Andreev magnetotransport in low-dimensional proximity structures: Spin-dependent conductance enhancement

Grygoriy Tkachov and Klaus Richter
Institute for Theoretical Physics, Regensburg University, 93040 Regensburg, Germany

(Dated: May 23, 2007)

We study the excess conductance due to the superconducting proximity effect in a ballistic two-dimensional electron system subject to an in-plane magnetic field. We show that under certain conditions the interplay of the Zeeman spin splitting and the effect of a screening supercurrent gives rise to a spin-selective Andreev enhancement of the conductance and anomalies in its voltage, temperature and magnetic field characteristics. The magnetic-field influence on Andreev reflection is discussed in the context of using superconducting hybrid junctions for spin detection.

PACS numbers: 74.45.+c, 73.23.Ad, 72.25.Dc

Advances in nanotechnology of semiconductor-superconductor junctions[1] have created a unique opportunity to investigate the interplay between superconducting phase-coherence and various electronic properties of low-dimensional semiconductors. Recent developments in this field include the realization of a long-range Josephson coupling mediated by Andreev reflection[2] of ballistic two-dimensional electrons and controlled by the injection of hot carriers[3,4], the observation of a giant proximity-induced enhancement of the conductance of two-dimensional electron systems (2DES)[5], theoretical[6] and experimental[7] studies of Andreev billiards, classical[8] and quantum[9,10] Andreev edge states in high magnetic fields.

In this paper we discuss a possibility of using the superconducting proximity effect for detecting the spin of transport carriers in low-dimensional systems, a problem closely related to the ongoing work on spin injection in semiconductors[11]. In nonmagnetic normal metal-superconductor (NS) junctions spin resolving transport measurements were first reported in Ref. 12 where the magnetic field spin splitting of the quasiparticle density of states in thin superconducting films served as an electron filter. Applied to ferromagnet-superconductor systems, this idea has developed into a sensitive technique of analyzing the spin polarization of ferromagnetic metals[13]. We note that the findings of Ref. 12 are specific to low-transparency tunnel junctions where the superconducting proximity effect is negligible and hence the electron transport is predominantly a quasiparticle one.

In structures with improved interfacial quality the penetration of the superconducting order parameter into the normal system is accompanied by the conversion of a quasiparticle current into a supercurrent via Andreev reflection which manifests itself as a low-bias conductance enhancement[14,15,16,17,18,19,20,21,22]. One of the most striking examples of the proximity effect occurs in ballistic semiconductor quantum wells (2DES) with a lateral superconducting contact [see Fig. 1(a)]. In this case the conductance in the plane of the quantum well can nearly twice exceed the normal-state value[5,7,10] due to a proximity-induced mixing of particle and hole states characterized by a superconducting minigap E_g in the excitation spectrum of the 2DES[18]. For such proximity structures we study the spin dependence of Andreev reflection arising from the Zeeman splitting of the gapped states in the 2DES subject to an in-plane magnetic field **B**.

Normally, a noticeable spin splitting in superconductors requires rather strong magnetic fields (above 1T)[12] of the order of the field $B_c \approx \Delta/\mu_B$ corresponding to the paramagnetic limit[23] (Δ and μ_B are the gap energy in the superconductor and the Bohr magneton). The advantage of the low-dimensional proximity structures is that the scale of relevant fields is set by the minigap, $B_g \approx (2E_g/g\Delta)B_c$, and hence can be much smaller than B_c both due to $E_g \ll \Delta$ and large electron g-factors (e.g. $|g| \approx 10 - 14$ in InAs/AlSb quantum wells[1,24]).

Apart from the smaller splitting fields, the magnetic field influence on ballistic proximity systems has one more specific feature. Because of the screening supercurrent generated by the field, the quasiparticles in the 2DES can acquire a significant Galilean energy shift arising from a finite Cooper pair momentum at the superconductor surface[25]. In ballistic quantum wells with weak momentum scattering the Galilean supercurrent effect does not average out, unlike in conventional diffusive superconductors[12,26,27], and therefore must be taken into account along with the Zeeman splitting. Here we discuss the influence of these two competing magnetic field effects on the voltage, temperature and magnetic-field characteristics of Andreev transport.

We consider a 2DES coupled to a superconducting film via a barrier of low transparency $\mathcal{T} \ll 1$ [Fig. 1(a)]. The film thickness d is assumed much smaller than both the superconducting coherence length and the London penetration depth in a parallel field $\mathbf{B} = [0; 0; B]$. In a ballistic quantum well the motion of particles and holes with energies smaller than Δ is coupled in the proximity region [$z > 0$ in Fig. 1(a)] via multiple Andreev reflections. In the quantum limit (2DES) the correlated particle-hole motion can be described in terms of the effective pairing energy which gives rise to the quasiparticle minigap $E_g \approx (v_F/v_{F_S})\mathcal{T}E_0 \ll \Delta$[18]. It depends on the energy E_0 of the lowest occupied subband, the transparency $\mathcal{T}$ and the ratio of the Fermi velocities in the normal (v_F)

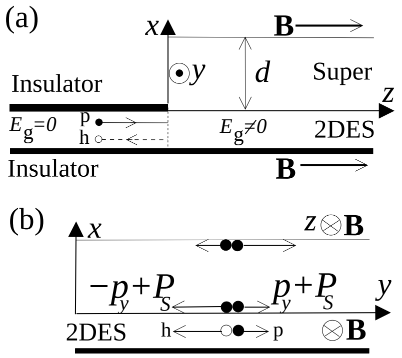

FIG. 1: (a) Cross section of a planar superconductor-2DES contact. At energies below the minigap E_g a particle (p) incident from the normal part of the 2DES is reflected as a hole (h) giving rise to conductance enhancement. (b) Schematic of the supercurrent flow in the xy plane. The particle and hole momenta in the 2DES are both shifted by $P_S = -(e/c)Bd/2$ in order to match the Cooper pair momentum $2P_S$.

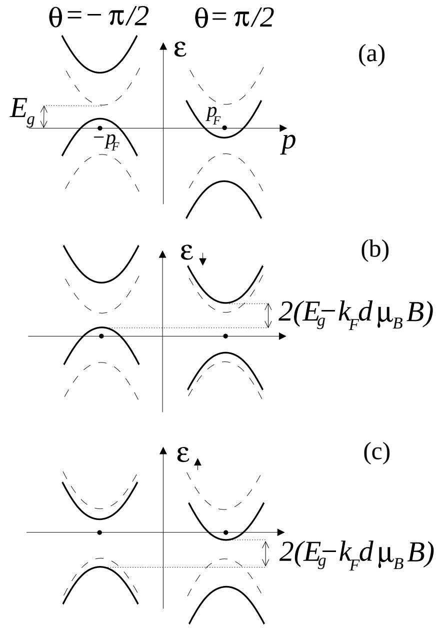

FIG. 2: Schematic view of the excitation spectrum (3) in the direction ($\theta = \pi/2$) and opposite ($\theta = -\pi/2$) to the supercurrent at $B > B_g$: (a) - for spinless quasiparticles ($\tilde{g} = 0$), (b) and (c) - for spin-down and spin-up ones with $\tilde{g} > 1$. Dashed curves correspond to $B = 0$.

and superconducting (v_{F_S}) systems. Accordingly, electron scattering from the proximity region can be treated using effective Bogolubov-de Gennes equations for the electron $u_\alpha(z)$ and hole $v_{-\alpha}(z)$ wave functions averaged over the thickness of the 2DES[28]:

$$\begin{bmatrix} \tilde{\epsilon} + \frac{\hbar^2\partial_z^2 + \tilde{p}_F^2}{2m} & -E_g\Theta(z) \\ E_g\Theta(z) & -\tilde{\epsilon} + \frac{\hbar^2\partial_z^2 + \tilde{p}_F^2}{2m} \end{bmatrix} \begin{bmatrix} u_\alpha(z) \\ v_{-\alpha}(z) \end{bmatrix} = 0. \quad (1)$$

Here ∂_z stands for a derivative; m, p_F, and $\alpha = \pm 1/2$ are respectively the electron mass, Fermi momentum, and spin; $\tilde{p}_F = (p_F^2 - p_y^2)^{1/2}$ where the parallel momentum $p_y = p_F \sin\theta$ depends on the angle $|\theta| \leq \pi/2$ measured from the z axis in the 2DES plane. As E_g scales with the transparency $\mathcal{T}$, there is virtually no "leaking" of electron pairing into the "normal" region [$z < 0$ in Fig. 1(a)] because it would require Cooper pair tunneling through the thick insulator. Hence we assume the step-like pairing energy $E_g\Theta(z)$ with $\Theta(z)$ being the Heaviside function.

A weak magnetic field can be taken into account by an energy shift:

$$\tilde{\epsilon} = \epsilon + \alpha g \mu_B B - v_F P_S \sin\theta, \quad P_S = -(e/c)Bd/2, \quad (2)$$

where the second term is the Zeeman energy whereas the third one is a Galilean energy arising from the shift of both particle and hole momenta which accounts for a screening supercurrent generated in the superconductor by a parallel magnetic field [see Fig. 1(b)]. As the thickness of the 2DES is considered negligible compared to d, the momentum shift of electrons and holes in the 2DES can be taken equal to the surface Cooper pair momentum per electron, P_S, in Eq. (2) ($e > 0$ is the elementary charge). It is proportional to half of the film thickness reflecting the fact that the field fully penetrates the film and generates an antisymmetric (linear) distribution of the supercurrent density with respect to its middle plane.

For $|\tilde{\epsilon}| \ll E_F = p_F^2/2m$ the magnetic field effect on the quasiparticle energies is important only in the proximity region. Indeed, from Eqs. (1) and (2) one finds the excitation spectrum in this region as

$$\epsilon^\pm_{\alpha p \theta} = -(\alpha g + k_F d \sin\theta)\mu_B B \pm [v_F^2(p - p_F)^2 + E_g^2]^{\frac{1}{2}}, \quad (3)$$

where p is the absolute value of the momentum, and $k_F = p_F/\hbar$. The magnetic field shifts the energies (3) with respect to the Fermi level so that at a certain field

$$B_g = E_g/[\mu_B k_F d(\tilde{g} + 1)], \qquad \tilde{g} = g/2k_F d, \quad (4)$$

the excitations with momenta (anti)parallel to the supercurrent ($\theta = \pm\pi/2$) become gapless. We point out that in Eqs. (3) and (4) the interplay of the Zeeman and supercurrent effects is controlled by a single material- and geometry-dependent parameter $\tilde{g}$. This is shown in Fig. 2 where we sketch the p-dependence of the energies $\epsilon^\pm_{\alpha p \theta}$ near the Fermi points $\pm p_F$ in the direction ($\theta = \pi/2$) and opposite ($\theta = -\pi/2$) to the supercurrent. For $\tilde{g} \ll 1$ (thick superconductors or small g-factors) the quasiparticles can be treated as spinless, and the minigap vanishes as soon as the dispersion curves cross the Fermi energy simultaneously at $\pm p_F$ [Fig. 2(a)]. For $\tilde{g} > 1$ (thin superconductors and/or large g-factors) the Zeeman splitting preserves the minigap at finite energies even after the appearance of gapless excitations at the Fermi level [Figs. 2(b) and (c)]. The gap remains open in the field range $1 < B/B_g < 1 + \tilde{g}$, rather broad for $\tilde{g} > 1$.

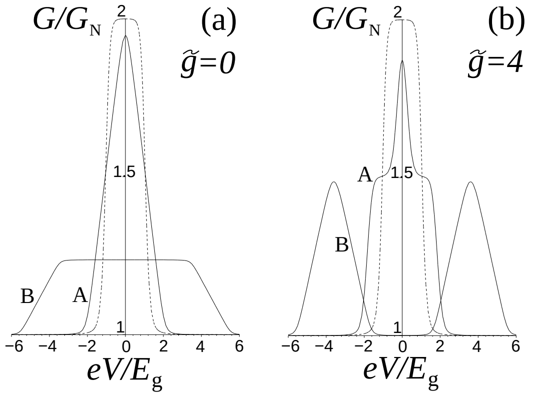

FIG. 3: Conductance versus voltage for (a) spinless and (b) spin-polarized electrons at magnetic fields B/B_g: 0 (dashed), 1 (A) and 4.5 (B); $k_BT/E_g = 0.1$.

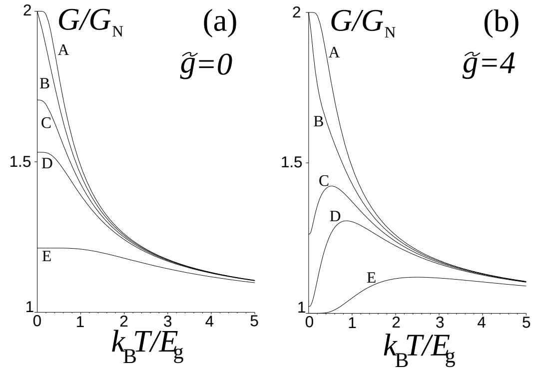

FIG. 4: Zero-bias conductance versus temperature for (a) spinless and (b) spin-polarized electrons at magnetic fields B/B_g: 0 (A), 1 (B), 1.5 (C), 2 (D), and 5 (E).

Due to the superconducting proximity effect the current of quasiparticles entering from the normal region [$z < 0$ in Fig. 1(a)] can be converted into a supercurrent via Andreev reflection at low energies $|\tilde{\epsilon}| < E_g$. The efficiency of the Andreev conversion is insured by the lack of a barrier between the normal and proximity-affected regions of the 2DES. That is why for calculating the conductance we can use the solution of the scattering problem of Ref. 29 applied to an ideal NS interface. In reality, normal reflection does occur because the contact to a metal slightly modifies the quantum well resulting in a mismatch of the Fermi momenta[7] at $z = 0$. However, for $\mathcal{T} \ll 1$ this mismatch is rather weak. We have checked that it has a small effect on the zero-field conductance while we have found no significant difference at finite fields $B \sim B_g$, the regime we are interested in.

In the absence of normal scattering the probability $A(\tilde{\epsilon})$ for a particle to be reflected from the proximity region as a hole is given within the usual approximation $|\tilde{\epsilon}|, E_g \ll E_F \cos^2\theta$ by[29]

$$A(\tilde{\epsilon}) = \Theta\left(E_g - |\tilde{\epsilon}|\right) + \Theta\left(|\tilde{\epsilon}| - E_g\right) \frac{|\tilde{\epsilon}| - [\tilde{\epsilon}^2 - E_g^2]^{\frac{1}{2}}}{|\tilde{\epsilon}| + [\tilde{\epsilon}^2 - E_g^2]^{\frac{1}{2}}}. \quad (5)$$

The expression for the conductance reads[30]

$$\frac{G(V,B,T)}{G_N} = 1 + \frac{1}{4}\sum_{\alpha}\int d\epsilon \left(-\frac{\partial f(\epsilon - eV)}{\partial \epsilon}\right) \times$$
$$\times \int_{-\pi/2}^{\pi/2} d\theta \cos\theta A\left[\epsilon + (\alpha g + k_F d \sin\theta)\mu_B B\right], \quad (6)$$

where G_N is the conductance of the normal 2DES and $f(\epsilon - eV)$ is the Fermi distribution of the incident electrons at bias energy eV. Since the θ-dependence enters through the energy shift in A, the double integral in Eq. (6) can be easily reduced to a single one:

$$\frac{G(V,B,T)}{G_N} = 1 + \sum_{\alpha}\int_{-\infty}^{\infty} \frac{d\tilde{\epsilon}\, A(\tilde{\epsilon})}{4k_F d\mu_B B} \times \quad (7)$$
$$\times \left[f\left(\tilde{\epsilon} - eV - (\alpha g + k_F d)\mu_B B\right) - \right.$$
$$\left. - f\left(\tilde{\epsilon} - eV - (\alpha g - k_F d)\mu_B B\right)\right].$$

Figure 3(a) shows the suppression of the excess conductance peak due to the vanishing of the minigap for spinless electrons as the field exceeds B_g. In contrast to this, for spin-polarized electrons the zero-bias anomaly splits into two twice smaller peaks [Fig. 3(b)] because the spin-dependent minigap in the excitation spectrum, Eq. (3), still supports Andreev reflection of spin-down particles for $V > 0$ and spin-up holes for $V < 0$. In other words, for a given voltage only quasiparticles of one spin-orientation give rise to the conductance enhancement. The spin-dependent Andreev process persists until the minigap closes at $B = B_g(1 + \tilde{g})$ followed by an overall decrease in the excess conductance at higher fields.

The temperature dependence of the zero-bias conductance, shown in Fig. 4(b), reveals one more manifesta-

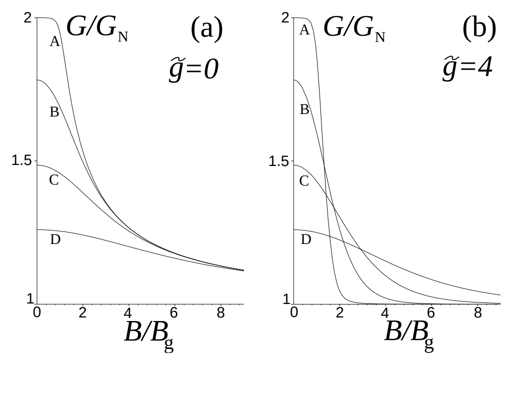

FIG. 5: Zero-bias magnetoconductance for (a) spinless and (b) spin-polarized electrons at temperatures $T/(k_B^{-1}E_g)$: 0.1 (A), 0.5 (B), 1 (C), and 2 (D).

tion of the magnetic spin splitting: a maximum in $G(T)$ in contrast to a monotonic decrease for spinless electrons [Fig. 4(a)]. It is due to the shift of the minigap to finite energies where Andreev reflection is mediated by thermally excited quasiparticles. The corresponding behaviour of the zero-bias magnetoconductance at different temperatures is shown in Figs. 5(a) and (b). The energy and B-field anomalies discussed above are already well-resolved when $\tilde{g}$ exceeds 1.

To conclude we have demonstrated that the magnetic field spin splitting of the quasiparticle states in proximity S-2DES nanostructures results in a pronounced spin-dependent Andreev reflection and anomalous singlet-pair magnetotransport as opposed to conventional NS junctions[12] where the spin splitting affects single-particle tunneling. The predicted bias-energy separation between Andreev enhancement peaks for spin-up and spin-down quasiparticles could be implemented for detection of ballistic spin-polarized carriers injected from a ferromagnetic source at low temperatures $T < E_g/k_B$. We note that the energy and magnetic field behaviour of the excess conductance discussed here for thick superconductors ($\tilde{g} \ll 1$) is consistent with the experimental findings (see, Refs. 5,7,10). To observe the predicted spin-dependent effects the superconducting film must be sufficiently thin. Taking $d = 5\,nm$ (as in the experiment of Ref. 12) and $k_F \approx 10^6\,cm^{-1}$ typical for high-mobility 2DES one finds the requirement for efficient spin splitting to be $\tilde{g} \approx g > 1$. It can be met in InAs heterostructures where the g-factor is much larger than 1.

We thank G. Fagas, V.I. Fal'ko, D. Ryndyk, C. Strunk, and D. Weiss for valuable discussions. The work was supported by the Deutsche Forschungsgemeinschaft (Forschergruppe 370 "Ferromagnet-Halbleiter-Nanostrukturen").

1 H. Kroemer and E. Hu, in: *Nanotechnology*, edited by G.L. Timp (Springer, Berlin 1999).

2 A.F. Andreev, Zh. Exp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].

3 A. F. Morpurgo, S. Holl, B. J. van Wees, T. M. Klapwijk, and G. Borghs, Phys. Rev. Lett. **78**, 2636-2639 (1997); K. W. Lehnert, N. Argaman, H.-R. Blank, K. C. Wong, S. J. Allen, E. L. Hu, and H. Kroemer, *ibid.* **82**, 1265 (1999); H. Takayanagi, T. Akazaki, E. Toyoda, and H. Nakano, Physica C **352**, 95 (2001).

4 Th. Schäpers, V. A. Guzenko, R. P. Müller, A. A. Golubov, A. Brinkman, G. Crecelius, A. Kaluza, and H. Lüth, Phys. Rev. B **67**, 014522 (2003).

5 C. Nguyen, H. Kroemer, and E.L. Hu, Phys. Rev. Lett. **69**, 2847 (1992); F. Rahman and T. J. Thornton, Superlat. Microstr., **25** 767 (1999).

6 I. Kosztin, D. L. Maslov, and P. M. Goldbart, Phys. Rev. Lett. **75**, 1735 (1995); J.A. Melsen, P.W. Brouwer, K.M. Frahm, and C.W.J. Beenakker, Europhys. Lett. **35**, 7 (1996); W. Ihra, M. Leadbeater, J.L. Vega, and K. Richter, Euro. Phys. J. B. **21**, 425 (2001); P. Jacquod, H. Schomerus, and C.W.J. Beenakker, Phys. Rev. Lett. **90**, 207004 (2003); J. Cserti, P. Polinak, G. Palla, U. Zülicke, and C. J. Lambert, Phys. Rev. B **69**, 134514 (2004).

7 J. Eroms, M. Tolkiehn, D. Weiss, U. Rössler, J. DeBoeck, and S. Borghs, Europhys. Lett. **58**, 569 (2002).

8 D. Uhlisch, S. G. Lachenmann, Th. Schäpers, A. I. Braginski, H. Lüth, J. Appenzeller, A. A. Golubov, and A. V. Ustinov, Phys. Rev. B **61**, 12463 (2000).

9 H. Takayanagi and T. Akazaki, Physica B **249-251**, 462 (1998); T.D. Moore and D.A. Williams, Phys. Rev. B **59**, 7308 (1999); H. Hoppe, U. Zülicke, and G. Schön, Phys. Rev. Lett. **84**, 1804 (2000).

10 J. Eroms, D. Weiss, J. De Boeck, and G. Borghs, cond-mat/0404323.

11 See, e.g. R. Fiederling, M. Keim, G. Reuscher, W. Ossau, G. Schmidt, A. Waag, L. W. Molenkamp, *Nature* **402**, 787 (1999); Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura, H. Ohno, D. D. Awschalom, *ibid.* **402**, 790 (1999).

12 R. Meservey, P. M. Tedrow, and P. Fulde, Phys. Rev. Lett. **25**, 1270 (1970).

13 R. Meservey and P. M. Tedrow, Phys. Reports **238**, 173 (1994).

14 A. Kastalsky, A. W. Kleinsasser, L. H. Greene, R. Bhat, F. P. Milliken and J. P. Harbison, Phys. Rev. Lett. **67**, 3026 (1991).

15 B. J. van Wees, P. de Vries, P. Magnee, and T. M. Klapwijk, Phys. Rev. Lett. **69**, 510-513 (1992); S. G. den Hartog, B. J. van Wees, Yu. V. Nazarov, T. M. Klapwijk, and G. Borghs, *ibid.* **79**, 3250 (1997).

16 V.T. Petrashov, V.N. Antonov, P. Delsing, and R. Claeson, Phys. Rev. Lett. **70**, 347-350 (1993).

17 A.F. Volkov, Phys. Lett. A **174**, 144 (1993).

18 A.F. Volkov, P.H.C. Magnee, B.J. van Wees, and T.M. Klapwijk, Physica C **242**, 261 (1995).

19 H. Courtois, Ph. Gandit, D. Mailly, and B. Pannetier, Phys. Rev. Lett. **76**, 130-133 (1996).

20 B. A. Aminov, A. A. Golubov, and M. Yu. Kupriyanov. Phys. Rev. B **53**, 365 (1996); W. Belzig, C. Bruder, and G. Schön *ibid.* **54**, 9443 (1996); A. Altland, B.D. Simons, and D. Taras-Semchuk, Adv. Phys. **49**, 321 (2000).

21 C. W. J. Beenakker, Rev. Mod. Phys. **69**, 731 (1997).

22 M. Schechter, Y. Imry, and Y. Levinson, Phys. Rev. B **64**, 224513 (2001).

23 A. M. Clogston, Phys. Rev. Lett. **5** (1962) 464; A. I. Larkin, Zh. Eksp. Teor. Fiz. **48** (1965) 232 [Sov. Phys. JETP **21** (1965) 153].

24 S. Brosig, K. Ensslin, A. G. Jansen, C. Nguyen, B. Brar, M. Thomas, and H. Kroemer, Phys. Rev. B **61**, 13045 (2000).

25 G. Tkachov and V. I. Fal'ko Phys. Rev. B **69**, 092503 (2004).

26 K. Maki, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, V.2, 1969).

27 A. Anthore, H. Pothier, and D. Esteve, Phys. Rev. Lett. **90** (2003) 127001.

28 The off-diagonal terms in the particle-hole space arise from the tunneling superconducting self-energy as for instance shown in W.L. McMillan, Phys. Rev. **175**, 537 (1968).

29 G.E. Blonder, M. Tinkham, and T.M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).

[30] Although the Andreev approximation $|\tilde{\epsilon}|, E_g \ll E_F \cos^2\theta$ fails for quasiparticles with vanishing normal momenta ($|\theta| \to \pi/2$), their contribution to the angle integral in Eq. (6) is rather small. For this reason, the 1D formula (5) of Ref. 29 remains a good approximation for $A(\tilde{\epsilon})$ in the 2D case as discussed in detail in N.A. Mortensen, K. Flensberg, and A.-P. Jauho, Phys. Rev. B **59**, 10176 (1999).