A Game-Theoretic Foundation for Competitive Equilibria in the Stiglitz-Weiss Model

Lutz G. Arnold*
University of Regensburg
Department of Economics
93 040 Regensburg, Germany
Phone: +49-941-943-2705
Fax: +49-941-943-1971
E-mail: lutz.arnold@wiwi.uni-r.de

Abstract

Financial intermediaries are, by definition, engaged in two-sided competition. Despite the well-known problems of achieving competitive solutions under two-sided price competition, models of financial intermediation are commonly solved for competitive equilibria. This paper provides a game-theoretic foundation for competitive equilibria in one of the most important models of financial intermediation, the seminal Stiglitz-Weiss (1981) adverse selection model of the credit market with a continuum of borrower types. The approach can readily be adapted to other models of financial intermediation as well.

JEL classification: D82, E51, G21
Key words: financial intermediation, asymmetric information, credit rationing.

*I am indebted to Martin Hellwig and John Riley. Hellwig suggested to analyze equilibria in the Stiglitz-Weiss model using the double-Bertrand approach. Riley and I jointly analyzed the version of the model with price-taking behavior in the deposit market.

A Game-Theoretic Foundation for Competitive Equilibria in the Stiglitz-Weiss Model

Abstract

Financial intermediaries are, by definition, engaged in two-sided competition. Despite the well-known problems of achieving competitive solutions under two-sided price competition, models of financial intermediation are commonly solved for competitive equilibria. This paper provides a game-theoretic foundation for competitive equilibria in one of the most important models of financial intermediation, the seminal Stiglitz-Weiss (1981) adverse selection model of the credit market with a continuum of borrower types. The approach can readily be adapted to other models of financial intermediation as well.

1 Introduction

Financial intermediaries are, by definition, engaged in competition in input (deposit) markets and output (loan) markets. As noted by Stahl (1988) and Yanelle (1989, 1997), two-sided price competition does not necessarily give rise to perfectly competitive equilibria, i.e., to the market outcome that would arise with price-taking behavior. This casts doubt on the common practice of solving models of financial intermediation for perfectly competitive equilibria. The seminal Stiglitz-Weiss (1981) (henceforth: "SW") model is a case in point. The purpose of the present paper is to provide a game-theoretic foundation for the SW model that yields the perfectly competitive solution as the outcome of two-sided price competition and that generalizes to other models of financial intermediation as well.

To motivate the problem, consider the two-stage game analyzed by Stahl (1988, pp. 195-196), the terminology adapted to the operation of banks rather than "merchants". Let sufficiently well-behaved supply and demand functions for loanable funds be given. The perfectly competitive solution entails that both the deposit rate and the interest rate are equal to the rate that equates supply of and demand for funds. At stage one, two banks bid for deposits. The higher bidder gets the whole supply of deposits ("winner-take-all"). In the case of equal bids, each bank gets the total supply with probability one-half ("random tie-breaking"). At stage two, the single bank endowed with loanable funds is a monopolist in the loan market. When the demand elasticity evaluated at the market-clearing rate is sufficiently high such that an increase in the interest rate reduces total repayment (principal plus interest paid by all firms), the perfectly competitive solution arises as a subgame-perfect equilibrium of the two-stage game: both banks offer the market-clearing rate to depositors, and the bank selected by the random tie-breaking rule sets the same rate in the credit subgame. The bank active at stage two has no incentive to raise the interest rate because of the ensuing drop in total repayment. Lowering the bid rate at stage one is not profitable because, due to winner-take-all competition, there is no supply left. Bidding higher at stage one leads to losses because the revenue-maximizing loan rate then falls short of the bid rate. However, the perfectly competitive solution does not arise if the demand for capital is inelastic, in that an increase in the interest rate raises total repayment. This is because a bank that acquires funds at the market-clearing rate at stage one has an incentive to set an interest rate above the market-clearing rate and hold back funds at stage two then.

Stahl (1988, pp. 196 ff.) proceeds to show that the perfectly competitive solution generally arises if the credit subgame precedes the deposit subgame and banks must not default on their deposit obligations: assuming random tie-breaking in the credit subgame, both banks set the market-clearing rate at stage one, and the bank selected by the tie-breaking rule bids the same rate to depositors. It is not feasible to get the required funds at a lower deposit rate at stage two. Deviating with a higher interest rate at stage one is not profitable, because there is no residual demand left. Setting a lower rate at stage one yields losses, because this forces a bank pay a deposit rate above the market-clearing level in order to meet its obligations at stage two.

This second double-Bertrand game, in which competition in the "output" market precedes competition in the "input market" appears of particular relevance for financial intermediation. For one thing, as pointed out above, financial intermediation is a prime example of two-sided competition (see Freixas and Rochet, 1997, Section 3.4). For another, the order of the stages is a natural way to express the fact that banks make long-term commitments by rolling over short-term debt. Thus, the reinterpretation of the Stahl (1988) model as a model of the capital market provides a sound game-theoretic foundation for perfectly competitive financial intermediation *in a frictionless environment*.

Matters are more complicated in markets with informational frictions, however. Two of the startling features of capital markets with asymmetric information are rationing and two-price equilibria, i.e., the "Repeal of the Law of Supply and Demand" and the "Repeal of the Law of the Single Price" (Stiglitz, 1987, pp. 4 and 7, resp.). The SW model is at the center of our analysis for the reason that it potentially gives rise to either of these two types of equilibria: Coco (1997) and Arnold and Riley (2009) show that a two-price allocation is an equilibrium of the model with price-taking behavior in the deposit market when the return on lending is a non-monotonic function of the interest rate and there is excess demand at the local maximum (and there is no market-clearing interest rate that yields a higher return).[1] If the model is modified such that riskier projects have a lower expected return, the return function may be hump-shaped and pure credit rationing may arise, as stressed by SW. This raises the question of whether these types of equilibria, like the perfectly competitive solution in the frictionless case, can also be given a sound game-theoretic foundation.

The Stahl (1988) model does not lend itself directly to this setup with informational frictions. It assumes that banks satisfy the entire market demand at the quoted interest rate. This is inconsistent with either credit rationing or a two-price allocation with excess demand at the lower rate. So, to make the Stahl (1988) approach applicable to rationing or two-price equilibria, we assume that a bank's strategy space consists of an interest rate and a limit on the amount of credit offered (a "credit limit"[2]) in the credit subgame and of a deposit rate and a maximum demand for funds in the deposit subgame. This modified double-Bertrand approach yields the desired game-theoretic foundation for the respective types of equilibria: the market-clearing, rationing, and two-price equilibria arise as subgame-perfect Nash equilibria (SPNE) of the two-stage game under the respective assumptions about the return function.[3] We also show how the modified double-Bertrand approach carries over to alternative models of financial intermediation as well. Thus, our modified double-Bertrand

[1]Related papers which cast doubt on the theoretical importance of credit rationing in SW-type models include Bester (1985), Riley (1987), De Meza and Webb (1987, 2006), and Lensink and Sterken (2002).

[2]Notice that the term "credit limit" does not refer to the size of an individual loan but to the "number" (mass) of loans a bank makes. De Meza and Webb's (2006) work shows that the distinction is crucial: with variable loan size rationing phenomena disappear altogether.

[3]This is our pragmatic definition of a "game-theoretic foundation for competitive equilibria": a specific game structure that yields the "equilibria" described in the literature as an SPNE (cf., by contrast, the "Foundations of Competitive Equilibria" in Mas-Colell et al. (1995, ch. 18).

approach yields a solid game-theoretic foundation for the common practice of solving models of financial intermediation such as the SW model for perfectly competitive equilibria.

Building this foundation for competitive equilibria in the SW model requires making several choices, e.g., with regard to how many loans different banks charging the same interest rate make and which amount of deposits banks bidding the same rate receive. While we are confident that our approach is robust to modifications of the rules of the game (except that, as explained above, the sequencing of the two subgames is crucial), one might validly object that we merely present *one* specification that yields the perfectly competitive outcome as an SPNE. However, the main motivation for the present analysis is that there is *no* such game in the existing literature. So the present analysis provides one way of filling this big gap in the literature.

There are two other resolutions to the problem that two-sided price competition does not necessarily give rise to a perfectly competitive equilibrium. First, Allen and Hellwig (1986) show that the equilibrium prices of the mixed-strategy game converge in distribution to competitive prices as the number of firms becomes large. Relatedly, Dixon (1987) demonstrates that as the economy is replicated a sufficiently large number of times, a competitive approximate equilibrium exists. This approach is not helpful for our purposes, however, because input prices (i.e., in our context, the deposit rate) are taken as given. Moreover, competitive solutions in Dixon's model require efficient rationing, which is incompatible with the information structure assumed by SW. Second, Gersbach (2002, 2008) derives a competitive double-Bertrand equilibrium in a financial market in which the deposit subgame precedes the credit subgame and with endogenous market-side switching (i.e., as the interest rate rises, entrepreneurs supply their endowments to the market rather than demand additional funds in order to invest). This approach is not suitable for our purposes either, because, though there is rationing out of equilibrium, it relies on market clearing in equilibrium.

The paper is organized as follows. Section 2 briefly recapitulates the assumptions of the SW model and explains double-Bertrand competition in detail. Section 3 proves that the the two-interest allocation is the unique subgame-perfect equilibrium of the double-Bertrand game. Section 4 is concerned with the SW model with a monotonic or a hump-shaped return function. Alternative models of financial intermediation are considered in Section 5. Section 6 concludes. Details of the proofs of theorems are delegated to a technical appendix.

2 Model

This section introduces the model. We first briefly recapitulate the assumptions of the well-known SW adverse selection model with a continuum of borrower types.[4] We then state our assumptions with regard to price competition in the markets for loans and deposits.

Firms and projects

The model covers two time periods, 1 and 2. There is a continuum of length N (> 0) of firms of different types, θ. The distribution of firm types, $G(\theta)$, is strictly increasing and continuous and has (bounded) support $[0, \theta^{max}]$. In period 1, each firm has access to one indivisible investment project with uncertain payoff R (≥ 0) in period 2. The distribution of returns on the projects of type-θ firms is denoted as $F(R|\theta)$. $F(R|\theta)$ is continuous in R for all θ. The returns on different projects are independent. All types of projects have the same expected return, $\bar{R}$: $E_R(R|\theta) = \bar{R}$ for all θ. It is assumed that if $\theta' > \theta$, the distribution $F(R|\theta')$ is a mean-preserving spread of $F(R|\theta)$: $\int_0^x F(R|\theta')dR > \int_0^x F(R|\theta)dR$ for all $x > 0$. In this sense, the higher θ, the riskier the project. Each project requires a capital input B ($0 < B < \bar{R}$). Banks collect deposits and make fixed-interest loans to firms. This process involves no factor cost. There is asymmetric information: firms observe their own type θ, while banks do not. As a result, the interest rate(s) charged r cannot be made contingent on borrower types θ. Firms are risk-neutral and apply for credit if their project yields a non-negative expected profit. The supply of capital $L^S(\rho)$ is a continuous and strictly increasing function of the interest rate paid on deposits ρ with $L^S(0) \geq 0$ and $L^S(\rho) \to \infty$ for $\rho \to \infty$.

Double-Bertrand competition

Banks are distinguished by an index $k \in \{1, \ldots, K\} = \mathcal{K}$. The number of banks is at least four (i.e., two per interest rate in a two-interest rate equilibrium): $K > 4$. The banks play a two-stage game, first stage the credit subgame, then the deposit subgame. In the credit subgame, bank k's strategy is a pair $(r_k, \lambda_k) \in \mathbb{R}^2_+$. r_k is the interest rate k sets and λ_k a credit limit (explained below). If there are several equilibrium interest rates, firms apply for credit at the lowest interest rate first and turn to the next-highest interest rate if rationed. If several banks charge the same interest rate r_k, the bank with the highest credit limit λ_k alone faces the market demand at r_k. If several banks choose the maximum credit limit at

[4] The analysis is very similar with a finite set of borrower types.

r_k, then a tie-breaking rule randomly selects one of them, which then serves the market demand at r_k alone. For the sake of simplicity, we assume that the tie-breaking rule assigns the same probability to each bank setting the maximum credit limit.[5] Hence, at each interest rate r_k the (residual) demand is served by a single bank k. Its credit limit λ_k obliges this bank to supply the minimum of λ_k and (residual) demand at r_k to the market. In the deposit subgame, bank k's strategy is a pair $(\rho_k, \delta_k) \in \mathbb{R}^2_+$. δ_k is the amount of deposits k demands at the bid rate ρ_k.[6] The bids are served in the order of decreasing deposit rates. If the (residual) supply of deposits does not fall short of demand at a given deposit rate ρ_k, then each bank k which bids ρ_k gets δ_k and lenders are rationed randomly. If the demand for deposits exceeds (residual) supply at ρ_k, then the banks k which bid ρ_k share the supply in proportion to the volumes δ_k they demand.[7] There is no secondary market for deposits. Following Stahl (1988, pp. 196-197), we assume that a bank's payoff in the case of default (i.e., if it fails to refinance its credit given) is a negative constant $\underline{\pi}$ (< 0).[8]

3 Equilibrium

Return function

As shown by SW, the credit market equilibrium is characterized by adverse selection: firms apply for credit if and only if their project is sufficiently risky. Formally, let $\pi(R, r) = \max\{R - (1 + r)B, -C\}$ denote a firm's profit. A firm with a type-θ project applies for credit if and only if $E_R[\pi(R, r)|\theta] \geq 0$. Given the mean-preserving spread assumption,

[5]More generally, we could assume that it assigns an arbitrary strictly positive probability to each bank. The analysis is unaffected.

[6]Competition is not "winner-take-all".

[7]This kind of coordination is needed in order to avoid default caused by the mechanism that determines the allocation of deposits to banks (cf. Stahl, 1988, p. 198).

[8]The assumption that a bank defaults if it does not raise deposits equal to or greater than the amount of loans it makes implies that banks have no alternative source of funds in period 1. On the other hand, negative payoffs may occur for banks (off the equilibrium path), which assumes that banks have funds they can use to cover losses in period 2. So we implicitly assume that banks have period-2 income but cannot borrow against this in period 1. An alternative interpretation is that banks are intermediaries for trade in physical, not financial, capital. Clearly, banks then have to default if the amount of funds raised falls short of credit given.

this condition is satisfied for $\theta \geq \vartheta(r)$, where $\vartheta(r)$ is an increasing function defined by $E_R[\pi(R,r)|\vartheta(r)] = 0$. The maximum interest rate above which no firm demands a loan is denoted r^{max}: $E_R[\pi(R,r^{max})|\theta^{max}] = 0$.

Each bank's pool of borrowers has the same risk characteristics, so the rate of return on lending can be expressed as a function of the interest rate alone:

$$\varrho(r) = \frac{\bar{R} - E_{R,\theta}[\pi(R,r)|\theta \geq \vartheta(r)]}{B} - 1, \tag{1}$$

SW observe that the return function $\varrho(r)$ is not necessarily monotonic. This is because an increase in the interest rate means that those borrowers who repay repay more, but since the risk pool worsens, the proportion of borrowers who do repay falls. This means that a small increase in the interest rate may be harmful, rather than beneficial, to the banks.

Coco (1997) and Arnold and Riley (2009) show, however, that $\varrho(r)$ attains its unique global maximum $\bar{R}/B - 1$ at r^{max} (see the upper panel of Figure 1). This follows from (1) and the observations that

$$E_{R,\theta}[\pi(R,r^{max})|\theta \geq \vartheta(r^{max})] = E_R[\pi(R,r^{max})|\theta^{max}] = 0$$

and

$$E_{R,\theta}[\pi(R,r)|\theta \geq \vartheta(r)] > 0, \qquad \text{for } r < r^{max}$$

(since $E_R[\pi(R,r)|\theta^{max} > 0)$. Intuitively, whenever $r < r^{max}$, the riskiest firms make positive expected profit, so the banks' return on lending falls short of the expected rate of return of the investment projects. When $r = r^{max}$, only the riskiest borrower class remains in the market and makes zero expected profit. This is the only possibility for banks to generate a rate of return equal to the expected rate of return of the investment projects. In particular, this means that the return function cannot have the hump shape assumed by SW.

For the sake of expositional convenience, we assume in what follows that $\varrho(r)$ is continuous, has a unique interior local maximum ρ^* at the interest rate r^*, and is positive-valued for all $r \geq r^*$ (see the upper panel of Figure 1).[9] It follows that there is a unique interest rate $r^{**} > r^*$ such that $\varrho(r^{**}) = \rho^*$ and that $\varrho(r)$ increases for $r > r^{**}$.

[9] Arnold and Riley (2009) show that $\varrho(r)$ is discontinuous if the lower bound of the support of $F(R|\theta)$ is identical for a positive mass of borrower types θ.

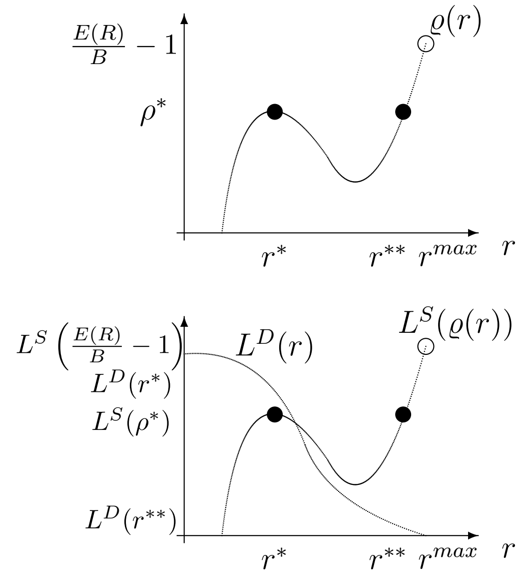

Figure 1: Return function and credit market

Supply and demand

To focus on non-market-clearing equilibria, we assume that

$$L^D(r^{**}) < L^S(\rho^*) < L^D(r^*)$$

(see the lower panel of Figure 1). This rules out several types of equilibria.[10] First, there is not a market-clearing equilibrium with $r < r^*$, since there is excess demand. Second, there is a market-clearing interest rate above r^*. But this interest rate cannot arise in an equilibrium with zero profit for banks either, because banks could raise expected profit by underbidding with the interest rate r^*. Moreover, there cannot be a pure rationing equilibrium in which banks set the interest rate r^* and pay ρ^* to depositors. This is because it would be profitable to deviate with an interest rate close to r^{max}: there is positive residual demand due to rationing at r^*; the return on lending rises, since $\varrho(r)$ attains its global maximum at r^{max}; and if the amount of loans made is sufficiently small, the deposit rate rises only slightly.

Existence of a two-price equilibrium

The clue to finding an equilibrium is in found SW (pp. 398-399) in their discussion of a return function with multiple humps: we have to look for an equilibrium with two interest

[10]The arguments put forward in this paragraph are somewhat heuristic. They will be made precise in the proofs of Theorems 1 and 2 below.

rates and equality of residual supply and residual demand at the higher rate. Accordingly, let

$$L^* = \frac{L^S(\rho^*) - L^D(r^{**})}{L^D(r^*) - L^D(r^{**})} L^D(r^*). \tag{2}$$

Suppose banks give credit L^* at r^* and $L^S(\rho^*) - L^*$ at r^{**} and pay the deposit rate ρ^*. Since $\varrho(r^*) = \varrho(r^{**}) = \rho^*$, they make zero profit. Residual supply and demand at r^{**} are $L^S(\rho^*) - L^*$ and $[1 - L^*/L^D(r^*)]L^D(r^{**})$. From (2), residual supply equals residual demand. The following theorem states that this two-price allocation is the outcome of the double-Bertrand competition in the markets for credit and deposits.

THEOREM 1: *The following pure strategies represent an SPNE:*

$\hookrightarrow$ *in the credit subgame,* $(r_k, \lambda_k) = (r^*, L^*)$ *for two banks* k*,* $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L^*)$ *for two banks* k*, and* $(r_k, \lambda_k) = (0, 0)$ *for the other banks* $k \in \mathcal{K}$*;*

$\hookrightarrow$ *in the deposit subgame, the bank* k *setting* r^* *and selected by the tie-breaking rule chooses* $(\rho_k, \delta_k) = (\rho^*, L^*)$*, the bank* k *setting* r^{**} *and selected by the tie-breaking rule chooses* $(\rho_k, \delta_k) = (\rho^*, L^S(\rho^*) - L^*)$*, and all other banks* $k \in \mathcal{K}$ *choose* $(\rho_k, \delta_k) = (0, 0)$*.*

Clearly, the strategies described in Theorem 1 yield the two-price allocation, with zero profit for all banks. Since there is no residual demand, it is not possible to make positive profit with $r > r^{**}$. The proof of Theorem 1 requires a careful analysis of the residual supply and demand functions, which is delegated to Appendix A. Appendix B provides a rigorous proof of the theorem. Here we give a non-technical sketch. To simplify matters, we let $\underline{\pi} = -\infty$ here. This rules out default by banks (in and out of equilibrium) by assumption.[11]

Solving the model backwards, consider the first the deposit subgame. Let l_k denote credit given by bank k at stage one and

$$\rho = (L^S)^{-1}\left(\sum_{k \in \mathcal{K}} l_k\right) \tag{3}$$

the deposit rate that is just sufficient to raise the required amount of funds. The following lemma summarizes banks' behavior in the deposit subgame.

LEMMA 1: *The following strategies represent a Nash equilibrium of the deposit subgame:*

[11] In banking theory, the no-default assumption has been popularized by Ben Bernanke and Mark Gertler (1987, p. 96). At a practical level, it can be motivated by the various solvency regulations unique to the banking sector.

$(\rho_k, \delta_k) = (\rho, l_k)$ *for each bank* k *with* $l_k > 0$, *and* $(\rho_k, \delta_k) = (0, 0)$ *for each bank* k *with* $l_k = 0$.

For banks with $l_k > 0$, bidding lower or reducing the demand for funds means default: deposits are insufficient to make the promised loans (and the payoff is $\underline{\pi} = -\infty$). Bidding higher or increasing the demand for funds also reduces profit, since interest income is already determined in the credit subgame. For banks with $l_k = 0$, attracting funds is unprofitable because they do not lend. For future reference, we note that the equilibrium deposit rate ρ in (3) is a continuous and strictly increasing function of aggregate credit given $\sum_{k\in\mathcal{K}} l_k$.
Given the credit-subgame strategies in Theorem 1, we have $\sum_{k\in\mathcal{K}} l_k = L^* + [L^S(\rho^*) - L^*] = L^S(\rho^*)$, so $\rho^* = (L^S)^{-1}[L^S(\rho^*)] = (L^S)^{-1}(\sum_{k\in\mathcal{K}} l_k)$. It follows from Lemma 1 that the strategies for the deposit subgame described in the theorem constitute a Nash equilibrium of the resulting deposit subgame.
Turning to the credit subgame (i.e., stage one of the double-Bertrand game), consider a given strategy profile, $\{(r_k, \lambda_k)\}_{k\in\mathcal{K}}$. The set of market interest rates is $\{r |\ r_k = r \text{ and } l_k > 0 \text{ for some } k \in \mathcal{K}\}$.

LEMMA 2: *Suppose the strategy profile in the credit subgame changes such that one of the following two situations arises:*

$\hookrightarrow$ *credit given becomes positive at one interest rate formerly not contained in the set of market interest rates; for all initial market interest rates, the maximum credit limit* λ_k *remains the same;*

$\hookrightarrow$ *the set of market interest rates remains unchanged; credit given rises for exactly one market interest rate; for all other initial market interest rates, the maximum credit limit* λ_k *remains the same;*

then aggregate credit given rises.

The proof of the lemma is tedious, but the assertion made is straightforward: if some bank makes additional loans at some interest rate, the total amount of loans made increases. Together with Lemma 1 and (3), it follows that the deposit rate also increases. Bearing this in mind, it is straightforward to show that it is not profitable to deviate from the strategies in Theorem 1 in the credit subgame. Strategies with an interest rate above r^{**} do not attract firms, since there is no residual demand. By offering credit in excess of $L^S(\rho^*) - L^*$ at r^{**} a bank overbids the highest credit limit, thereby capturing the residual demand. However, as there is no excess demand at r^{**}, the amount of credit given does not rise, so that zero profit ensues. Firms can also be attracted by offering more funds at r^* or by setting an interest rate below r^{**} other than r^*. However, this yields losses because the return on lending is no greater than ρ^* and the additional demand for deposits raises the deposit rate above ρ^*. This proves Theorem 1.

"Uniqueness"

The SPNE in Theorem 1 is not unique: in the deposit subgame, banks k with $l_k = 0$ can play any strategy that leads to zero deposits; and in the credit subgame, any strategy that yields $l_k = 0$ with certainty also yields zero profit. However, all SPNE strategies lead to the same market outcome:

THEOREM 2: *In any pure-strategy SPNE, r^* and r^{**} are the only market interest rates; credit given at these two interest rates equals L^* and $L^S(\rho^*)$, respectively; the single market deposit rate is ρ^*; and the supply of deposits is $L^S(\rho^*)$.*

The proof is in Appendix C. Again, we give an informal sketch here, maintaining the assumption that banks avoid default under all circumstances (i.e., $\underline{\pi} = -\infty$).

Consider first the deposit subgame. Let d_k denote bank k's deposits. In any Nash equilibrium of the deposit subgame, each bank k must get $d_k \geq l_k$ in order to avoid default. If $d_k > l_k$, k can raise its profit by reducing δ_k and, therefore, d_k. So $d_k = l_k$ for all k. Suppose a bank with $l_k > 0$ bids $\rho_k < \rho$ (with ρ given by (3)) and gets the funds it needs. Since $\rho_k \leq \rho$ for all k implies that the amount of deposits is insufficient to refinance aggregate credit given, we must have $\rho_k > \rho$ for some other bank. This cannot happen in a Nash equilibrium, since this latter bank could then get its funds more cheaply. So $\rho_k \geq \rho$ for all k with $l_k > 0$. But then again, a bank with $\rho_k > \rho$ could get the deposits it needs more cheaply. It follows that $d_k = l_k$ for all k and $\rho_k = \rho$ for all k such that $l_k > 0$ in any Nash equilibrium of the deposit subgame. Thus, for given loan volumes l_k determined in the credit subgame, the amount of deposits and the market deposit rates are uniquely determined. For future reference, we note that from $d_k = l_k$ and $\rho_k = \rho$, bank k's payoff is

$$\pi_k = [\varrho(r_k) - \rho]l_k.$$

Turning to the credit subgame, we show first that all banks make zero expected profit $E\pi_k$ (expectations being taken when the random tie-breaking rule determines which banks make loans). This follows from standard Bertrand arguments. It suffices to argue that not all banks make positive expected profit, for if some banks make profits and others do not, the latter have an incentive to adopt one of the former's strategies. If all banks make positive expected profit, then each bank sets a different interest rate. This is because each of two banks charging the same interest rate has an incentive to push its direct competitor out of the market, by either raising the credit limit (if there is excess demand) or undercutting the competitor's interest rate slightly (if there is no excess demand). Moreover, there cannot be excess demand at the highest interest rate if all firms make positive expected profit. If excess demand prevailed, then the bank charging the highest interest rate, K say, could increase its expected profit by changing r_K such that its return rises holding credit given constant (i.e., without affecting the deposit rate). Thus, either no bank makes positive expected profit, or else all banks make positive expected profit, each setting a different interest rate, and supply equals residual demand at the highest interest rate. Consider the two banks, K and $K-1$, say, setting the highest and second-highest interest rates, r_K and r_{K-1}, respectively. By offering slightly more funds than $K-1$ at r_{K-1}, bank K pushes $K-1$ out of the market. As credit given at r_{K-1} increases slightly and credit given at r_K drops to zero, the deposit rate ρ falls, and K's resulting profit $(1+\Delta)(E\pi_K)$ is higher than $K-1$'s expected profit was: $(1+\Delta)(E\pi_K) > E\pi_{K-1}$. Conversely, let $K-1$ withdraw its credit offer at r_{K-1} and fully satisfy the residual demand at an interest rate slightly below r_K. This implies that K drops out of the market, and $K-1$'s credit given is higher than K's was. Since the average interest rate at which the residual demand that prevails at r_{K-1} is satisfied rises (from a weighted average of r_{K-1} and r_K to slightly less than r_K), aggregate credit given falls by a non-infinitesimal amount and so does the equilibrium deposit rate ρ. As $K-1$'s return is close to $\varrho(r_K)$, the deposit rate jumps downward, and credit given exceeds l_K, it follows that $K-1$'s ensuing expected profit $(1+\Delta)(E\pi_{K-1})$ is higher than K's was: $(1+\Delta)(E\pi_{K-1}) > E\pi_K$. This contradicts the definition of a Nash equilibrium (which entails $E\pi_k \geq (1+\Delta)(E\pi_k)$ for $k \in \{K-1, K\}$):

$$E\pi_K \geq (1+\Delta)(E\pi_K) > E\pi_{K-1} \geq (1+\Delta)(E\pi_{K-1}) > E\pi_K.$$

So any equilibrium is characterized by zero expected profit for all banks.

Given zero profit, we can rule out the possibility of an equilibrium with credit given at a single interest rate, r_1 say (this implies non-existence of market-clearing or pure-rationing equilibria, as mentioned in Section 3). Excess demand for credit in a single-interest rate equilibrium implies that banks can make positive profit by making a small "number" of loans at an interest rate close to r^{max}. So credit given in a single-interest rate equilibrium is $L^D(r_1)$, and, from (3), the bank which gives credit at r_1 bids the deposit rate $\rho_1 = (L^S)^{-1}[L^D(r_1)]$. Together with zero expected profit (i.e., $\varrho(r_1) = \rho_1$), it follows that $L^S[\varrho(r_1)] = L^D(r_1)$. Due to the assumed shape of the return, demand, and supply functions (with $L^D(r^{**}) < L^S[\varrho(r^{**})] = L^S[\varrho(r^*)] < L^D(r^*)$), this implies that r_1 is in the interval (r^*, r^{**}), so there is positive demand at r^* (see Figure 1). Together with $\varrho(r_1) < \rho^*$, it follows further that making a small "number" of loans at r^* is profitable, since the ensuing return, ρ^*, is higher than $\varrho(r_1)$ and the deposit rate ρ rises only slightly.

r^* and r^{**} are the only market interest rates. Suppose not. Given that all banks make the same expected profit (viz., zero), the fact that the number of market interest rates exceeds one, and the assumed shape of the return function, there are two or three interest rates, each yields a return strictly less than ρ^*, and one market interest rate is in the interval (r^*, r^{**}) (see the upper panel of Figure 1). But then again it is possible to make positive expected profit by offering a small amount of credit at r^*.

At least two banks set the maximum credit limit at r^*. Otherwise the single bank setting the maximum credit limit at r^* can make positive expected profit by decreasing its credit limit slightly, so that the deposit rate ρ falls, while the return on lending ρ^* is unaffected. And at least two banks set a credit limit at r^{**} that is at least as large as the residual demand. If all credit limits at r^{**} fall short of residual demand, then there is positive residual demand for all interest rates up to r^{max}, so it is profitable to make loans at close to r^{max}. If only one bank sets a credit limit equal to or greater than residual demand, then this bank gains from increasing the interest rate it charges slightly and satisfying the residual demand at that interest rate: as it withdraws its credit offer at r^{**}, it generates positive residual demand above r^{**}, so that expected returns above ρ^* can be achieved. At the same time, as the residual demand which prevails at r^{**} is satisfied at a higher interest rate on average, credit given and the deposit rate ρ fall.

Total credit given at r^* and r^{**} equals $L^S(\rho^*)$. Otherwise the equilibrium deposit rate deviates from ρ^*, which contradicts zero expected profit. A decrease in credit given at r^* implies a less than one-for-one decrease in the residual demand at r^{**}. This is because only the sufficiently risky proportion of the additionally rationed borrowers demand loans at r^{**}. So aggregate credit given at r^* and r^{**} falls below $L^S(\rho^*)$ when credit given at r^* falls below $L^S(\rho^*)$, a contradiction. Conversely, when credit exceeds L^* at r^*, residual demand at r^{**} falls less than one-for-one, so there is excess demand at at r^{**}. This is inconsistent with equilibrium, since banks could make positive profit by making a small "number" of loans at an interest rate close to r^{max}.

4 Variations of the model

Theorems 1 and 2 state that double-Bertrand competition in the markets for credit (stage one) and deposits (stage two) uniquely gives rise to a two-price equilibrium when the return on lending is a non-monotonic function of the interest rate with excess demand at the local maximum and there is no market-clearing interest rate that yields a higher return. This section argues that this modified double-Bertrand competition is a natural approach to finding competitive equilibria in the SW model, because it gives rise to the usual kinds of equilibria under alternative assumptions about the shape of the return function, viz., market clearing when the return function is monotonic and credit rationing when it is hump-shaped and there is excess demand at the return-maximizing rate.

Monotonic return function

Suppose the return function $\varrho(r)$ is continuous and monotonic, so that the composite loan supply function $L^S[\varrho(r)]$ is also monotonically increasing. Suppose further that there is a market-clearing interest rate $\bar{r}$ (see the left panel of Figure 2). Let $\bar{\rho} \equiv \varrho(\bar{r})$ and $\bar{L} \equiv L^S(\bar{\rho})$. The following theorem states that equality of supply and demand then holds true generally:

THEOREM 3: *Suppose $\varrho(r)$ is continuous and monotonic and there is $\bar{r}$ such that $L^S(\varrho(\bar{r})) = L^D(\bar{r})$. Then the following pure strategies represent an SPNE:*

$\hookrightarrow$ *in the credit subgame, $(r_k, \lambda_k) = (\bar{r}, \bar{L})$ for two banks k and $(r_k, \lambda_k) = (0, 0)$ for the other banks $k \in \mathcal{K}$;*

$\hookrightarrow$ *in the deposit subgame, the bank k setting $\bar{r}$ and selected by the tie-breaking rule chooses $(\rho_k, \delta_k) = (\bar{\rho}, \bar{L})$ and all other banks $k \in \mathcal{K}$ choose $(\rho_k, \delta_k) = (0, 0)$.*

The deposit rate, interest rate, and amounts of deposits and credit are the same in any pure-strategy SPNE.

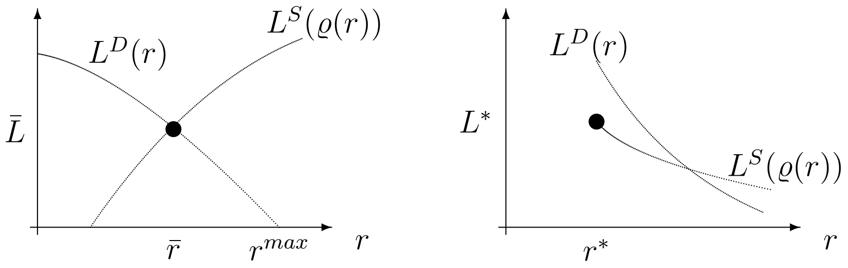

Figure 2: Credit market with monotonic or hump-shaped return function

The first part of the theorem is clear enough. There is no residual demand at interest rates above the market-clearing level. And setting an interest rate r below $\bar{r}$ is unprofitable, because this yields $\varrho(r) < \bar{\rho}$ and, from Lemmas 1 and 2, raises the deposit rate. As for the "uniqueness" part, we merely have to adapt a few steps in the proofs of Theorem 2. Details are in Appendix D. The arguments used to rule out positive expected profit in equilibrium in the proof of Theorem 2 apply here as well. Together with the fact that all banks pay the same deposit rate in equilibrium, it follows that all banks that make loans earn the same rate of return. Given the monotonicity of the return function, this means that there is a single market interest rate. If the single market interest rate is above the market-clearing level $\bar{r}$, there is excess supply of deposits, so it would be profitable to acquire funds at a deposit rate below $\bar{\rho}$ and lend at an interest rate slightly below $\bar{r}$. Conversely, if the market interest rate falls short of the market-clearing level, there is excess demand in the credit market and banks can make positive profit by lending at a rate slightly below r^{max}.

Hump-shaped return function

From the analysis in Section 3, it follows that in order to obtain a hump-shaped return function, we have to modify the assumptions made in Section 2. Accordingly, let the projects' expected return $E_R(R|\theta)$ be lower for projects with higher θ. That is, high-θ projects are not only more risky (in terms of second-order stochastic dominance) but worse in terms of expected return. Then we can assume that the return function $\varrho(r)$ takes on a unique interior maximum ρ^*, at a "bank-optimal" interest rate r^*, say.[12] Suppose further there is

[12]For instance, let type-θ projects succeed with probability $1-\theta$ and fail otherwise. The payoff is $\bar{R}/(1-\theta)+\gamma$ $(\bar{R}+\gamma-B > B-C > \gamma > 0)$ in case of success and zero otherwise. $G(\theta)$ is uniform on $[0,1]$. Expected profit is zero for type $\vartheta(r) = [(1+r)B-\gamma-\bar{R}]/[(1+r)B-\gamma-C]$. For $r \leq (\gamma+\bar{R})/B-1$ $(\equiv r^*)$,

excess demand at the bank-optimal rate: $L^D(r^*) > L^S(\rho^*)$ ($\equiv L^*$) (see the right panel of Figure 2). In this case, the double-Bertrand price-setting game uniquely yields the SW credit rationing equilibrium:

THEOREM 4: *Suppose $\varrho(r)$ is hump-shaped and $L^D(r^*) > L^S(\rho^*)$. Then the following pure strategies represent an SPNE:*

$\hookrightarrow$ *in the credit subgame, $(r_k, \lambda_k) = (r^*, L^*)$ for two banks k and $(r_k, \lambda_k) = (0, 0)$ for the other banks $k \in \mathcal{K}$;*

$\hookrightarrow$ *in the deposit subgame, the bank k setting r^* and selected by the tie-breaking rule chooses $(\rho_k, \delta_k) = (\rho^*, L^*)$ and all other banks $k \in \mathcal{K}$ choose $(\rho_k, \delta_k) = (0, 0)$.*

The deposit rate, interest rate, and amounts of deposits and credit are the same in any pure-strategy SPNE.

Again, the first part of the theorem is obvious: from Lemmas 1 and 2, the deposit rate rises above ρ^* if entry into the loan market is successful, while the expected rate of return cannot exceed ρ^*. The same arguments as above prove that banks make zero expected profit in equilibrium. Given that banks pay the same deposit rate, it follows that all market interest rates yield the same expected return. So given the hump shape of the return function, either there is a single market interest rate, or else there are two equilibrium interest rates, one below and the other above r^*. In the latter case, as there is positive residual demand at r^*, it would be profitable to make a small "number" of loans at r^*. So all loans are made at the same interest rate. If this rate exceeds r^*, there is positive residual demand at r^*, and it is profitable to make a small 'number" of loans at r^*. From the assumption that there is excess demand at r^*, it follows that there is excess demand at lower rates. So if the single market interest rate is lower than r^*, there is positive residual demand at r^*, and again it is profitable to make a small "number" of loans at r^*.

---

we have $\vartheta(r) \leq 0$, so all firms demand loans. The return function $\varrho(r) = C/B - 1 + [(1+r)B - C]/(2B)$ is upward-sloping in this case. Due to the parameter constraint, $\varrho(r^*) = (\bar{R} + \gamma + C)/(2B) - 1 > 0$. For $r > r^*$, the constraint $\bar{R} > C$ ensures $\vartheta'(r) > 0$, i.e., there is adverse selection. The return function $\varrho(r) = C/B - 1 + [(\bar{R} - C)/(2B)][(1+r)B - C]/[(1+r)B - \gamma - C]$ is downward-sloping.

5 Application to alternative models

In the preceding section, we argued that the modified double-Bertrand approach provides a natural foundation for competitive equilibria in the SW model, because it gives rise to the usual kinds of equilibria, depending on the shape of the return function. In this section, we argue that, more generally, the modified double-Bertrand approach provides a rigorous game-theoretic foundation for competitive equilibria in other models of financial intermediation as well. We substantiate this claim by incorporating modified double-Bertrand competition into several simple models of financial intermediation with different informational frictions (evidently, this endeavor cannot aim at being exhaustive). This requires only minor modifications of the analysis in the preceding sections. This is because, like the SW adverse selection model, standard credit market models give rise to a (return) function $\varrho(r) : \mathbb{R}_+ \to \mathbb{R}$ that relates the return on lending to the interest rate r *independently of other endogenous variables*. Equilibrium in the deposit and loan markets can then be analyzed using demand $L^D(r)$ and (composite) supply $L^S(\varrho(r))$ following exactly the same steps as in the SW model.

First-order dominance

Suppose projects of type θ yield a certain payoff $R(\theta)$. $R(\theta)$ is monotonically decreasing, so low-θ projects first-order (stochastically) dominate high-θ projects (cf. DeMeza and Webb, 1987). Firms demand capital if $R(\theta) \geq (1+r)B$, i.e., if $\theta \leq R^{-1}((1+r)B)$. The demand for capital is thus $L^D(r) \equiv G(R^{-1}((1+r)B))$. As there is no default risk, the return on lending is given by the return function

$$\varrho(r) = r.$$

Capital supply $L^S(\varrho(r)) = L^S(r)$ increases monotonically with the interest rate r. By the reasoning put forward in the SW model with a monotonic return function in Section 4, the equilibrium is uniquely characterized by the interest rate $\bar{r}$ that equates supply and demand (i.e., $L^S(\bar{r}) = L^D(\bar{r})$).

Moral hazard

Consider N (> 0) firms endowed with one project per capita. Each project requires input B (> 0). If it succeeds, it yields R (> 0). If it fails, it yields nothing. The success probability $p(e)$ is a continuously differentiable function of effort e (with $p'(e) > 0$ and $0 \leq p(e) \leq 1$). Effort has a cost $c(e)$ (twice continuously differentiable with $c'(e) > 0$ and $c''(e) \geq 0$).

Ignoring collateral, a firm's expected profit is $E(\pi(r,e)) = p(e)[R - (1+r)B]$. Let $e(r) = \arg\max_e E[\pi(r,e)]$. Firms demand a loan and choose $e = e(r)$ if $E[\pi(r,e(r))] \geq 0$. Otherwise, they choose not to carry out their project. That is, the demand for capital is NB for r up r^{max}, where $E[\pi(r^{max}, e(r^{max}))] = 0$. We assume that the banks' return on lending

$$\varrho(r) = p(e(r))(1+r) - 1$$

is a hump-shaped function of the interest rate with a positive maximum, at r^* say.[13] Finally, we assume that the supply of capital at r^* falls short of supply: $S(\varrho(r^*)) < NB$. By the same arguments as in Section 4, the equilibrium is a rationing equilibrium.

Limited enforcement

Finally, assume that lenders cannot perfectly enforce repayment, even if they can observe R: what they can do is impose a non-pecuniary penalty $\phi(R)$ on a borrower with payoff R who decides not to repay, where $\phi(R)$ is assumed strictly increasing. The distribution of returns $F(R)$ is identical for each firm and has bounded support $[0,\, R^{max}]$. Borrowers choose to repay if, any only if, $R \geq \phi^{-1}((1+r)B)$. The return on lending is given by the return function

$$\varrho(r) = (1+r)[1 - F(\phi^{-1}((1+r)B)] - 1.$$

Let r^{max} denote the interest rate at which the probability of repayment becomes zero: $r^{max} = \phi(R^{max})/B - 1$. The return function takes on its minimum value at this interest rate: $\varrho(r^{max}) = -1$. So it has an interior maximum at some r^*.[14] If there is excess demand at r^*, then by the same arguments as in Section 4, the equilibrium is a rationing equilibrium.

6 Conclusion

Financial intermediation, by definition, leads to two-sided competition. Generally, double-Bertrand competition in the markets for credit and deposits possibly gives rise to existence

[13] For instance, let $R > 2B$ and $p(e) = e$ and $c(e) = e^2/2$ for $0 \leq e \leq 1$. Then $e(r) = R - (1+r)B$, firms demand a loan if $r \leq R/B - 1$, and the return function $\varrho(r) = [R - (1+r)B](1+r) - 1$ attains an interior maximum of $R^2/(4B) - 1$ at $r = R/(2B) - 1$.

[14] For instance, let $\phi(R) = \alpha R$ $(0 < \alpha < 1)$ and $F(R) = R/R^{max}$ for $R \in [0,\, R^{max}]$. Then the return function is $\varrho(r) = r - (1+r)^2 B/(\alpha R^{max})$ and takes on its maximum value $\phi(\alpha R^{max}/(2B) - 1) = \alpha R^{max}/(4B)$ at $r = \alpha R^{max}/(2B) - 1$.

problems and non-competitive equilibria. We show that in the SW adverse selection model with a continuum of borrower types the usual types of competitive equilibria (with two prices or with market clearing or rationing at a single price) emerge in any subgame-prefect equilibrium of the double-Bertrand game if the credit subgame precedes the deposit subgame and banks can set limits on the amounts of credit they offer and deposits they take. The analysis is readily adapted to other models of financial intermediation as well. Thus, the modified double-Bertrand approach yields a solid game-theoretic foundation for the common practice of solving models of financial intermediation for perfectly competitive equilibria.

References

Allen, Beth, and Martin Hellwig (1986) Bertrand-Edgeworth Oligopoly in Large Markets, *Review of Economic Studies* 53, 175-204.

Arnold, Lutz G., and John G. Riley (2009) On the Possibility of Credit Rationing in the Stiglitz-Weiss Model, *American Economic Review* 99, 2012-2021.

Bernanke, Ben S., and Mark Gertler, Mark (1987) Banking and macroeconomic equilibrium, in William A. Barnett and Kenneth J. Singleton, eds.: *New Approaches to Monetary Economics* (Cambridge: Cambridge University Press), 89-111.

Bester, Helmut (1985) Screening vs. Rationing in Credit Markets with Imperfect Information, *American Economic Review* 75, 1985, 850-5.

Coco, Giuseppe (1997) Credit rationing and the welfare gain from usury laws, Discussion Paper 97/15, University of Exeter.

De Meza, David, and David C. Webb (1987) Too Much Investment: A Problem of Asymmetric Information, *Quarterly Journal of Economics* 102, 281-92.

De Meza, David, and David C. Webb (2006) Credit Rationing: Something's Gotta Give, *Economica* 73, 563-78.

Dixon, Huw (1987) Approximate Bertrand Equilibria in a Replicated Industry, *Review of Economic Studies* 54, 47-62.

Freixas, Xavier, and Jean-Charles Rochet (1997) *Microeconomics of Banking* (Cambridge, MA: MIT Press).

Gersbach, Hans (2002) Financial Intermediation and the Creation of Macroeconomic Risks, *CESifo Working Papers*, No, 695.

Gersbach, Hans (2008) Banking with Contingent Contracts, Macroeconomic Risks, and Banking Crises, *CER-ETH Working Paper*, No. 08/93.

Hellwig, Martin (1987) Some Recent Developments in the Theory of Competition in Markets with Adverse Selection, *European Economic Review* 31, 319-25.

Lensink, Robert, and Elmer Sterken (2002) The Option to Wait to Invest and Equilibrium Credit Rationing, *Journal of Money, Credit, and Banking* 34, 221-5.

Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green (1995) *Microeconomic Theory* (Oxford: Oxford University Press).

Riley, John G. (1987) Credit Rationing: A Further Remark, *American Economic Review* 77, 224-7.

Stahl, Dale O. (1988) Bertrand Competition for Inputs and Walrasian Outcomes, *American Economic Review* 78, 189-201.

Stiglitz, Joseph E. (1987) The Causes and Consequences of the Dependence of Quality on Price, *Journal of Economic Literature* 25, 1-48.

Stiglitz, Joseph E., and Andrew Weiss (1981) Credit Rationing in Markets with Imperfect Information, *American Economic Review* 71, 393-410.

Yanelle, Marie-Odile (1989) The strategic analysis of intermediation, *European Economic Review* 33, 294-301.

Yanelle, Marie-Odile (1997) Banking Competition and Market Efficiency, *Review of Economic Studies* 64, 215-239.

A Game-Theoretic Foundation for Competitive Equilibria in the Stiglitz-Weiss Model: Technical Appendix

Appendix A. Residual supply, residual demand, and payoffs

A.1 Residual supply

In the deposit subgame, bank k's strategy is (ρ_k, δ_k). Let d_k denote the amount of deposits bank $k \in \mathcal{K}$ gets. Given that the bids are served in the order of decreasing rates ρ_k, $d_k = 0$ for banks with ρ_k below some marginal rate. Let $\mathcal{D} \equiv \{\rho | \, \rho_k = \rho \text{ and } d_k > 0 \text{ for some } k \in \mathcal{K}\}$ be the set of market deposit rates, i.e., of rates such that some bank raises a positive amount of deposits. Let the market deposit rates $\rho_n \in \mathcal{D}$ be ordered such that $\rho_1 > \rho_2 > \cdots > \rho_N$, where N is the number of market deposit rates. Let D_n be the total amount of deposits taken by banks at the deposit rate ρ_n. Let $l_n^S(\rho)$ denote the residual supply at $\rho > \rho_{n-1}$. Because of random rationing, residual supply is determined recursively by

$$l_{n+1}^S(\rho) = \left[1 - \frac{D_n}{l_n^S(\rho_n)}\right] l_n^S(\rho), \quad \rho > \rho_n, \tag{A.1}$$

for $n \in \mathcal{N} \backslash \{N\}$, where $\mathcal{N} \equiv \{1, \ldots, N\}$.

LEMMA A.1: *Residual supply satisfies* $l_1^S(\rho_1) = L^S(\rho_1)$ *and*

$$l_n^S(\rho) = \left[1 - \sum_{n'=1}^{n-1} \frac{D_{n'}}{L^S(\rho_{n'})}\right] L^S(\rho), \quad \rho > \rho_{n-1}, \tag{A.2}$$

for $n \in \mathcal{N} \backslash \{1\}$.

Proof: The assertion is evidently true for $n = 1$. The validity of (A.2) for $n > 1$ is proved by induction, by substituting from the induction hypothesis (A.2) into (A.1):

$$\begin{aligned}
l_{n+1}^S(\rho) &= \left[1 - \frac{D_n}{l_n^S(\rho_n)}\right] l_n^S(\rho) \\
&= l_n^S(\rho) - D_n \frac{l_n^S(\rho)}{l_n^S(\rho_n)} \\
&= \left[1 - \sum_{n'=1}^{n-1} \frac{D_{n'}}{L^S(\rho_{n'})}\right] L^S(\rho) - \frac{D_n}{L^S(\rho_n)} L^S(\rho) \\
&= \left[1 - \sum_{n'=1}^{n} \frac{D_{n'}}{L^S(\rho_{n'})}\right] L^S(\rho). \quad ///
\end{aligned}$$

For $n \in \mathcal{N}$, let $\mathcal{K}_n = \{k \in \mathcal{K} | \, \rho_k = \rho_n\}$ denote the set of banks which play ρ_n in the deposit subgame. The amount of deposits raised by a bank $k \in \mathcal{K}_n$ is

$$d_k = \min\left\{\delta_k, \, \frac{\delta_k}{\sum_{k' \in \mathcal{K}_n} \delta_{k'}} l_n^S(\rho_n)\right\}. \tag{A.3}$$

$d_k = 0$ if $k \notin \mathcal{K}_n$ for all $n \in \mathcal{N}$. The total amount of deposits with deposit rate ρ_n is

$$D_n = \sum_{k \in \mathcal{K}_n} d_k. \tag{A.4}$$

The fact that there is positive (residual) supply at ρ_n implies that there is excess supply at the lower market deposit rates:

$$l_n^S(\rho_n) > \sum_{k' \in \mathcal{K}_n} \delta_{k'}$$

for $n \in \mathcal{N} \backslash \{N\}$. So from (A.3), $d_k = \delta_k$ for $k \in \mathcal{K}_n$ and

$$D_n = \sum_{k \in \mathcal{K}_n} \delta_k \tag{A.5}$$

for $n \in \mathcal{N} \backslash \{N\}$.

LEMMA A.2: *If $\rho_n \leq \rho$ for all $n \in \mathcal{N}$ and $\rho_N < \rho$, then*

$$\sum_{n=1}^{N} D_n < L^S(\rho).$$

Proof: Since $L^S(\rho)$ is strictly increasing, we have $L^S(\rho_n) \leq L^S(\rho)$ for all $n \in \mathcal{N}$ and $L^S(\rho_N) < L^S(\rho)$. Using $D_N \leq l_N^S(\rho_N)$ and (A.2), we obtain:

$$\begin{aligned}
D_N &\leq l_N^S(\rho_N) \\
&= \left[1 - \sum_{n'=1}^{N-1} \frac{D_{n'}}{L^S(\rho_{n'})}\right] L^S(\rho_N) \\
&< \left[1 - \sum_{n'=1}^{N-1} \frac{D_{n'}}{L^S(\rho_{n'})}\right] L^S(\rho) \\
&= L^S(\rho) - \sum_{n'=1}^{N-1} \frac{L^S(\rho)}{L^S(\rho_{n'})} D_{n'} \\
&\leq L^S(\rho) - \sum_{n'=1}^{N-1} D_{n'} \\
\sum_{n=1}^{N} D_n &< L^S(\rho). \quad ///
\end{aligned}$$

Starting from a given strategy profile, let Δx denote the change in a variable x induced by a change in the strategies, so that $(1+\Delta)x$ is the new value this variable takes on. As a direct corollary to Lemma A.1, we obtain from (A.2):

LEMMA A.3: *If the amount of deposits raised at ρ_{n-1} changes by ΔD_{n-1}, the residual supply at ρ_n changes by*

$$\Delta l_n^S(\rho_n) = -\frac{L^S(\rho_n)}{L^S(\rho_{n-1})}\Delta D_{n-1}$$

($n \in \mathcal{N}\backslash\{1\}$). $\Delta l_n^S(\rho_n) \to -\Delta D_{n-1}$ as $\rho_n \to \rho_{n-1}$.

A.2 Residual demand

In the credit subgame, bank k's strategy is (r_k, λ_k). Let l_k denote the amount of loans actually made by bank $k \in \mathcal{K}$. Given that firms apply for credit at the lowest rates first, $l_k = 0$ for banks with r_k above some marginal rate. Let $\mathcal{R} \equiv \{r|\ r_k = r \text{ and } l_k > 0 \text{ for some } k \in \mathcal{K}\}$ be the set of market interest rates, i.e., at which the amount of loans made is positive. Let the interest rates in $\mathcal{R}$ be ordered such that $r_1 < r_2 < \cdots < r_M$, where M is the number of market interest rates. Let $\mathcal{M} \equiv \{1, 2, \ldots, M\}$. For each $m \in \mathcal{M}$, denote the amount of credit given at $r_m \in \mathcal{R}$ as L_m (> 0). Let $l_m^D(r)$ denote the residual demand at r. Due to random rationing, residual demand is determined recursively by

$$l_{m+1}^D(r) = \left[1 - \frac{L_m}{l_m^D(r_m)}\right] l_m^D(r), \quad r > r_m, \tag{A.6}$$

for $m \in \mathcal{M}\backslash\{M\}$.

LEMMA A.4: *Residual demand satisfies $l_1^D(r) = L^D(r)$ and*

$$l_m^D(r) = \left[1 - \sum_{m'=1}^{m-1} \frac{L_{m'}}{L^D(r_{m'})}\right] L^D(r), \quad r > r_{m-1}, \tag{A.7}$$

for $m \in \mathcal{M}\backslash\{1\}$.

Proof: The proof parallels that of Lemma A.1. $l_1^D(r) = L^D(r)$ is obvious. We prove the validity of (A.7) by induction on m. Suppose (A.7) holds for m. Substituting for $l_m^D(r_m)$ and $l_m^D(r)$ from (A.7) in (A.6) proves the validity of (A.7) for $m+1$:

$$\begin{aligned} l^D_{m+1}(r) &= \left[1 - \frac{L_m}{l^D_m(r_m)}\right] l^D_m(r) \\ &= l^D_m(r) - L_m \frac{l^D_m(r)}{l^D_m(r_m)} \\ &= \left[1 - \sum_{m'=1}^{m-1} \frac{L_{m'}}{L^D(r_{m'})}\right] L^D(r) - L_m \frac{L^D(r)}{L^D(r_m)} \\ &= \left[1 - \sum_{m'=1}^{m} \frac{L_{m'}}{L^D(r_{m'})}\right] L^D(r). \quad /// \end{aligned}$$

LEMMA A.5: *Residual demand satisfies*

$$l^D_m(r_m) > \sum_{m'=m}^{M} L_{m'}$$

for $m \in \mathcal{M}\backslash\{M\}$.

Proof: From the fact that $l^D_m(r)$ is strictly decreasing, $L_m < l^D_m(r_m)$ for $m \in \mathcal{M}\backslash\{M\}$, the ordering convention $r_m < r_{m+1}$, and (A.6),

$$\begin{aligned} l^D_m(r_m) &= L_m + \left[1 - \frac{L_m}{l^D_m(r_m)}\right] l^D_m(r_m) \\ &> L_m + \left[1 - \frac{L_m}{l^D_m(r_m)}\right] l^D_m(r_{m+1}) \\ &= L_m + l^D_{m+1}(r_{m+1}) \end{aligned} \tag{A.8}$$

for $m \in \mathcal{M}\backslash\{M\}$. For given $m \in \mathcal{M}\backslash\{M-1, M-2\}$, we assert that

$$l^D_m(r_m) > \sum_{m'=m}^{m''} L_{m'} + l^D_{m''+1}(r_{m''+1}) \tag{A.9}$$

holds true for all $m'' \in \mathcal{M}\backslash\{1, \ldots, m-1, M\}$. (A.8) proves the validity of (A.9) for $m'' = m$. We prove (A.9) by induction on m''. From the induction hypothesis and $l^D_{m''+1}(r_{m''+1}) - L_{m''+1} > l^D_{m''+2}(r_{m''+2})$ (from (A.8)),

$$\begin{aligned} l^D_m(r_m) &> \sum_{m'=m}^{m''} L_{m'} + l^D_{m''+1}(r_{m''+1}) \\ &= \sum_{m'=m}^{m''+1} L_{m'} - L_{m''+1} + l^D_{m''+1}(r_{m''+1}) \\ &> \sum_{m'=m}^{m''+1} L_{m'} + l^D_{m''+2}(r_{m''+2}). \end{aligned}$$

This proves the validity of (A.9) for $m''+1$ and, hence, for all $m'' \in \mathcal{M}\backslash\{1,\dots,m-1,M\}$. Setting $m''=M-1$ and using $L_M \leq l_M^D(r_M)$ yields

$$\begin{aligned} l_m^D(r_m) &> \sum_{m'=m}^{M-1} L_{m'} + l_M^D(r_M) \\ &\geq \sum_{m'=m}^{M} L_{m'}. \quad /// \end{aligned}$$

For each $m \in \mathcal{M}$, let $\Lambda_m = \max\{\lambda_k | \, k \in \mathcal{K}, r_k = r_m\}$ denote the maximum credit limit of those banks k which set r_m. Given that (one of) the bank(s) with the highest credit limit alone serves the entire market demand, the amount of credit given at r_m is

$$L_m = \min\left\{\Lambda_m, l_m^D(r_m)\right\}. \tag{A.10}$$

Starting from a strategy profile $\{(r_k,\lambda_k)\}_{k\in\mathcal{K}}$, let the strategies change in such a way that one of the following two situations arises.

(a) Credit given becomes positive for exactly one interest rate, and credit supply remains unchanged at all market interest rates:

$\hookrightarrow$ $(1+\Delta)L_{m^d} = \Delta L_{m^d} > 0$ at one $r_{m^d} \notin \mathcal{R}$;

$\hookrightarrow$ $\Delta\Lambda_m = 0$ for all $r_m \in \mathcal{R}$.

(b) Credit given increases at exactly one interest rate, and credit supply remains unchanged at all other market interest rates:

$\hookrightarrow$ $\Delta L_{m^d} > 0$ at one $r_{m^d} \in \mathcal{R}$;

$\hookrightarrow$ $\Delta\Lambda_m = 0$ for all $r_m \in \mathcal{R}\backslash\{r_{m^d}\}$.

Let $m^d = 0$ and $\mathcal{M}^d = \mathcal{M}\cup\{0\}$ in case (a) and $\mathcal{M}^d = \mathcal{M}$ in case (b).

LEMMA A.6: $\Delta(\sum_{m\in\mathcal{M}^d} L_m) > 0$ *in both cases (a) and (b).* $\Delta(\sum_{m\in\mathcal{M}^d} L_m) \to 0$ *as* $\Delta L_{m^d} \to 0$.

Proof: We focus on case (a). [The necessary changes in case (b) are inserted in brackets.]

To begin with, let $r_{m^d} > r_M$ [$r_{m^d} = r_M$ in case (b)]. From (A.7) and (A.10), residual demands and credit given at all interest rates r_m, $m \in \mathcal{M}$ [$m \in \mathcal{M}\backslash\{M\}$ in case (b)] are unaffected:

$$\Delta L_m = 0 \quad \text{for } m \in \mathcal{M}$$

[for $m \in \mathcal{M}\backslash\{M\}$ in case (b)]. So $\Delta L_{m^d} = \Delta L_0 > 0$ [$\Delta L_{m^d} = \Delta L_M > 0$ in case (b)] yields

$$\Delta\left(\sum_{m\in\mathcal{M}^d} L_m\right) = \Delta L_{m^d} > 0.$$

Obviously, $\Delta(\sum_{m\in\mathcal{M}^d} L_m) \to 0$ as $\Delta L_{m^d} \to 0$.

If, on the other hand, $r_{m^d} < r_M$, let $r_{m'}$ denote the lowest interest rate above r_{m^d}: $r_{m'-1} < r_{m^d} < r_{m'}$ [$m' = m^d$ in case (b)]. From (A.7) and (A.10), ΔL_{m^d} does not affect residual demand or credit given at lower interest rates:

$$\Delta L_m = 0 \quad \text{for } m \in \{1, \ldots, m'-1\}. \tag{A.11}$$

The fact that $r_{m^d} < r_M$ implies that for the initial strategy profile, there is excess demand at r_{m^d} and, therefore, positive residual demand at higher interest rates r_m, $m \in \{m', \ldots, M\}$ [here and throughout the remainder of the proof, replace m' with $m'+1$ in case (b)]. As $\Delta L_{m^d} > 0$, (A.7) implies that these residual demands fall. We have to distinguish three cases.

(1) $(1+\Delta)L_{m^d} < l^D_{m^d}(r_{m^d})$ and $\Lambda_m < (1+\Delta)l^D_m(r_m)$ for all $m \in \{m', \ldots, M-1\}$:

That is, ΔL_{m^d} is small enough such that the excess demands at interest rates up to r_{M-1} remain positive. This is satisfied for $\Delta L_{m^d} \to 0$. We then have

$$\Delta L_m = 0 \quad \text{for } m \in \{m', \ldots, M-1\} \tag{A.12}$$

[in case (b), if $r_{m^d} = r_{M-1}$, then (A.12) drops out]. Furthermore, from (A.7), (A.11), and (A.12),

$$\Delta l^D_M(r_M) = -\frac{\Delta L_{m^d}}{L^D(r_{m^d})} L^D(r_M). \tag{A.13}$$

If $L_M = l^D_M(r_M)$, then $\Lambda_M \geq l^D_M(r_M)$. Using (A.13), we have

$$\begin{aligned} \Delta L_M &= \Delta l^D_M(r_M) \\ &= -\frac{L^D(r_M)}{L^D(r_{m^d})} \Delta L_{m^d}. \end{aligned} \tag{A.14}$$

Adding up (A.11), ΔL_{m^d}, (A.12), (A.14) and using $r_{m^d} < r_M$ and $\Delta L_{m^d} > 0$, we obtain

$$\Delta\left(\sum_{m\in\mathcal{M}^d} L_m\right) = \left[1 - \frac{L^D(r_M)}{L^D(r_{m^d})}\right] \Delta L_{m^d} > 0.$$

Obviously, $\Delta(\sum_{m\in\mathcal{M}^d} L_m) \to 0$ as $\Delta L_{m^d} \to 0$.

If, on the other hand, $L_M < l^D_M(r_M)$, then $\Lambda_M = L_M$. Using (A.10), the case distinction $L_M <$

$l_M^D(r_M)$, and (A.13), it follows that

$$\begin{aligned}
\Delta L_M &= \min\left\{\Lambda_M,\, (1+\Delta)l_M^D(r_M)\right\} - L_M \\
&= \min\left\{\Lambda_M - L_M,\, \Delta l_M^D(r_M) + l_M^D(r_M) - L_M\right\} \\
&= \min\left\{0,\, \Delta l_M^D(r_M) + l_M^D(r_M) - L_M\right\} \\
&\geq \min\left\{0,\, \Delta l_M^D(r_M)\right\} \qquad (A.15)\\
&= \min\left\{0,\, -\frac{L^D(r_M)}{L^D(r_{m^d})}\Delta L_{m^d}\right\} \\
&= -\frac{L^D(r_M)}{L^D(r_{m^d})}\Delta L_{m^d}.
\end{aligned}$$

Hence, from (A.11), (A.12), $r_{m^d} < r_M$, and $\Delta L_{m^d} > 0$,

$$\Delta\left(\sum_{m\in\mathcal{M}^d} L_m\right) \geq \left[1 - \frac{L^D(r_M)}{L^D(r_{m^d})}\right]\Delta L_{m^d} > 0. \qquad (A.16)$$

As $\Delta L_{m^d} \to 0$, we have, from (A.13), $\Delta l_M^D(r_M) \to 0$. Together with the case distinction $L_M < l_M^D(r_M)$, it follows that the equality sign holds in (A.15) and, hence, in the former inequality in (A.16). So $\Delta(\sum_{m\in\mathcal{M}^d} L_m) \to 0$ as $\Delta L_{m^d} \to 0$.

(2) $(1+\Delta)L_{m^d} < l_{m^d}^D(r_{m^d})$*, but* $\Lambda_m < (1+\Delta)l_m^D(r_m)$ *does not hold true for all* $m \in \{m', \ldots, M-1\}$*:* In this case [which cannot occur if $r_{m^d} = r_{M-1}$ in case (b)], there is $m'' \in \{m', \ldots, M-1\}$ such that $\Lambda_{m''} \geq (1+\Delta)l_{m''}^D(r_{m''}) > 0$. The fact that there is positive residual demand at $r_{m''}$ (as $(1+\Delta)l_{m''}^D(r_{m''}) > 0$) implies $\Lambda_m < (1+\Delta)l_m^D(r_m)$ for $m \in \{m', \ldots, m''-1\}$. Hence, from (A.7) and (A.10),

$$\Delta L_m = 0 \text{ for } m \in \{m', \ldots, m''-1\}. \qquad (A.17)$$

From (A.7), (A.11), and (A.17),

$$\Delta l_{m''}^D(r_{m''}) = -\frac{\Delta L_{m^d}}{L^D(r_{m^d})}L^D(r_{m''}).$$

Using $(1+\Delta)L_{m''} = (1+\Delta)l_{m''}^D(r_{m''})$, we have

$$\begin{aligned}
\Delta L_{m''} &= (1+\Delta)l_{m''}^D(r_{m''}) - L_{m''} \\
&= -\frac{L^D(r_{m''})}{L^D(r_{m^d})}\Delta L_{m^d} + l_{m''}^D(r_{m''}) - L_{m''}. \qquad (A.18)
\end{aligned}$$

There is no residual demand at r_m for $m \in \{m''+1, \ldots, M\}$ (as $\Lambda_{m''} \geq (1+\Delta)l_{m''}^D(r_{m''})$). So

$$\Delta L_m = -L_m \text{ for } m \in \{m''+1, \ldots, M\}. \qquad (A.19)$$

Adding up (A.11), ΔL_{m^d}, and (A.17)-(A.19) yields

$$\Delta\left(\sum_{m\in\mathcal{M}^d} L_m\right) = \left[1 - \frac{L^D(r_{m''})}{L^D(r_{m^d})}\right]\Delta L_{m^d} + l^D_{m''}(r_{m''}) - \sum_{m=m''}^{M} L_m > 0.$$

The inequality sign follows $r_{m^d} < r_{m''}$, $\Delta L_{m^d} > 0$, and $l^D_{m''}(r_{m''}) > \sum_{m=m''}^{M} L_m$ (from Lemma A.5).

(3) $(1+\Delta)L_{m^d} = l^D_{m^d}(r_{m^d})$*:*

In this case,

$$\Delta L_{m^d} = l^D_{m^d}(r_{m^d}) \tag{A.20}$$

[$\Delta L_{m^d} = l^D_{m^d}(r_{m^d}) - L_{m^d}$ in case (b)]. Since there is no residual demand at interest rates above r_{m^d},

$$\Delta L_m = -L_m \quad \text{for } m \in \{m', \ldots, M\}. \tag{A.21}$$

Adding up (A.11), (A.20), and (A.21) yields

$$\Delta\left(\sum_{m\in\mathcal{M}^d} L_m\right) = l^D_{m^d}(r_{m^d}) - \left(L_{m^d} + \sum_{m=m'}^{M} L_m\right) > 0.$$

The inequality sign follows from the fact that $l^D_{m^d}(r_{m^d}) > L_{m^d} + \sum_{m=m'}^{M} L_m$ (from Lemma A.5). ///

This is Lemma 2 in the main text.

A.3 Payoff functions

LEMMA A.7: $\varrho(r_m)$ *gives the return on lending for all* $r_m \in \mathcal{R}$*.*

Proof: At each interest rate $r_m \in \mathcal{R}$ rationing, if it occurs, is random (the probability of receiving funds $L_m/l^D_m(r_m)$ is uniform across types). Consequently, the relative frequencies of types $\theta \in [\vartheta(r_m), \theta^{max}]$ which still have not received credit and, therefore, the relative frequencies of those types $\theta \in [\vartheta(r_{m+1}), \theta^{max}]$ which demand credit at r_{m+1} do not change. Applying this reasoning recursively, starting at $m = 1$, proves the lemma. ///

For each $r_m \in \mathcal{R}$, let $\mathcal{K}_m = \{k \in \mathcal{K} | (r_k, \lambda_k) = (r_m, \Lambda_m)\}$ denote the set of banks which set the maximum credit limit. Let $\mathcal{K}^+ = \{k \in \mathcal{K} | l_k > 0\}$ denote the set of banks which give credit in equilibrium. If $\mathcal{K}_m$ contains a single bank k, then $k \in \mathcal{K}^+$. If $\mathcal{K}_m$ contains several banks k, according to the tie-breaking rule, the probability of being in $\mathcal{K}^+$ is $(\#\mathcal{K}_m)^{-1}$ for each of these banks. Since at each market interest rate $r_m \in \mathcal{R}$ only one bank k serves the market demand, we can relabel the banks k in $\mathcal{K}^+$ such that $k = m$ for $m \in \mathcal{M}$, reinterpret $l^D_k(r_k) = l^D_m(r_m)$ as the residual demand faced by bank k, and let $l_k = L_m$ denote credit given by bank $k \in \mathcal{K}^+$. From (A.7) and (A.10), we then have

$$l_k^D(r_k) = \left[1 - \sum_{k'=1}^{k-1} \frac{l_{k'}}{L^D(r_{k'})}\right] L^D(r_k) \tag{A.22}$$

and

$$l_k = \min\{\lambda_k, l_k^D(r_k)\} \tag{A.23}$$

for $k \in \mathcal{K}^+$. $l_k = 0$ for $k \notin \mathcal{K}^+$. Using Lemma A.7, bank k's profit is

$$\pi_k = \begin{cases} [1 + \varrho(r_k)]l_k - (1 + \rho_k)d_k; & \text{for } d_k \geq l_k \\ \underline{\pi}; & \text{for } d_k < l_k \end{cases} \tag{A.24}$$

for $k \in \mathcal{K}$. This equation maps strategy profiles $\{(r_k, \lambda_k), (\rho_k, \delta_k)\}_{k \in \mathcal{K}}$ to expected payoffs $E\pi_k$ for $k \in \mathcal{K}$. In the credit subgame, the choices $\{(r_k, \lambda_k)\}_{k \in \mathcal{K}}$ together with the tie-breaking rule determine $\mathcal{K}^+$. While $l_k = 0$ for $k \notin \mathcal{K}^+$, (A.22) and (A.23) determine l_k for $k \in \mathcal{K}^+$. In the deposit subgame, the choices $\{(\rho_k, \delta_k)\}_{k \in \mathcal{K}}$ determine d_k via (A.2), (A.3), and (A.4).

Appendix B: Proof of Theorem 1

Given the strategies in Theorem 1, the tie-breaking rule determines one bank, $k = 1$ say, which serves credit demand at r^* and another bank, $k = 2$ say, which serves the residual demand at r^{**}. From (A.23), these banks give credit $l_1 = \min\{L^*, L^D(r^*)\} = L^*$ and $l_2 = \min\{L^S(\rho^*) - L^*, l_2^D(r^{**})\}$, respectively. $l_k = 0$ for all other banks k so $\mathcal{K}^+ = \{1, 2\}$. By bidding $\rho_1 = \rho_2 = \rho^*$, the two banks in $\mathcal{K}^+$ raise deposits $L^S(\rho^*)$. From the definition of x and Lemma A.4, it follows that the residual demand at r^{**}, $l_2^D(r^{**}) = (1 - x)L^D(r^{**})$, equals supply $l_2 = L^S(\rho^*) - L^*$. So aggregate credit given is $\sum_{k \in \mathcal{K}} l_k = l_1 + l_2 = L^S(\rho^*)$. From (A.3), $d_k = l_k$ for both banks $k \in \mathcal{K}^+$. $\varrho(r^*) = \varrho(r^{**}) = \rho^*$ and Lemma A.7 imply zero expected profit for both banks ($E\pi_1 = E\pi_2 = 0$). To prove Theorem 1, we have to show that deviations from the strategies in the theorem are not profitable.

Throughout we make the following two labeling conventions. First, when starting from a given strategy profile, banks k are labeled as explained in Appendix A.3, they keep their label after a change in the strategy profile. For instance, if K is thy only bank setting the maximum market interest rate r_K initially, we denote the post-change interest rate it sets as $(1 + \Delta)r_K$ even if there are now other banks with higher interest rates. Second, when more than one bank set r_k, we label the bank that is determined by the random tie-breaking rule as k, although a different bank may be chosen by the tie-breaking rule after a change in the strategy profile.

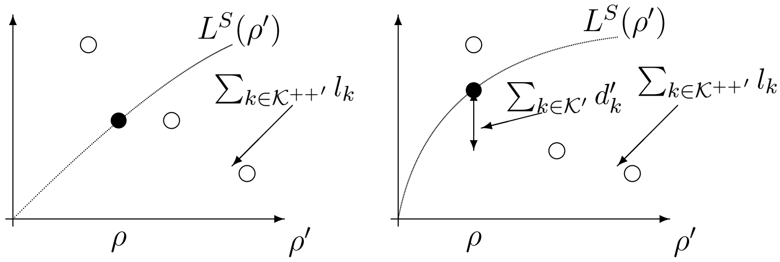

Figure B.1: Equilibrium deposit rate

B.1 Deposit subgame

If a bank $k \in \mathcal{K}^+$ raises deposits d_k equal to the amount of loans it makes l_k its payoff (A.24) simplifies to

$$\pi_k = [\varrho(r_k) - \rho_k]\, l_k, \quad k \in \mathcal{K}^+. \tag{B.1}$$

For a given deposit rate ρ', let $\mathcal{K}^{++\prime}$ be the subset of banks $k \in \mathcal{K}^+$ which (weakly) prefer raising $d_k = l_k$ to default: $\mathcal{K}^{++\prime} = \{k \in \mathcal{K} |\, k \in \mathcal{K}^+, [\varrho(r_k) - \rho']l_k \geq \underline{\pi}\}$. As ρ' rises, elements of $\mathcal{K}^{++\prime}$ successively drop out. So $\sum_{k\in\mathcal{K}^{++\prime}} l_k$ is a decreasing step function (see Figure B.1). (a) If there is a solution ρ' to $\sum_{k\in\mathcal{K}^{++\prime}} l_k = L^S(\rho')$, denote it as ρ (see the left panel of Figure B.1). (b) Otherwise, let ρ be the deposit rate ρ' such that $\sum_{k\in\mathcal{K}^{++\prime}} l_k > L^S(\rho')$ for $\rho' = \rho$ and $\sum_{k\in\mathcal{K}^{++\prime}} l_k < L^S(\rho')$ for $\rho' > \rho$ (see the right panel of Figure B.1). Let $\mathcal{K}'$ be the subset of banks k which are indifferent between raising $d_k = l_k$ at ρ and default in this case: $\mathcal{K}' = \{k \in \mathcal{K} |\, k \in \mathcal{K}^{++}, [\varrho(r_k) - \rho]l_k = \underline{\pi}\}$. Finally, let $\{d'_k\}_{k\in\mathcal{K}'}$ be any set of deposits for $k \in \mathcal{K}'$ such that $d'_k < l_k$ for all $k \in \mathcal{K}'$ and

$$\sum_{k\in\mathcal{K}'} d'_k = L^S(\rho) - \sum_{k\in\mathcal{K}^{++}\setminus\mathcal{K}'} l_k. \tag{B.2}$$

The definition of ρ implies that such a set of deposits exists. Letting $\mathcal{K}' = \emptyset$ in case (a), we can treat both cases using the same notation. Notice that if $\mathcal{K}^{++} = \mathcal{K}^+$ and $\mathcal{K}' = \emptyset$, then, using $l_k = 0$ for $k \notin \mathcal{K}^+$, (B.2) becomes

$$\rho = (L^S)^{-1}\left(\sum_{k\in\mathcal{K}} l_k\right) \tag{B.3}$$

(this holds true, for example, if one assumes $\underline{\pi} = -\infty$, so that all banks in $\mathcal{K}^+$ strictly prefer any loss to default and $\mathcal{K}' = \emptyset$).

LEMMA B.1: *Suppose d'_k satisfies (B.2) for $k \in \mathcal{K}'$, and let ρ be given by (B.3). Then the following strategies are a Nash equilibrium of the deposit subgame: $(\rho_k, \delta_k) = (\rho, l_k)$ for $k \in \mathcal{K}^{++}\setminus\mathcal{K}'$, $(\rho_k, \delta_k) = (\rho, d'_k)$ for $k \in \mathcal{K}'$, and $(\rho_k, \delta_k) = (0, 0)$ for $k \notin \mathcal{K}^{++}$.*

Proof: Banks $k \in \mathcal{K}^{++}$ acquire funds by paying a common deposit rate ρ. From (B.2), demand equals supply in the market for deposits:

$$\begin{aligned}\sum_{k\in\mathcal{K}} \delta_k &= \sum_{k\in\mathcal{K}^{++}\backslash\mathcal{K}'} l_k + \sum_{k\in\mathcal{K}'} d'_k \\ &= L^S(\rho).\end{aligned}$$

So, from (A.3), $d_k = \delta_k$ for all $k \in \mathcal{K}^{++}$. We now consider the three groups of banks distinguished in the lemma successively. We have to show that no bank can raise its payoff by deviating with $\Delta(\rho_k, \delta_k) \neq (0,0)$.

Consider first a bank $k^d \in \mathcal{K}^{++}\backslash\mathcal{K}'$. From the definition of $\mathcal{K}^{++}$, the fact that $d_{k^d} = l_{k^d}$, and (B.1), we have $\pi_{k^d} \geq \underline{\pi}$ (with strict inequality in case (b)). As $d_{k^d} = l_{k^d}$, any deviating strategy $(1+\Delta)(\rho_{k^d}, \delta_{k^d})$ which gives rise to $\Delta d_{k^d} < 0$ implies default and, therefore, is unprofitable: $(1+\Delta)\pi_{k^d} = \underline{\pi} \leq \pi_{k^d}$. Clearly, this occurs for $\Delta\delta_{k^d} < 0$. The same also holds true for bids with $\Delta\rho_{k^d} < 0$. To see this, notice that Lemma A.2 implies that the supply of deposits falls below $L^S(\rho)$ if $\Delta\rho_{k^d} < 0$, so that $\Delta d_{k^d} < 0$, since all other banks bid a higher rate than k^d. So only deviating strategies with $\Delta d_{k^d} \geq 0$ and $\Delta\rho_{k^d} \geq 0$ and with one inequality strict avoid default. However, this implies $\Delta[(1+\rho_{k^d})d_{k^d}] > 0$ and, from (A.24) and the fact that r_{k^d} and l_{k^d} are determined in the credit subgame already, $\Delta\pi_{k^d} < 0$. Hence, any deviating strategy, whether it leads to default or not, reduces profit.

Next, consider a bank $k^d \in \mathcal{K}'$ (which is non-empty in case (b) only). The strategy in the lemma implies default: $\pi_{k^d} = \underline{\pi}$, since $\delta_k = d'_k < l_k$. Any deviating strategy that also leads to default is equally profitable: $(1+\Delta)\pi_{k^d} = \underline{\pi}$. A deviating strategy that avoids default necessarily satisfies $(1+\Delta)d_{k^d} \geq l_{k^d} > 0$. Since supply equals demand in the market for deposits and the supply is shared in proportion to the bids made in the case of excess demand for deposits, this necessitates $(1+\Delta)\rho_{k^d} > \rho$. Together with (A.24), this implies

$$\begin{aligned}(1+\Delta)\pi_{k^d} &= [1+\varrho(r_{k^d})]l_{k^d} - [1+(1+\Delta)\rho_{k^d}](1+\Delta)d_{k^d} \\ &< [1+\varrho(r_{k^d})]l_{k^d} - (1+\rho)l_{k^d} \\ &= [\varrho(r_{k^d}) - \rho]l_{k^d} \\ &= \underline{\pi}. \end{aligned} \tag{B.4}$$

So deviating with a non-default strategy decreases profit below the default level.

Banks $k \notin \mathcal{K}^{++}$ can be subdivided into banks $k \in \mathcal{K}^{+}\backslash\mathcal{K}^{++}$ and banks $k \notin \mathcal{K}^{+}$. Banks $k \in \mathcal{K}^{+}\backslash\mathcal{K}^{++}$ default, for they have chosen $l_k > 0$ but do not get deposits (since $\delta_k = 0$). By the same reasoning as above, in order to improve upon the default payoff $\underline{\pi}$, a bank $k^d \in \mathcal{K}^{+}\backslash\mathcal{K}^{++}$ has to deviate such that $(1+\Delta)d_{k^d} \geq l_{k^d} > 0$ and $(1+\Delta)\rho_{k^d} > \rho$. But from (B.4), this is unprofitable: $(1+\Delta)\pi_{k^d} < \underline{\pi}$. Finally, banks $k \notin \mathcal{K}^+$ make zero profit. Since they do not generate returns in the credit subgame, there is no way of improving upon this outcome.

Notice that this holds true even if there is only one bank active in the credit market (i.e., $\mathcal{K}^+ = \{1\}$). In this case, from (B.3), $\rho = (L^S)^{-1}(l_1)$. ///

This is Lemma 1 in the main text. From the lemma, it follows that the bidding strategies in Theorem 1 constitute a Nash equilibrium in the deposit subgame. $\mathcal{K}^{++} = \mathcal{K}^+ = \{1, 2\}$ and $\rho = \rho^* = \varrho(r^*) = \varrho(r^{**})$ satisfy the definitions of $\mathcal{K}^{++}$ and ρ (case (a)): $\mathcal{K}^{++} = \{k \in \mathcal{K} | \, k \in \mathcal{K}^+, [\varrho(r_k) - \rho]l_k \geq \underline{\pi}\}$ and $\sum_{k \in \mathcal{K}^{++}} l_k = L^S(\rho)$. So $(\rho_1, \delta_1) = (\rho, l_1) = (\rho^*, L^*)$, $(\rho_2, \delta_2) = (\rho, l_2) = (\rho^*, L^S(\rho^*) - L^*)$, and $(\rho_k, \delta_k) = (0, 0)$ for $k \notin \{1, 2\}$ constitute a Nash equilibrium.

B.2 Credit subgame

To complete the proof of Theorem 1, it remains for us to show that, given the Nash equilibrium strategies in the deposit subgame described in Lemma B.1, it is not possible for any bank $k^d \in \mathcal{K}$ to make positive expected profit $(1+\Delta)(E\pi_{k^d}) > 0$ with a deviating strategy in the credit subgame, i.e., with $\Delta(r_{k^d}, \lambda_{k^d}) \neq (0, 0)$.

Setting an interest rate $(1+\Delta)r_{k^d} > r^{**}$ is not profitable because residual demand is zero. This follows from the fact that supply equals residual demand at r^{**}. So we focus on strategies $(1+\Delta)(r_{k^d}, \lambda_{k^d})$ with $(1+\Delta)r_{k^d} \leq r^{**}$.

Since at each interest rate, only the banks with the highest credit limit have a positive probability of making loans, $(1+\Delta)r_{k^d} = r^{**}$ and $(1+\Delta)\lambda_{k^d} < L^S(\rho^*) - L^*$ does not generate profit. Since, by assumption, two banks choose $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L^*)$ initially, this holds true even if k^d is one of these two banks. If some other bank k^d plays $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r^{**}, L^S(\rho^*) - L^*)$, it faces a positive probability of being selected by the tie-breaking rule. However, even then it makes zero profit. By setting a credit limit $(1+\Delta)\lambda_{k^d} > L^S(\rho^*) - L^*$ bank k^d captures the market at r^{**} with certainty. But since there is no excess demand at r^{**}, bank k^d merely adopts the role of bank 2, serves the residual demand $l_2^D(r^{**})$ $(= L^S(\rho^*) - L^*)$, and makes zero profit.

If k^d chooses $(1+\Delta)r_{k^d} = r^*$ and $(1+\Delta)\lambda_{k^d} > L^*$, it captures the entire demand at r^* and makes a return $\varrho((1+\Delta)r_{k^d}) = \rho^*$. Since demand at r^* exceeds L^*, bank k^d makes loans $(1+\Delta)l_{k^d} > L^*$. By assumption, two banks choose $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L^*)$ initially. So even if k^d is one of these banks, the supply of credit remains unchanged at the higher interest rate, while credit given rises at the lower rate. From Lemma A.6 (case (b)), it follows that $\Delta(\sum_{k \in \mathcal{K}} l_k) > 0$, even though $\Delta l_2 < 0$. Hence, as illustrated in Figure B.2, $\Delta(\sum_{k \in \mathcal{K}^{++\prime}} l_k) > 0$ for ρ' small enough such that

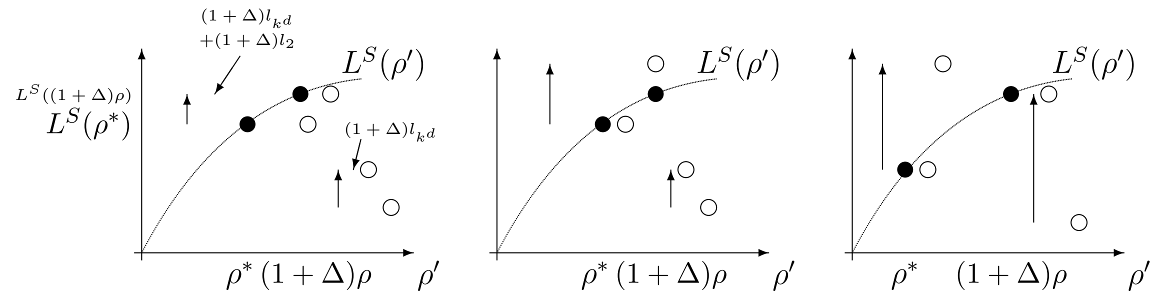

Figure B.2: Change in the equilibrium deposit rate when a bank deviates in the credit subgame

all $k \in \mathcal{K}^+$ prefer $d_k = l_k$ over default. Suppose k^d's deviation is profitable: $(1+\Delta)\pi_{k^d} > 0$. We will derive a contradiction. Suppose first that the deposit rate ρ' above which 2 prefers default over $(1+\Delta)d_2 = (1+\Delta)l_2$ does not exceed the deposit rate above which k^d prefers default over $(1+\Delta)d_{k^d} = (1+\Delta)l_{k^d}$. We have to distinguish three cases. (a) There is $(1+\Delta)\rho > \rho^*$ such that $(1+\Delta)l_{k^d} + (1+\Delta)l_2 = L^S((1+\Delta)\rho)$ (see the left panel of Figure B.2). From Lemma B.1, $(1+\Delta)\rho$ is the equilibrium deposit rate. Since $(1+\Delta)\rho > \rho^*$, this contradicts the fact that k^d makes a profit:

$$(1+\Delta)\pi_{k^d} = [\rho^* - (1+\Delta)\rho](1+\Delta)l_{k^d} < 0.$$

In the other two cases, (b) and (c), since $\Delta l_2 < 0$, the deposit rate ρ' above which bank 2 prefers default over $(1+\Delta)d_2 = (1+\Delta)l_2$ rises. Given the premise that k^d does not default, it follows that the first step of the function $(1+\Delta)(\sum_{k\in\mathcal{K}^{++\prime}} l_k)$ occurs at a higher deposit rate ρ' than before, in particular at a deposit rate greater than ρ^*. (b) Suppose there is $(1+\Delta)\rho$ such that $(1+\Delta)l_{k^d} + (1+\Delta)l_2 > L^S((1+\Delta)\rho) > (1+\Delta)l_{k^d}$ (see the middle panel of Figure B.2). From Lemma B.1, $(1+\Delta)\rho$ is the equilibrium deposit rate. It follows that $(1+\Delta)\rho > \rho^*$, which again contradicts $(1+\Delta)\pi_{k^d} > 0$. (c) Otherwise there is $(1+\Delta)\rho > \rho^*$ such that $(1+\Delta)l_{k^d} = L^S((1+\Delta)\rho)$ (see the right panel of Figure B.2). From Lemma B.1, $(1+\Delta)\rho$ is the equilibrium interest rate and $(1+\Delta)\pi_{k^d} < 0$, a contradiction. If the deposit rate above which 2 prefers default over $(1+\Delta)d_2 = (1+\Delta)l_2$ does exceed the analogous deposit rate for k^d, case (a) is treated analogously. Otherwise it is bank k^d that ceases to prefer $(1+\Delta)d_{k^d} = (1+\Delta)l_{k^d}$ at the first downward discontinuity of the step function $(1+\Delta)(\sum_{k\in\mathcal{K}^{++\prime}} l_k)$. From the case distinction made, $(1+\Delta)\rho$ is no less than the ρ'-value at which this discontinuity occurs. So k^d defaults in the deposit market equilibrium, again contradicting $(1+\Delta)\pi_{k^d} > 0$.

The remaining possibility is a deviating strategy $(1+\Delta)(r_{k^d}, \lambda_{k^d})$ with $(1+\Delta)r_{k^d} < r^{**}$, $(1+\Delta)r_{k^d} \neq r^*$, and $(1+\Delta)\lambda_{k^d} > 0$. Due to the assumed shape of the return function, we have $\varrho((1+\Delta)r_{k^d}) < \rho^*$. Market clearing at r^{**} implies positive residual demand for all $(1+\Delta)r_{k^d} < r^{**}$. So $(1+\Delta)l_{k^d} >$

0 and, from Lemma A.6 (case (a)), $\Delta(\sum_{k\in\mathcal{K}} l_k) > 0$. Moreover, $\Delta l_k < 0$ for $k \in \{1,2\}$ when $(1+\Delta)r_{k^d} < r^*$, and $\Delta l_2 < 0 = \Delta l_1$ when $r^* < (1+\Delta)r_{k^d} < r^{**}$. Similarly as above, suppose first that the deposit rates ρ' above which $k = 1$ and $k = 2$ prefer default over $(1+\Delta)d_k = (1+\Delta)l_k$ do not exceed the deposit rate above which k^d prefers default over $(1+\Delta)d_{k^d} = (1+\Delta)l_{k^d}$. (a) If there is $(1+\Delta)\rho > \rho^*$ such that $(1+\Delta)l_{k^d} + (1+\Delta)l_1 + (1+\Delta)l_2 = L^S((1+\Delta)\rho)$, then $(1+\Delta)\rho$ is the equilibrium deposit rate. Since $(1+\Delta)\rho > \rho^*$, k^d makes a loss:

$$(1+\Delta)\pi_{k^d} = [\varrho((1+\Delta)r_{k^d}) - (1+\Delta)\rho](1+\Delta)l_{k^d} < 0.$$

In the other two cases, (b) and (c), since $\Delta l_k \leq 0$ for $k \in \{1,2\}$, the deposit rates ρ' above which banks $k = 1$ and $k = 2$ prefer default over $(1+\Delta)d_k = (1+\Delta)l_k$ do not fall. Since $\Delta l_k < 0$ for at least one $k \in \{1,2\}$, the rate rises for at least one k. So the first step of the function $(1+\Delta)(\sum_{k\in\mathcal{K}^{++\prime}} l_k)$ occurs at a higher deposit rate ρ' than before. By the same reasoning as in the preceding paragraph, irrespective of whether (b) $(1+\Delta)(\sum_{k\in\mathcal{K}^{++\prime}} l_k)$ intersects $L^S(\rho')$ at some ρ or (c) not, the equilibrium interest rate ρ exceeds ρ^*, so k^d's profit is negative. Next, suppose first that the deposit rate ρ' above which k^d prefers default over $(1+\Delta)d_{k^d} = (1+\Delta)l_{k^d}$ lies in between the corresponding rates for banks 1 and 2. As noted above, since $\Delta l_k \leq 0$ for $k \in \{1,2\}$, no bank prefers default at deposit rates up to ρ^*, for $(1+\Delta)(\sum_{k\in\mathcal{K}^{++\prime}} l_k) > L^S(\rho')$ at $\rho' = \rho^*$, so that $(1+\Delta)\rho > \rho^*$ and $(1+\Delta)\pi_{k^d} < 0$. Finally, suppose k^d is the first bank to prefer default as ρ' rises. If $(1+\Delta)(\sum_{k\in\mathcal{K}^{++\prime}} l_k)$ intersects $L^S(\rho')$ at some $\rho' = (1+\Delta)\rho$ lower than the rate at which k^d prefers default, $(1+\Delta)\rho > \rho^*$ is the equilibrium interest rate, and $(1+\Delta)\pi_{k^d} < 0$. Otherwise the deposit market equilibrium occurs at a rate ρ' at which k^d prefers to default, so that $(1+\Delta)\pi_{k^d} = \underline{\pi} < 0$. Hence, in each case, k^d's profit from the deviating strategy $(1+\Delta)(r_{k^d}, \lambda_{k^d})$ is negative. This completes the proof of Theorem 1. ///

Appendix C. Proof of Theorem 2

C.1 Deposit subgame

Observe, to begin with, that default cannot occur in equilibrium. To see this, suppose there is a bank $k \in \mathcal{K}^+$ (i.e., with $l_k > 0$) which defaults. The only other thing that could have happened is that the bank would not have been selected by the tie-breaking rule (if there are other banks setting the same interest rate and credit limit), so that zero profit would have ensued. So expected profit $E\pi_k$ is negative. The fact that zero profit with certainty is possible rules out this possibility. Consequently, $\mathcal{K}^{++} = \mathcal{K}^+$ and $\mathcal{K}' = \emptyset$, and ρ is given by (B.3).

It follows that in equilibrium banks $k \in \mathcal{K}^+$ choose (ρ_k, δ_k) such that $d_k \geq l_k$, since otherwise they suffer default. Suppose a bank $k^d \in \mathcal{K}^+$ chooses $(\rho_{k^d}, \delta_{k^d})$ such that $d_{k^d} > l_{k^d}$. From (A.3), d_k is continuous and increasing in δ_k, and $d_k = 0$ for $\delta_k = 0$. Thus, there exists a $\Delta\delta_{k^d} < 0$ such that $(1 + \Delta)d_{k^d} = l_{k^d}$ for $(1 + \Delta)(\rho_{k^d}, \delta_{k^d}) = (\rho_{k^d}, (1 + \Delta)\delta_{k^d})$. As r_{k^d} and l_{k^d} are determined in the credit subgame already, (A.24) implies $\Delta\pi_{k^d} > 0$. So $d_k = l_k$ for all $k \in \mathcal{K}^+$.

Banks $k \notin \mathcal{K}^+$ make zero profit if $d_k = 0$. Any bid (ρ_k, δ_k) such that $d_k > 0$ entails $\pi_k < 0$, which cannot occur in equilibrium. So $d_k = l_k$ holds true for $k \notin \mathcal{K}^+$ as well. Consequently, payoffs are given by (B.1), and, from Lemma B.1, the amount of deposits made equals the amount of credit given: $\sum_{n=1}^{N} D_n = \sum_{k\in\mathcal{K}} l_k$.

Suppose $\rho_k < \rho$, where ρ is given by (B.3), for some $k \in \mathcal{K}^+$. Lemma A.2 then rules out $\rho_k \leq \rho$ for all $k \in \mathcal{K}^+$, for this would entail default for some bank, as aggregate deposits fall short of credit given:

$$\begin{aligned} \sum_{n=1}^{N} D_n &< L^S(\rho) \\ &= \sum_{k\in\mathcal{K}} l_k. \end{aligned}$$

So if $\rho_k < \rho$ for some $k \in \mathcal{K}^+$, then the highest market deposit rate ρ_1 satisfies $\rho_1 > \rho$. Suppose $\mathcal{K}_1$, the set of banks bidding ρ_1, contains only one bank, k^d say. The fact that a bank k bidding $\rho_k < \rho$ faces positive residual supply (which is implied by $d_k = l_k > 0$ for all $k \in \mathcal{K}^+$) means that k^d can decrease ρ_{k^d} slightly and still raise deposits l_{k^d}. So a Nash equilibrium does not prevail. If, on the other hand, $\mathcal{K}_1$ contains more than one bank $k \in \mathcal{K}$, then $d_k = \delta_k$ for all $k \in \mathcal{K}_1$ (since $d_k > 0$ for some bank k with $\rho_k < \rho$). Let a bank $k^d \in \mathcal{K}_1$ choose $(1 + \Delta)(\rho_{k^d}, \delta_{k^d})$ with $\Delta\rho_{k^d}$ negative and small in absolute value and $\Delta\delta_{k^d} = 0$. The demand for deposits at ρ_1 falls by $\delta_{k^d} = d_{k^d} = l_{k^d}$. From Lemma A.3, the residual supply at $(1 + \Delta)\rho_{k^d}$ rises by approximately l_{k^d}. The fact that the residual supply at ρ_k ($< \rho$) is positive implies that the residual supply at $(1 + \Delta)\rho_{k^d}$ is sufficient so as to raise l_{k^d}. So we can rule out $\rho_k < \rho$ for some $k \in \mathcal{K}^+$ in a Nash equilibrium. So $\rho_k \geq \rho$ for all $k \in \mathcal{K}^+$. In order to prove that $\rho_k = \rho$ for all $k \in \mathcal{K}^+$ it remains for us to rule out $\rho_k > \rho$ for some $k \in \mathcal{K}^+$.

If there is more than one market deposit rate (i.e., $N > 1$), the arguments put forward in the preceding paragraph prove that it is possible for each bank $k \in \mathcal{K}_1$ setting the highest market deposit rate ρ_1 to raise profit by decreasing the deposit rate slightly. So consider the case of a single deposit rate (i.e., $N = 1$), $\rho_1 > \rho$. $\mathcal{K}_1 = \mathcal{K}^+$ in this case. From $d_k = l_k$ for all $k \in \mathcal{K}$, (B.3), and the fact that $L^S(\rho)$ is strictly increasing, we have

$$\begin{aligned}
\sum_{k\in\mathcal{K}_1} d_k &= \sum_{k\in\mathcal{K}} d_k \\
&= \sum_{k\in\mathcal{K}} l_k \\
&= L^S(\rho) \\
&< L^S(\rho_1). \qquad \text{(C.1)}
\end{aligned}$$

From (A.3) with $n = 1$, $l_1^S(\rho_1) = L^S(\rho_1)$, and (C.1), we have

$$\begin{aligned}
\sum_{k\in\mathcal{K}_1} d_k &= \sum_{k\in\mathcal{K}_1} \min\left\{\delta_k,\, \frac{\delta_k}{\sum_{k'\in\mathcal{K}_1}\delta_{k'}} L^S(\rho_1)\right\} \\
&= \sum_{k\in\mathcal{K}_1} \delta_k \min\left\{1,\, \frac{1}{\sum_{k'\in\mathcal{K}_1}\delta_{k'}} L^S(\rho_1)\right\} \\
&= \min\left\{1,\, \frac{1}{\sum_{k'\in\mathcal{K}_1}\delta_{k'}} L^S(\rho_1)\right\} \sum_{k\in\mathcal{K}_1} \delta_k \\
&= \min\left\{\sum_{k\in\mathcal{K}_1}\delta_k,\, L^S(\rho_1)\right\} \\
&= \sum_{k\in\mathcal{K}_1}\delta_k.
\end{aligned}$$

From the fact that $d_k \le \delta_k$ for all $k \in \mathcal{K}_1$, it follows that $d_k = \delta_k$ and, hence, $l_k = \delta_k$ for all $k \in \mathcal{K}_1$. Suppose a bank $k^d \in \mathcal{K}_1$ deviates with $\Delta\rho_{k^d}$ negative and small in absolute value. The amount of deposits raised at ρ_1 falls by l_{k^d}. Lemma A.3 implies that demand at $(1+\Delta)\rho_{k^d}$ increases by approximately the same amount. Together with the fact that there is excess supply at ρ_1 (cf. (C.1)), it follows that k^d faces sufficient residual supply at $(1+\Delta)\rho_{k^d}$ so as to raise l_{k^d}. So a Nash equilibrium does not prevail. This completes the proof that $d_k = l_k$ for all $k \in \mathcal{K}$ and $\rho_k = \rho$ (with ρ given by (B.3)) for all $k \in \mathcal{K}^+$ in any Nash equilibrium of the deposit subgame.

C.2 Credit subgame

Let $\mathcal{K}^{+++}$ be the subset of banks with strictly positive expected payoff as of stage one in equilibrium: $\mathcal{K}^{+++} = \{k \in \mathcal{K} | \, E\pi_k > 0\}$.

LEMMA C.1: *$\mathcal{K}^{+++} = \emptyset$ or $\mathcal{K}^{+++} = \mathcal{K}$.*

Proof: Suppose to the contrary that $E\pi_k > 0$ and $E\pi_{k^d} = 0$ for $\{k, k^d\} \subset \mathcal{K}$. k^d can also make a positive expected profit by choosing $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r_k, \lambda_k)$. ///

LEMMA C.2: $r_k \neq r_{k^d}$ *if* $\{k, k^d\} \subset \mathcal{K}^{+++}$.

Proof: Suppose the contrary: $r_k = r_{k^d}$ for $\{k, k^d\} \subset \mathcal{K}^{+++}$. This implies $\lambda_k = \lambda_{k^d}$. We have to distinguish two cases: (a) there is excess demand at r_{k^d} or (b) not.
(a) Suppose there is excess demand at r_{k^d}: $\lambda_k = \lambda_{k^d} < l^D_{k^d}(r_{k^d})$. Let k^d deviate with $\Delta r_{k^d} = 0$ and $\Delta\lambda_{k^d} = \epsilon$, where $\epsilon > 0$. Given the random tie-breaking rule, k^d's probability of serving the market rises from $(\#\{k|\, r_k = r_{k^d}\})^{-1}$ (< 1) to unity. Excess demand at r_{k^d} implies $\Delta l_{k^d} = \Delta\lambda_{k^d} = \epsilon$ for ϵ small enough. From Lemma A.6 (case (b)) and (B.3), $\Delta\rho$ rises, and the increase converges to zero as $\epsilon \to 0$. As the probability of serving the demand for loans jumps upward, while $\varrho(r_{k^d})$ is unchanged and Δl_{k^d} and $\Delta\rho$ go to zero as $\epsilon \to 0$, it follows from (B.1) that $\Delta(E\pi_{k^d}) > 0$.
(b) Next, suppose there is no excess demand at r_{k^d}: $\lambda_k = \lambda_{k^d} \geq l^D_{k^d}(r_{k^d})$. This implies that r_{k^d} is the highest market interest rate. Suppose k^d deviates by making loans to all firms that demand credit at $(1+\Delta)r_{k^d}$, where $\Delta r_{k^d} = \epsilon$ is negative and small in absolute value. As above, the probability of facing positive demand jumps upward. All other arguments of the payoff function (B.1) go to zero as $\epsilon \to 0$: $\varrho(r_{k^d} + \epsilon) \to \varrho(r_{k^d})$ because of continuity of the return function; $(1+\Delta)l_{k^d} \to l^D_{k^d}(r_{k^d})$ because of the continuity of the residual demand functions; from Lemma A.6, $\Delta(\sum_{k\in\mathcal{K}} l_k) \to 0$ because $\Delta(\sum_{k\in\mathcal{K}} l_k) = (1+\Delta)l_{k^d} - l^D_{k^d}(r_{k^d})$, as three is no residual demand left at r_{k^d}; from (B.3), $\Delta\rho \to 0$. So from (B.1), $\Delta(E\pi_{k^d}) > 0$. ///

LEMMA C.3: $l^D_K(r_K) = l_K$ *if* $\mathcal{K}^{+++} = \mathcal{K}$.

Proof: $\mathcal{K}^{+++} = \mathcal{K}$ and Lemma C.2 imply that one bank K alone sets the maximum market interest rate r_K. Therefore, $l_K > 0$ with probability one. Suppose $l^D_K(r_K) > l_K$, i.e., K does not satisfy the total residual demand it faces. Let K deviate with $\Delta r_K = \epsilon$, where $r_K + \epsilon > r_{K-1}$ and $\varrho(r_K + \epsilon) > \varrho(r_K)$ (i.e., $\epsilon > 0$ if $\varrho(r)$ is increasing at r_K and $\epsilon < 0$ if $\varrho(r)$ is decreasing at r_K), and $\Delta\lambda_K = 0$. Because of excess demand, $\Delta l_K = \Delta\lambda_K = 0$ for ϵ small enough. Given that l_K is constant, so is $\sum_{k\in\mathcal{K}} l_k$. From (B.3), $\Delta\rho = 0$. From (B.1), $\Delta(E\pi_K) > 0$. ///

LEMMA C.4: $\mathcal{K}^{+++} = \emptyset$.

Proof: Suppose not. Then, from Lemmas C.1-C.3, $E\pi_K > 0$, $E\pi_{K-1} > 0$, $r_K > r_{K-1}$, and $l^D_K(r_K) = l_K$.
(a) Let K deviate with $(1+\Delta)(r_K, \lambda_K) = (r_{K-1}, \lambda_{K-1} + \epsilon)$, where $\epsilon > 0$. Analogously to the proof of Lemma A.6 (which does not apply directly because the supply of credit falls at r_K), we have

$$\Delta l_k = 0 \text{ for } k \in \{1, \ldots, K-2\}. \tag{C.2}$$

$K \in \mathcal{K}^{+++}$ implies excess demand at r_{K-1} and, hence, $l_{K-1} = \lambda_{K-1}$. So bank $K-1$ is pushed out of the market:

$$\Delta l_{K-1} = -l_{K-1} \tag{C.3}$$

Furthermore, excess demand at r_{K-1} implies $(1+\Delta)l_K = l_{K-1} + \epsilon$, i.e.,

$$\Delta l_K = l_{K-1} + \epsilon - l_K \tag{C.4}$$

for ϵ small enough. Since, moreover, no bank sets an interest rate above r_K (this would imply zero profit, contradicting $\mathcal{K}^{+++} = \mathcal{K}$), adding up (C.2)-(C.4) yields

$$\Delta \left(\sum_{k \in \mathcal{K}} l_k \right) = \epsilon - l_K < 0$$

for ϵ small enough. That is, aggregate credit given and, from (B.3), the deposit rate jump downward. From (B.1), we have

$$\begin{aligned} (1+\Delta)(E\pi_K) &= [\varrho(r_{K-1}) - (1+\Delta)\rho](l_{K-1} + \epsilon) \\ &> [\varrho(r_{K-1}) - \rho] l_{K-1} \\ &= E\pi_{K-1} \end{aligned} \tag{C.5}$$

for ϵ small enough.

(b) Next, let $K-1$ deviate with $(1+\Delta)(r_{K-1}, \lambda_{K-1}) = (r_K + \epsilon, l^D_{K-1}(r_K + \epsilon))$, where ϵ is negative and small enough in absolute value such that $r_K + \epsilon > r_{K-1}$. (C.2) holds true. Bank $K-1$'s credit given changes by

$$\Delta l_{K-1} = l^D_{K-1}(r_K + \epsilon) - l_{K-1}. \tag{C.6}$$

Since there is no excess demand left at $(1+\Delta)r_{K-1} = r_K + \epsilon$,

$$\Delta l_K = -l_K. \tag{C.7}$$

Adding up (C.2), (C.6), and (C.7) and letting $\epsilon \to 0$ yields

$$\begin{aligned} \Delta \left(\sum_{k \in \mathcal{K}} l_k \right) &= l^D_{K-1}(r_K + \epsilon) - (l_{K-1} + l_K) \\ &\to l^D_{K-1}(r_K) - (l_{K-1} + l_K) \end{aligned} \tag{C.8}$$

as $\epsilon \to 0$. From (A.6) (with $m = K-1$, $r = r_K$, and $L_m = l_{K-1}$),

$$l^D_K(r_K) = \left[1 - \frac{l_{K-1}}{l^D_{K-1}(r_{K-1})} \right] l^D_{K-1}(r_K).$$

Hence, using $l^D_K(r_K) = l_K$ and the fact that $r_K > r_{K-1}$,

$$\begin{aligned} l_{K-1} + l_K &= l_{K-1}\left[1 - \frac{l^D_{K-1}(r_K)}{l^D_{K-1}(r_{K-1})}\right] + l^D_{K-1}(r_K) \\ &> l^D_{K-1}(r_K). \end{aligned}$$

From (C.8), it follows that, for $\epsilon \to 0$, the deviating strategy $(1+\Delta)(r_{K-1}, \lambda_{K-1}) = (r_K + \epsilon, l^D_{K-1}(r_K + \epsilon))$ causes a downward jump in $\sum_{k\in\mathcal{K}} l_k$. From (B.3), the deposit rate falls discontinuously: $\Delta\rho < 0$. Together with the fact that the return function $\varrho(r)$ is continuous and $l^D_{K-1}(r_K) > l^D_K(r_K) = l_K$, it follows from (B.1) that

$$\begin{aligned} (1+\Delta)(E\pi_{K-1}) &= [\varrho(r_K + \epsilon) - (1+\Delta)\rho] l^D_{K-1}(r_K + \epsilon) \\ &> [\varrho(r_K) - \rho] l_K \\ &= E\pi_K \end{aligned}$$

for $\epsilon \to 0$.

So we have $(1+\Delta)(E\pi_K) > E\pi_{K-1}$ and $(1+\Delta)(E\pi_{K-1}) > E\pi_K$. As shown in the main text, this is not consistent with the definition of a Nash equilibrium. ///

Notice that the analysis so far holds true independently of the shape of the return function.

LEMMA C.5: *$r_1 = r^*$ and $r_2 = r^{**}$ are the only market interest rates.*

Proof: Clearly, $\mathcal{M} = \emptyset$ (no credit given) cannot arise in equilibrium. $\mathcal{M} = \emptyset$ implies $r_k \geq r^{max}$ or $\lambda_k = 0$ for all $k \in \mathcal{K}$. Any $(1+\Delta)(r_{k^d}, \lambda_{k^d})$ such that $\varrho(r_{k^d}) > (L^S)^{-1}(l_{k^d})$ yields $(1+\Delta)(E\pi_{k^d}) > 0$. Suppose next that there is a single equilibrium interest rate r_1, so that $\mathcal{M} = \{1\}$ and, given the convention $k = m$, $\mathcal{K}^+ = \{1\}$. Consider a bank $k^d \in \mathcal{K}\backslash\{1\}$. Suppose there is excess demand at r_1 (i.e., $l^D_1(r_1) = L^D(r_1) > l_1$). Then, the fact that $\mathcal{K}^+ = \{1\}$ implies that for all $k \in \mathcal{K}\backslash\{1\}$, $r_k = r_1$ and $\lambda_k \leq \lambda_1$ or $r_k \geq r^{max}$ or $\lambda_k = 0$. $k^d \in \mathcal{K}\backslash\{1\}$ can make positive expected profit by choosing $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r^{max} + \eta, \epsilon)$ with η negative and small in absolute value and ϵ positive and small. Because of excess demand at r_1, $(1+\Delta)l_{k^d} = \epsilon$ with probability one for ϵ small. By virtue of Lemma A.6 and (B.3), $\Delta\rho \to 0$ and thus $\varrho(r^{max} + \eta) > (1+\Delta)\rho$ as $\epsilon \to 0$ and $\eta \to 0$. From (B.1), $(1+\Delta)\pi_{k^d} > 0$. This rules out excess demand in a single-interest equilibrium.

So $l_1 = l^D_1(r_1) = L^D(r_1)$. From Lemma B.1, in the deposit subgame, bank 1 bids $\rho_1 = (L^S)^{-1}(L^D(r_1))$ and gets $d_1 = L^D(r_1)$. Hence, $L^S(\rho_1) = L^D(r_1)$. According to Lemma C.4, bank 1 makes zero profit: $\rho_1 = \varrho(r_1)$. So $L^S(\varrho(r_1)) = L^D(r_1)$. Given the assumed continuity and shape of the return, demand, and supply functions (with $L^D(r^*) > L^S(\varrho(r^*))$ and $L^D(r^{**}) < L^S(\varrho(r^{**}))$ and with $L^D(r) - L^S(\varrho(r))$ decreasing for $r < r^*$ and for $r > r^{**}$), this implies $r^* < r_1 < r^{**}$.

It follows that the demand for credit is positive at r^* and that $\varrho(r_1) < \rho^*$. Suppose k^d chooses $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r^*, \epsilon)$, where ϵ is positive and small. The fact that the demand for credit is positive at r^* implies $(1+\Delta)l_{k^d} = \epsilon$ with probability one. From Lemma A.6 and (B.3), $\Delta\rho$ is small. It follows that $\varrho(r^*) = \rho^* > (1+\Delta)\rho$. From (B.1), $(1+\Delta)(E\pi_{k^d}) > 0$. This rules out a single-interest rate equilibrium.

Given that all banks $k \in \mathcal{K}^+$ get deposits $d_k = l_k$ (> 0) and pay a common deposit rate ρ (Lemma B.1) and make zero expected profit (Lemma C.4), (B.1) implies that all banks $k \in \mathcal{K}^+$ realize the same return $\varrho(r_k) = \rho$. Given the assumed shape of the return function, identical returns at all, and at at least two, market interest rates implies that either (as asserted by the lemma) the equilibrium is a two-interest rate equilibrium with credit given positive for r^* and r^{**} and for no other interest rate, or else $\mathcal{K}^+$ consists of two or three banks, $\varrho(r_k) < \rho^*$ for all $k \in \mathcal{K}^+$, and $r_2 > r^*$. We now rule out the latter case. Consider a bank k^d. If for each r_k, $k \in \mathcal{K}^+$, there is a single bank choosing (r_k, Λ_k), then let $k^d \in \mathcal{K}\backslash\mathcal{K}^+$ (the fact that there are two or three banks in $\mathcal{K}^+$ and at least four banks in $\mathcal{K}$ ensures that $\mathcal{K}\backslash\mathcal{K}^+ \neq \emptyset$). Otherwise, consider a bank k^d either in $\mathcal{K}\backslash\mathcal{K}^+$ or such that there is another bank $k' \in \mathcal{K}$ with $(r_{k'}, \lambda_{k'}) = (r_{k^d}, \lambda_{k^d})$. This choice of k^densures that Lemma A.6 is applicable when k^d makes loans at a new interest rate. Let k^d set $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r^*, \epsilon)$, where ϵ is small and positive. $r_2 > r^*$ implies that there is positive residual demand at r^*, so that $(1+\Delta)l_{k^d} = \epsilon$ with probability one. By virtue of Lemma A.6 and (B.3), $\Delta\rho \to 0$ as $\epsilon \to 0$. So $(1+\Delta)(E\pi_{k^d}) = [\rho^* - (1+\Delta)\rho](1+\Delta)l_{k^d} > 0$. This proves that the equilibrium is a two-interest rate equilibrium with credit given positive for r^* and r^{**}. ///

LEMMA C.6: *At least two banks $k \in \mathcal{K}$ set $(r_k, \lambda_k) = (r_1, \Lambda_1)$, and at least two banks $k \in \mathcal{K}$ set $(r_k, \lambda_k) = (r_2, \lambda_k)$ with $\lambda_k \geq l_2^D(r_2)$.*

Proof: Suppose $(r_k, \lambda_k) = (r_1, \Lambda_1)$ for only one bank $k = 1$. From Lemma C.5, there is positive residual demand at r^{**}, which implies $l_1 = \Lambda_1$. Let bank 1 choose $(1+\Delta)(r_1, \lambda_1) = (r^*, l_1 + \epsilon)$, where ϵ is negative and small enough in absolute value such that bank 1 continues to serve the market at r^* (i.e., there is no $k \in \mathcal{K}$ such that $r_k = r^*$ and $l_1 + \epsilon \leq \lambda_k < l_1$). If there is excess demand at r^{**} (i.e., $l_2 = \Lambda_2 < l_2^D(r^{**}) = [1 - l_1/L^D(r^*)]L^D(r^{**})$), then $\Delta l_2 = 0$ and, using $\Delta l_1 = \epsilon$,

$$\begin{aligned} \Delta\left(\sum_{k\in\mathcal{K}} l_k\right) &= \epsilon \\ &< 0. \end{aligned}$$

If there is no excess demand at r^{**}, then $\Delta l_2 = \Delta l_2^D(r^{**}) = -[L^D(r^{**})/L^D(r^*)]\Delta l_1$ and, using

$\Delta l_1 = \epsilon$,

$$\Delta\left(\sum_{k\in\mathcal{K}} l_k\right) = \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right]\epsilon$$
$$< 0$$

for ϵ small enough small enough in absolute value. $\Delta\rho < 0$ in both cases. Since the return $\varrho(r^*) = \rho^*$ is unaffected and $(1+\Delta)l_1 = l_1 + \epsilon$ with probability one, we have, from (B.1), $(1+\Delta)(E\pi_1) > 0$. Turning to the higher equilibrium interest rate $r_2 = r^{**}$, suppose no bank k sets $\lambda_k \geq l_2^D(r_2)$. Then, given Lemma C.5, there is positive residual demand for all interest rates in the interval (r^{**}, r^{max}), so that any bank k^d not setting r_1 can make positive profit with $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r^{max} + \eta, \epsilon)$, where η is negative and small in absolute value and ϵ is positive and small.

So suppose there is exactly one bank $k = 2$ which chooses $(r_k, \lambda_k) = (r_2, \lambda_k)$ with $\lambda_k \geq l_2^D(r_2)$. This implies that there is no excess demand at $r_2 = r^{**}$: $l_2 = l_2^D(r^{**})$. Let bank 2 deviate with $(1+\Delta)r_2 = r^{**} + \eta$, where η is positive, so that $\varrho(r^{**} + \eta) > \rho^*$. The fact that $k = 2$ is the only bank which chooses $\lambda_k \geq l_2^D(r^{**})$ at r^{**} implies that credit given at r^{**} by the bank with the next-highest credit limit, k' say (notice that $l_{k'} = 0$), satisfies $(1+\Delta)l_{k'} < l_2^D(r^{**})$, so the residual demand is positive at $r^{**} + \eta$. Let η be small enough such that there is no bank k with $r^{**} < r_k \leq r^{**} + \eta$. Then bank 2 faces positive residual demand $(1+\Delta)l_2^D(r^{**} + \eta)$. Suppose it sets $(1+\Delta)\lambda_2 = (1+\Delta)l_2^D(r^{**} + \eta)$. Then $(1+\Delta)l_2 = (1+\Delta)l_2^D(r^{**} + \eta)$, and using $r^{**} + \eta > r^{**}$ and Lemma A.5,

$$\begin{aligned} (1+\Delta)l_{k'} + (1+\Delta)l_2 &< l_2^D(r^{**}) \\ &= l_2. \end{aligned}$$

Hence, using $l_{k'} = 0$,

$$\begin{aligned} \Delta\left(\sum_{k\in\mathcal{K}} l_k\right) &= (1+\Delta)l_{k'} + (1+\Delta)l_2 - (l_2 + l_{k'}) \\ &< l_2 - (l_2 + l_{k'}) \\ &= 0. \end{aligned}$$

From (B.3), $\Delta\rho < 0$. As $\varrho(r^{**} + \eta) > \rho^* = \rho$, and $\Delta\rho < 0$, bank 2's expected profit, as given by (B.1), becomes positive: $\Delta(E\pi_2) > 0$. ///

LEMMA C.7: $l_1 = L^*$ *and* $l_2 = L^S(\rho^*) - L^*$.

Proof: Aggregate credit given obeys

$$l_1 + l_2 = L^S(\rho^*). \tag{C.9}$$

Suppose not. Then, from Lemma B.1 and (B.3),

$$\begin{aligned} \rho &= (L^S)^{-1}(l_1 + l_2) \\ &\neq (L^S)^{-1}(L^S(\rho^*)) \\ &= \rho^*. \end{aligned}$$

From Lemma C.5 and $\varrho(r^*) = \varrho(r^{**}) = \rho^*$, it then follows that $E\pi_k \neq 0$ for $k \in \{1, 2\}$. This contradicts Lemma C.4. Given that aggregate credit given is $L^S(\rho^*)$, from (A.22), the residual demand at r^{**} is

$$l_2^D(r^{**}) = \left[1 - \frac{l_1}{L^D(r^*)}\right] L^D(r^{**}). \tag{C.10}$$

Suppose $l_1 < L^*$. From $l_2 \leq l_2^D(r^{**})$, (C.10), $L^D(r^{**}) < L^D(r^*)$, and the definition of L^*, we obtain

$$\begin{aligned} l_1 + l_2 &\leq l_1 + l_2^D(r^{**}) \\ &= l_1 + \left[1 - \frac{l_1}{L^D(r^*)}\right] L^D(r^{**}) \\ &= \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right] l_1 + L^D(r^{**}) \\ &< \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right] L^* + L^D(r^{**}) \\ &= L^* + \left[1 - \frac{L^*}{L^D(r^*)}\right] L^D(r^{**}) \\ &= L^S(\rho^*). \end{aligned}$$

This contradicts (C.9).

Suppose $l_1 > L^*$. Then the same formulas as employed above and (C.9) yield

$$\begin{aligned} l_2^D(r^{**}) - l_2 &= \left[1 - \frac{l_1}{L^D(r^*)}\right] L^D(r^{**}) - [L^S(\rho^*) - l_1] \\ &= \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right] l_1 + L^D(r^{**}) - L^S(\rho^*) \\ &> \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right] L^* + L^D(r^{**}) - L^S(\rho^*) \\ &= \left[1 - \frac{L^*}{L^D(r^*)}\right] L^D(r^{**}) - [L^S(\rho^*) - L^*] \\ &= 0. \end{aligned}$$

That is, there is excess demand at r^{**}. But then $(1+\Delta)(r_{k^d}, \lambda_{k^d}) = (r^{max}+\eta, \epsilon)$, where η is negative and small in absolute value and ϵ is positive and small, yields $(1 + \Delta)(E\pi_{k^d}) > 0$. This contradicts Lemma C.4, thereby completing the proof of Lemma C.7 and Theorem 2. ///

Appendix D: Proof of Theorem 3

The analysis of residual demands and supplies in Appendix A and of the deposit subgame in Appendix B.1 goes through without modification. Because of market clearing at $\bar{r}$, if k^d sets $(1+\Delta)r_{k^d} > \bar{r}$ in the credit subgame, it does not attract firms. Likewise, $(1+\Delta)r_{k^d} = \bar{r}$ and $(1+\Delta)\lambda_{k^d} < \bar{L}$ implies $(1+\Delta)l_{k^d} = 0$. With $(1+\Delta)r_{k^d} = \bar{r}$ and $(1+\Delta)\lambda_{k^d} > \bar{L}$, k^d takes on the role of bank 1 (that serves the market before k^d's deviation) and makes zero profit. Suppose k^d deviates with $(1+\Delta)(r_{k^d}, \lambda_{k^d})$ where $(1+\Delta)r_{k^d} < \bar{r}$ and $(1+\Delta)\lambda_{k^d} > 0$. $\Delta l_1 < 0$ but, from Lemma A.6, $\Delta(\sum_{k\in\mathcal{K}} l_k) > 0$. As explained in Appendix B.2, if the deposit rate ρ' above which 1 prefers default over $(1+\Delta)d_1 = (1+\Delta)l_1$ does not exceed the deposit rate above which k^d prefers default over $(1+\Delta)d_{k^d} = (1+\Delta)l_{k^d}$, the deposit rises, so $(1+\Delta)\pi_{k^d} < 0$. And if the deposit rate above which 1 prefers default over $(1+\Delta)d_1 = (1+\Delta)l_1$ exceeds the analogous deposit rate for k^d, either the deposit rises or else k^d defaults. $(1+\Delta)\pi_{k^d} < 0$ in both cases.

Turning to the "uniqueness" part, the proofs of Lemmas C.1-C.4 go through without modification, so $\mathcal{K}^{+++} = \emptyset$, i.e., there is no bank $k \in \mathcal{K}$ that makes positive expected profit $E\pi_k$. By the same reasoning as in the proof of Lemma C.5, $\mathcal{M} \neq \emptyset$. As shown in Appendix C.1, all banks $k \in \mathcal{K}^+$ pay the same deposit rate ρ and acquire deposits $d_k = l_k$. It follows that $\varrho(r_k) = \rho$ for all $k \in \mathcal{K}^+$. Given the assumed monotonicity of $\varrho(r)$, this means that there is a single market interest rate (i.e., $\mathcal{M} = \{1\}$).

Suppose the single market interest rate satisfies $r_1 > \bar{r}$, so that $L^D(r_1) < L^S(\varrho(r_1))$. Given that the single active bank 1 acquires deposits $d_1 = l_1 \leq L^D(r_1)$, it follows that there is excess supply in the market for deposits (i.e., $d_1 < L^S(\varrho(r_1))$) and, therefore, positive residual supply at lower rates. So a bank k^d can lend $(1+\Delta)l_{k^d} > 0$ at $(1+\Delta)r_{k^d}$ slightly below $\bar{r}$ and raise deposits $(1+\Delta)d_{k^d} = (1+\Delta)l_{k^d}$ at a rate below $\bar{\rho}$, thereby making a positive profit $(1+\Delta)\pi_{k^d}$.

In the opposite case $r_1 < \bar{r}$, we have $L^S(\varrho(r_1)) < L^D(r_1)$ and $l_1 \leq L^S(\rho_1)$, since otherwise bank 1 would default. Using $d_1 = l_1$ and zero profit, it follows that $l_1 < L^D(r_1)$. But that means that there is excess demand in the credit market. So a bank k^d can make a positive profit $(1+\Delta)\pi_{k^d}$ by offering a small amount of credit at interest rate slightly below r^{max}. ///