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Testing for Codependence of Non-Stationary Variables*

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Abstract

We analyze non-stationary time series that do not only trend together in the long run, but restore the equilibrium immediately in the period following a deviation. While this represents a common serial correlation feature, the framework is extended to codependence, allowing for delayed adjustment. We show which restrictions are implied for VECMs and lay out a likelihood ratio test. In addition, due to identification problems in codependent VECMs a GMM test approach is proposed. We apply the concept to US and European interest rate data, examining the capability of the Fed and ECB to control overnight money market rates.

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1 Introduction

In this paper we test whether dynamic adjustment processes in cointegrated systems are finished after a limited number of periods, or even immediately. Such a situation would be characterized by short-lived deviations from long-run equilibria being restored rather fast subsequently to a shock. Important examples can be found for financial variables. So, the absence of persisting arbitrage opportunities between similar assets in flexible markets requires prices to promptly incorporate economic disturbances. This would guarantee that the market quickly returns to the no-arbitrage equilibrium. Another field of applications refers to control of certain target variables by economic policy using specific instruments.

Concerning the latter point we will analyze whether central banks are able to control overnight interest rates. Monetary authorities like the Federal Reserve Bank (Fed) or the European Central Bank (ECB) try to keep short-term interest rates close to the announced target values. Hence, if central banks can sufficiently control overnight rates, then overnight and target rates should be cointegrated and deviations of the overnight rate from the target should be relatively short-lived. In other words, the deviations may ideally be white noise or serially correlated of low maximum order such that they are completely eliminated after few periods. We will argue that the Fed was much more successful in controlling overnight rates than the ECB in the recent decade.

The presence of a cointegration error implies a certain serial correlation structure in the system variables. As will be shown, immediate adjustment to equilibrium deviations is equivalent to common serial dependence in the levels of these non-stationary variables. This represents a special case of serial correlation common features (SCCFs) as introduced by Engle & Kozicki (1993). An SCCF exists if a linear combination of serially correlated variables containing a common factor does not exhibit any serial correlation itself. However, completely identical serial correlation is often a too strong assumption for empirical data. Therefore, we additionally focus on constellations that allow for delays in adjustment. Based on Gourieroux & Peaucelle (1988, 1992), Vahid & Engle (1997) generalize SCCF to the concept of codependence for cases where the serial correlation of the linear combination drops to zero after q lags.

In relation to variables integrated of order one, $I(1)$, SCCF and codependence had originally been introduced for the *first differences* by Vahid & Engle (1993) and Vahid & Engle (1997), respectively. Schleicher (2007) discusses in detail SCCF and codependence related to cointegrated variables within the vector error correction model (VECM) framework. As mentioned above, we apply the concept of codependence to the *levels* of $I(1)$ variables. Since codependence implies that a linear combination of the variables has a (stationary) finite-order serial correlation structure, the variables must be cointegrated. Indeed, any vector containing the weights of such a linear combination must lie in the cointegration space. Hence, the vector does not only eliminate the common trend but also the common cyclical movements after q lags. Paruolo (2003) and Franchi & Paruolo (2010) contain a general treatment of common serial dependence in non-stationary systems. We deal with an important special case, propose testing procedures, and clarify the connection of the concept of codependent cointegrated variables to equilibrium adjustment, which motivates our application. In the same vein, we link codependence to the so-called persistence profiles of Pesaran & Shin (1996).

Trenkler & Weber (2010) point out that (cointegrated) vector autoregressions (VARs) with codependence restrictions are not generally identified, at least for codependence orders larger than one. Then, maximum likelihood (ML) estimation and likelihood ratio (LR) testing cannot be applied. Thus, we do not exclusively rely on LR tests for inferring the presence of codependence. Instead, we show how the testing problem can be solved even if an underlying codependent VECM cannot be identified. In detail, we consider a test for a cut-off in the serial correlation of the cointegration error. This test is motivated by a GMM estimation approach that has been proposed by Vahid & Engle (1997) and makes use of the VEC framework only as an approximation of the infinite vector moving average representation.

The plan for the paper is as follows. In the next section we present the methodology comprising the model framework, the codependence restrictions and statistical testing. Using the codependence approach we explore in section 3 whether the Fed and the ECB could control overnight rates. Finally, the last section concludes.

2 Methodology

2.1 MA Representation and Level Codependence

Let the $n \times 1$-dimensional time series variable x_t be integrated of order one, $I(1)$, such that its first difference Δx_t is $I(0)$. Accordingly, Δx_t has the following Wold representation, compare e.g. Johansen (1995, Definition 3.2):

$$\Delta x_t = C(L)\varepsilon_t, \tag{2.1}$$

where $C(L) = \sum_{i=0}^{\infty} C_i L^i$ with L being the lag operator and

$$C_0 = I_n, C(1) \neq 0, \text{ and } \sum_{j=1}^{\infty} j|C_j| < \infty.$$

Moreover, the $n \times 1$ error term vector ε_t is assumed to be i.i.d.$(0, \Omega)$ with positive definite covariance matrix Ω and finite fourth moments. To simplify the exposition we do not consider deterministic terms. Note, however, that adding e.g. a constant vector to (2.1) will generate a linear trend in the levels x_t.

As pointed out by Vahid & Engle (1993), one can rewrite equation (2.1) as

$$\Delta x_t = C(1)\varepsilon_t + C^*(L)\Delta\varepsilon_t, \tag{2.2}$$

where $C^*(z) = \dfrac{C(z) - C(1)}{1 - z} = \sum_{i=0}^{\infty} C_i^* z^i$ with $C_i^* = -\sum_{j=i+1}^{\infty} C_j$, in particular $C_0^* = I_n - C(1)$. Then, integrating both sides of (2.2) gives the multivariate version of the Beveridge-Nelson decomposition for x_t, Beveridge & Nelson (1981),

$$x_t = C(1)\sum_{s=0}^{\infty}\varepsilon_{t-s} + C^*(L)\varepsilon_t. \tag{2.3}$$

The matrix $C(1)$ is assumed to have rank $n - r$, $0 < r < n$, such that there exists an $n \times r$ matrix $\beta \neq 0$ with $\beta' C(1) = 0$. Then, $\beta' x_t = \beta' C^*(L)\varepsilon_t = \sum_{i=0}^{\infty} \beta' C_i^* \varepsilon_{t-i}$ is $I(0)$ and, thus, x_t is cointegrated of rank r, compare Vahid & Engle (1993, Definition 1). The cointegration matrix $\beta = (\beta_1, \beta_2, \ldots, \beta_r)$ contains the r linearly independent cointegration vectors β_j, $j = 1, 2, \ldots, r$. Note that these are not unique but only identified up to an invertible transformation.

If x_t is codependent of order q, then a linear combination of x_t, say $\delta' x_t$, must exist that is uncorrelated with all lags of x_t beyond q, compare Vahid & Engle (1997). Hence, $\delta' x_t$ must have a finite serial correlation structure of order q. Accordingly, this linear combination has to be stationary and, thus, the $n \times 1$ vector δ must lie in the space spanned by the cointegration vectors. The latter implies that $\delta' C(1) = 0$. In general, there may exist $s \leq r$ vectors that generate codependence of k different orders in x_t, compare e.g. Schleicher (2007). These considerations give rise to the following definition.

Definition 1. Let x_t be an n-dimensional I(1) process with cointegrating rank r, $0 < r < n$, that has the representation (2.3). Let s be the maximal number of linearly independent $n \times 1$ vectors collected in k nonzero $n \times s_j$ matrices $\delta_{0,j}$, $j = q_1, q_2, \ldots, q_k$, with $\delta_{0,j}' C(1) = 0$ for all j, $s = s_{q_1} + s_{q_2} + \cdots + s_{q_k} \leq r$, $q_1 < q_2 < \cdots < q_k$, and

$$\delta_{0,j}' C_i^* = 0 \text{ for all } i > j \text{ and } \delta_{0,j}' C_j^* \neq 0, \tag{2.4}$$

where $\mathrm{rk}(\delta_{0,j}' C_j) = s_j$ for all $j = q_1, q_2, \ldots, q_k$. Then x_t is codependent of orders $(q_1, q_2, \ldots, q_k)$.

Some remarks on Definition 1 are in order. The s vectors represented by the columns of $D_0 = [\delta_{0,q_1} : \delta_{0,q_2} : \cdots : \delta_{0,q_k}]$ are labeled as codependence vectors, a term introduced by

Gourieroux & Peaucelle (1988, 1992). The assumption rk$(\delta_{0,j}' C_j) = s_j$ for all $j = q_1, q_2, \ldots, q_k$ rules out that codependence vectors associated with the same order can be linearly combined such that a smaller order than their common codependence order is obtained. Therefore, the full row rank assumption on $\delta_{0,j}' C_j$ for all $j = q_1, q_2, \ldots, q_k$ implies (i) that $(q_1, q_2, \ldots, q_k)$ represents the full set of minimal codependence orders and (ii) that the numbers of codependence vectors for each order, $s_{q_1}, s_{q_2}, \ldots, s_{q_k}$, are maximal in the sense that no s_j can be increased by lowering s_i for $j < i$. From now on, the terminology *set of minimal codependence orders* will refer to both implications (i) and (ii).

Clearly, Definition 1 implies that $\delta_{0,j}' x_t = \delta_{0,j}' C^*(L)\varepsilon_t = \delta_j'(L)\varepsilon_t = \sum_{i=0}^{j} \delta_{i,j}' \varepsilon_{t-i}$, where $\delta_{i,j} = C_i^{*\prime} \delta_{0,j}$.[1] We will label any vector which achieves such a finite-order serial correlation structure with respect to x_t as a codependence vector even if it is not a vector associated with the set of minimal codependence orders. The reasons are that any such vector must be a linear combination of the codependence vectors covered by Definition 1 and that the association of a certain vector with the minimal orders also depends on the specific identification structure applied to D_0. Note that the vectors in $\delta_{0,j}$, $j = q_1, q_2, \ldots, q_k$, are only identified up to an invertible transformation. Moreover, the design of an identification scheme for D_0 that maintains the composition of the different codependence orders may not be a trivial task, compare Trenkler & Weber (2010). We will comment on the issue of identification regarding the codependence vectors at several instances.

To distinguish codependence in levels from codependence in first differences of $I(1)$ variables, we introduce the terminology of level codependence of order q, abbreviated as LCO(q). Accordingly, a vector generating level codependence of order q will be labeled as LCO(q) vector. If $q = 0$, a level serial correlation common feature (LSCCF) is present and the corresponding codependence vector(s) are named LSCCF vector(s).

In the following, we link the established framework to different approaches in the literature.

1. For a single codependence order setup, Definition 1 is a special case of the codependence setup considered by Paruolo (2003) and Franchi & Paruolo (2010) in relation to cointegrated vector I(1) process that have a VAR(p) representation. Let x_t follow such a cointegrated I(1)-VAR(p) processes as defined in the next subsection and be $\beta_\perp$ the orthogonal complement to β. Then, $Y_t \equiv (x_t'\beta : \Delta x_t'\beta_\perp)'$ follows a stable VAR(p), as shown by Paruolo (2003) and Franchi & Paruolo (2010). They consider linear combinations of the form $b'Y_t$ with the $n \times s$ matrix $b \equiv (b_0' : b_1)'$ in order to analyze codependence. Thus, Definition 1 refers to the special case $b_1 = 0$ but in relation to $I(1)$ processes with general MA representation. Moreover, Definition

[1]The full row rank assumption on $\delta_{0,j}' C_j$ together with the linear independence of the codependence vectors ensures that the polynomials $\delta_j'(L)$, $j = q_1, q_2, \ldots, q_k$, are of full-column rank in the sense of Franchi & Paruolo (2010).

1 simultaneously covers the case of several codependence orders.

2. If a vector δ satisfies (2.4) for $j = q$, then we have $\delta' x_t = \sum_{i=0}^{q} \delta' C_i^* \varepsilon_{t-i}$. Hence, $\delta' x_t$ can be regarded as a linear combination of a multivariate MA(q) process,[2] which is a special case of a scalar component model (SCM), see Vahid & Engle (1997). According to Tiao & Tsay (1989), a non-zero linear combination $v_0' x_t$ of an n-dimensional process x_t follows an SCM(p,q) structure, if one can write

$$v_0' x_t + \sum_{j=1}^{p} v_j' x_{t-j} = v_0' \varepsilon_t + \sum_{j=1}^{q} h_j' \varepsilon_{t-j}$$

for a set of n-dimensional vectors $\{v_j\}_{j=1}^{p}$ and $\{h_j\}_{j=1}^{q}$ with $v_p \neq 0$ and $h_q \neq 0$. Thus, codependence of order q with respect to x_t results in an SCM$(0, q)$, where $q = 0$ represents the case of an SCCF.

3. To analyze common and codependent cycles with respect to the first differences of $I(1)$ variables, Vahid & Engle (1993, 1997) used the following common trend representation of Stock & Watson (1988), which can be obtained from (2.3):

$$\begin{aligned} x_t &= \gamma \tau_t + c_t \\ \tau_t &= \tau_{t-1} + \varphi' \varepsilon_t \end{aligned}$$

where γ and φ are $n \times (n-r)$ parameter matrices with $C(1) = \gamma \varphi'$ such that $\tau_t = \varphi' \sum_{s=0}^{\infty} \varepsilon_{t-s}$ are linear combinations of $n - r$ random walks representing the trend part and $c_t = C^*(L)\varepsilon_t$ is the cyclical part.

Vahid & Engle (1993) show that an SCCF with respect to Δx_t leads to a common cycle, i.e. an SCCF vector δ with $\delta' \Delta x_t = \delta' \varepsilon_t$ eliminates the cycles such that $\delta' c_t = 0$. Vahid & Engle (1997) generalize this result. If Δx_t is codependent of order q with a codependence vector δ' so that $\delta' \Delta x_t$ is an SCM$(0, q)$, then $\delta' c_t$ is an SCM$(0, q - 1)$. Thus, there exists a codependent cycle of order $q - 1$. Schleicher (2007) extended the work of Vahid & Engle (1993, 1997) by analyzing codependence regarding Δx_t within the VECM framework.

Thus, if x_t satisfies the LCO(q) constraints for δ such that $\delta' x_t$ is an SCM(0,q), then $\delta' \Delta x_t = \delta' x_t - \delta' x_{t-1}$ follows an SCM(0,$q + 1$). Accordingly, a codependence vector with respect to the first differences of $I(1)$ variables might also be related to codependence structures in the levels. Moreover, it follows that $\delta' c_t$ is an SCM(0,q). Hence, the vector δ, which lies in the cointegration space, does not only eliminate the common trend but also produces a codependent cycle of order q.[3]

[2] Note that $\delta' x_t$ can be given a univariate MA(q^*) representation with $q^* \leq q$ according to Lütkepohl (2005, Proposition 11.1). However, the error term in this representation does not equal $\delta' \varepsilon_t$.

[3] A codependence vector δ lying in the cointegration space has to satisfy additional restrictions compared to the

Finally, note that the SCCF and codependence setup with respect to the first differences of cointegrated I(1) variables analyzed by Vahid & Engle (1993) and Schleicher (2007) is another special case of the codependence framework considered by Paruolo (2003) and Franchi & Paruolo (2010). Here, the restriction $b_0 = 0$ is assumed and the codependence vectors studied in Vahid & Engle (1993) and Schleicher (2007) are linear combinations of $\beta_\perp$.

4. The adjustment properties implied by level codependence relate our framework to Pesaran & Shin (1996). They propose the persistence profile approach to analyze the dynamics of adjustment towards the cointegration equilibrium. Persistence profiles represent the effect of systems-wide shocks on the cointegration relations over time. For the case of a single cointegration relation we can define the persistence profile of $z_t = \beta' x_t$ in our notation as

$$H_z(j) = \frac{\beta' C_j^* \Omega C_j^{*\prime} \beta}{\beta' \Omega \beta}, \quad \text{for } j = 0, 1, 2, \ldots. \tag{2.5}$$

If only a single cointegration vector is assumed, $H_z(j)$ can be interpreted as the square of the impulse response function of z_t to a unit composite shock in the error $u_t = \beta' \varepsilon_t$. From (2.5) it is clear that an LCO(q) constraint requires the persistence profiles $H_z(j)$ to be zero for $j > q$. Using an (approximating) VAR representation for x_t, Pesaran & Shin (1996) provided the limiting distributions of the ML estimators of $H_z(j)$ for each j individually. Hence, significance testing is done pointwise as it is standard in the literature on impulse response analysis. In contrast, the test procedures in section 2.3 aim at testing the joint hypothesis $H_z(j) = 0$ for all $j > q$.

Concerning the last approach of Pesaran & Shin (1996) it is important to note that their original focus is not on codependence itself but on equilibrium adjustment. A similar motivation underlies our application that addresses the adjustment dynamics of interest rates towards their cointegration relation. To be precise, we want to analyze whether adjustment towards equilibria is finished after a finite number of periods. Formally, a cointegration error $\beta_j' x_t = \sum_{i=0}^{\infty} \beta_j' C_i^* \varepsilon_{t-i}$, $j = 1, \ldots, r$, would then have a finite-order serial correlation structure. Thereby, we have in mind that the cointegration matrix is identified in an economically meaningful way such that cointegration vectors have an appropriate definition. Therefore, we explore directly whether a cointegration vector β_j is also a codependence vector. In other words, we are testing for codependence conditional on a set of potential codependence vectors given by the cointegration vectors. Hence, we only consider cointegration vectors with their specific identifying structure for D_0, which, otherwise, had to be identified directly. Concretely, we say that

type of codependence vectors analyzed in Vahid & Engle (1997) or Schleicher (2007). In detail, it must produce a specific SCM structure with respect to Δx_t that can be rewritten as $\delta' \Delta x_t = \delta' \Delta \varepsilon_t + \delta' C_1^* \Delta \varepsilon_{t-1} + \cdots + \delta' C_{q_j}^* \Delta \varepsilon_{t-q_j}$. Moreover, $\delta' x_t$ does not need to be $I(1)$ if δ is a codependence vector with respect to Δx_t in contrast to the statement in Schleicher (2007, Appendix A).

β_j generates codependence of order q_j with respect to x_t, if

$$\beta_j' C_i^* = 0 \text{ for all } i > q_j \text{ and } \beta_j' C_{q_j}^* \neq 0.$$

Obviously, the condition $\beta_j' C(1) = 0$ is automatically satisfied.

Note that it can turn out that the cointegration vectors do not capture the set of minimal codependence orders. This would be the case if combining several β_j produces smaller codependence orders. Then, (some of) the codependence vectors generating the minimal codependence orders represent linear combinations of the cointegration vectors. Clearly, this cannot happen but for cointegration vectors associated with identical codependence orders according to the corresponding discussion of Definition 1. Thus, it follows that the concepts of level codependence and codependence conditional on the set of codependence vectors given by β coincide if no $q_j \geq 1$ results from two or more cointegration vectors. Two special cases in this respect are immediate equilibrium adjustment (i.e., LSCCF) and $r = 1$ as given in our application.

2.2 VAR Representation and Level Codependence

It is quite common to approximate the MA representation of x_t by a finite-order VAR process or to directly assume that x_t follows a VAR process in order to analyze SCCF and codependence, see e.g. Vahid & Engle (1993, 1997), Paruolo (2003), Schleicher (2007), Franchi & Paruolo (2010). However, Trenkler & Weber (2010) showed that codependent VAR models are not generally identified. In fact, only restrictions due to codependence of order one generated by at most one codependence vector and SCCF restrictions can be uniquely imposed on a VAR. This applies both to stationary and (cointegrated) $I(1)$ systems and, therefore, also to VECMs typically used in case of cointegration. Hence, the VAR framework is of limited use for analyzing general codependence restrictions. Accordingly, ML estimation of codependent VARs and conventional LR testing for codependence cannot be applied but in few special cases. Therefore, we only briefly discuss the VAR model setup in relation to the LSCCF and LCO(1) frameworks.

If the coefficient matrices in (2.3) satisfy specific restrictions, x_t follows the VAR(p)

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + \cdots + A_p x_{t-p} + \varepsilon_t, \tag{2.6}$$

where A_j, $j = 1, 2, \ldots, p$, are $(n \times n)$ coefficient matrices and the error term ε_t satisfies the same assumptions as above. Defining $\Pi = -(I_n - A_1 - \cdots - A_p)$ and $\Gamma_j = -(A_{j+1} + \cdots + A_p)$, $j = 1, \ldots, p-1$, we can re-write (2.6) in the vector error correction form

$$\Delta x_t = \Pi x_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta x_{t-j} + \varepsilon_t, \quad t = 1, 2, \ldots. \tag{2.7}$$

The relationship of the VAR and VECM representations can be compactly described by $A(z) = I_n - A_1 z - \cdots - A_p z^p = I_n \Delta - \Pi z - \Gamma_1 \Delta z - \cdots - \Gamma_{p-1} \Delta z^{p-1}$. Typically, the following assumptions are made in this context, compare e.g. Johansen (1995, Theorem 4.2): The roots of $A(z)$ are either $|z| > 1$ or $z = 1$. The matrix Π has reduced rank $r < n$, i.e. the matrix Π can be written as $\Pi = \alpha\beta'$, where α and β are $n \times r$ matrices with $\text{rk}(\alpha) = \text{rk}(\beta) = r$. Finally, the matrix $\alpha'_\perp \Gamma \beta_\perp$ has full rank, where $\Gamma = I_n - \sum_{j=1}^{p-1} \Gamma_j$ and where $\alpha_\perp$ and $\beta_\perp$ are the orthogonal complements to α and β.

Then, we have a special case of (2.3) with $C(1) = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ and the coefficients of $C^*(L)$ given by the recursive formula

$$\Delta C_i^* = \Pi C_{i-1}^* + \sum_{j=1}^{p-1} \Gamma_j \Delta C_{i-j}^*, \quad i = 1, 2, \ldots, \tag{2.8}$$

with $C_{-1}^* = \cdots = C_{-p+1}^* = -C(1)$, see Hansen (2005, Theorem 1).

We directly focus on the cointegration vectors β_j, $j = 1, 2, \ldots, r$, being the potential set of codependence vector when describing the restrictions on the VECM implied by level codependence. For an LSCCF generated by a cointegration vector β_j, i.e. for $q_j = 0$, one can easily deduce from the VECM (2.7) that $\beta'_j \Pi = -\beta'_j$ and $\beta'_j \Gamma_j = 0_{1\times n}$ for $j = 1, \ldots, p-1$ is required. This makes up $n(p-1) + r$ restrictions per vector. Note that $\beta'_j \Pi = -\beta'_j$ implies that $\beta'_j \alpha$ is a $(1 \times r)$ vector of which the j-th entry is equal to -1 while the other entries are equal to 0. In the LCO(1) case one can refer to the recursion (2.8) in order to derive the restrictions on the VECM parameters. First, note that (2.8) can be equivalently written as $\Delta C_i^* = C_{i-1}^* \Pi + \sum_{j=1}^{p-1} \Delta C_{i-j}^* \Gamma_j, i = 1, 2, \ldots$. Then, tedious algebra shows that LCO(1) implies $\beta'_j(\Pi + \Gamma_1)^2 + \beta'_j \Gamma_1 \Pi = \beta'_j(\Pi + \Gamma_1 + I)\Gamma_1 + \beta'_j \Gamma_2 = \beta'_j(\Pi + \Gamma_1 + I)\Gamma_2 + \beta'_j \Gamma_3 = \cdots = \beta'_j(\Pi + \Gamma_1 + I)\Gamma_{p-2} + \beta'_j \Gamma_{p-1} = \beta'_j(\Pi + \Gamma_1 + I)\Gamma_{p-1} = 0$. Obviously, the parameter restrictions underlying an LCO(1)-VECM are nonlinear. In general, it is more convenient to use a state-space representation based on the companion form of the VECM in order to describe the restrictions on the VECM parameters, compare Schleicher (2007). It follows from Trenkler & Weber (2010), that we have again $n(p-1) + r$ constraints associated with LCO(1) noting that the codependence vector is known in our framework.[4]

We make two final remarks on codependent VECMs. First, the VECM framework is only valid if the above discussed restrictions on (2.3) hold. If they are not exactly fulfilled, the VECM may nonetheless serve as an approximation of the true process (2.3). Second, the upper bound for the LCO order in a VECM is given by $q_{max} = (n-1)p - (n-r)$, compare Trenkler & Weber (2010, Theorem 1). Note, however, that a VECM with such a codependence order is typically not identified, unless $q_{max} \leq 1$.

[4]For joint imposition of several codependence vectors on a VECM see section 2.3.3.

2.3 Testing Approaches

As regards the definition of level codependence we have treated the cointegration matrix β as given. In applied work, however, β is unknown and has to be replaced by some estimator. We assume in the following that a superconsistent estimator $\hat{\beta}$, e.g. a reduced rank estimator based on the VECM (2.7), is available. This ensures that the limiting distributions of the considered test statistics do not change, i.e. inference can be conducted as if β were known; compare the references given below.

Accordingly, the test procedures considered in all the following three subsections are conditional on the estimator $\hat{\beta}$. Of course, the tests can also be applied conditional on some prespecified cointegration matrix. Such a setup is typically justified in cases of strong (economic) priors and statistical evidence for a particular cointegration vector. This applies e.g. to arbitrage implying a cointegration vector $\beta = (1, -1)'$ if a bivariate setup is considered. Another example is the controllability of overnight interest rates by central banks as analyzed in section 3.

In the following, we discuss GMM and LR tests for codependence conditional on β representing the set of codependence vectors. Subsequently, a general test procedure for detecting the set of minimal codependence orders is described.

2.3.1 GMM-type Test

If x_t is level codependent of order q_j due to the cointegration vector β_j, then $z_{t,j} = \beta_j' x_t$ should be uncorrelated with all its lags beyond q_j. Assuming that Δx_t follows the VECM (2.7), it is sufficient to focus on zero correlations between $z_{t,j}$ and $X_{t-q_j-1} = (z_{t-q_j-1}, \Delta x'_{t-q_j-1}, \Delta x'_{t-q_j-2}, \dots, \Delta x'_{t-q_j-p+1})'$, where $z_t = \beta' x_t$. This statement follows because $\beta' x_t = (I_r + \beta'\alpha)\beta' x_{t-1} + \sum_{i=1}^{p-1} \beta' \Gamma_i \Delta x_{t-i} + \beta' \varepsilon_t$. In their corresponding frameworks, Vahid & Engle (1997) and Schleicher (2007) have used such zero correlations as moment conditions for GMM estimation of the codependence vector. Based on the GMM approach, overidentifying restrictions can then be tested.

We do not apply the GMM principle to estimate β since a superconsistent estimator $\hat{\beta}$ is needed and assumed to be available. However, we use the corresponding test approach in order to test for a cut-off in the serial correlation of the cointegration errors after q_j lags. Following Vahid & Engle (1997) and Schleicher (2007), we test the null hypothesis

$$H_0 : g(\beta_j) = \mathrm{E}(z_{t,j} X_{t-q_j-1}) = 0_{[(n(p-1)+r)\times 1]} \qquad (2.9)$$

by considering the statistic

$$Z_T = g_T(\hat{\beta}_j)' P_T(\hat{\beta}_j) g_T(\hat{\beta}_j), \qquad (2.10)$$

with $g_T(\hat{\beta}_j) = \frac{1}{\sqrt{T-p-q_j}} \sum_{t=p+q_j+1}^{T} \hat{z}_{t,j} \hat{X}_{t-q_j-1}$, where $\hat{z}_{t,j} = \hat{\beta}_j' x_t$ and $\hat{X}_t$ is the same as X_t with z_t replaced by $\hat{z}_t = \hat{\beta}' x_t$. Moreover, $P_T(\hat{\beta}_j) = \left(\hat{\sigma}^2 \left(\frac{1}{T-p-q_j} \sum_{t=p+q_j+1}^{T} \hat{X}_{t-q_j-1} \hat{X}_{t-q_j-1}' \right) + \sum_{i=1}^{q_j} \hat{\gamma}_i \left(\frac{1}{T-p-q_j} \sum_{t=p+q_j+i+1}^{T} (\hat{X}_{t-q_j-1} \hat{X}_{t-q_j-1-i}' + \hat{X}_{t-q_j-1-i} \hat{X}_{t-q_j-1}') \right) \right)^{-1}$, which is the weighting matrix with $\hat{\sigma}^2$ and $\hat{\gamma}_i$ being consistent estimators of the variance and autocovariances of z_t.

The GMM statistic (2.10) has an asymptotic χ^2-distribution with $n(p-1)+r$ degrees of freedom. Note in this respect that using a superconsistent estimator $\hat{\beta}$ ensures that $P_T(\hat{\beta}_j)$ is a consistent estimator of $(\lim_{T\to\infty} \mathrm{E}(g_T(\beta_j) g_T(\beta_j)'/T))^{-1}$, compare also Brüggemann, Lütkepohl & Saikkonen (2006).

As already pointed out by Schleicher (2007), the choice of the instrument set X_t makes the GMM test depend on the VECM framework. In other words, this approach can only be interpreted as a test for LCO(q_j) or, to be more precise, the null hypothesis (2.9) only represents the LCO(q_j) constraints if the VECM provides a correct representation of Δx_t. This link has two important implications. First, the upper bound for the codependence order should also be applied with respect to the GMM test. Second, if the VECM is only regarded as an approximation of the process Δx_t, then the null hypothesis $g(\beta_j) = 0_{[(n(p-1)+1)\times 1]}$ is only covering a subset of the restrictions implied by LCO(q_j). Nevertheless, evidence *against* LCO(q_j) can still be collected since rejection of $g(\beta_j) = 0_{[(n(p-1)+r)\times 1]}$ implies also rejection of the LCO(q_j) constraints. In this respect, the GMM-type test is a useful procedure, even if the VECM is only an approximation of the MA process for Δx_t.

2.3.2 LR Test

If an identified codependent VECM is used we can apply the LR principle to test the null of LCO(q). That is, we estimate the unrestricted and restricted model by (nonlinear) ML and take twice the log-likelihood difference. In line with section 2.2, the LR test statistic is asymptotically χ^2-distributed with $n(p-1)+r$ degrees of freedom if a single codependence vector is considered. As already mentioned, only VEC models with an LSCCF or LCO(1) are uniquely identified for single codependence vector setups. Moreover, it is possible to uniquely impose several LSCCFs, potentially combined with one LCO(1) vector. Therefore, the applicability of the LR test is limited to these cases. Situations with a joint imposition of several codependence vectors are discussed in the next subsection.

If the focus is on LSCCFs, one can use the framework of Paruolo (2003, Section 6) to estimate the restricted VECM. The estimation is based on reduced rank techniques taking into account the restriction that only $\beta' x_t$ is involved in the LSCCF constraints. This requires to accordingly restrict the matrix b in Paruolo (2003, Section 6) that generates the LSCCFs. Alternatively, one may numerically maximize the likelihood function of the VECM (2.7). The latter approach has to be chosen in case of estimating VECMs with LCO(1) due to the nonlinear parameter restrictions. Therefore, we decided to use numerical methods in order to estimate the restricted VECM for both the LSCCF and LCO(1) setups. Due to the results in Paruolo (2003), replacing β by a superconsistent estimator does not change the asymptotic distribution of the LR test statistic. Moreover, in the case of a pre-specified cointegration matrix, the LSCCF restrictions on the VECM parameters could also be tested by F- or Wald-tests, which have their usual asymptotic distributions.

Finally, we make some comparing remarks on the GMM and LR tests. The GMM test can be used in situations where the LR test cannot be applied due to non-identification of the VECM. Furthermore, the GMM test alleviates the sensitivity to model misspecification and circumvents the potentially demanding numerical optimization under the LR null hypothesis, see Schleicher (2007). Nevertheless, we have a preference for applying the LR test for identified model setups. This is due to the results of the simulation study in Schleicher (2007), which indicates an obvious advantage of the LR over the GMM test in terms of small sample power.

2.3.3 Test for Minimal Codependence Orders

As mentioned in section 2.1, using the cointegration vectors as a set of potential codependence vectors does not necessarily obtain the set of minimal codependence orders. If the latter is of interest, one can apply the sequential GMM test procedures outlined below. We note that the application of such a test procedure is not necessary for empirical setups with a single cointegration vector as considered in the next section. Obviously, in a single cointegration vector case the cointegration vector is a codependence vector of minimal order if the process x_t is codependent. Moreover, if the focus is on the analysis of the adjustment dynamics in a multiple cointegration vector case, one does not need to use the sequential test procedure either. It is then sufficient to refer to the GMM and LR tests discussed in the previous sections 2.3.1 and 2.3.2.

The codependence vectors introduced by Definition 1 have to be linear combinations of the cointegration vectors. Hence, one has to test for the existence of these linear combinations in order to determine the set of minimal codependence orders. Note that there can be up to r such linear combinations of the cointegration vectors. In the following, we first present a sequential GMM test procedure that can be generally applied. Afterwards, we discuss the (limited) usefulness of an LR test approach to detect the set of minimal codependence orders.

In contrast to section 2.3.1, we now consider linear combinations of $\beta' x_t$ in the relevant moment conditions. Let γ be the $r \times s$ matrix containing the weights of the linear combinations generating the set of minimal codependence orders. Then, the columns of $D_0 = \beta\gamma$ represent the corresponding $s \leq r$ level codependence vectors. Since the parameters in γ are unknown, they have to be estimated in the GMM approach. Therefore, one has to subtract the number of unknown parameters in γ from the number of tested moment conditions in order to obtain the number of degrees of freedom of the corresponding GMM tests. In other words, we are now considering a typical test for overidentifying restrictions based on estimated parameters compared to the test setup in section 2.3.1, where all parameters are given. Note in this respect, that we again condition on some superconsistent estimator $\hat{\beta}$ or some pre-specified cointegration matrix β. Hence, the error-correction term $\beta' x_{t-1}$ or the estimator of it is regarded as given.

The GMM test sequence works as follows. First, one tests the null hypothesis of $s_0^0 = 1$ LSCCF vector: $H_0 : s_0^0 = 1$, i.e. one tests whether one appropriate linear combination of β exists. If the according null hypothesis is not rejected, then one continues to test for $H_0 : s_0^0 = 2$ against no LSCCF vector and so forth until H_0 is rejected the first time. Let γ_0 denote the version of γ considered under H_0, then the null hypotheses tested in this sequence can be formally represented by

$$H_0 : g(\gamma^0, \beta) = \mathrm{E}\left(\mathrm{vec}\left(\gamma^{0\prime} z_t \otimes X_{t-1}\right)\right) = 0_{[(s_0^0(n(p-1)+r))\times 1]}, \tag{2.11}$$

where the instrument set X_{t-1} is as defined in section 2.3.1. The corresponding GMM statistics are appropriately adjusted versions of (2.10). We note, however, that the application of these statistics requires to impose an identifying structure on γ^0. Here, we apply $\gamma^0 = [I_{s_0^0} : \gamma_0^{0\prime}]'$, where γ_0^0 is an $(r - s_0^0) \times s_0^0$ matrix of free parameters to be estimated. Hence, the GMM test statistic for (2.11) is χ^2 distributed with $s_0^0(n(p-1) + s_0^0) = s_0^0(n(p-1) + r) - s_0^0(r - s_0^0)$ degrees of freedom.

Let s_0 be the largest number of LSCCF vectors that is not rejected. If $s_0 < r$, one continues to test for the number of LCO(1) vectors. Since the s_0 LSCCF vectors are also LCO(1) vectors, it is sufficient to start with the null hypothesis of $s_1^0 = s_0 + 1$ LCO(1) vectors, $H_0 : s_1^0 = s_0 + 1$, against the alternative $s_1^0 = 0$. If this null hypothesis is not rejected, the sequence continues with $s_1^0 = s_0 + 2$, $s_1^0 = s_0 + 3$ and so forth according to the LSCCF setup. One has to pay particular attention to the identification scheme applied to γ^0 in order to maintain the combination of LSCCF and LCO(1) vectors under H_0. Results from Trenkler & Weber (2010) suggest to use

$$\gamma^0 = \begin{bmatrix} I_{s_0} & 0_{s_0 \times (s_1^0 - s_0)} \\ \gamma_0^0 & \left(I_{s_1^0 - s_0} : \gamma_1^{0\prime}\right)' \end{bmatrix},$$

where γ_1^0 is an $(r - s_1^0) \times (s_1^0 - s_0)$ matrix and γ_1^0 is as defined above. Accordingly, γ^0 contains $s_0(s_1^0 - s_0) + s_1^0(r - s_1^0)$ free parameters. Thus, the corresponding GMM test for $H_0 : s_1^0 = s_0 + 1$, which explicitly takes into account that there are s_0 LSCCF vectors, has $s_1^0(n(p-1)+s_1^0) - s_0(s_1^0 - s_0)$ degrees of freedom. The null hypotheses can be written as

$$H_0 : g(\gamma^0, \beta) = \mathrm{E}\left(\operatorname{vec}\begin{bmatrix} \gamma_{[s_0]}^{0\prime} z_t \otimes X_{t-1} \\ \gamma_{[s_0^1 - s_0]}^{0\prime} z_t \otimes X_{t-2} \end{bmatrix}\right) = 0_{[(s_1^0(n(p-1)+r))\times 1]},$$

where $\gamma_{[s_0]}$ and $\gamma_{[s_0^1 - s_0]}$ represent the first s_0 and last $s_0^1 - s_0$ columns of γ^0, respectively.

If s_1 is the largest number of LCO(1) vectors which is not rejected, then the test sequence continues if $s_1 < r$ in the same spirit as above. Thus, when a larger codependence order is reached one uses explicitly the information that a specific number of codependence vectors of lower orders has not been rejected beforehand. According to the previous discussion, this both refers to the identification scheme applied to γ^0 and the determination of the degrees of freedom as well as to number of codependence vectors tested first under H_0 for a new codependence order. This test sequence stops if r codependence vectors are not rejected or if the upper bound $q_{max} = (n-1)p - (n-r)$ for the LCO order within a VECM is reached. The latter stopping criterion follows from the discussion in section 2.3.1.[5]

We now turn to the discussion of a potential LR test sequence. Since only LSCCF vectors and at most one LCO(1) vector can be uniquely imposed on a VECM, it is generally not possible to impose all codependence vectors generating the set of minimal codependence orders. Only the setups with r LSCCF vectors or $r-1$ LSCCF vectors and one LCO(1) vector tested under H_0 are known to be cases that cover the full set of minimal codependence orders. If $s < r$ codependence vectors are imposed, then there may be up to $r - s$ additional vectors that induce further LCO(1) restrictions or orders beyond one. However, these cases cannot be uniquely considered within a VECM and can, therefore, not be tested for by applying an LR test. Hence, there can exist uncertainty concerning the full set of minimal codependence orders. Nevertheless, one may wish test for a subset of the minimal codependence orders or for the special cases just mentioned using a sequential LR test approach.

The LR test sequence proceeds as the corresponding GMM test sequence. However, it may stop earlier due to restrictions mentioned in the previous paragraph. The alternative hypothesis never imposes codependence restrictions on the VECM. Hence, the VECM is estimated unrestrictedly via standard ML under the alternative in order to obtain the unrestricted log-likelihood value. Under the null hypothesis one has to jointly impose the relevant number of LSCCF vectors and, potentially, one LCO(1) vector. Due to conditioning on β or on a superconsistent estimate $\hat{\beta}$, it is again sufficient to estimate the matrix γ containing the weights of the relevant

[5] As an alternative to the GMM test, one may use a test based on canonical correlations that was suggested by Tiao & Tsay (1989). However, this procedure does not allow to impose the specific composition of codependence orders tested under the null hypothesis. Therefore, we do not consider the procedure of Tiao & Tsay (1989).

linear combinations of β instead of estimating the corresponding codependence vectors. The VECM has to be estimated by nonlinear ML under H_0 in order to compute the restricted log-likelihood value. As for the single codependence vector case discussed in section 2.3.1, one can follow Paruolo (2003) to estimate VECMs on which only LSCCFs are imposed or one uses a nonlinear numerical estimation approach. The latter has to be chosen if an LCO(1) vector is involved. Based on the restricted and unrestricted log-likelihood values, the corresponding LR statistic is formulated in the usual way. The LR statistics are asymptotically χ^2 distributed. Since the same identification scheme as for the GMM tests is applied to γ under H_0, one also obtains the same number of degrees of freedom as above, compare also Trenkler & Weber (2010) in this respect.

3 Can Central Banks Control Overnight Rates?

Herein, we examine to which extent overnight money market rates are controllable by monetary policy makers. For this purpose, the LSCCF and LCO concept and the proposed statistical tests are applied. We look in turn at the world's most prominent central banks, the Fed and the ECB.

3.1 Federal Reserve Bank

Over decades, the Fed has developed an institutional framework for effectuating its monetary policy. An important change occurred in February 1994: Since then, the Fed has announced changes in the federal funds target rate immediately after the decision. Such transparency is likely to contribute to low persistence of deviations of the federal funds rate from its target, see Nautz & Schmidt (2009). The same presumably holds true for the forward-looking assessment of inflationary pressure and economic slowdown, which complements the Federal Open Market Committee (FOMC) statements since January 2000.

The Fed requires commercial banks to hold a certain average amount of reserves during each maintenance period of two weeks. Therefore, we argue that a natural frequency for the empirical analysis is provided by biweekly data. In this, while the maintenance periods end on the so-called Settlement Wednesdays, we measure the interest rates on the Wednesdays in between. Doing so has the further advantage to avoid dealing with predictable day-of-the-week effects or the Settlement Wednesday tightness (see Hamilton 1996), which is rendered innocuous by sampling at the midpoints of the maintenance periods. The sample is chosen as 06/28/2000-12/03/2008, where the starting point ensures consistency with the European case discussed below. The end date is determined by the fact that the Fed replaced its target rate by a target *range* (initially from 0 to 0.25) on 16 December 2008; thereafter, the federal funds rate stayed within the range, but not particularly close neither to the upper or lower limit nor to the midpoint. In total, we have 221 observations. Figure 1 shows the federal funds and the target rate as well as the policy spread.

Figure 1. Federal funds rate (solid line), target rate (dotted line) and spread (dashed line)

Evidently, the overnight rate closely follows the target, so that the spread reveals no long-lasting swings. The correlogram of the spread can be seen in Figure 2. The spread yields mostly small serial correlations that would be judged insignificant applying the asymptotic standard error $1/\sqrt{T}$. However, the first two autocorrelations do not seem to be necessarily negligible. Therefore, as discussed above, testing within the underlying VECM is preferred in order to jointly consider all relevant lags.

The VECM for the federal funds rate i_t and the target rate i_t^* is specified with a restricted constant and four lags in first differences, as suggested by the AIC and HQ criteria. For a definition of these information criteria see e.g. Lütkepohl (2005). The Johansen trace test easily confirms cointegration with a test statistic of 35.27 (p-value = 0.02%). The Portmanteau test for non-autocorrelated residuals, compare Lütkepohl (2005), is clearly insignificant at all lags. Thus, the model seems to be adequate in the sense of picking up the complete dynamics from the data. In contrast, reducing the lag length would leave pronounced autocorrelation in the residuals. Concerning the cointegration vector, we have a strong theoretical prior for $\beta = (1 \quad -1)'$. Empirically, this restriction is not rejected given an LR p-value of 27.6%. This test was repeated after the LSCCF restrictions had been imposed, with the same result. The estimated VECM takes the following form:

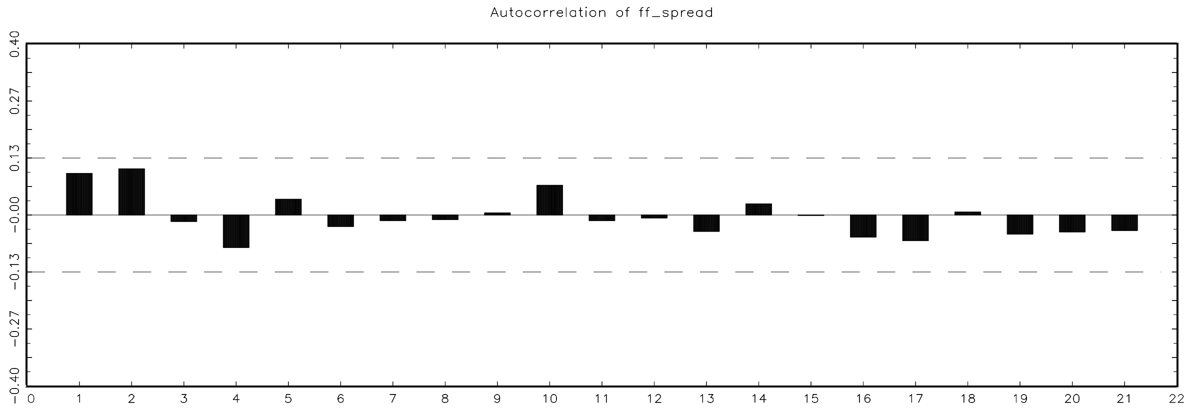


Figure 2. Autocorrelations of federal funds spread

$$\begin{pmatrix}\Delta i_t\\ \Delta i_t^*\end{pmatrix} = \begin{pmatrix}\underset{(0.225)}{-0.766}\\ \underset{(0.140)}{0.168}\end{pmatrix}(i_{t-1} - i_{t-1}^* + \underset{(0.014)}{0.023}) + \begin{pmatrix}\underset{(0.204)}{0.042} & \underset{(0.215)}{-0.036}\\ \underset{(0.127)}{0.025} & \underset{(0.134)}{-0.126}\end{pmatrix}\begin{pmatrix}\Delta i_{t-1}\\ \Delta i_{t-1}^*\end{pmatrix} + \begin{pmatrix}\underset{(0.177)}{0.047} & \underset{(0.189)}{0.213}\\ \underset{(0.110)}{-0.070} & \underset{(0.118)}{0.170}\end{pmatrix}\begin{pmatrix}\Delta i_{t-2}\\ \Delta i_{t-2}^*\end{pmatrix}$$
$$+ \begin{pmatrix}\underset{(0.148)}{0.072} & \underset{(0.168)}{0.236}\\ \underset{(0.092)}{0.021} & \underset{(0.105)}{0.295}\end{pmatrix}\begin{pmatrix}\Delta i_{t-3}\\ \Delta i_{t-3}^*\end{pmatrix} + \begin{pmatrix}\underset{(0.108)}{0.060} & \underset{(0.137)}{0.147}\\ \underset{(0.067)}{0.113} & \underset{(0.085)}{0.195}\end{pmatrix}\begin{pmatrix}\Delta i_{t-4}\\ \Delta i_{t-4}^*\end{pmatrix} + \begin{pmatrix}\hat{u}_{1t}\\ \hat{u}_{2t}\end{pmatrix}$$

In this model, LSCCF would be given if $\beta'\alpha = -1$ and $\beta'\Gamma_j = 0$, $j = 1, \ldots, 4$, compare section 2.2. Indeed, the difference of the adjustment coefficients, $\alpha_2 - \alpha_1$, lies near one. Among the parameters in the short-run dynamics, there are pairs that are more and some that are less equal. Even though most single coefficients are estimated relatively imprecisely, the model has been carefully specified and is unlikely to provide an incorrect representation of the true data generating process (as far as any model can be correct, of course). Rather, the large standard errors are more a consequence of natural multicollinearity in VARs than signs of true expendability. In sum, the impression from institutional facts, visual inspection and preliminary statistical analysis suggests a close controllability of the federal funds rate by the Fed. Indeed, applying the nine LSCCF restrictions to the VECM, the likelihood does not shrink dramatically. The p-value of the according LR test with nine degrees of freedom amounts to 34.1%. Moreover, the corresponding GMM test results in a p-value of 17.9%. Therefore, we can conclude that on average, deviations of the federal funds rate from the target are corrected at least within one maintenance period. The Fed was obviously able to control the short end of the money market.

3.2 European Central Bank

We now turn to the European case, where we also allude to critical points connected to model specification and codependence testing. The ECB provides liquidity to the European banking sector through weekly main refinancing operations (MROs). The relevant market and target rates are the Euro Overnight Index Average (Eonia) and the minimum bid rate (MBR). Since June 2000, the date chosen as our starting point, the MROs are conducted as variable rate tenders, see Hassler & Nautz (2008).[6] Furthermore, the ECB shortened the MRO maturity from two weeks to one week in March 2004. The considerable rise in spread persistence, as established by Hassler & Nautz (2008), could then be explained by higher costs and risk of refinancing. Consequently, we split the European data into two sub-samples in order to accommodate a potential structural break. These sub-samples have 97 (June 2000 - February 2004) and 124 (March 2004 - December 2008) observations, respectively. We keep the frequency of the US data. Figure 3 plots the Eonia and the MBR as well as the European spread.

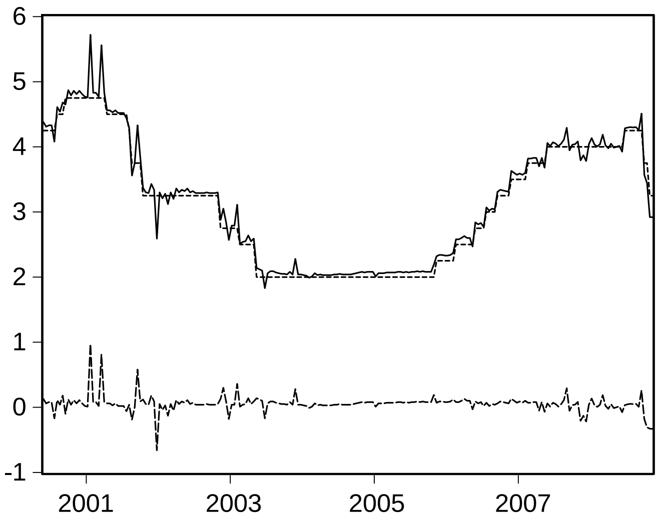
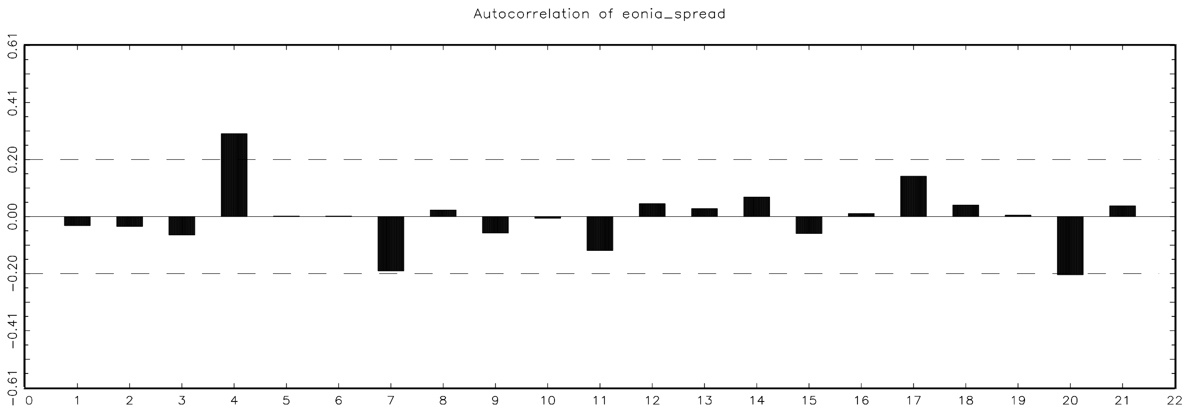


Figure 3. Eonia (solid line), minimum bid rate (dotted line) and spread (dashed line)

Even though the spreads are still small compared to the level of the interest rates, the deviations do not feature the white-noise character from the US case. Backing the visual impression, Figure 4 presents the autocorrelations for both sub-samples.

[6]On 15 October 2008 the ECB switched to fixed rate tenders. So, the last few, but indecisive, MBR observations in our sample effectively equal the fixed rate. The end point is chosen as in the US case for reasons of comparability.

Panel A: 1st sub-sample: June 2000 - February 2004



Panel B: 2nd sub-sample: March 2000 - December 2008

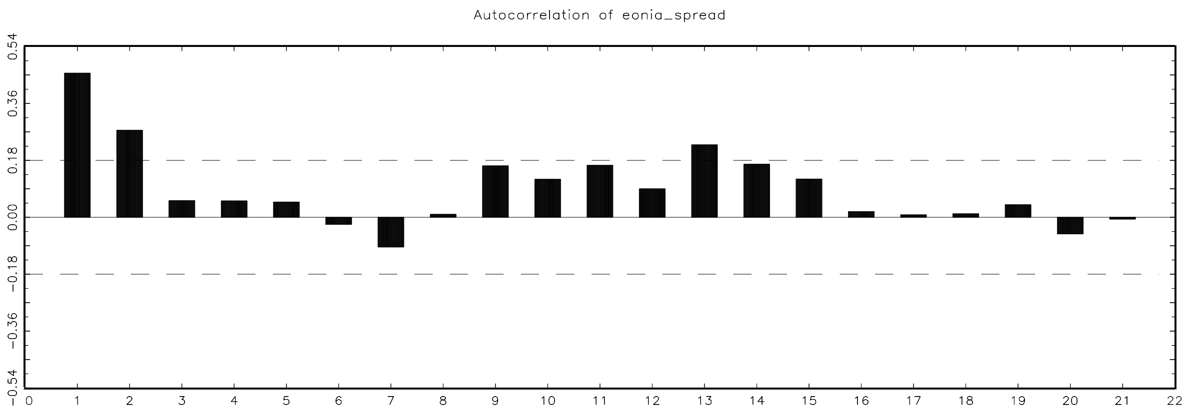


Figure 4. Autocorrelations of Eonia Spread

Most serial correlations of the spread are rather negligible. However, lag four in the first and various lags in the second sub-period cast the LSCCF hypothesis into doubt. In the second period, Hassler & Nautz (2008) have established fractional integration (long memory) for the spread using daily data. In general, long-memory behavior should not change when sampling at different frequencies (e.g. Chambers 1998). Indeed, Panel B of Figure 4 reveals a typical pattern of persistent serial correlations, even if most of them individually do not reach significance due to the relatively low number of observations.

For the first sub-period VECM, all information criteria suggest a lag length of zero, i.e. $p = 1$. One cointegrating relation is significant with a trace statistic of 74.90, and the $\beta = (1 \ -1)'$ restriction passes with a p-value of 25.8%. The resulting model is

$$\begin{pmatrix} \Delta i_t \\ \Delta i_t^* \end{pmatrix} = \begin{pmatrix} \underset{(0.125)}{-1.059} \\ \underset{(0.070)}{-0.026} \end{pmatrix} (i_t - i_t^* - \underset{(0.017)}{0.068}) + \begin{pmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \end{pmatrix} .$$

One can assert at first sight that the LSCCF restriction $\alpha_1 - \alpha_2 = -1$ is empirically acceptable. Indeed, the LR and GMM tests produce p-values of 75.6% and 75.2%, respectively. However, the Portmanteau test for no residual autocorrelation is significant from lag nine onwards. That is, despite the unanimous decision of all information criteria, the model seems to be misspecified. Presumably, the VECM with no lags in first differences, VECM(0), has not taken into account the 30% autocorrelation on the fourth lag of the spread (Figure 4, Panel A).

This last conjecture can be supported when estimating a VECM(3), equivalent to VAR(4), which yields good Portmanteau results. The seven LSCCF constraints in this model cannot be rejected with p-values of 19.2% for the LR test and 25.7% for the GMM test. Nevertheless, the bulk of the coefficients introduced by the higher model order is insignificant and superfluous, as it is but the fourth VAR lag that matters for the Eonia dynamics. Indeed, the second and third lag can be excluded from the VAR by conventional Wald tests. This reduces the number of restrictions coming from LSCCF by four - two for each excluded 2×2 matrix. When applying the LR test with $7 - 4 = 3$ restrictions to the VECM derived from the accordingly restricted VAR(4), we obtain a p-value of 2.8%.[7] This suggests that the fully general VECM(3) had inflated the number of degrees of freedom, lowering the power of the LR test. In conclusion, it is an advantage to have a closed modeling framework and a clear-cut test at hand, but correct model specification and power issues are to be carefully dealt with.

In the second sub-period, both AIC and HQ choose two lags. The Portmanteau tests are quite favorable until lag 22, but for a few tens of lags from 23 upwards, the p-values do not reach more than 3% to 4%. The trace statistic of 31.67 is clearly significant, whereas the evidence against $\beta = (1, -1)$ is somewhat stronger than before with a p-value of 1.5%. Nonetheless, we proceed with $\beta_2 = -1$, because restricting the freely estimated parameter of -0.975 is not going to affect the LSCCF test outcome. The VECM results as

$$\begin{pmatrix} \Delta i_t \\ \Delta i_t^* \end{pmatrix} = \begin{pmatrix} \underset{(0.227)}{-0.531} \\ \underset{(0.153)}{0.036} \end{pmatrix} (i_t - i_t^* - \underset{(0.011)}{0.029}) + \begin{pmatrix} \underset{(0.219)}{-0.259} & \underset{(0.256)}{0.598} \\ \underset{(0.147)}{-0.119} & \underset{(0.172)}{0.166} \end{pmatrix} \begin{pmatrix} \Delta i_{t-1} \\ \Delta i_{t-1}^* \end{pmatrix} + \begin{pmatrix} \underset{(0.191)}{0.275} & \underset{(0.280)}{0.148} \\ \underset{(0.128)}{0.200} & \underset{(0.188)}{0.029} \end{pmatrix} \begin{pmatrix} \Delta i_{t-2} \\ \Delta i_{t-2}^* \end{pmatrix} + \begin{pmatrix} \hat{u}_{1t} \\ \hat{u}_{2t} \end{pmatrix} .$$

As might be suspected in view of the estimates for the adjustment coefficients, both the LSCCF-LR and LSCCF-GMM tests reject the null hypothesis, implying five degrees of freedom, with p-values close to zero. However, Figure 4, Panel B might suggest that this rejection

[7]The GMM test cannot be adjusted to the restricted VAR model.

is primarily triggered by the significant autocorrelations at lag one and two. In other words, the adjustment process would be finished in the third period. However, a VECM with LCO(2) is not uniquely identified, as noted in section 2.2. Accordingly, an LR test cannot be applied. Applying an LR test for LCO(1) instead leads to a clear rejection with a p-value close to zero. The same result is obtained using the GMM-type test. Notwithstanding, the latter test can also be used to test for level codependence order of two. Again we have to reject the null hypothesis due to a p-value of 2.4%.

Hence, the tests seem to pick up the later non-negligible autocorrelations in the EONIA spread, judging them as evidence against serial correlation of maximal finite order 1 or 2. Indeed, exactly this decision was to be expected, recalling the long-memory result of Hassler & Nautz (2008). Thus, our test succeeds in discriminating between different degrees of interest rate controllability both through different countries and time periods. The change in the operational conduct of monetary policy seems to have impaired interest rate controllability by the ECB.

4 Conclusions

While cointegration denotes the commonality of non-stationary components among different variables, we combine it with the concept of common serial correlation. Time series obeying the according restrictions move in parallel in the sense that a specific linear combination is free of any autocorrelation structure. Concerning cointegration adjustment, this implies that any deviation from the equilibrium is corrected within a single period. In order to allow cointegration relations to be restored only with a delay of q periods, the framework is extended to codependence. We discuss the close connection of this concept to equilibrium adjustment, the subject of interest.

For both LSCCF and codependence, we derive the constraints to be fulfilled in VECMs. Regarding statistical inference, we propose LR testing for codependent VECMs that are identified. In non-identified cases, we consider a flexible GMM test for a cut-off in the serial correlation of the cointegration error. For both methods a test sequence is discussed to detect the set of minimal orders of codependence among multiple variables.

Important applications of the developed framework arise whenever economic reasoning suggests that variables stay in close contact over time. Such a development may be generated by processes of financial arbitrage. In our empirical section, we examine the question of controllability of interest rates by central banks. In particular, we examine whether the Fed and the ECB succeeded in making overnight money market rates closely follow their target rates. Results for the US are quite favorable in this regard, since LR and GMM tests yield no evidence against the LSCCF hypothesis. The European case delivers contrary results, even though in the 2000-2004 sub-period, an LSCCF might be present. However, since a change in the operational monetary policy framework in 2004, neither LSCCF nor the weaker concept of codependence can be empirically confirmed.

In conclusion, this paper offers both an innovative and a cautious perspective: On the one hand, common serial correlation in levels provides an intuitive and useful enhancement of the literatures of common cycles, cointegration and adjustment speed. On the other hand, we critically evaluate the scope of VECM-based common serial correlation analyses, pointing at conceptual and empirical problems. Nevertheless, based on appropriate statistical tools as proposed in this study, we believe that an appreciable potential of the underlying methodology can be exploited in future research.

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