Conditions for Adiabatic Spin Transport in Disordered Systems

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We address the controversy concerning the necessary conditions for the observation of Berry phases in disordered mesoscopic conductors. For this purpose we calculate the spin-dependent conductance of disordered two-dimensional structures in the presence of inhomogeneous magnetic fields. Our numerical results show that for both, the overall conductance and quantum corrections, the relevant parameter defining adiabatic spin transport scales with the square root of the number of scattering events, in generalization of Stern's original proposal [1]. This could hinder a clear-cut experimental observation of Berry phase effects in diffusive metallic rings.

In contrast to phenomena related to Aharonov-Bohm (AB) phases [2] for charge carriers, the corresponding observation of Berry phases [3] due to the coupling of a spin to an orientationally nonuniform magnetic field B requires the limit of *adiabatic* spin evolution. In mesoscopic conductors, such a limit corresponds to the situation where the carrier spin can follow the spatially varying field during transport through the system. In terms of time scales, the adiabatic limit is reached when the Larmor frequency of spin precession, $\omega_{\rm s} = 2\mu B/\hbar$, is large compared to the reciprocal of a characteristic time scale $t_{\rm c}$ on which, from the point of view of the spin, the direction of the field has changed significantly during motion. There is consensus that for *ballistic* (disorder-free) systems with magnetic field configurations commonly theoretically considered [1,4–6] and experimentally realized [7], it holds $t_{\rm c} \sim L/v_{\rm F}$, where $v_{\rm F}$ denotes the Fermi velocity of the carriers, and L is the characteristic length scale of the system over which the field changes. For one-dimensional (1d) ballistic systems the condition for adiabaticity, $\omega_{\rm s} \gg 2\pi/t_{\rm c}$, therefore reads [8]

$$Q_{\rm 1d} \equiv \frac{\omega_{\rm s}}{2\pi v_{\rm F}/L} \gg 1 \,, \qquad (1)$$

where we introduced the adiabaticity parameter $Q_{\rm 1d}$.

However, in the case of *disordered* systems there are two candidates for the characteristic time $t_{\rm c}$: (i) the mean elastic scattering time τ and (ii) the Thouless time $t_{\rm Th} = (L/\ell)^2\tau$, with $\ell = v_{\rm F}\tau$ the elastic mean free path. The issue which of these two time scales is the relevant one has recently led to a controversial discussion [9,10].

In his proposal for 1d diffusive rings Stern [1] perturbatively calculated the lifetime of the adiabatic eigenstates and compared it to $t_{\rm Th}$. He arrived at the condition

$$Q_{\rm 1d} \gg L/\ell \qquad (2)$$

for adiabatic spin transport (Eq. (7) in Ref. [1]). This corresponds to setting $t_c = \tau$. Comparing Eqs. (1) and (2) one recognizes that in the diffusive regime, $L \gg \ell$, the adiabatic limit would require a magnetic field L/ℓ times larger than in the ballistic case. This "pessimistic criterion", which later has also been advocated by van Langen *et al.* [9], would imply field strengths in the quantum Hall regime that let an experimental observation of Berry phases in diffusive metallic rings appear rather unlikely.

Alternatively, in analogy to the ballistic travelling time L/v_F, it appears convincing to associate $t_{\rm c}$ for diffusive systems with $t_{\rm Th}$, the time it takes the electron to *diffuse* through the structure. This argumentation has been put forward by Loss *et al.* [4,10,11]. By calculating the quantum corrections of the conductance in diffusive 1d rings they predicted clear signatures of Berry's phase to be observable in a regime given by

$$Q_{\rm 1d} \gg \ell/L \,. \qquad (3)$$

This condition for adiabaticity differs from criterion (2) by a factor $(L/\ell)^2$ and predicts for the observability of Berry phases a field strength above 20 mT [11], which is well in reach of modern experimental techniques [7].

In view of various recent experimental efforts to observe Berry phases in the magneto conductance of mesoscopic rings [7,12–14], a clarification of this issue of the relevant time scale is desirable. The derivations of the conditions (2) and (3) were based on diagrammatic and semiclassical techniques. Here we choose a different approach and study numerically the spin-dependent conductance of ballistic and disordered mesoscopic systems in the presence of a spatially varying magnetic field $\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$. The Hamiltonian for noninteracting electrons with effective mass m^* and charge $-e$ reads

$$H = \frac{1}{2m^*}\left[\vec{p} + \frac{e}{c}\vec{A}(\vec{r})\right]^2 + V(\vec{r}) + \mu\vec{B}(\vec{r})\cdot\vec{\sigma}. \qquad (4)$$

The nontrivial coupling of the spin to the magnetic field enters via the Zeeman term $\mu\vec{B}(\vec{r})\cdot\vec{\sigma}$, where $\vec{\sigma}$ is the Pauli spin vector and $\mu = g^*e\hbar/(4m_0c)$ the magnetic moment with g^* the gyromagnetic ratio. The electrostatic potential $V(\vec{r})$ includes the confinement and the potential of random impurities in the disordered case. At $T = 0$ the spin-dependent conductance of a mesoscopic system with two attached leads is given by the Landauer formula [15]

$$G = \frac{e^2}{h} \sum_{s',s=\pm 1} T_{s's} = \frac{e^2}{h} \sum_{m',m=1}^{M} \sum_{s',s=\pm 1} |t_{s's}^{m'm}|^2 \ . \quad (5)$$

Here $t_{s's}^{m'm}$ is the transmission amplitude from an incoming channel m with spin s to an outgoing channel (m', s'). We calculate $t_{s's}^{m'm}$ by projecting the corresponding Green function matrix onto the asymptotic spinors in the leads. We compute the Green function for the Hamiltonian (4) numerically, using a generalized version of the recursive Green function technique based on a tight-binding model [15] including spin [16]. We model the (non-magnetic) disorder potential leading to elastic scattering within an Anderson model by chosing random delta-like scatterers with amplitudes following a box distribution. The spin-dependent conductance is then obtained from ensemble averages over independent disorder configurations [17].

We now turn to the subject of interest and study how adiabaticity is approached in mesoscopic spin quantum transport. For this purpose we introduce a model system consisting of a 2d strip with a rotating in-plane magnetic field between two ballistic leads, see Fig. 1. This system can also be regarded as a model for transport through magnetic domain walls. We assume incoming electrons with spin-down polarization in the $-y$ direction [18], injected from the left with Fermi wave number $k_{\rm F} = 2\pi/\lambda_{\rm F}$.
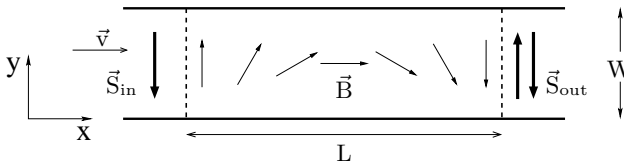

FIG. 1. 2d strip configuration used for the calculations of the spin-dependent conductance. The magnetic field $\vec{B}(x)$ performs a $180°$-rotation within the plane of the strip. Spin states $(\vec{S}_{\rm in}, \vec{S}_{\rm out})$ are defined with respect to the y-axis.

We first consider the overall conductance. In the ballistic case, it is feasible to derive an analytical expression for the spin-resolved transmission of this system using a transfer matrix approach. The normalized transmission $T_{\downarrow\downarrow}$ for spin-down polarized incoming electrons to exit the system with spin-down polarization reads [19]

$$T_{\downarrow\downarrow} \equiv \frac{1}{M} \sum_{m',m=1}^{M} |t_{-1,-1}^{m'm}|^2 = \sum_{m=1}^{M} \frac{\sin^2\left(\frac{\pi}{2}\sqrt{1+Q_m^2}\right)}{M(1+Q_m^2)} \quad (6)$$

with the generalized adiabaticity parameter (Eq. (1)),

$$Q_m \equiv \frac{g^*}{k_{\rm F}W} \frac{m^*}{m_0} \left(\frac{LWB}{hc/e}\right) \left[1 - \left(\frac{m\pi}{k_{\rm F}W}\right)^2\right]^{-1/2} , \quad (7)$$

for the m-th propagating mode in a 2d strip of length L and width W. Summing over all transverse modes in Eq. (6) we find that the overall dependence of the

ballistic transmission (dashed lines in Fig. 2) is given by a Lorentzian $T_{\downarrow\downarrow} \simeq 1/(1+Q^2)$ (dotted lines). This defines an 'effective' adiabaticity parameter $Q \sim B$ for the 2d strip, with $Q_1 < Q < Q_M$ and $Q \sim 1.4\, Q_{\rm 1d}$. This allows us to introduce a quantity that solely characterizes the adiabatic regime in the case of several open channels.
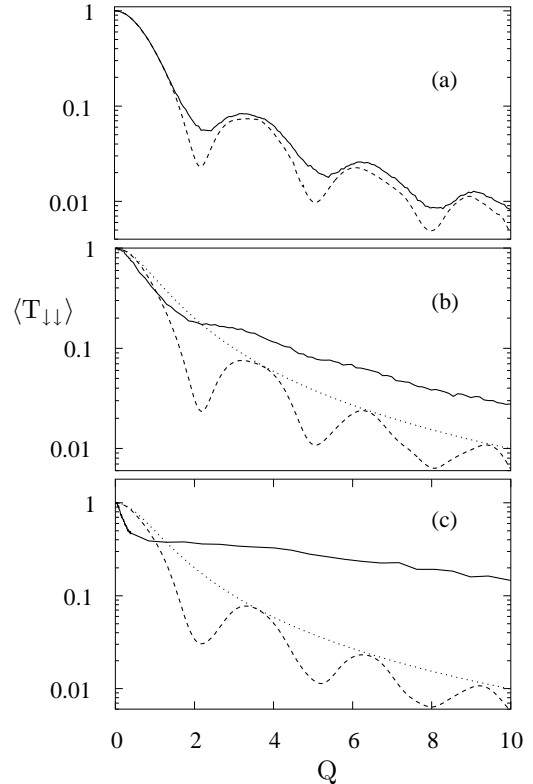

FIG. 2. Ensemble averaged normalized transmission $\langle T_{\downarrow\downarrow}\rangle$ for a disordered (solid lines) and ballistic (dashed) 2d strip (Fig. 1) as a function of the adiabaticity parameter $Q \sim B$. The panels correspond to different disorder strengths: (a) quasi-ballistic: $L/\ell = 0.5$ ($L/W = 4.4$), (b) moderate: $L/\ell = 3$ ($L/W = 7.8$), and (c) diffusive: $L/\ell = 10$ ($L/W = 13.9$). The ballistic curves, Eq. (6), differ slightly from each other since they correspond to different Fermi wave vectors (ranging from $k_{\rm F}W/\pi = 7.7$ to 11.6) but show the same overall Lorentzian decay with Q (dotted).

For $B \to 0$ the spin direction is preserved and $T_{\downarrow\downarrow}$ is maximal. In the limit of a strong B-field the spin stays adiabatically aligned with the orientationally inhomogeneous field during transport, minimizing the probability of leaving the conductor in Fig. 1 in a spin-down state. The Lorentzian dependence of $T_{\downarrow\downarrow}$ on $Q \sim B$ reflects this behavior and appears as the natural measure for the crossover from the non-adiabatic ($T_{\downarrow\downarrow} \to 1, Q \ll 1$) to the adiabatic ($T_{\downarrow\downarrow} \to 0, Q \gg 1$) regime.

To find a proper condition for adiabaticity in the disordered case we compute the ensemble averaged transmission $\langle T_{\downarrow\downarrow}\rangle$ in the presence of elastic scattering for $\lambda_{\rm F} \ll \ell$ as a function of Q and compare it to the ballistic result (6). Our results for different ratios L/ℓ are shown as

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the solid lines in Fig. 2 exhibiting the following features: (i) The oscillations in the ballistic transmission are averaged out with increasing disorder. (ii) For $Q \gg 1$, the normalized $\langle T_{\downarrow\downarrow}\rangle$ is larger in the disordered than in the ballistic case. This means that in the presence of elastic scattering a stronger scaled field Q is required for acceding to the adiabatic regime of $\langle T_{\downarrow\downarrow}\rangle \approx 0$. (iii) For $Q \ll 1$ we observe the opposite behavior: The non-adiabatic limit of almost maximum transmission $\langle T_{\downarrow\downarrow}\rangle$ is restricted to lower magnetic fields compared to the ballistic case. (iv) The crossover region $Q \sim 1$ is characterized by a transmission plateau, which approaches $\langle T_{\downarrow\downarrow}\rangle \approx \langle T_{\uparrow\downarrow}\rangle \approx 0.5$ with increasing diffusiveness. Here, the non-magnetic disorder acts as a spin randomizer of the originally spin-polarized current. The features (i)-(iv) already begin to appear in the quasi-ballistic regime (Fig. 2(a)) and become more pronounced with increasing degree of diffusiveness given by the ratio L/ℓ (Fig. 2(c)).

After this qualitative discussion we now derive a quantitative condition for adiabaticity in the disordered strip. In analogy to the ballistic case we expect the disorder averaged transmission $\langle T_{\downarrow\downarrow}\rangle$ to exhibit a scaled Lorentzian dependence in the limits of small and large Q. Indeed, in the adiabatic limit the Lorentz function is an excellent fit to the diffusive curve $\langle T_{\downarrow\downarrow}\rangle$ in Fig. 2(c), if the ballistic parameter Q is replaced by $Q/\sqrt{N_{\text{ad}}}$ with N_{ad} being fitted (left inset Fig. 3). Corresponding results hold for the nonadiabatic limit where we use the scaling $Q\sqrt{N_{\text{nad}}}$ (right inset in Fig. 3). We further determined N_{ad} and N_{nad} for various ratios L/ℓ and obtain power-law dependences illustrated in Fig. 3. We hence can formulate as a necessary condition for adiabatic spin transport through the disordered 2d strip: $Q \gg (L/\ell)^{0.95}$. Comparing this with Eq. (2) we obtain a little smaller exponent. To explain this deviation we note that Eq. (2) can be written in the more general form $Q \gg \sqrt{\langle N\rangle}$, with the average number of scattering events $\langle N\rangle = \langle t_{\text{Th}}\rangle/\tau = (L/\ell)^2$. This suggests to associate N_{ad} and N_{nad} with the number of scattering events the electron has to undergo upon traversing the microstructure. Due to the strong coupling of the finite-size 2d strip to the ballistic leads we expect the diffusion time to be smaller than the Thouless time t_{Th}, thus reducing the number of scattering events.

To confirm the above arguments we independently checked numerically the dependence of $\langle N\rangle$ on the scaled length L/ℓ of a finite disordered conducting strip. To this end we used a 1d [20] random walk model taking explicitly into account the interface between disordered and ballistic regions. We find that $\langle N\rangle$ obeys a power law in L/ℓ with exponent 1.91 which, as expected, is lower than two (dotted line in Fig. 3). In Fig. 3, besides small deviations for small L/ℓ in the non-diffusive limit, there is good agreement with the fitted straight lines for $L/\ell \gtrsim 5$, indicating diffusive behavior. Within the given error tolerance all three curves in Fig. 3 exhibit identical exponents and deviate only in the prefactor of order one.

We conclude from our numerical, quantum mechanical results, together with the expression for $\langle N\rangle$ from the independent random-walk model, that the adiabaticity parameter scales with $\sqrt{\langle N\rangle}$. This enables us to formulate a *general* (system-independent) adiabaticity condition for diffusive systems, that only depends on the corresponding adiabaticity parameter Q of the ballistic system and the mean number of scattering events $\langle N\rangle$ [21]:

$$Q \gg \sqrt{\langle N\rangle}\,. \tag{8}$$

For diffusive 1d rings this criterion is in perfect agreement with Stern's original condition (2).
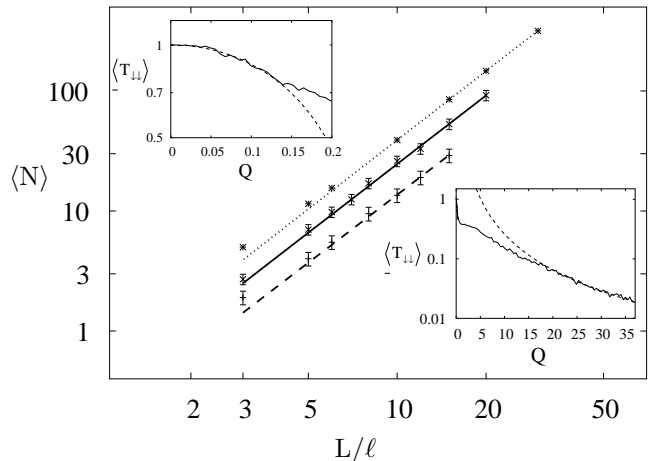

FIG. 3. Functional dependence of fit parameters N_{ad} (solid line) and N_{nad} (dashed) on the scaled length L/ℓ of the disordered strip. Linear regression yields $N_{\text{ad}} = 0.31\,(L/\ell)^{1.9}$ (solid line) and $N_{\text{nad}} = 0.18\,(L/\ell)^{1.88}$ (dashed). For comparison the mean number of scattering events $\langle N\rangle = 0.48\,(L/\ell)^{1.91}$ is also shown (dotted) obtained from an independent 1d random walk model. Insets: Transmission $\langle T_{\downarrow\downarrow}\rangle$ and fitted Lorentzians in the nonadiabatic (left) and adiabatic (right) limit for strip with $L/\ell = 10$. (Error bars include uncertainty from fitting procedure.)

So far we considered the total conductance which is dominated by the Boltzmann contribution. However, signatures of Berry phases in diffusive conductors appear only in the phase coherent part of the conductance, i.e. quantum corrections such as Aharonov-Bohm (AB) oscillations and universal conductance fluctuations (UCFs). To decide whether distinct Berry phase effects, e.g. in diffusive rings, can be observed at realistic magnetic field strengths, one has to check if an adiabaticity condition different from (8) holds for the quantum corrections. For this purpose, spin-resolved UCFs represent a suitable quantity, defined as $\delta g_{s's} = \sqrt{\langle T_{s's}^2\rangle - \langle T_{s's}\rangle^2}$ in units of e^2/h. We calculated $\delta g_{s's}$ numerically as a function of Q for a diffusive 2d strip with $L/\ell = 15$. The results are depicted in Fig. 4 in terms of the normalized difference $(\delta g_{\downarrow\downarrow} - \delta g_{\uparrow\downarrow})/(\delta g_{\downarrow\downarrow} + \delta g_{\uparrow\downarrow})$ which can be regarded as a polarization. We note that the UCFs exhibit precisely the

same scaling behavior as the corresponding quantity for the total conductance, $(T_{\downarrow\downarrow} - T_{\uparrow\downarrow})/(T_{\downarrow\downarrow} + T_{\uparrow\downarrow})$, and consequently obey condition (8). Fig. 4 also illustrates the important fact that in a wide region $\ell/L \lesssim Q \lesssim L/\ell$ the respective adiabatic ($T_{\uparrow\downarrow}$, $\delta g_{\uparrow\downarrow}$) and nonadiabatic ($T_{\downarrow\downarrow}$, $\delta g_{\downarrow\downarrow}$) components are comparable in magnitude.
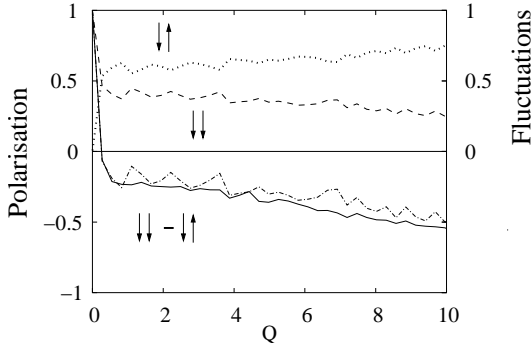

FIG. 4. Quantum fluctuations $\delta g_{\downarrow\downarrow}/(\delta g_{\downarrow\downarrow} + \delta g_{\uparrow\downarrow})$ (dashed) and $\delta g_{\uparrow\downarrow}/(\delta g_{\downarrow\downarrow} + \delta g_{\uparrow\downarrow})$ (dotted) as a function of Q for a diffusive strip with $L/\ell = 15$. For comparison we also show $(\delta g_{\downarrow\downarrow} - \delta g_{\uparrow\downarrow})/(\delta g_{\downarrow\downarrow} + \delta g_{\uparrow\downarrow})$ (dashed-dotted) and the polarization $(\langle T_{\downarrow\downarrow}\rangle - \langle T_{\uparrow\downarrow}\rangle)/(\langle T_{\downarrow\downarrow}\rangle + \langle T_{\uparrow\downarrow}\rangle)$ (solid).

We further note that numerical quantum calculations of the spin-dependent magneto-conductance in disordered rings subject to a circular inhomogeneous B-field show that signatures of Berry phases appear only in the AB-oscillations of the adiabatic components ($\langle T_{\uparrow\downarrow}\rangle$ and $\delta g_{\uparrow\downarrow}$), which are dominated by electrons with spin always aligned with the local field [22]. In view of Fig. 4 we hence can conclude for diffusive rings that in the experimentally relevant plateau region $\ell/L \lesssim Q < 1$ the adiabatic components, which show Berry phase signatures are of the same magnitude as the nonadiabatic components. Berry phase effects in the AB-oscillations of the total conductance and the UCF's are hence masked by the regular, nonadiabatic contribution [23]. However, in this broad plateau region one still finds effects of the inhomogeneous magnetic field that can be ascribed to the nonadiabatic generalization of the Berry phase, the Aharonov-Anandan phase [24]. Our numerical results imply that to observe clear Berry phase effects such as the 'magic angles' found by Engel *et al.* [11] in the magneto conductance and UCF's of diffusive rings one has to go to the truly adiabatic regime given by condition (8).

For a typical experimental AB setup based on copper rings with radius $r_0 = 500$ nm and $\ell = 15$ nm [12] strict application of the criterion (8) corresponds to B-field strengths larger than 10^3 T. On the other hand, according to the condition $Q \ll 1/\sqrt{\langle N\rangle}$, the opposite non-adiabatic regime ($\langle T_{\downarrow\downarrow}\rangle \sim 1$) is restricted to fields smaller than 0.1 T. In the broad intermediate B-field range, covering four orders of magnitude, the magneto conductance is expected to show at most signatures of the Aharonov-Anandan phase.

Recently, imprints of Berry's phase in AB oscillations have been reported for holes in quasi-ballistic 2d GaAs rings with strong spin-orbit interaction [14]. A rough estimate of the system parameters suggests that the experimental conditions may fulfill the adiabaticity criterion (8) with B replaced by an effective Rashba field strength.

To summarize we studied spin-dependent quantum transport through 2d disordered geometries. We showed that the relevant parameter defining the adiabatic limit both for the total conductance and the quantum corrections scales with the square root of the number of scattering events. This can be cast into a generalized criterion for adiabaticity for both, ballistic and disordered systems. It appears as a severe obstacle for direct experimental observation of Berry phases in the conductance through diffusive metal rings. Our numerical findings indicate that elastic scattering due to non-magnetic impurities in the presence of a spatially varying magnetic field exhibits features similar to those in systems with spin-flips associated with the scattering process as for magnetic impurities or Rashba spin orbit coupling.

[1] A. Stern, Phys. Rev. Lett. **68**, 1022 (1992).
[2] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[3] M. V. Berry, Proc. Roy. Soc. London A **392**, 45 (1984).
[4] D. Loss, H. Schöller, and P.M. Goldbart, Phys. Rev. B **48**, 15218 (1993).
[5] D. Frustaglia and K. Richter, Found. Phys. **31**, 399 (2001).
[6] D. Frustaglia, M. Hentschel, and K. Richter, Phys. Rev. Lett. **87**, 256602 (2001).
[7] Transport through ballistic and diffusive rings surrounding micromagnets has been reported by P.D. Ye, S. Tarucha, and D. Weiss, *Proceedings of the 24th ICPS, Jerusalem* (World Scientific, Singapore, 1998) and by T.M. Jacobs and N. Giordano, Superlattices Microstruct., **23**, 635 (1998), respectively.
[8] See Ref. [5] for adiabaticity in 2d ballistic structures.
[9] S.A. van Langen, H.P.A. Knops, J.C.J. Paasschens, and C.W.J. Beenakker, Phys. Rev. B **59**, 2102 (1999).
[10] D. Loss, H. Schöller, and P.M. Goldbart, Phys. Rev. B **59**, 13328 (1999).
[11] H.-A. Engel and D. Loss, Phys. Rev. B **62**, 10238 (2000).
[12] R. Häußler, PhD thesis, Universität Karlsruhe (1999).
[13] A. F. Morpurgo, J. P. Heida, T. M. Klapwijk, B. J. van Wees, and G. Borghs, *Phys. Rev. Lett.* **80**, 1050 (1998).
[14] J.-B. Yau, E.P. De Poortere, and M. Shayegan, *Phys. Rev. Lett.* **88**, 146801 (2002).
[15] See e.g. S. Ferry and S.M. Goodnick, *Transport in Nanostructures* (Cambridge Univ. Press, Cambridge, 1997).
[16] This has proven to be an efficient method for studying spin-dependent quantum transport, see Ref. [6] and D. Frustaglia, PhD thesis, TU Dresden (2001).

[17] Testing this model on a narrow wire we find quantitative agreement with predictions for quantum conductance corrections (weak localization, universal conductance fluctuations) and an exponential decrease of the conductance when approaching the localized regime.

[18] Equivalent results are obtained for spin-up electrons.

[19] See Ref. [6] for a related derivation and M. Popp, diploma thesis, Universität Regensburg (2002).

[20] The restriction to a 1d model is justified since we are dealing with narrow strips of width $W \lesssim \ell$.

[21] To gain further insight into the scaling law (8) we developed a phenomenological model based on a perturbative approach valid in the far adiabatic and non-adiabatic limit. Assuming that spin flips before and after a scattering event are independent processes we can derive a scaling behavior which coincides with condition (8) [22]. The ballistic limit then corresponds to $N = 1$.

[22] M. Popp, D. Frustaglia, and K. Richter, unpublished.

[23] This has been numerically confirmed for the magneto conductance in disordered ($L/\ell \approx 4$) rings subject to an inhomogeneous field [22].

[24] Y. Aharonov and J. Anandan, *Phys. Rev. Lett.* **58**, 1593 (1987).