i\hbar} \int d\omega \int d\omega' \frac{1}{\omega - \beta\mu' + i\eta} \frac{1}{\omega + \beta(\Delta - \mu) + i\eta} \frac{f^-(\omega) f^-(\omega')}{\omega + \omega' - \beta\Delta + i\eta} \right], \quad \rightsquigarrow -\lim_{\eta \rightarrow 0} \operatorname{Re} \left[\frac{\beta}{i\hbar} \int d\omega \int d\omega' \frac{1}{\omega - \beta\mu + i\eta} \frac{1}{\omega + \beta\mu' - i\eta} \frac{f^-(\omega) f^-(\omega')}{\omega + \omega' - \beta\Delta + i\eta} \right],$$

is to be considered. To ensure convergence for the secular case $\mu + \mu' = \Delta$, the expression is regularised in the following way [53, 54]:

$$\int d\omega \frac{g(\omega)}{\omega^2 + \eta^2} = \int d\omega \frac{g(0)}{\omega^2 + \eta^2} + \int d\omega \frac{g(\omega) - g(0)}{\omega^2 + \eta^2} = \underbrace{\frac{\pi}{\eta} g(0)}_{=\mathcal{O}(\eta^{-1})} + \int' d\omega \frac{g(\omega)}{\omega^2 + \eta^2},$$

where \int' denotes a principal part integration. The contribution $\propto 1/\eta$ is assigned to sequential tunnelling processes and hence must be disregarded. One obtains

$$\begin{aligned} \tilde{C}_{(2)}^{\text{Tmat}}(\Delta - \mu', \mu', \Delta) \\ = \frac{\beta}{\hbar} \operatorname{Re} \left(i \int d\omega \int d\omega' \frac{1}{\omega - \beta\mu' + i\eta} \frac{1}{\omega - \beta\mu' - i\eta} \frac{f^-(\omega) f^-(\omega')}{\omega + \omega' - \beta\Delta + i\eta} \right) - \mathcal{O}(\eta^{-1}) = \end{aligned}$$

is to be considered. To ensure convergence for the secular case $\mu + \mu' = 0$, the expression is regularised in the following way [53, 54]:

$$\int d\omega \frac{g(\omega)}{\omega^2 + \eta^2} = \int d\omega \frac{g(0)}{\omega^2 + \eta^2} + \int d\omega \frac{g(\omega) - g(0)}{\omega^2 + \eta^2} = \underbrace{\frac{\pi}{\eta} g(0)}_{=\mathcal{O}(\eta^{-1})} + \int' d\omega \frac{g(\omega)}{\omega^2 + \eta^2},$$

where \int' denotes a principal part integration. The contribution $\propto 1/\eta$ is assigned to sequential tunnelling processes and hence must be disregarded. One obtains

$$\begin{aligned} \tilde{C}_{(2)}^{\text{Tmat}}(\mu, -\mu, \Delta) \\ \rightsquigarrow \rightsquigarrow \frac{\beta}{\hbar} \operatorname{Re} \left(i \int d\omega \int d\omega' \frac{1}{\omega - \beta\mu + i\eta} \frac{1}{\omega - \beta\mu - i\eta} \frac{f^-(\omega) f^-(\omega')}{\omega + \omega' - \beta\Delta + i\eta} \right) - \mathcal{O}(\eta^{-1}) = \end{aligned}$$

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$$\begin{aligned} &= \frac{\pi\beta}{\hbar} \int' d\omega \frac{f^-(\omega) f^+(\omega - \beta\Delta)}{(\omega - \beta\mu')^2 + \eta^2} = \frac{\pi\beta}{\hbar} b^-(\beta\Delta) \frac{d}{d(\beta\mu')} \int' d\omega \frac{1 - f^-(\omega) - f^+(\omega - \beta\Delta)}{\omega - \beta\mu'} \\ &= -\frac{\beta}{2\hbar} b^-(\beta\Delta) \left(\operatorname{Im} \Psi^{(1)} \left(\frac{1}{2} + \frac{i\beta\mu'}{2\pi} \right) + \operatorname{Im} \Psi^{(1)} \left(\frac{1}{2} + \frac{i\beta(\Delta - \mu')}{2\pi} \right) \right). \quad (47a) \end{aligned}$$

This result has to be compared with the correct expression for $\tilde{C}_{(2)}$ as given in Eq. (130). In the limit $\mu + \mu' = \Delta$.

$$\begin{aligned} \hbar \tilde{C}_{(2)}(\Delta - \mu', \mu', \Delta) &= -\frac{\beta}{2} (b^-(\beta\Delta) + f^-(\beta\Delta - \beta\mu')) \operatorname{Im} \Psi^{(1)} \left(\frac{1}{2} + \frac{i\beta\mu'}{2\pi} \right) \\ &\quad - \frac{\beta}{2} (b^-(\beta\Delta) + f^-(\beta\mu')) \operatorname{Im} \Psi^{(1)} \left(\frac{1}{2} + \frac{i\beta(\Delta - \mu')}{2\pi} \right). \quad (47b) \end{aligned}$$

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$$\begin{aligned} &= \frac{\pi\beta}{\hbar} \int' d\omega \frac{f^-(\omega) f^+(\omega - \beta\Delta)}{(\omega - \beta\mu)^2 + \eta^2} = \frac{\pi\beta}{\hbar} b^-(\beta\Delta) \frac{d}{d(\beta\mu)} \int' d\omega \frac{1 - f^-(\omega) - f^+(\omega - \beta\Delta)}{\omega - \beta\mu} \\ &= -\frac{\beta}{2\hbar} b^-(\beta\Delta) \left(\operatorname{Im} \Psi^{(1)} \left(\frac{1}{2} + \frac{i\beta\mu}{2\pi} \right) + \operatorname{Im} \Psi^{(1)} \left(\frac{1}{2} + \frac{i\beta(\Delta - \mu)}{2\pi} \right) \right). \quad (47a) \end{aligned}$$

This result has to be compared with the correct expression for $\tilde{C}_{(2)}$ as given in Eq. (130). In the limit $\mu + \mu' = 0$:

$$\begin{aligned} \hbar \tilde{C}_{(2)}(\mu, -\mu, \Delta) &= \beta^+ X_{++}^{--}(\Delta - \mu, \mu, \Delta) \\ \rightsquigarrow &= -\frac{\beta}{2} \left(b^-(\beta\Delta) \{ \tilde{\Psi}_1(\mu) + \tilde{\Psi}_1(\Delta - \mu) \} + f^-(\beta\Delta - \beta\mu) \tilde{\Psi}_1(\mu) + f^-(\beta\mu) \tilde{\Psi}_1(\Delta - \mu) \right). \quad (47b) \end{aligned}$$