Andreev magnetotransport in low-dimensional proximity structures: Spin-dependent conductance enhancement

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We study the excess conductance due to the superconducting proximity effect in a ballistic two-dimensional electron system subject to an in-plane magnetic field. We show that under certain conditions the interplay of the Zeeman spin splitting and the effect of a screening supercurrent gives rise to a spin-selective Andreev enhancement of the conductance and anomalies in its voltage, temperature and magnetic field characteristics. The magnetic-field influence on Andreev reflection is discussed in the context of using superconducting hybrid junctions for spin detection.

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Advances in nanotechnology of semiconductor-superconductor junctions[1] have created a unique opportunity to investigate the interplay between superconducting phase-coherence and various electronic properties of low-dimensional semiconductors. Recent developments in this field include the realization of a long-range Josephson coupling mediated by Andreev reflection[2] of ballistic two-dimensional electrons and controlled by the injection of hot carriers,[3,4] the observation of a giant proximity-induced enhancement of the conductance of two-dimensional electron systems (2DES),[5] theoretical[6] and experimental[7] studies of Andreev billiards, and classical[8] and quantum[9,10] Andreev edge states in high magnetic fields.

In this paper we discuss a possibility of using the superconducting proximity effect for detecting the spin of transport carriers in low-dimensional systems, a problem closely related to the ongoing work on spin injection in semiconductors.[11] In nonmagnetic normal metal-superconductor (NS) junctions spin resolving transport measurements were first reported in Ref. 12 where the magnetic field spin splitting of the quasiparticle density of states in thin superconducting films served as an electron filter. Applied to ferromagnet-superconductor systems, this idea has developed into a sensitive technique of analyzing the spin polarization of ferromagnetic metals.[13] We note that the findings of Ref. 12 are specific to low-transparency tunnel junctions where the superconducting proximity effect is negligible and hence the electron transport is predominantly a quasiparticle one.

In structures with improved interfacial quality the penetration of the superconducting order parameter into the normal system is accompanied by the conversion of a quasiparticle current into a supercurrent via Andreev reflection which manifests itself as a low-bias conductance enhancement.[14–22] One of the most striking examples of the proximity effect occurs in ballistic semiconductor quantum wells (2DES) with a lateral superconducting contact [see Fig. 1(a)]. In this case the conductance in the plane of the quantum well can nearly twice exceed the normal-state value[5,7,10] due to a proximity-induced mixing of particle and hole states characterized by a superconducting minigap E_g in the excitation spectrum of the 2DES.[18] For such proximity structures we study the spin dependence of Andreev reflection arising from the Zeeman splitting of the gapped states in the 2DES subject to an in-plane magnetic field **B**.

Normally, a noticeable spin splitting in superconductors requires rather strong magnetic fields (above 1 T) (Ref. 12) of the order of the field $B_c \approx \Delta/\mu_B$ corresponding to the paramagnetic limit[23] (Δ and μ_B are the gap energy in the superconductor and the Bohr magneton). The advantage of the low-dimensional proximity structures is that the scale of relevant fields is set by the minigap, $B_g \approx (2E_g/g\Delta)B_c$, and hence can be much smaller than B_c both due to $E_g \ll \Delta$ and large electron g-factors (e.g., $|g| \approx 10-14$ in InAs/AlSb quantum wells[1,24]).

Apart from the smaller splitting fields, the magnetic field influence on ballistic proximity systems has one more specific feature. Because of the screening supercurrent generated by the field, the quasiparticles in the 2DES can acquire a significant Galilean energy shift arising from a finite Cooper pair momentum at the superconductor surface.[25] In ballistic quantum wells with weak momentum scattering the Galilean supercurrent effect does not average out, unlike in conventional diffusive superconductors,[12,26,27] and therefore must be taken into account along with the Zeeman splitting. Here we discuss the influence of these two competing magnetic field

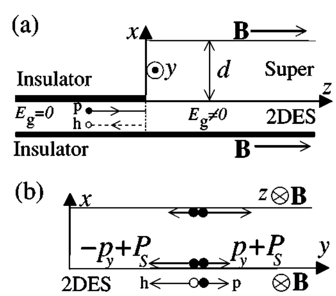

FIG. 1. (a) Cross section of a planar superconductor-2DES contact. At energies below the minigap E_g a particle (p) incident from the normal part of the 2DES is reflected as a hole (h) giving rise to conductance enhancement. (b) Schematic of the supercurrent flow in the xy plane. The particle and hole momenta in the 2DES are both shifted by $P_S=-(e/c)Bd/2$ in order to match the Cooper pair momentum $2P_S$.

effects on the voltage, temperature, and magnetic-field characteristics of Andreev transport.

We consider a 2DES coupled to a superconducting film via a barrier of low transparency $\mathcal{T} \ll 1$ [Fig. 1(a)]. The film thickness d is assumed much smaller than both the superconducting coherence length and the London penetration depth in a parallel field $\mathbf{B}=[0;0;B]$. In a ballistic quantum well the motion of particles and holes with energies smaller than Δ is coupled in the proximity region [$z>0$ in Fig. 1(a)] via multiple Andreev reflections. In the quantum limit (2DES) the correlated particle-hole motion can be described in terms of the effective pairing energy which gives rise to the quasiparticle minigap $E_g \approx (v_F/v_{F_S})\mathcal{T}E_0 \ll \Delta$.[18] It depends on the energy E_0 of the lowest occupied subband, the transparency $\mathcal{T}$ and the ratio of the Fermi velocities in the normal (v_F) and superconducting (v_{F_S}) systems. Accordingly, electron scattering from the proximity region can be treated using effective Bogoliubov–de Gennes equations for the electron $u_\alpha(z)$ and hole $v_{-\alpha}(z)$ wave functions averaged over the thickness of the 2DES,[28]

$$\begin{bmatrix} \tilde{\epsilon} + \dfrac{\hbar^2\partial_z^2 + \tilde{p}_F^2}{2m} & -E_g\Theta(z) \\ E_g\Theta(z) & -\tilde{\epsilon} + \dfrac{\hbar^2\partial_z^2 + \tilde{p}_F^2}{2m} \end{bmatrix} \begin{bmatrix} u_\alpha(z) \\ v_{-\alpha}(z) \end{bmatrix} = 0. \qquad (1)$$

Here ∂_z stands for a derivative; m, p_F, and $\alpha = \pm 1/2$ are, respectively, the electron mass, Fermi momentum, and spin; $\tilde{p}_F = (p_F^2 - p_y^2)^{1/2}$ where the parallel momentum $p_y = p_F \sin\theta$ depends on the angle $|\theta| \leq \pi/2$ measured from the z axis in the 2DES plane. As E_g scales with the transparency $\mathcal{T}$, there is virtually no "leaking" of electron pairing into the "normal" region [$z<0$ in Fig. 1(a)] because it would require Cooper pair tunneling through the thick insulator. Hence we assume the steplike pairing energy $E_g\Theta(z)$ with $\Theta(z)$ being the Heaviside function.

A weak magnetic field can be taken into account by an energy shift,

$$\tilde{\epsilon} = \epsilon + \alpha g \mu_B B - v_F P_S \sin\theta, \quad P_S = -(e/c)Bd/2, \qquad (2)$$

where the second term is the Zeeman energy whereas the third one is a Galilean energy arising from the shift of both particle and hole momenta which accounts for a screening supercurrent generated in the superconductor by a parallel magnetic field [see Fig. 1(b)]. As the thickness of the 2DES is considered negligible compared to d, the momentum shift of electrons and holes in the 2DES can be taken equal to the surface Cooper pair momentum per electron, P_S, in Eq. (2) ($e>0$ is the elementary charge). It is proportional to half of the film thickness reflecting the fact that the field fully penetrates the film and generates an antisymmetric (linear) distribution of the supercurrent density with respect to its middle plane.

For $|\tilde{\epsilon}| \ll E_F = p_F^2/2m$ the magnetic field effect on the quasiparticle energies is important only in the proximity region. Indeed, from Eqs. (1) and (2) one finds the excitation spectrum in this region as

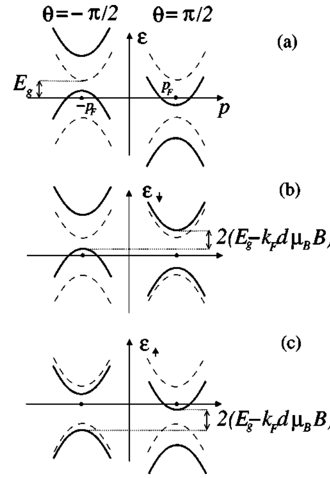

FIG. 2. Schematic view of the excitation spectrum (3) in the direction ($\theta=\pi/2$) and opposite ($\theta=-\pi/2$) to the supercurrent at $B>B_g$, (a) for spinless quasiparticles ($\tilde{g}=0$), (b) and (c) for spin-down and spin-up ones with $\tilde{g}>1$. Dashed curves correspond to $B=0$.

$$\epsilon^{\pm}_{\alpha p \theta} = -(\alpha g + k_F d \sin\theta)\mu_B B \pm [v_F^2(p-p_F)^2 + E_g^2]^{1/2}, \quad (3)$$

where p is the absolute value of the momentum, and $k_F = p_F/\hbar$. The magnetic field shifts the energies (3) with respect to the Fermi level so that at a certain field

$$B_g = E_g/[\mu_B k_F d(\tilde{g}+1)], \quad \tilde{g} = g/2k_F d, \qquad (4)$$

the excitations with momenta (anti)parallel to the supercurrent ($\theta = \pm\pi/2$) become gapless. We point out that in Eqs. (3) and (4) the interplay of the Zeeman and supercurrent effects is controlled by a single material- and geometry-dependent parameter $\tilde{g}$. This is shown in Fig. 2 where we sketch the p dependence of the energies $\epsilon^{\pm}_{\alpha p \theta}$ near the Fermi points $\pm p_F$ in the direction ($\theta=\pi/2$) and opposite ($\theta=-\pi/2$) to the supercurrent. For $\tilde{g} \ll 1$ (thick superconductors or small g factors) the quasiparticles can be treated as spinless, and the minigap vanishes as soon as the dispersion curves cross the Fermi energy simultaneously at $\pm p_F$ [Fig. 2(a)]. For $\tilde{g}>1$ (thin superconductors and/or large g factors) the Zeeman splitting preserves the minigap at finite energies even after the appearance of gapless excitations at the Fermi level [Figs. 2(b) and 2(c)]. The gap remains open in the field range $1 < B/B_g < 1+\tilde{g}$, rather broad for $\tilde{g}>1$.

Due to the superconducting proximity effect the current of quasiparticles entering from the normal region [$z<0$ in Fig. 1(a)] can be converted into a supercurrent via Andreev reflection at low energies $|\tilde{\epsilon}| < E_g$. The efficiency of the Andreev conversion is insured by the lack of a barrier between the normal and proximity-affected regions of the 2DES. That is why for calculating the conductance we can use the solution of the scattering problem of Ref. 29 applied to an ideal NS interface. In reality, normal reflection does occur because the contact to a metal slightly modifies the quantum well resulting in a mismatch of the Fermi momenta[7] at $z=0$. However, for $\mathcal{T} \ll 1$ this mismatch is rather weak. We have checked that it has a small effect on the zero-field conduc-

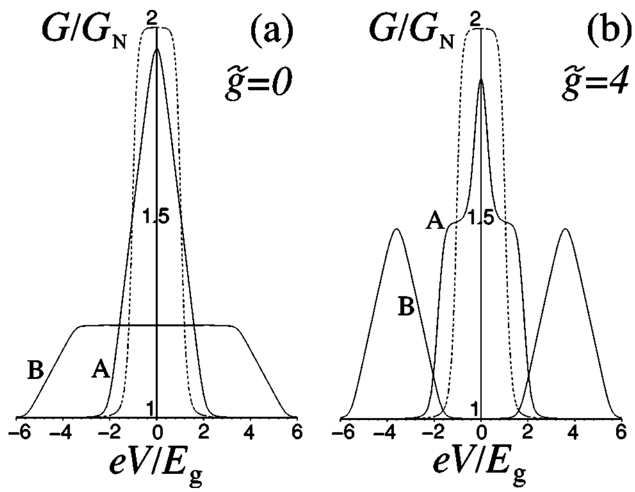

FIG. 3. Conductance versus voltage for (a) spinless and (b) spin-polarized electrons at magnetic fields B/B_g, 0 (dashed), 1 (A) and 4.5 (B); $k_BT/E_g=0.1$.

tance while we have found no significant difference at finite fields $B\sim B_g$, the regime we are interested in.

In the absence of normal scattering the probability $A(\tilde{\epsilon})$ for a particle to be reflected from the proximity region as a hole is given within the usual approximation $|\tilde{\epsilon}|, E_g \ll E_F \cos^2\theta$ by[29]

$$A(\tilde{\epsilon}) = \Theta(E_g - |\tilde{\epsilon}|) + \Theta(|\tilde{\epsilon}| - E_g)\frac{|\tilde{\epsilon}| - [\tilde{\epsilon}^2 - E_g^2]^{1/2}}{|\tilde{\epsilon}| + [\tilde{\epsilon}^2 - E_g^2]^{1/2}}. \quad (5)$$

The expression for the conductance reads[30]

$$\frac{G(V,B,T)}{G_N} = 1 + \frac{1}{4}\sum_{\alpha}\int d\epsilon\left(-\frac{\partial f(\epsilon - eV)}{\partial \epsilon}\right) \times \int_{-\pi/2}^{\pi/2} d\theta \cos\theta A[\epsilon + (\alpha g + k_F d \sin\theta)\mu_B B], \quad (6)$$

where G_N is the conductance of the normal 2DES and $f(\epsilon - eV)$ is the Fermi distribution of the incident electrons at bias energy eV. Since the θ dependence enters through the energy shift in A, the double integral in Eq. (6) can be easily reduced to a single one,

$$\frac{G(V,B,T)}{G_N} = 1 + \sum_{\alpha}\int_{-\infty}^{\infty}\frac{d\tilde{\epsilon}A(\tilde{\epsilon})}{4k_F d\mu_B B} \times \{f[\tilde{\epsilon} - eV - (\alpha g + k_F d)\mu_B B] - f[\tilde{\epsilon} - eV - (\alpha g - k_F d)\mu_B B]\}. \quad (7)$$

Figure 3(a) shows the suppression of the excess conductance peak due to the vanishing of the minigap for spinless electrons as the field exceeds B_g. In contrast to this, for spin-polarized electrons the zero-bias anomaly splits into two twice smaller peaks [Fig. 3(b)] because the spin-dependent minigap in the excitation spectrum, Eq. (3), still supports Andreev reflection of spin-down particles for $V>0$ and spin-up holes for $V<0$. In other words, for a given voltage only quasiparticles of one spin orientation give rise to the conductance enhancement. The spin-dependent Andreev process persists until the minigap closes at $B=B_g(1+\tilde{g})$ followed by an overall decrease in the excess conductance at higher fields.

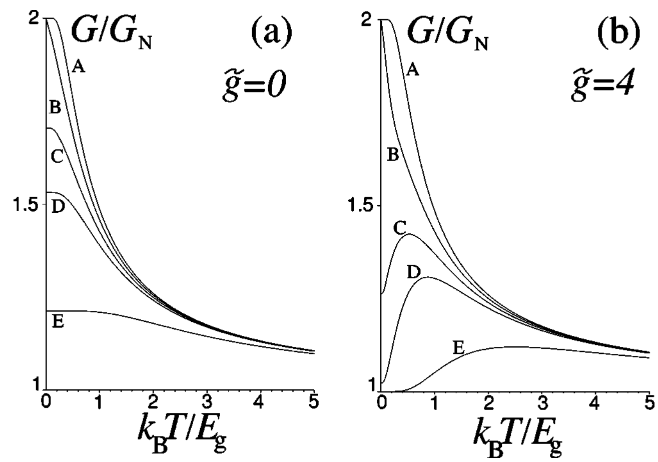

FIG. 4. Zero-bias conductance versus temperature for (a) spinless and (b) spin-polarized electrons at magnetic fields B/B_g, 0 (A), 1 (B), 1.5 (C), 2 (D), and 5 (E).

The temperature dependence of the zero-bias conductance, shown in Fig. 4(b), reveals one more manifestation of the magnetic spin splitting: a maximum in $G(T)$ in contrast to a monotonic decrease for spinless electrons [Fig. 4(a)]. It is due to the shift of the minigap to finite energies where Andreev reflection is mediated by thermally excited quasiparticles. The corresponding behavior of the zero-bias magnetoconductance at different temperatures is shown in Figs. 5(a) and 5(b). The energy and B-field anomalies discussed above are already well-resolved when $\tilde{g}$ exceeds 1.

To conclude we have demonstrated that the magnetic field spin splitting of the quasiparticle states in proximity S-2DES nanostructures results in a pronounced spin-dependent Andreev reflection and anomalous singlet-pair magnetotransport as opposed to conventional NS junctions[12] where the spin

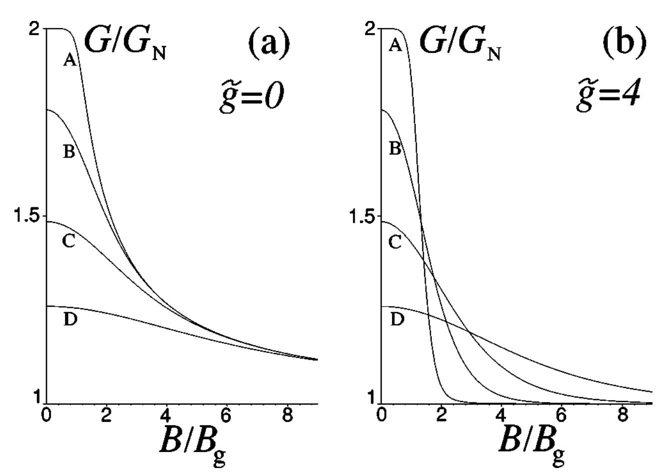

FIG. 5. Zero-bias magnetoconductance for (a) spinless and (b) spin-polarized electrons at temperatures $T/(k_B^{-1}E_g)$, 0.1 (A), 0.5 (B), 1 (C), and 2(D).

splitting affects single-particle tunneling. The predicted bias-energy separation between Andreev enhancement peaks for spin-up and spin-down quasiparticles could be implemented for detection of ballistic spin-polarized carriers injected from a ferromagnetic source at low temperatures $T < E_g / k_B$. We note that the energy and magnetic field behavior of the excess conductance discussed here for thick superconductors ($\tilde{g} \ll 1$) is consistent with the experimental findings (see, Refs. 5, 7, and 10). To observe the predicted spin-dependent effects the superconducting film must be sufficiently thin. Taking $d = 5$ nm (as in the experiment of Ref. 12) and $k_F \approx 10^6$ cm^{-1} typical for high-mobility 2DES one finds the requirement for efficient spin splitting to be $\tilde{g} \approx g > 1$. It can be met in narrow-gap semiconductors (e.g., InAs) where electron g factors in the conduction band can be significantly enhanced due to the strong coupling to the spin-orbit split valence band.[31]

In general, spin-orbit interactions, arising from the lack of the inversion symmetry of the host crystal or a built-in electric field across asymmetric quantum wells, can influence the proximity effect also via a mixing of the spin states of the conduction electrons in a 2DES. However, the strength of this mixing does not seem to have any apparent relation to the mechanism of the g-factor enhancement. For instance, in some large g-factor InAs wells transport measurements reveal no observable spin-orbit interaction for conduction electrons.[32] In some cases it appears only when an additional back-gate voltage is applied (see, e.g., Ref. 33). In our analysis we assumed the spin-orbit scattering length to be much greater than the induced coherence length $\xi_N \approx \hbar v_F / E_g$, allowing us to neglect the spin mixing in the conduction band. To discuss the latter effect in detail more material- and sample-specific studies are required.

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