Real Estate Investment Dynamics

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Contents

List of Figures 4

I Housing, Time-to-Build, and the Business Cycle 8

1 Introduction 9

2 Real Business Cycle Models 15

2.1 The Prototype RBC Model . . . 19
2.1.1 Households . . . 19
2.1.2 Firms . . . 20
2.1.3 Equilibrium and First Order Necessary Conditions . . . 21
2.1.4 The RBC Model Formulated as a Ramsey Problem . . . 25
2.1.5 Balanced Growth and Restrictions on Technology and Preferences . 28
2.1.6 Stationary Economies and Functional Forms . . . 30
2.1.7 An Analytical Solution . . . 32
2.1.8 Steady States and Convergence . . . 33
2.1.9 Numerical Solution and Calibration . . . 36
2.2 Multi-Sector RBC Models . . . 39

3 A Multi-Sector RBC Model with Time-to-Build 49

3.1 Economic Environment . . . 49
3.1.1 Final Goods Production . . . 50
3.1.2 Productivity . . . 51
3.1.3 The Housing Sector . . . 51
3.1.4 Time-to-Build . . . 52
3.1.5 Government . . . 54
3.1.6 Households . . . 55
3.2 The Recursive Competitive Equilibrium . . . 57
3.2.1 The Household Sector . . . 58
3.2.2 The Production Sectors . . . 59
3.2.3 Definition . . . 61

3.3 Balanced Growth and Solution Method 62
3.4 Calibration and Data 64
3.4.1 Consumption 64
3.4.2 House Prices 65
3.4.3 Total Fixed Investment 65
3.4.4 Aggregate Stocks and Depreciation of Capital and Housing 65
3.4.5 Productivity and Preference Parameters 66
3.4.6 Tax Rates 67
3.4.7 Solow Residual 67
3.4.8 Time-to-Build Technology 69

4 Results **70**
4.1 First Moments 71
4.2 Second Moments 71
4.3 Impulse Responses 74

5 Conclusions **77**

II German Commercial Property Prices and Bank Lending **80**

6 Introduction **81**

7 Data and Related Literature **85**
7.1 Data Description 85
7.2 German Commercial Property Development 86
7.3 Macroeconomics, Bank Lending and Real Estate Linkage 88

8 Vector Autoregression (VAR) and Empirical Analysis **91**
8.1 VAR 91
8.2 Empirical Results and Implications 96
8.2.1 Germany: **Impulse Response Analyses and Variance Decomposition** 97
8.2.2 Bavaria: **Impulse Response Analyses and Variance Decomposition** 99
8.2.3 Nordrhein - Westfalen: **Impulse Response Analyses and Variance Decomposition** 101

9 Conclusion **104**

10 Appendix Part I **106**
10.1 Appendix A 107
10.1.1 Data 107
10.1.2 Solow Residual 108
10.2 Appendix B 111
10.2.1 First Order Conditions 111

10.2.2 FOC with Efficiency Conditions from Goods Production 114
10.2.3 Steady State Equations 115
10.3 Appendix C 116
10.3.1 Growth Rates 116
10.4 Appendix D 123
10.4.1 Solution Procedure 123
10.4.2 The Reduced System 126
10.4.3 Policy Functions for $\boldsymbol{\pi}_t$ 129
10.4.4 Policy Functions for $\mathbf{x}_{t+1}$ 130
10.4.5 Policy Functions for $\mathbf{u}_t$ 130
10.5 Appendix E 132
10.5.1 Matlab Code for the DSGE Model 132

Bibliography **147**

List of Figures

0.1 U.S. Business Investment (Non-RESI), Residental Investment (RESI) and House Prices (Ph) 6

1.1 Correlation GDP - RESI 10
1.2 Correlation RESI - Non-RESI 11
1.3 Correlation GDP - Ph 12

2.1 Convergence of k 35
2.2 Convergence of c 35

4.1 Impulse Responses I 74
4.2 Impulse Responses II 75

7.1 Germany - Plot of Commercial Property Prices (PP) against Bank Lending (CR), Investment in Construction (INV), GDP 86
7.2 Bavaria - Plot of Commercial Property Prices (PP) against Bank Lending (CR), Investment in Construction (INV), GDP. 87
7.3 NRW - Plot of Commercial Property Prices (PP) against Bank Lending (CR), Investment in Construction (INV), and GDP 87

8.1 Germany - Impulse Responses from SVAR(1) 98
8.2 Germany - Variance Decompositon from SVAR(1) 99
8.3 Bavaria - Impulse Responses from SVAR(1) 100
8.4 Bavaria - Variance Decompositon from SVAR(1) 100
8.5 NRW - Impulse Responses from SVAR(2) 102
8.6 NRW - Varaince Decomposition from SVAR(2) 103

Preface

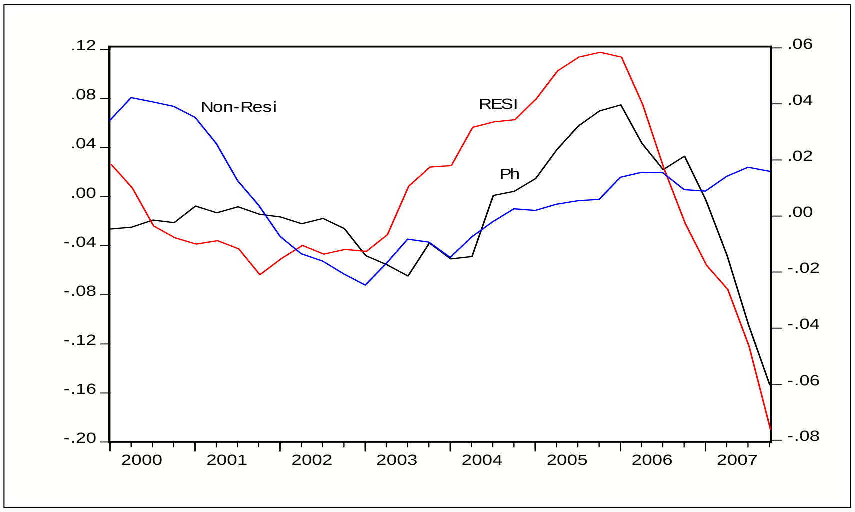
This thesis is motivated by the steadily increasing interest in the dynamic relationship between the macroeconomy and the real estate sector. One of the main issues in this respect is to study the investment dynamics in general and real estate investment dynamics in particular. The bursting of the U.S. housing bubble in 2006 is identified as the point of origin of the so called *subprime crises* which led to the collapse of the U.S. financial system and caused negative consequences for the entire global economy. The speculative bubble in the U.S. housing market, on the other hand, was the result of irrational public enthusiasm for housing investments. The accelerating growth in home prices made it attractive to build homes which lead to an investment boom into real estate. As can be seen in Figure 0.1 the growth of residential investment increased steadily since the end of 2003 until it peaked in the last quarter of 2005 where residential investment reached a share of 6.3% of the U.S. GDP, the highest levels since the 1950-51 housing boom.[1,2] An oversupply of new homes was the consequence of the investment boom in the housing sector. The originated disequilibrium in the real estate market brought the home price appreciation to an halt and prices started to decline in mid-2006. Price declines accelerated in 2007, reaching rates close to 20 percent at the end of 2008.[3] As a result, the boom in the U.S. home construction industry collapsed. The chronology of the ongoing crisis again indicates the relevance of the relationship between the housing market and the macroeconomy. Residential investment

[1]The data are real chain-weight values, logged and Hodrick-Prescott filtered. Both investment series are on the left hand scale, the price series is on the right hand scale. For details see the Data Appendix.

[2]see also Schiller (2008) or Krainer (2009)

[3]This rate is measured by the annual percentage change of the S&P/Case-Shiller Composite-20 US home price index, see also Fratianni and Marchionne (2009)

Figure 0.1: U.S. Business Investment (Non-RESI), Residental Investment (RESI) and House Prices (Ph)



is the main economic indicator for the quantity of new housing supplied.[4] As described in the U.S. national income and product accounts, residential investment consists of new construction put in place, expenditures on maintenance and home improvement, equipment purchased for use in residential structures and brokerage commissions. The quantity of residental investment in the U.S. economy is enormous. It has accounted for nearly 30% of gross private investment and approximately 5% of U.S. GDP in the period 1970 - 2008.

To investigate the dynamics of residential investment and its relationship to the overall economy, a dynamic stochastic general equilibrium (DSGE) model is introduced in which a consumption good sector and a housing sector are incorporated. As described in more detail in Part I, the model is brought to the data in order to evaluate whether it can account for stylized facts of the U.S. housing economy.

Another much talked real estate topic with respect to the current financial crisis is

[4]see also Krainer (2006)

the relationship between bank lending, property prices and economic activity. To that end, the second part of my thesis examines the potential effects of macro-policy and bank lending shocks on the German real estate sector. In particular, the importance of macroeconomic factors like credit to real estate construction, residential investment, and gross domestic product for the dynamics of German commercial real estate prices are analyzed. Since it is well known, that a single equation setup potentially suffer from simultaneity problems, a recursive vector autoregression (VAR) model is employed. The VAR estimation is conduct for both, aggregate Germany and the largest regional states of Bavaria and Nordrhein-Westfalen for the period 1975 to 2004.

Part I

Housing, Time-to-Build, and the Business Cycle

Chapter 1

Introduction

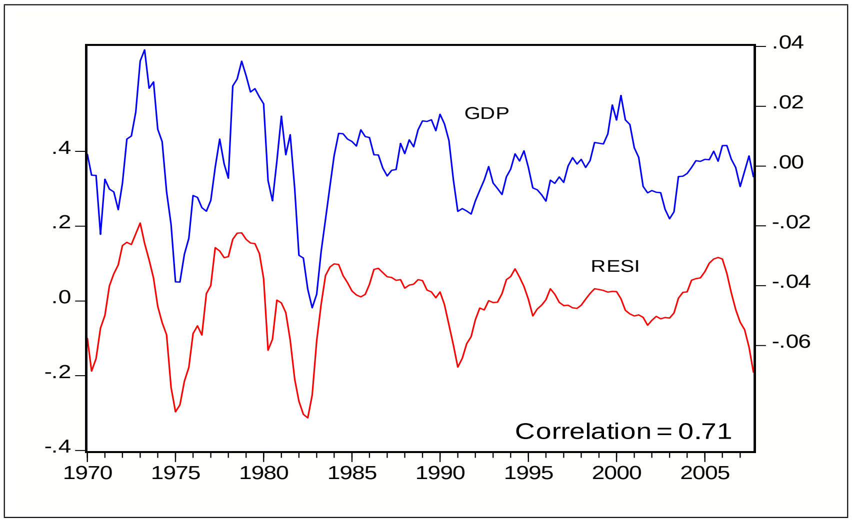
The breakdown of the U.S. financial system at the end of 2008 is leading an economic downturn with an expected contraction of the world economy of 2.7% in 2009, the worst global recession in 80 years.[1] The collapse of the U.S. real estate market is identified as the point of origin of the financial crisis which led to this unexpected dramatic consequences.[2] U.S. house prices have risen three times as fast as real income in the period 1999 - 2006, and have nearly doubled in nominal terms between 2002 - 2006. The halt of house price inflation in the mid-2006 together with increasing interest rates led to severe troubles then in the subprime mortgage market and the connected residential mortgage backed security sector. Rapidly increased mortgage default rates induced the Federal Reserve to pump enormous amounts of liquidity into the banking sector, which could however not prevent the collapse of Fannie Mae and Freddie Mac, the two mortgage giants in the U.S.[3]

[1] see also: The Economist (2009)

[2] Dean Baker (2002) is an exception in this respect. He predicted the collapse of the U.S. housing market and severe consequences for the world economy.

[3] see also e.g. The Economist (2007a,b, 2008)

Figure 1.1: Correlation GDP - RESI



These dramatic events once again demonstrate the close relationship between the real estate sector and the macroeconomy. Whether the house price bubble, the creation of complex, nontransparent assets, or the failure of rating agencies to properly evaluate the risk of such assets weights more heavily for the acceleration of the crisis is still an open issue[4].

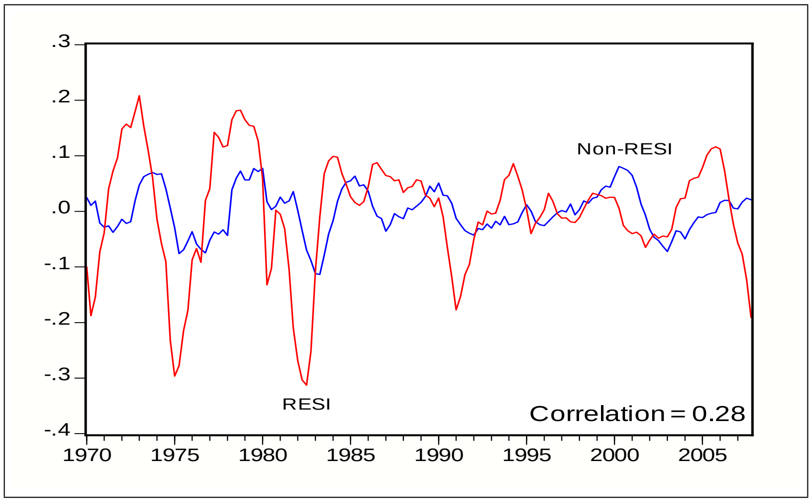
However, positive and high correlations between residential investment (RESI) and output (GDP) is robust and concrete. A critical analyses of the relationship between the housing market and the economy is essential in detecting the dynamic forces of the boom-bust cycles in the real estate sector as well as in understanding the sources of investment volatility. Moreover, since property investment is highly correlated with GDP, see also Figure 1.1, this should also help us to better understand the sources of investment volatility in other economic sectors as well[5].

To that end, I analyze the behavior of property investment and house prices

[4]see also Fisher and Quayyum (2006)

[5]The data are quarterly real data from NIPA tables which are logged and Hodrick-Prescott filtered. The left scale is for RESI and the right scale for GDP, for more details see the Data Appendix.

Figure 1.2: Correlation RESI - Non-RESI

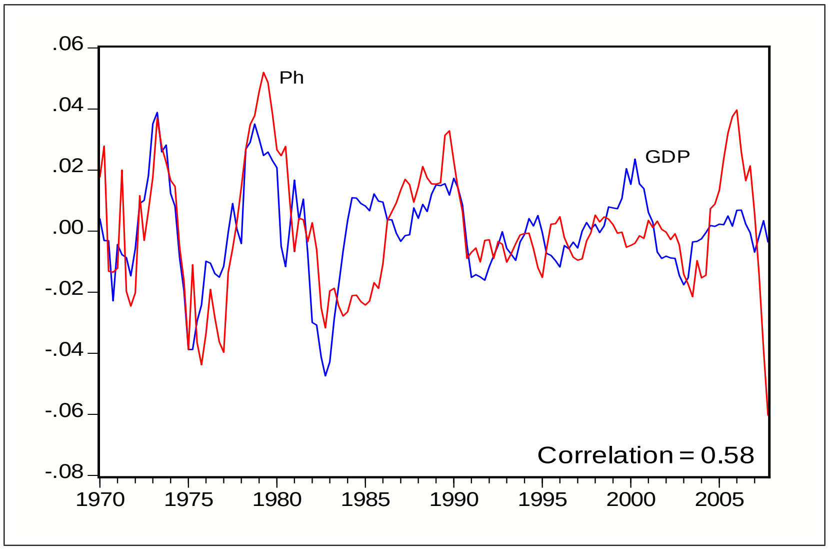


in this part of my thesis. The traditional framework of earlier works on investment and price volatility in the property sector is a partial equilibrium setup in which investment is determined by supply conditions (see e.g. Poterba (1984) or Topel and Rosen (1988)). Real estate developers invest more if prices are high. On the other hand, demand is assumed to be perfectly elastic since real estate assets must earn a return similar to the market return. A demand shock will affect future expected returns and move the demand curve along the supply schedule to a new equilibrium. A drawback of this approach is that it cannot account for the following stylized facts that are well known especially from U.S. data.

First, residential investment co-moves with investment in business capital (Non-RESI). In Figure 1.2 we find this co-movement for the time span 1970 to 2007. Second, residential investment is more than twice as volatile as nonresidential investment, a fact which can be observed in Figure 1.2 as well.

For residential investment we can measure a standard deviation from real, logged

Figure 1.3: Correlation GDP - Ph



and Hodrick-Prescott filtered U.S. data of 10.5 percent, whereas for nonresidential investment this standard deviation is only 4.1 percent. A third fact, also observable in the figures, is the lead of residential investment via business investment and via GDP. By comparing contemporaneous correlations and two period lead and lag correlations we can measure a 10 percent increase from 0.7 contemporaneous correlation of RESI and GDP to a correlation of 0.8 if RESI has a two quarter lead via GDP. On the other hand, if RESI has a two period lead to Non-RESI the correlation is increasing from 0.28 (the contemporaneous correlation) to 0.63.

And lastly, house prices (Ph) are procyclical and more volatile then output as shown in Figure 1.3. What we can also observe in this picture too is the bursting of the real estate bubble in 2006 after a sharp price increase the years ahead.

The objective of this part of my thesis is to build a neoclassical multisector stochastic growth model which is able to account for the business cycle dynamics of the U.S.

economy in general and for the stylized facts of residential investment and house prices in particular. In this model a consumption/capital investment good is produced in the first sector. The second sector produces residential structures which are combined with newly available land to establish the real estate good, i.e. houses. Final goods are produced with capital and labor rented from a representative household. The output of these goods is stochastic since final goods production is augmented by exogenous productivity shocks. On the demand side, consumption, housing services and leisure enter the households' utility function. A representative household maximizes his discounted lifetime utility by deciding each period how much to work and consume and how to spend savings. Savings can be invested in a new housing project or in physical capital.

Following Kydland and Prescott (1982), investment in a new housing project takes several periods (quarters) until it will contribute to the actual housing stock, that is, the household has to account for a time-to-build (or gestation) period in the real estate sector before he can occupy a new house. This time-to-build technology is also consistent with the empirical evidence that it takes around six month to finish a single unit house and around nine month for multi family houses on average in the U.S., see also Table 1.1

Table 1.1

Months of Construction for New Privately Owned Residental Building - Averages from 1971 - 2007

Buildings with 1 unit				Buildings with 2 units or more				
Total	Purpose of construction			Total	Number of units in building			
	Built for sale	Contractor-built	Owner-built		2 to 4	5 to 9	10 to 19	20 or more
6.1	5.5	5.6	9.4	9.5	7.6	9.0	9.7	12.3

Source: own calculations based on U.S. Census Bureau Construction Statistics data

In the consumption good sector investment adds to the existing capital stock after one period. This asymmetric time-to-build structure is motivated by the fact that capacities in the business sector can be adjusted without building new plants in the short run if there is a positive supply shock. My modeling is consistent with the available data on the supply side of the economy in matching length and frequency. I calibrate the model using aggregate U.S. data from the business and housing sector and produce artificial data from the numerical solutions of the model.

In the next section I give an overview on Real Business Cycle (RBC) models. After a short description of the evolution of this model class I present the prototype one-sector RBC model and the model equilibrium. Then I describe the equivalence of the centralized and decentralized model formulation and restrictions on preferences and technology to obtain balanced growth. Afterwards, technicalities like functional forms, steady states and convergence of variables towards equilibrium are considered. Since my model is dealing with more then one sector, I give a short overview on multisector RBC models in the second part of Chapter 2. In Chapter 3, I start with the presentation of my model economy. Subsequently, I define the recursive competitive equilibrium and describe the balanced growth path and the solution method used. Afterwards, data are described and the calibration procedure is presented. Results from numerical solutions of the model are presented in Chapter 4. Initially, steady state values of the model are compared with their real world equivalences. Afterwards, second moments obtained from the averages of 600 simulations are presented. Finally, impulse response functions of the model are considered to analyses the behavior of the model economy as reaction to a transitory technology shock. The last chapter concludes.

Chapter 2

Real Business Cycle Models

The point of departure for the so called Real Business Cycle research program is the interest in analyzing recurrent expansions and contractions (or business cycles) in aggregate economic activity due to shocks which are real - as opposed to "monetary" - in origin. In particular, the primary driving force is taken to be shocks to technology, rather then monetary and fiscal policy disturbances.

Business cycles where an active field of interest prior to the Great Depression. Economists such as Ragnar Frisch (1933), Eugen Slutsky (1937) or Joseph Schumpeter (1942) developed models able to produce short term fluctuations as a result of random shocks (Frisch and Schlutzky) or technological innovations (Schumpeter). However, the Keynesian revolution that followed the publication of Keynes' *General Theory (1936)* shifted away the basic interest from cycles. Instead, the explanation of economic forces determining real economic aggregates at a point in time, conditional an past economic history became the major research topic in mainstream macroeconomics.

Simultaneous with the appearance of Keynesian macroeconomics a renewed interest in understanding the long-run growth patterns of modern economies came along. Along with two works of Roy Harrod (1939) and Evsey Domar (1946), Robert Solow (1956) and Trevor Swan (1956) have developed what is now called the neoclassical growth model. In the Solow-Swan model three main sources of dynamic growth appear: population growth, productivity growth and capital formation. The main result of the aforementioned works is that technological progress is the key factor of long-run growth.

While the trend component of economic activity was explained by growth models, the cyclical component was analyzed with Keynesian models at the time. In this perspective, short-run fluctuations in output and employment are mainly driven by variations in aggregate demand, i.e. in investors' willingness to invest and the consumers' willingness to consume. Macroeconomic stabilization policy should then focus on the control of aggregate demand so as to fluctuations in economic output. However, a major drawback of this approach is the possible influence of economic policy on the relationships between macroeconomic variables. Estimated reduced-form relationships, the applied versions of Keynesian business cycle models, could not be expected to be robust to changes in policy regimes or in the macroeconomic environment.

This critique, initially formulated by Robert Lucas (1976), initiated a revival in equilibrium business cycle analyses. Lucas called for an alternative to the Keynesian paradigm in which macroeconomic policy should be analyzed on the base of an explicit microeconomic structure. Only by carefully modeling consumers and their preferences, firms and their technology, the information sets of agents, markets of interaction, and the like, would it be possible to derive robust conclusions regarding privat-sector responses to changes in economic policy. These so called *deep parameters* are not likely to be affected by changes in fiscal or monetary policy regimes or in the macroeconomic environment and hence, quantitative analyses based on microeconomic underpinnings is more robust to environmental or regime changes.

Finn Kydland and Edward Prescott (1982) provide the microfoundation approach to computation in macroeconomics. In a pathbreaking contribution, the authors integrate economic growth and business cycles into one framework. In their version of a neoclassical growth model, stochastic technological progress is assumed as the main source of short-run output variations, since they hypothesize that technological growth might be an important determinant, not only of long-run development.

Their approach allows the model economy, on the one hand, to grow on average at a constant exogenous rate. On the other hand, technology shocks induce aggregate economic variables to fluctuate around their long-run steady state growth path. In contrast to the Keynesian tradition they rely on Walrasian microeconomic mechanisms with prices, wages, and interest rates adjusting to clear markets. They thus argue that short periods of low output growth need not be a result of market failures, but could simply follow from temporarily slow growth in production technologies. Business cycles are not a disequilibrium outcome then but an optimal reaction in response to fluctuations in productivity growth.

Kydland and Prescott (1982) demonstrate not only the ability of dynamic general equilibrium models to account for business cycles, but that one can go way beyond the qualitative comparison of model properties with stylized facts, the dominant strategy in theoretical macroeconomics so far.[1] In their "quantitative approach" a model is formulated in terms of the deep parameters. These parameters necessary to solve the model numerically are drawn to match a subset of moments in the data. In particular, they choose parameter values to match certain long-run macroeconomic statistics, such as average postwar interest rates and average capital-output ratios, and microeconomic data to parametrize preferences. This *calibrated* model is used then to produce artificial data, i.e. allocation rules for the model are computed, using these policy functions, a large number of artificial time series - having the same length as the real world data set to be compared with - are computed.[2] Each simulation corresponds to a randomly generated series with disturbances drawn from a known distribution mirroring technological shocks in the model economy. To measure the models' accuracy, first and second moments of the simulated data are compared with moments from real data.

Kydland and Prescott's baseline model generates output fluctuations that amount to around 70 percent of those observed in the postwar U.S. data. However, their 1982 paper transformed the academic research on business cycles and initiated an extensive research program. Successively more sophisticated dynamic models of business cycles have been formulated and numerical analysis of economic models has evolved into a subfield of its own in economic research.[3]

[1] "Stylized facts" of economic growth, a term labeled by Nicholas Kaldor (1957) became the benchmarks of the theory of economic growth. Some of these "stylized facts" of growth are: (i) Real output grows at a more or less constant rate. (ii) The stock of real capital grows at a constant rate greater than the rate of growth of labor input. (iii) The growth rate of output and the stock of capital tend to be about the same.

[2] For a complete description on the numerical solution procedure see e.g. Hansen and Prescott (1995)

[3] For references and an overview , see Amman, Kendrick and Rust (1996). For an applied treatment on dynamic general equilibrium models see Heer and Maussner (2005)

2.1 The Prototype RBC Model

The standard real business cycle model is based upon an economy populated by identical infinitely-lived households and firms.[4] This economy is characterized by the absence of any frictions and by perfect competition in both, output and factor markets. Each of the households has an endowment of time for each period, t, which it must divide between leisure, L_t, and work, N_t. The households' time endowment is normalized to unity, that is $L_t + N_t = 1$. In addition, households own an initial stock of capital, K_0, which they rent to firms and may augment through investment.[5]

2.1.1 Households

Households in this economy are faced with a complex decision problem; given their initial capital stock, agents have to decide how much labor to supply and how much consumption and investment to purchase, i.e. a representative consumer chooses infinite sequences of consumption $\{C_t\}_{t=0}^{\infty}$, labor, $\{N_t\}_{t=0}^{\infty}$, and investment, $\{I_t\}_{t=0}^{\infty}$, in order to maximize expected lifetime utility

$$U[C(\cdot), N(\cdot)] = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u[C_t, 1 - N_t] \right\} \tag{2.1}$$

with $0 < \beta < 1$ the agents discount factor. $u(\cdot)$ is assumed to be strictly increasing, concave, twice continuously differentiable and to satisfy Inada-type conditions that ensure that the optimal solution for C_t and N_t is always (if feasible) interior[6]. Note also, that the utility is assumed to be time-separable; that is the choices of consumption and labor at time t do not

[4] To be precise, all households live on a continuum of mass 1.
[5] The basic model presented here is without government.
[6] For a specific definition of the Inada condtions see e.g. King et al. (1988)

affect the marginal utilities of consumption and leisure in any other time period. Agents maximize their utility subject to the following budget constraint

$$C_t + I_t \leq w_t N_t + r_t K_t \tag{2.2}$$

where w_t and r_t represents the real wage and the real rental rate of capital in units of output, respectively. It is assumed that consumers augment their stock of capital by investing some amount of real output each period, such that investment in period t produces productive capital in period $t+1$, i.e.

$$K_{t+1} = I_t + (1-\delta)K_t \tag{2.3}$$

where δ is the depreciation rate for capital.

2.1.2 Firms

Each period firms choose capital and labor to maximize profits

$$\max_{K_t, N_t \geq 0} \{Y_t - r_t K_t - w_t N_t\} \tag{2.4}$$

subject to a constant-returns-to-scale production function

$$Y_t = Z_t F(K_t, X_t N_t) \tag{2.5}$$

here Y_t is the level of output, Z_t is a random shock to total factor productivity, and X_t is *"trend"* growth restricted to be labor augmenting for reasons to be discussed below.[7] The effects of stochastic technology shocks are the basis for RBC Theory. There are several common ways of defining the stochastic process for technology. One is to assume that the

[7] The price of output is normalized to one.

logarithm of Z_t follows a first-order autoregressive process

$$\begin{aligned} \ln Z_t &= \rho \ln Z_{t-1} + \varepsilon_t \;\Leftrightarrow\; Z_t = Z_{t-1}^{\rho} e^{\varepsilon_t}, \\ 0 &< \rho < 1 \end{aligned} \tag{2.6}$$

with ε_t an exogenous *i.i.d.*[8], standard normally distributed disturbance term with standard deviation σ_ε, and ρ the measure of persistence for the process.

2.1.3 Equilibrium and First Order Necessary Conditions

To describe an equilibrium in this economy the recursive competitive equilibrium concept as first proposed by Prescott and Mehra (1980) has proven very useful for this class of models. Firms and households are seen as decision making entities where firms have to solve a static maximization problem and individual households a dynamic infinite horizon maximization problem. Since utility is assumed to be time-seperable the nature of the households problem is the same every period and one can solve their infinite horizon problem by utilizing its recursive structure: given the beginning-of-period capital stock and the current productivity shock, choose consumption, labor and investment.

In this decentralized setting we distinguish between individual variables and aggregate variables, the later denoted by a underline bar.

The state variables for the household are $S_t = (Z_t, K_t, \underline{K}_t)$, and the aggregate state variables are $\underline{S}_t = (Z_t, \underline{K}_t)$. In equilibrium, we know that K_t and $\underline{K}_t$ will have to be equal, otherwise the representative consumer would not be representative. But this equality cannot be imposed on the consumer; prices have to move to make this equality desirable to

[8]Independent and identically-distributed

him.

The recursive structure of the households problem fits naturally into the dynamic programming approach which is used extensively to solve this kind of optimization problems.[9] Using the established notation the maximization problem of the household can be stated as the following dynamic programming problem:

$$
\begin{aligned}
V(Z_t, K_t, \underline{K}_t) &= \max_{C_t, N_t, I_t} \{u(C_t, 1-N_t) + \beta E_t[V(Z_{t+1}, K_{t+1}, \underline{K}_{t+1})|Z_t]\} \\
s.t. \quad C_t + I_t &\leq r_t(Z_t, \underline{K}_t)K_t + w_t(Z_t, \underline{K}_t)N_t, \\
K_{t+1} &= I_t + (1-\delta)K_t \\
\underline{K}_{t+1} &= \underline{I}_t(Z_t, \underline{K}_t) + (1-\delta)\underline{K}_t \\
Z_t &= Z_{t-1}^{\rho} e^{\varepsilon_t} \\
C_t &\geq 0, 0 \leq N_t \leq 1
\end{aligned} \tag{2.7}
$$

The value function $V(Z_t, K_t, \underline{K}_t)$ is the households maximum obtainable expected return over all feasible plans in this economy.

A *recursive competitive equilibrium* for this economy then consists of

(i) factor price functions $w_t(Z_t, \underline{K}_t)$ and $r_t(Z_t, \underline{K}_t)$;

(ii) a set of decision rule for households, $C_t(Z_t, K_t, \underline{K}_t)$, $N_t(Z_t, K_t, \underline{K}_t)$, and $I_t(Z_t, K_t, \underline{K}_t)$;

(iii) a corresponding set of aggregate per capita decision rules, $\underline{C}_t(Z_t, \underline{K}_t)$, $\underline{N}_t(Z_t, \underline{K}_t)$, and $\underline{I}_t(Z_t, \underline{K}_t)$,

[9] An extensive treatment of this equilibrium concept can be found in Stocky and Lucas (1989). A excelent introduction to dynamic programming is from Adda and Cooper (2003).

(iv) and a value function $V(Z_t, K_t, \underline{K}_t)$;

such that

(a) firms solve problem (2.4) subject to (2.5);

(b) households solve problem (2.7);

(c) the consistency of individual and aggregate decisions is given, that is

$C_t(Z_t, K_t, \underline{K}_t)) = \underline{C}_t(Z_t, \underline{K}_t),\ N_t(Z_t, K_t, \underline{K}_t) = \underline{N}_t(Z_t, \underline{K}_t)$, and $I_t(Z_t, K_t, \underline{K}_t) = \underline{I}_t(Z_t, \underline{K}_t)$, $\forall\ t$;

(d) and the aggregate resource constraint, $\underline{C}_t(Z_t, \underline{K}_t)+\underline{I}_t(Z_t, \underline{K}_t) = \underline{Y}_t(Z_t, \underline{K}_t)$, is satisfied $\forall\ t$.

A solution to maximization problem (2.7) must satisfy the following necessary conditions and resource constraint:

$$u_N(C_t, 1-N_t) = u_C(C_t, 1-N_t)w_t \tag{2.8}$$

$$u_C(C_t, 1-N_t) = \beta E_t\{u_C(C_{t+1}, 1-N_{t+1})[r_{t+1} + (1-\delta)]\} \tag{2.9}$$

$$C_t + I_t = r_t K_t + w_t N_t, \tag{2.10}$$

u_A and F_A represents the partial derivatives of u and F with respect to variable A.[10] Equation (2.8) is also known as the intratemporal efficiency condition and (2.9) as the intertemporal efficiency condition.

[10] $I_t = K_{t+1} - (1-\delta)K_t$ is substituted in the resource constraint. The choice variable I_t is replaced by K_{t+1}.

The optimization problem of firms yield the following first order necessary conditions:

$$r_t = Z_t F_K(K_t, X_t N_t) \tag{2.11}$$

$$w_t = Z_t X_t F_N(K_t, X_t N_t) \tag{2.12}$$

i.e. factor prices (stated in terms of output) are equal to the marginal products of factor inputs. Given constant returns to scale in production, in equilibrium, profits of firms are clearly equal to zero.

Under most specifications of preferences and production functions the set of efficiency conditions 2.8 to 2.10 can not be solved analytically[11]. Consequently, one has to work with approximation procedures. These procedures typically results in policy rules that are linear in the state variables K_t and Z_t. The first step towards an approximate solution is to choose points to approximate around. The natural choice is the set of points where the system is in long-run equilibrium - the stable steady state values.

The second step is to express the first order conditions in terms of percentage deviations from the steady state values and then take a linear approximation to each condition. This results in a set of linear difference equations in percentage deviations from the steady state. There are related ways to state and solve such systems. Two of the classical methods are from Blanchard and Kahn (1980) - using a Jordan decomposition - and King and Watson (1998), (2002) - using a Generalized Schur decomposition[12].

[11]There are only few exceptions, one is a log-linear utility function and full depriciation of capital each period. The analytical solution for this case is shown below.

[12]Other classical references are Farmer (1993), Uhlig (1999), or Klein (2000).

2.1.4 The RBC Model Formulated as a Ramsey Problem

The decentralized economy presented above is characterized by perfect competition and is free of frictions. It is straightforward for such a setting to develop a Ramsey (or social planner) problem with an equivalent equilibrium outcome in terms of Pareto efficiency. In the static theory of general equilibrium with a finite dimensional commodity space the correspondence between a competitive equilibrium and a Pareto efficient allocation of resources is stated in the Two Fundamental Theorems of Welfare Economics.[13] Our infinite horizon model has infinitely many commodities. Nevertheless, as shown by Debreu (1954), the Pareto optimum as characterized by the optimal infinite sequences for consumption, labor, and capital will be identical to that in a competitive equilibrium. Furthermore, factor prices are determined by the marginal products of capital and labor evaluated at the equilibrium quantities[14].

To determine the Pareto optimum, the RBC model from the last section is recast as the following Ramsey model:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t u[C_t, 1 - N_t] \right]$$

$$\begin{aligned} \text{subject to} &: \\ C_t + I_t &= Z_t F(K_t, X_t N_t) = Y_t \qquad (2.13) \\ K_{t+1} &= I_t + (1-\delta) K_t \\ Z_t &= Z_{t-1}^{\rho} e^{\varepsilon_t} \end{aligned}$$

[13]see e.g. Mas-Colell et al. (1995)

[14]For a detailed treatment of the relationship between Pareto optimum and competitive equilibrium for RBC models, see e.g. Cooley and Prescott (1995).

with K_0 given and ε_t having the same properties as in equation 2.6. Since, by assumption a benevolent social planner is choosing allocations we can set up the following value function

$$\begin{aligned} V(Z_t, K_t) &= \max_{C_t, N_t, K_{t+1}} \{u(C_t, 1-N_t) + \beta E_t[V(Z_{t+1}, K_{t+1})]\} \\ s.t.\ \ C_t + K_{t+1} &\leq Z_t F(K_t, X_t N_t) + (1-\delta) K_t \\ Z_t &= Z_{t-1}^{\rho} e^{\varepsilon_t} \end{aligned} \tag{2.14}$$

Note that investment has been eliminated again by using the law of motion for the capital stock and the control variable I_t is replaced by K_{t+1}. A solution to this problem must satisfy the following necessary conditions and resource constraint:

$$u_N(C_t, 1-N_t) = u_C(C_t, 1-N_t) Z_t X_t F_N(K_t, X_t N_t) \tag{2.15}$$

$$\begin{aligned} u_C(C_t, 1-N_t) &= \beta E_t\{u_C(C_{t+1}, 1-N_{t+1})[Z_{t+1} F_K(K_{t+1}, X_{t+1} N_{t+1}) \\ &\quad +(1-\delta)]\} \end{aligned} \tag{2.16}$$

$$K_{t+1} = Z_t F(K_{t,} X_t N_t) + K_t(1-\delta) - C_t \tag{2.17}$$

Condition (2.15) represents the consumption - leisure tradeoff of the representative consumer, the intra-temporal efficiency condition. It implies that the marginal rate of substitution between labor and consumption must equal the marginal product of labor. A positive productivity shock (represented in Z_t) has two effects here. A substitution effect, i.e. a higher wage increases the incentive to work and thus leisure $(1-N)$ will decrease. And a wealth effect, that is, if people feel richer, they want to consume more of both, leisure and the consumption good. So consumption will increase and labor will decrease. The net effect of a shock therefore depends on both, the elasticity of substitution between labor and consumption and on the persistence of the shock.

Condition (2.16) represents the consumption-saving tradeoff, an inter-temporal efficiency condition. This Euler equation tells us for an equilibrium that marginal cost in terms of utility in investing in more capital (i.e. consuming less) must be equal the expected marginal utility gain. In other words, it gives the rate at which the consumer is willing to forego consumption in period t for consumption one period ahead. Condition (2.17) just states that the resource constraint has to be satisfied.

To see the relation between Pareto efficiency and intertemporal equilibrium we just have to consider the first-order condition of both, the centralized and the decentralized model. In the decentralized economy the factor market equilibrium conditions are given by

$$\begin{aligned} r_t &= Z_t F_K(K_t, X_t N_t) \\ w_t &= Z_t X_t F_N(K_t, X_t N_t) \end{aligned}$$

Using these conditions to substitute for w_t and r_{t+1} and applying the Euler theorem to $F(\cdot)$,

$$Y_t = Z_t F(K_t, X_t N_t) = Z_t F_K(K_t, X_t N_t) K_t + Z_t X_t F_N(K_t, X_t N_t) N_t$$

equations (2.8) to (2.10) reduce to the efficiency conditions of the Ramsey problem, equations (2.15) to (2.17). Furthermore, since households live on a continuum of mass 1 and the production technology is linear homogenous, it is guaranteed that there exists both, an aggregate production function and a representative household. Thus, a benevolent social planer who solved the Ramsey problem (2.13) could implement this solution in terms of a competitive equilibrium. On the other hand, the equilibrium allocations of the decentralized economy are optimal in the sense that it maximizes the utility of all households given the resource constraint of the economy.

2.1.5 Balanced Growth and Restrictions on Technology and Preferences

The distinguishing features of the basic RBC model - in contrast to the neoclassical growth model - are the household's labor-leisure choice and the presence of shocks to technology. These features were added to address a yet simple but fundamental question: does a model designed to be consistent with long-term economic growth produce the sort of fluctuations that we associate with the business cycle?

To answer this question we have to restrict the artificial economy described on certain dimensions. Variables like output per capita, investment per capita and consumption per capita all exhibit (roughly) constant growth rates over time in most industrialized countries. This fact is taken as evidence of balanced growth[15]. The concept of a balanced growth path is the counterpart to a stationary equilibrium in a deterministic Ramsey setting for stochastic growth models. It is a growth path where the growth rate of capital, output and consumption are constant.

Thus, the first dimension to restrict our artificial economy is the growth pattern, i.e. to have constant - but possibly different - growth rates on certain key variables. To achieve this goal, additional restrictions on preferences and technologies are required. Purely labor augmenting technological progress is the key restriction in the production sector. Using the common Cobb-Douglas technology this restriction is stated as

$$\begin{aligned} Y_t &= Z_t F(K_t, X_t N_t) \\ &= Z_t K_t^{\alpha} (X_t N_t)^{1-\alpha} \end{aligned} \tag{2.18}$$

with α referred to as the capital's share in output[16]. The production function (2.18) together

[15] see e.g. Cooley and Prescott (1995)

[16] That is, because if capital is paid its marginal product, it will earn that fraction in output since,

with the accumulation equation (2.3) and the resource constraint (2.2) then imply a constant growth rate of output, consumption, capital and investment which is equal to the growth rate of the labor augmenting technical progress

$$g_Y = g_C = g_K = g_I = g_X \tag{2.19}$$

with g_A denoting one plus the growth rate of a variable A (i.e. A_{t+1}/A_t). Since hours devoted to work N are bounded by the endowment of time, it cannot grow in equilibrium and the growth rate has to be zero or

$$g_N = 1. \tag{2.20}$$

The restriction to labor augmenting technological progress is not sufficient to guarantee the existence of a balanced growth path when labor supply is endogenous. Also equation (2.19) and (2.20) describe the technologically feasible growth rates for a balanced growth path, they will never be an equilibrium outcome if they do not fit with the efficiency conditions of the representative agents.

To insure compatibility with these conditions the following restrictions have to be imposed on preferences: (i) the one-period utility function u has to be restricted to the constant-elasticity functions with respect to consumption; (ii) the income and substitution effects with respect to the static labor supply decision must be exactly offsetting[17].

Since consumption, output, and capital all grow at a constant rate on the balanced growth path we know from equation (2.9) that the growth rate of the marginal utility, $\frac{u_{1,t+1}}{u_{1,t}}$, is constant on that path. Therefore, the intertemporal elasticity of substitution must be

$\alpha\frac{Y}{K} = r \Rightarrow \alpha Y = rK.$

[17] see also Heer and Maussner (2005)

constant and independent of the level of consumption[18].

The second condition is required since $g_N = 1$ in equilibrium, but the marginal product of labor increases in the long-run at rate g_Z. Thus, income and substitution effects of productivity growth must be exactly offsetting effects on labor supply.[19] Following these conditions the momentary utility function has to be restricted to the following class of admissible utility functions:

$$u(C, 1-N) = \begin{cases} C^{1-\sigma} v(1-N) \text{ if } \sigma \neq 1, \\ \ln C + v(1-N) \text{ if } \sigma = 1 \end{cases} \tag{2.21}$$

with $1/\sigma$ the constant intertemporal elasticity of substitution in consumption. The function v must be chosen such that $u(C, 1-N)$ is concave[20].

2.1.6 Stationary Economies and Functional Forms

Given the restrictions on technology and preferences it is possible now to transform the economy into a stationary one. The growth rates are used to take transformations of all variables in the model such that the transformed variables exhibit no trends. This is a standard procedure in the RBC literature to make the local dynamics around the steady state of the model economy more amenable to an analysis[21]. Also for computational purposes it is more convenient to work with stationary variables. Since all variables (except N_t) in the neoclassical model grow at the same rate as X_t in equilibrium, the transformation

[18] This can be seen dirctly by taking the differential of $\frac{u_{1,t+1}}{u_{1,t}}$ on the balance growth path with respect to c_0, and c_0 an arbitrary constant substituted for $c_t = c_o g_c^t$ in the growth rate of marginal utility.

[19] Also empirical observations show that time devoted to work is rather constant and real wages increased in the postwar period.

[20] Note that for $\sigma = 1$, and $v(1-N) = A\ln(1-N)$ we obtain the specification used by Hansen (1985) without indivisible labor and A a weight for leisure in the utility function. For $u(C, 1-N)$ to be concace v must be choosen such that $u_{ii} \leq 0$, and $(u_{11}u_{22} - u_{12}^2) \geq 0$.

[21] see e.g. King et al. (1988)

can be achieved by deflating these variables by X_t. By denoting transformed variables by lower case letters we get

$$y = Y/X,\; c = C/X,\; k = K/X,\; i = I/X$$

The economy with transformed variables is identical to an economy in which technological progress is absent and growth rates are zero in the steady state, with two exceptions. Since capital accumulation is expressed in difference equation form, this relation is altered as follows. In the non-transformed economy we have

$$K_{t+1} = I_t + (1-\delta)K_t = Z_t F(K_t, X_t N_t) - C_t + (1-\delta)K_t$$

and the transformation by division of X_t gives

$$g_X k_{t+1} = i_t + (1-\delta)k_t = Z_t F(k_t, N_t) - c_t + (1-\delta)k_t \tag{2.22}$$

The second relation to be altered potentially is the effective rate of time preference. That can be seen in the transformed lifetime utility

$$\begin{aligned} U &= \sum_{t=0}^{\infty} \beta^t u(C_t, 1-N_t) = \sum_{t=0}^{\infty} \beta^t u(c_t X_t, L_t) \\ &= \begin{cases} \left(X_0^{1-\sigma}\right) \sum_{t=0}^{\infty} (\beta^*)^t [(c_t^{1-\sigma} v(L_t)] & \text{for } \sigma \neq 1 \\ \\ \sum_{t=0}^{\infty} (\beta^*)^t [\log(c_t) + \log(X_t) + v(L_t)] & \text{for } \sigma = 1 \end{cases} \end{aligned} \tag{2.23}$$

with $\beta^* = \beta(g_X)^{1-\sigma}$, and $\beta^* < 1$ is required to guarantee finiteness of lifetime utility. Since the terms $\left(X_0^{1-\sigma}\right)$ and $\sum_{t=0}^{\infty} (\beta^*)^t \log(X_t)$ do not affect the preference orderings we can make $\sum_{t=0}^{\infty} (\beta^*)^t u[(c_t, L_t]$ the objective by suitable selection of X_0[22].

[22]e.g. one may set $X_0 = 1$ or $\sum_{t=0}^{\infty} (\beta^*)^t [\log(X_t)] = 0$ in the preceding expressions, see also King et al. (2002)

2.1.7 An Analytical Solution

Given functional forms for production technology and consumer preferences, it is possible to solve the households problem for the time path of the control variables, c_t, k_{t+1}, N_t. Since our focus is on business cycles we concentrate on shocks to technology and neglect productivity growth for now, i.e. we set $g_X = 1$. To obtain analytical solutions we will assume full depreciation of capital within a single period (i.e. $\delta = 1$), and use the following functional forms for production and consumer preferences

$$\begin{aligned} y_t &= Z_t F(k_t, N_t) = Z_t k_t^{\alpha} N_t^{(1-\alpha)} \\ u(c_t, 1 - N_t) &= \ln c_t + A(1 - N_t) \end{aligned}$$

These assumptions leads to the subsequent first order necessary conditions

$$c_t = \left[\frac{(1-\alpha)}{A} Z_t \left(\frac{k_t}{N_t} \right)^{\alpha} \right] \tag{2.24}$$

$$\frac{1}{c_t} = \beta E_t \left[\frac{\alpha}{c_{t+1}} Z_{t+1} \left(\frac{N_{t+1}}{k_{t+1}} \right)^{(1-\alpha)} \right] \tag{2.25}$$

$$c_t + k_{t+1} = Z_t k_t^{\alpha} N_t^{(1-\alpha)} \tag{2.26}$$

To obtain solution equations for this special system one has to note that with a utility function of the form chosen and complete depreciation, the income and substitution effects of a wage rate change will just offset each other. Thus, the leisure choice will be unaffected and consequently it is reasonable to conjecture that N_t will be a constant in the solution, i.e. $N_t = N$. Another guess, based on the manner in which Z_t and k_t enter the production function, is that c_t and k_{t+1} are proportional to the product $Z_t k_t^{\alpha}$. Both guesses are verified, namely N is a constant in equilibrium given by

$$N = \frac{(1-\alpha)}{(1-\alpha\beta)A} \tag{2.27}$$

and the solution equations for c_t and k_{t+1} are given by[23]

$$c_t = (1-\alpha\beta)Z_t k_t^{\alpha} N^{(1-\alpha)} \tag{2.28}$$

$$k_{t+1} = \alpha\beta Z_t k_t^{\alpha} N^{(1-\alpha)} \tag{2.29}$$

The solutions path for c_t, and k_{t+1} as functions of the models states, k_t and Z_t, are optimal decision rules in the sense that they satisfy the efficiency conditions of this model economy.

The core message of the policy functions is that temporary shocks to Z_t not only effects current consumption, but also alters the next period capital stock, which propagates the effects of the disturbance. That is, a positive shock to productivity will lead to an increase in k_t and c_t for several periods, causing the model to produce cyclical patterns. Furthermore, if the disturbances are given by a first order autoregressive (AR(1)) process, as assumed in (2.6), then, consumption and the capital stock will follow AR(2) processes. As noted by McCallum (1989, p. 23), this is a significant result since detrended quarterly time series of various macroeconomic variables are well described by AR(2) processes for the U.S. economy. However, a drawback of the simplification to obtain analytical solutions is the constancy of N, and hence the neutrality of labour via productivity shocks. This is of course not what we observe in the data.

2.1.8 Steady States and Convergence

A steady state equilibrium for this economy is one in which the technology shock is assumed to be constant, i.e. $Z_t = 1$ for all t so that there is no more uncertainty in the

[23] This strategy to find solutions is called the *method of guess and verify* or *method of undetermined coefficients*. For further details see e.g. McCallum (1989).

system, and the values of capital, labor and consumption are constant, i.e. $k_t = k, \;\; N_t = N$ and $c_t = c$ for all t[24]. Imposing these steady state conditions in system (2.24) to (2.26), the steady state values are found by solving the following steady state equilibrium conditions:

$$c = \left[\frac{(1-\alpha)}{A}\left(\frac{k_t}{N_t}\right)^{\alpha}\right] \tag{2.30}$$

$$1 = \beta\alpha\left(\frac{N}{k}\right)^{(1-\alpha)} \tag{2.31}$$

$$c + k = k^{\alpha}N^{(1-\alpha)} \tag{2.32}$$

From this equation system we obtain the following steady state equilibrium values:

$$\begin{aligned} k &= (\alpha\beta)^{\frac{1}{1-\alpha}}\frac{1-\alpha}{(1-\alpha\beta)A} \\ c &= \frac{1-\alpha}{A}(\alpha\beta)^{\frac{\alpha}{1-\alpha}} \\ N &= \frac{(1-\alpha)}{(1-\alpha\beta)A} \end{aligned}$$

The stability conditions of the positive steady states can be seen well from the deterministic versions of the policy functions (2.28) and (2.29)

$$c_t = g(k_t) = (1-\alpha\beta)k_t^{\alpha}N^{(1-\alpha)} \tag{2.33}$$

$$k_{t+1} = f(k_t) = \alpha\beta k_t^{\alpha}N^{(1-\alpha)} \tag{2.34}$$

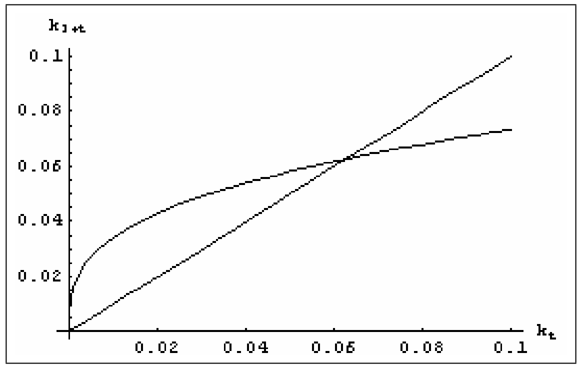
and are shown in Figure 2.1 and 2.2, respectively. Notice in Figure 2.1 that between 0 and the positive steady state $k = 0.0629$, the function $k_{t+1} = f(k_t)$ is above the 45 degree line, so that k_{t+1} is greater then k_t[25].

In this range, capital is growing and converges to the positive k. Above the positive steady-state, the value of the function, $f(k_t)$, is less then k, so that capital stock declines,

[24]For labor we know allready that it is constant in equilibrium, and of course, we could have obtained the steady-state values shown below from our solution equations. However, I like to demonstrate the procedure in general here and not only for the pedagogical relevant analytical solutions.

[25]The steady state values are calculated for parameter values, $\alpha = \frac{1}{3}, \;\; \beta = 0.99, A = 3$.

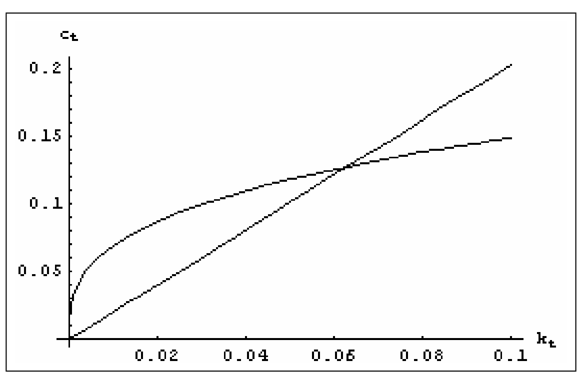
Figure 2.1: Convergence of k



converging to the positive k. Thus, regardless of where k_t starts, it converges to the steady-state k.

For consumption the story is somehow similar. In Figure 2.2, between 0 and the steady-state of k, the function $c_t = g(k_t)$ is above the 45 degree line, so that consumption is growing and converging to the positive steady state of k which is in that point equal to the positive steady state of $c = 0.1277$. Above the positive steady state, consumption is decreasing, converging to its deterministic equilibrium value in which $c = 2.0432 * k$.

Figure 2.2: Convergence of c



2.1.9 Numerical Solution and Calibration

As mentioned already, there is a limited amount of cases in which RBC models admit analytical solutions. Therefore, one has to work with numerical solutions. The approach frequently used to obtain numerical results is to take linear approximations of efficiency conditions (2.8) to (2.10) around stationary points[26]. The natural choice for that points are the one where the system is in long-run equilibrium - the steady state values as defined above. The linearization procedure most popular is a Taylor series expansion. The linear system obtained can then be solved for policy functions of the endogenous variables.

The next step towards a numerical solution is to assign specific values for the parameters of the model economy. This is done through *calibration*, which requires more economic theory and time series of real world data. The goal is to set the parameter values such that the steady-state behavior of the model is consistent with the long-run characteristics of economic aggregates and prices. In the given context, one has to choose values for the set of parameters $\{\alpha, \beta, \theta, \delta, \rho\}$ such that the model economy mimics the actual economy on the dimensions associated with long term growth[27].

Having assigned values to the deep parameters one then generates a set of artificial time series from the model. This involves generating many different series of values of disturbances to technology. Once the stochastic process for productivity is chosen, an AR(1) process in our basic model, this job is done by the random number generator of the solution software. One then feeds these series of normally distributed random numbers into the

[26] For alternative solution methods on this model class see e.g. Heer and Maussner (2005) or DeJong with Dave (2007). For a comparsion of various solution methods see e.g. Aruoba et al. (2006).

[27] An extensive describtion on how to calibrate various parameter values from U.S. data is in Cooley and Prescott (1995)

model to yield samples of artificial times series for the state variables and control variables of the model economy. One way to judge the performance of the model is to compare sample moments of artificial model-generated data with those of a real world actual data set. Typically, the focus of interest are the second moments of main economic aggregates as well as comovements of these series with output.

As mentioned above, Kydland and Prescott's (1982) contribution in advancing this methodology, was a trigger point for a new direction in applied dynamic macroeconomics. They used a prototype RBC model which departs from the basic setup in two important ways. First, they introduced a time-to-build restriction for the accumulation of capital. For the share of output which is not consumed but invested to accumulate, new capital is getting productive with a four period (one year) lag. An investment project, so to say, takes four quarters to be finished, and the costs are spread out evenly over this period[28]. An assumption which increases the dimension of the state space and creates larger persistence to the effects of technology shocks.

Second, they assume higher current utility flows of leisure the harder an agent has worked in the past. Using a non-time-separable utility function they consequently obtain a greater intertemporal substitutability of labor without altering the assumed intertemporal substitutability of consumption - *"something which is needed to explain aggregate movements in employment in an equilibrium model"* (Kydland and Prescott, 1982, p. 1351). In addition, they included inventories as a factor of production. This improves the match of the model's series correlation and allowed them to solve the model by linear quadratic

[28]The costs have not to be evenly spreded, Christiano and Todd (1996), for example, assume a lengthy planning phase, and the overwhelming part of the project's costs are spent in the construction phase.

approximation.

Due to these extensions, the propagation mechanism of shocks in Kydland and Prescott's (1982) model is much more pronounced then in the basic setup. Suppose a positive technology shock occurs, this will increase the current productivity of capital and labor. The current period becomes more attractive to work and produce, relative to conditions that are expected in future periods, so both, employment and output rise. It also *may* signal high productivity in subsequent periods. This will induce firms to initiate investment projects now. The projects started will increase employment and output until they are completed several periods later and this spreads the effects of the shock forward into the future. This will remain true even if it turns out for the productivity increase to be transient, since the investment decision is modeled to be bounded as equally sized tranches over the entire time-to-build period. Since the capital stock is - possible inappropriately - increased and workers will be less willing to supply labor in future periods due to extensions in the boom time, the contingency of a future downturn is already inherent in the investment decision.

For illustrative purpose, Table 2.1 reproduces some simulation results from that paper. In the table, sample moments with moments implied by the estimated model for deviations of the indicated series around a fitted trend are compared. In the simulation underlying these results, the variance of technology shocks was chosen so as to make the standard deviations (around trend) of real output for the model equal to its value for the post-war U.S. economy. Table 2.1 shows that the model captures the fact that investment is much more volatile then output and consumption is less volatile then output. However, like many RBC models it can not replicate the cyclical behavior of the employment series so well.

Table 2.1

Kydland and Prescott's (1982) Model				
	U.S. Data*		Model	
	Std**	Corr***	Std	Corr
Real Output	1.80		1.80	
Consumption	1.30	0.74	0.63	0.94
Investment	3.10	0.71	6.45	0.80
Hours	2.00	0.85	1.05	0.93

* Quarterly data from 1950:1 - 1979:2, loged and detrended using the Hodrick-Prescott filter
** Std is the standard deviation in percentages
*** Corr shows correlations with output

2.2 Multi-Sector RBC Models

Since the implementation of the early RBC models various modifications of the basic setup where introduced to better match business cycle stylized facts. Some of the more prominent examples in the context of a homogenous output are: the introduction of indivisible labor as in Hansen (1985), introducing money as initially done by Cooley and Hansen (1989), the modeling of government consumption shocks as in Christiano and Eichenbaum (1992), to consider heterogenous agents as in Ríos-Rull (1995), to treat product markets as imperfect as in Rotemberg and Woodford (1995), or to include habit persistence of consumers in order to account for the *equity premium puzzle,* as in Boldrin et al. (2001)[29].

However, in this section I like to consider multi-sector extensions of the basic RBC model. Two of the most important empirical shortcomings of one-sector models are: First, they do not contain a strong enough endogenous mechanism to propagate shocks over time. For the model to produce sustained fluctuations in output, consumption, investment, etc., the shocks to the model must themselves be sustained over time[30]. Second, the one-sector

[29]For a comprehensive survey on extensions of the basic RBC model see e.g. Stadler (1994), Cooley (1995), or Gaggl and Steindl (2007).

[30]This fact was pointed out, for example by Cogley and Nason (1995), and Baxter (1996).

model is unable to produce realistic comovement in output, investment and labor supply across different sectors of the economy.

The pioneering paper in this context is from Long and Plosser (1983). The authors propose a multi-sector model that exhibits strong sectoral comovement via a production structure in which any given commodity may be used as an input in the production of the other commodities, and production of any one commodity requires positive inputs of other commodities. Long and Plosser obtain an elegant analytical solution to their model by assuming full depreciation of capital every period and a log - linear momentary utility function.

The policy functions obtained from their analytical solution indicates that a positive productivity shock in one sector, leading to an unexpected high output of the commodity in this sector at a certain point in time, implies that all productive inputs in this sector will also be unexpected high. By the assumption of alternative employments for that commodity, this not only propagates the output shock forward in time, it also spreads the future effects of the shock across sectors. That is, the induced production possibilities hypothesis lead to co-movement in consumption, input and output series and to sustained fluctuations without highly autocorrelated shocks. However, many properties of the model do not generalize once moved away form the simplifying assumptions to obtain an analytical solution.

Two other examples with focus on sectoral comovements are from Baxter (1996) and from Hornstein and Praschnik (1997). Baxter investigates a two-sector, two-factor equilibrium model of a closed economy in which sector one produces a nondurable, pure consumption good and sector two produces a consumer durable as well as the capital good used in both sectors. The factors of production are homogenous labor which is perfectly mobile across sectors and capital which is subject to convex adjustment costs.

Baxter's simulation results indicate an internal propagation mechanism which is no stronger then in a one-sector model though the model does a good job in replicating the comovement of main aggregates with output and the positive cross-sectoral correlations of output, investment, hours and consumption.

Hornstein and Praschnik (1997) are modeling a two-sector model similar to Baxter's (1996), but without adjustment costs for capital. The linkage of the two sectors in their framework stems from the potential for nondurable goods to be employed as an intermediate input in durable goods production. Their model economy replicates the quantitative pattern of relative volatilities, and it displays high correlations of variables with GDP on most dimensions. Furthermore, they are successful in producing observed cross-sectoral correlations for the U.S. economy with two exceptions - business investment and labor productivity.

A contribution dealing with the afore mentioned lack of internal propagation of shocks is published by Benhabib, Perli and Sakellaris (2006). The propagation behavior of two multi-sector models, a one-capital-good, two-sector model, and a two-capital-goods, three-sector model are explored in this paper. Persistence is defined by the authors as an increase of output when a positive, non permanent shock first hits the economy, and continues to increase for a few periods after that. Such a hump-shaped impulse response of output growth is obtained for their two-sector model in which output takes to long back to the steady state and autocorrelations of outputs are all positive for the first ten lags, i.e. there is too much, rather then too little persistence.

The idea of having a second investment good to absorb some of the role played by consumption - for which an unrealistic high intertemporal elasticity of substitution is assumed in the two-sector model - is the base for an investigation of a three-sector model. The results presented indicate that this model is able to replicate business cycle stylized facts of the U.S. economy and to produce hump-shaped impulse responses of output growth. However, persistence in the three-sector model is still to large even though intertemporal elasticity of substitution of consumption is reduced to a standard level.

In standard RBC models all economic activity takes place in the market. An extension of the basic setup is dealing with nonmarket activity of consumers. The rationale for the hypothesis that nonmarket activity or *household production* deserves explicit attention in business cycle modeling stems from the fact that the household sector is sizable, both in terms of labor and capital inputs used in home production and in terms of home-produced output[31].

The basic idea behind household production in real business cycle theory is that individuals substitute between home goods and market goods depending on the wage rate. That is, the market wage measures the opportunity cost of engaging in household activity. An increase in the wage rate during an economic boom should be accompanied by an increase of both, market work and demand for market goods and a corresponding decrease in household production. Examples of goods eligible for substitution between market purchase and home production are food preparation, home maintenance (e.g. housecleaning,

[31]see also Greenwood et al. (1995)

gardening, repairs, and the like), child care, doing laundry, or financial services such as preparation of income tax returns.

Since Gary Becker (1988), in his 1987 American Economic Association presidential address, advocated the introduction of home production into macroeconomics, several authors followed that advice in subsequent publications. In particular, the teams of Greenwood - Hercowitz and Banhabib - Rogerson - Wright show how real business cycles models with explicit home production sectors are better able to account qualitatively and quantitatively for several patterns of aggregate economic time series.

Greenwood and Hercowitz (1991) emphasize in their RBC model the role of capital in household activities to study the allocation of capital and time across a business and a household sector. Economic activities are described by two production functions, one for market activities and the other for nonmarket activities. In the first, market goods are produced by a cooperation of market capital (equipment and structures) and labor. In the second, household capital (consumer durables and residence) and time left over is used to produce a nontradeable consumption good - the home good.

As standard in this literature, both technologies are subject to stochastic productivity shocks. Complete symmetry between sectors is skewed by the rule that capital goods can be produced in the market sector only. As an example of activity in the home sector one can imagine the production of the good entertainment by the interaction of, say a stereo and time left from work.

In their simulations the authors compare the performance of a model with unitary elasticity of substitution in both preferences and household production - the benchmark -

with a model in which the substitutability between factors in household production is decreased. The benchmark model predicts volatilities and autocorrelations in main economic aggregates which are too low. Furthermore, investments in both sectors react in opposite directions in contrast to observed comovement in real data. This impact is emerging from the asymmetries in the two types of capital, since there is a tendency to build up business capital first and household capital after, as a reaction to positive shocks.

However, with higher complementarity in home production the two opposing effects followed by a technological change - the increase of the marginal product of household capital and the decline in the shadow price of the home good - do not cancel out each other any more. Namely, the marginal productivity effect becomes more important relative to the relative price effect and this results in procyclical investments. Also variations and first-order correlation of the second model are much closer to reality as in the benchmark case.

Benhabib, Rogerson and Wright (1991) address issues with focus on the labor market, like procyclical hours allocated to the production of consumption goods. In contrast to Greenwood and Hercowitz, they incorporate leisure in the utility function. Leisure is defined as net of time allocated to market and nonmarket production. Apart from functional forms for technology shocks and abstracting from taxation, the setup is related to the Greewood and Hercowitz model: two production functions, each with factors capital and labor and a stochastic productivity parameter, investment goods are produced in the market sector only, and capital is free to move between sectors.

In addition to the motivation already mentioned, the authors refer to reported evidence[32] that the fraction of nonmarket time devoted for household production is large. What they are asking and investigating subsequently is the question, whether household production and market production interact in such a way that technology shocks effecting both sectors account for a larger fraction of the business cycle, then do those effecting the market sector alone.

Crucial to this question is the elasticity of substitution in preferences between the market good and the home good[33]. The authors set this elasticity equal to five, motivated by a work of Eichenbaum and Hansen (1990) who find little statistical evidence against a hypothesis of perfect elasticity between nondurables and durables. Benhabib et al. (1991) compare their multi-sector household production model with a standard RBC model and obtain volatility and correlation results which come close to the one of U.S. aggregate data. They obtain significant improvements especially for the volatility of output, hours (both were too low in the benchmark), and investment (which was too high in benchmark). However, the extended model predicts a correlation of output and hours spent in the production of market consumption goods of 0.1. Even though the authors show that this correlation can be turned from a large negative number into this small positive value, simply through the introduction of home production, the magnitude of this comovement seems much too low. Another failure of the model is to replicate the comovement of investment in market and nonmarket capital and the phase shift pattern of these two types of investment[34].

To overcome the shortcomings just mentioned, Gomme, Kydland and Rupert

[32]see Benhabib, Rogerson and Wright (1990)

[33]Other important parameters in this context are the standard deviation of the home technology shock and the degree of correlation between both shocks.

[34]U.S. data reveal that household investment leads the cycle by about one quarter whereas market investment lags about a quarter, see e.g. Gomme et al. (2001).

(2001) use a household production model à la Benhabib et al. (1991) and incorporate a time-to-build technology for the production of market capital. A positive shock to market productivity induces the future market output to increase due to an increase in current investment of market capital. But, current home investment will decrease and will rise in subsequent periods only. The authors argue, that incorporating a time to build technology will induce home investment to co-move with market investment via two effects.

First, only a fraction of total resources for market investment is needed on impact of a positive productivity shock with a gestation lag technology. This spreads the response of market investment over the complete construction period. Second, the costs (in terms of current consumption and leisure) of increasing the capital stock in the market over the long run are reduced by gestation lags in market investment. However, their model can replicate the comovement of the investment series, but can not produce the lead-lag pattern of business investment and household investment. Furthermore, business investment is more volatile then household investment in their model, the opposite what we observe in the data.

A multi-sector model closely related to the home - production framework is provided by Davis and Heathcote (2005). They employ a setup with intermediate goods in which construction, manufacturing and services are used to produce consumption, business investment, and structures. Structures are then combined with land to produce homes. Structures in the model are supplied elastically but not housing, since the availability of land constraints the production of new houses such that land forces a wedge between house price and structure costs. To achieve their main goal of making predictions about house prices and residential investment, Davis and Heathcote assume additionally a labor-intensive construction industry, a low depreciation rate for housing, and volatile shocks to the construction industry. On the demand side, consumers have preferences for consumption, for leisure and for housing owned. Using the model economy, the authors are able to replicate some of the features of U.S. aggregate data.

In particular, residential investment is positively correlated with consumption, nonresidential investment and GDP. By the utilization of sectoral technology shocks they can also explain the high volatility of housing investment in contrast to business investment. However, these shocks also yield the counterfactual prediction that house prices and housing investment are negatively correlated. The model can also not account for disproportionately high house prices and for the lead-lag pattern of business investment and household investment.

A recent extension of the Davis and Heathcote (2005) framework is from Dorofeenko, Lee, and Salyer (2009). The authors incorporate a financial sector with lending under time varying asymmetric information in a multi-sector real business cycle model with house production. The motivation to introduce bank lending under uncertainty in a multi-sector RBC model is twofold. On the one hand, the authors like to capture one of the basic characteristic of the current financial crises, namely, changes in the uncertainty associated with future events. Second, as the authors document, residential real estate loans account for approximately 50% of total lending by domestically chartered commercial banks in the U.S. over the time span from 1996 to 2007.

Dorofeenko et al. find that house prices in their model are affected by expected bankruptcies and the associated agency costs. Since these costs serve as time-varying markup, the volatility in this markup translates into increased volatility in house prices. To that end, the authors model is able to replicate the disproportionally high volatility of house prices, if the uncertainty in the real estate sector is assumed to be high. Furthermore, housing investment is more volatile then business investment, the magnitude of the difference depending on the assumed uncertainty in the housing sector. They also obtain positive correlations of main economic aggregates such as, GDP to consumption and house prices, and consumption to both types of investment. However, as in Davis and Heathcote (2005), the presented model can not replicate the comovement of house prices with real estate investment and the lead-lag pattern of business investment and housing investment.

Chapter 3

A Multi-Sector RBC Model with Time-to-Build

3.1 Economic Environment

In this section a multisector real business cycle model is presented in which I introduce a time to build technology in the real estate sector to better match the stylized facts well known in the US housing market. My model economy is populated by a constant number of households. For simplicity it is assumed that all household are a continuum of mass 1. Every household has the same one-period utility function and the same time $t = 0$ housing stock and capital stock, respectively. The size of the household grows at an exogenously given constant gross growth rate η. All variables of the model are expressed in per-capita terms. In a perfectly competitive market two goods, a consumption/capital investment (CI) good, and a residential investment (RESI) good are produced. A representative household supplies homogenous labor, n_{it}, and rents homogenous capital, k_{it}, to the CI good sector, $i = c$, and the RESI sector, $i = d$[1].

3.1.1 Final Goods Production

Both goods are produced with a Cobb-Douglas technology, with y_{it} denoting the quantity of output in sector i:

$$y_{ct} = k_{ct}^{\alpha}(z_t n_{ct})^{(1-\alpha)} \tag{3.1}$$

$$y_{dt} = k_{dt}^{\gamma}(z_t n_{dt})^{(1-\gamma)} \tag{3.2}$$

Although, technology is Cobb-Douglas in both sectors, production differs with respect to input shares in production. Namely, these shares differ across sectors. In fact, my calibration will impose $\alpha > \gamma$, reflecting the fact that the RESI sector is more labor intensive then the CI sector [2].

The price of the consumption good is normalized to one, each price p_{it} is then the price of good i in units of the final consumption good. Let w and r denote the rental rates of labor and capital, respectively, in the same units[3]. Since there is no link of successive periods on the production side, maximization of the firm's present value is equivalent to

[1]To consider a representative household is a consequence of the assumption stated initially. This assumption states that each individual household is assigend a unique real number m from the invervall $[0, 1]$. Since all households face the same path of outputs and factor prices by assumption they choose identical sequences of their decision variables. Let $x(m)$ denote such an arbitrary variable of household $m \in [0, 1]$ and put $x(m) = \bar{x} \ \forall m \in [0, 1]$. Since

$$\int_0^1 x(m)dm = \int_0^1 \bar{x}dm = \bar{x}$$

aggregate and individual variables are indentical.

[2]To have one sector which is more labor intensive is also a necessary condition to obtain a balanced growth path in two-sector growth models (see e.g. Uzawa 1961).

[3]Since the same depriciation rate, δ_k, is assumed for both types of capital it is consistent to use one interest rate, r (see also Burmeister and Dobell (1970)).

maximize one-period profits. For period t they are given by

$$\max_{\{k_{it},n_{it}\}_{i\in\{c,d\}}} \{y_{ct} + p_{dt}y_{dt} - r_t k_t - w_t n_t\} \tag{3.3}$$

subject to equations (3.1) and (3.2) and to the constraints

$$k_{ct} + k_{dt} \leq k_t \tag{3.4}$$

$$n_{ct} + n_{dt} \leq n_t \tag{3.5}$$

$$\{k_{it}, n_{it}\}_{i\in\{c,d\}} \geq 0 \;\forall\; t \tag{3.6}$$

3.1.2 Productivity

The variable z_t, represents labour-augmenting technological progress. It consists of an deterministic component, the trend gross growth rate g_z, and the deviations from trend, $\widetilde{z}_t$, the stochastic component. $\widetilde{z}_t$ follows an autoregressive process of order 1 (AR(1)), such that we obtain the following representation:

$$\ln z_t = t\ln g_z + \ln z_0 + \ln \widetilde{z}_t \tag{3.7}$$

$$\ln \widetilde{z}_t = b\ln \widetilde{z}_{t-1} + \varepsilon_t \tag{3.8}$$

with b the coefficient which captures the autoregressive structure of $\widetilde{z}$. ε_t are shocks drawn independently from a Normal distribution with mean 0 and variance σ_ε^2.

3.1.3 The Housing Sector

In modeling the housing sector I abstract here from issues concerning the supply of land. Instead it is assumed that the representative household is endowed each period with a constant lot of new land which is sold to the real estate developer. The size of the lot is normalized to 1 for simplicity. The real estate developer is combining this land, x_{lt}, with the quantity of new structures purchased, x_{dt}, to produce new houses according to the following Cobb-Douglas technology

$$y_{ht} = x_{dt}^{\theta} x_{lt}^{(1-\theta)} \tag{3.9}$$

θ denotes the share of residential structure and $(1-\theta)$ the share of land in the production of new homes. The real estate developer has to solve the following maximization problem at each period t

$$\max_{x_{dt} x_{lt}} \{p_{ht} y_{ht} - p_{dt} x_{dt} - p_{lt} x_{lt}\} \tag{3.10}$$

subject to (3.9).

3.1.4 Time-to-Build

The housing stock is accumulated according to a technology introduced by Kydland and Prescott (1982). In each period (quarter of a year) the representative household launches a new investment project in the housing sector. After J periods this project is completed and adds to the overall housing stock. The investment costs are spread out over the entire gestation period. More formally, let $s_{jt}, j = 1, 2, ...J$, denote the number of housing projects j periods from completion at time t and it requires the household to pay the fraction ω_i of its total costs. At any period, there are j unfinished housing projects and since output in the real estate sector is the only factor which expands the existing housing stock, total investment expenditures equal

$$y_{ht} = \sum_{j=1}^{J} \omega_j s_{jt} \tag{3.11}$$

The fraction of resources, ω_j, allocated to the housing investment project at the jth stage from completion are taken as exogenously determined fixed parameters such that

$$\sum_{j=1}^{J} \omega_j = 1 \tag{3.12}$$

Furthermore, the investment plan is considered as fixed, i.e.

$$s_{j,t+1} = s_{j+1,t}, \qquad j = 1, 2, ..., J-1 \tag{3.13}$$

In equations (3.9) and (3.11) we observe that y_{ht} has to fulfill two restrictions. On the one hand, the stock of new houses is equal to the amount of new properties produced via the combination of residental structures and land by real estate developers (equation 3.9). On the other hand, the stock of new houses can not be larger then the amount consumers decided to invest in this sector in different periods. Consider for example $J = 2$, i.e. two periods time to build, equation 3.11 reads then

$$y_{ht} = \omega_1 s_{1t} + \omega_2 s_{2t}$$

If we update this equation by two periods we get

$$y_{h,t+2} = \omega_1 s_{1,t+2} + \omega_2 s_{2,t+2}$$

By the fixed investment plan restriction this can be written as

$$y_{h,t+2} = \omega_1 s_{0,t+1} + \omega_2 s_{0t}$$

such that the stock of new housing in period $t+2$ consists of the amount of housing invested two peroids before (period t) and ready to occupy in $t+2$, plus the amount of housing invested one priod before (period $t+1$) and ready to occupy in $t+2$.

In equilibrium the two amounts of new housing, the production of and the investment in new properties have to coincide. The overall housing stock consist then of the housing stock allready existent minus depriciation plus the investment in new housing. This results in the following law of motion for the housing stock:

$$\eta h_{t+1} = s_{1t} + (1 - \delta_h)h_t \tag{3.14}$$

with δ_h the depreciation rate of the housing stock [4,5].

3.1.5 Government

There is a government which imposes proportional taxes on capital income, τ_k, net of depreciation, and on labor income, τ_n. The tax revenue is rebated by a lump-sum payment, υ, to households. Government spending G_t is given by:

$$G_t = \tau_n w_t n_t + \tau_k r_t k_t - \delta_k \tau_k k_t - \upsilon_t \tag{3.15}$$

For simplicity it is assumed from now on that all revenues are rebated back to consumers by the government, so that government expenditures are set to zero, i.e. $G_t = 0$.

[4] Because all variables are in per-capita terms, variables dated $t+i$ are multiplied with population growth η^i.

[5] It is assumed that structures depreciate once combined with land, this assumption is inherent in the depriciation rate of the housing stock, see also Davis and Heathcote (2005).

3.1.6 Households

Each period a representative household derives utility from per-capita consumption, c_t, from housing owned, h_t, and from leisure, $l_t = (1 - n_t)$. The amount of labor supplied per household-member plus leisure can not exceed the period endowment of time. The total time endowment will be normalized to 1. Utility per household member in period t is given by:

$$U(c_t, h_t, (1 - n_t)) = \frac{(c_t^{\mu_c} h_t^{\mu_h} (1 - n_t)^{(1-\mu_c-\mu_h)})^{1-\sigma}}{1 - \sigma} \tag{3.16}$$

The intertemporal elasticity of substitution is denoted by the parameter σ (> 0) and is given by $1/\sigma$[6].The relative weights in utility on consumption, housing and leisure are determined by μ_c and μ_h and are assumed to be constant[7].

In contrast to firms, households in this economy face a non-trivial problem because they have to form expectations over future prices. Households will choose consumption, spending on new capital, spending on new housing projects, and hours of work. They receive income from renting out capital and labor at rates r_t and w_t, respectively, and from selling land to developers at price, p_{lt}. At each date, households try to maximize the expected discounted value of utility, given their expectations over future prices, subject to sequences of budged constraints and different law of motions for capital stocks and housing stocks, respectively:

$$\max_{\{c_t, n_{ct}, n_{dt}, k_{c,t+1}, k_{d,t+1}, s_{Jt}\}} E_0 \sum_{t=0}^{\infty} \beta^t \eta^t U(c_t, h_t, (1 - n_t)) \tag{3.17}$$

[6]This is the reason why utility functions of the form of (3.16) are also called CES (constant elasticity of substitution) utility functions.

[7]Empirical evidence shows that hours worked have remaind constant in the post-World-War-II period (see e.g. Kydland (1995)). The Consumer Expenditure Survey reports a constant fraction of about 30% for housing expenditures between 1970 and 2005 in the U.S.

subject to

$$c_t + i_t + p_{ht}\left(\sum_{j=1}^{J}\omega_j s_{jt}\right) = \tag{3.18}$$

$$(1-\tau_n)w_t n_t + (1-\tau_k)r_t k_t + \delta_k \tau_k k_t + p_{lt}x_{lt} + \upsilon_t$$

and the constraints

$$\eta k_{ct+1} = i_{ct} + (1-\delta_k)k_{ct} \tag{3.19}$$

$$\eta k_{dt+1} = i_{dt} + (1-\delta_k)k_{dt} \tag{3.20}$$

$$\eta h_{t+1} = s_{1t} + (1-\delta_h)h_t \tag{3.21}$$

$$y_{ht} = \sum_{j=1}^{J}\omega_j s_{jt} \tag{3.22}$$

$$i_t = i_{ct} + i_{dt} \tag{3.23}$$

$$k_t \geq k_{ct} + k_{dt}$$

$$n_t \geq n_{ct} + n_{dt}$$

$$s_{j,t+1} = s_{j+1,t}, \;\; j = 1, 2, ..., J-1 \tag{3.24}$$

with $\beta < 1$ the discount factor, and δ_k the depreciation rate of business capital. Overall investment, i_t, consists of investment in the CI sector, i_{ct}, and the RESI sector, i_{dt}, respectively. At their decisions, households take as given a set of contingent prices and transfers, $p_{ht}, p_{dt}, p_{lt}, r_t, w_t, \upsilon_t$, tax rates τ_n, τ_k, the initial stocks of capital, k_{c0}, k_{d0}, the stock of housing, h_0, and housing investments, $s_{10}, ..., s_{J0}$, and a probability distribution over possible future states. In addition, they are faced with the following set of inequality constraints

$$c_t, k_t, n_t, h_t, s_{jt} \geq 0, \; n_t \leq 1, \;\; \forall j \text{ and } \forall t$$

and the transversality conditions

$$\lim_{t\to\infty} \beta^t E_t \lambda_t k_{t+1} = 0 \qquad (3.25)$$

$$\lim_{t\to\infty} \beta^t E_t \lambda_t h_{t+1} = 0 \qquad (3.26)$$

with λ the shadow price of consumption (the Lagrangian multiplier). Conditions (3.25) and (3.26) just state that the expected present values of both, the terminal business capital stock and the terminal housing stock must approach zero.

Inherent in the budget constraint (3.18) is the assumption that new business capital $(k_{c,t+1}, k_{d,t+1})$ can only be produced in the consumption good sector, an assumption important to break the symmetry between the CI sector and the RESI sector to get a dynamics different from the single-good case.

3.2 The Recursive Competitive Equilibrium

In this section a recursive competitive equilibrium is defined in the line of Stocky et al.(1989). Time subscripts are dropped in standard fashion, i.e. a prime for a variable denotes next periods value. Our economy is populated by a continuum of households of mass 1. Although we have this measure of households, we have to distinguish between economy wide state variables, $\{\Psi, z\}$, and decision variables, Φ, over which the household has no control, and individual state variables, ψ , and decision variables, ϕ, defined as

$$\begin{aligned}
\Psi &\equiv \{K_c, K_d, S_1, S_2, ..., S_{J-1}, H, X_l\} \\
\Phi &\equiv \{C, N_c, N_d, S_J, K_c', K_d'\} \\
\psi &\equiv \{k_c, k_d, s_1, s_2, ..., s_{J-1}, h, x_l\} \\
\phi &\equiv \{c, n_c, n_d, s_J, k_c', k_d'\}
\end{aligned}$$

In a recursive competitive equilibrium, prices[8] and transfers, $\{p_h, p_d, p_l, r, w, \upsilon\}$, and decision variables are functions of the economy-wide state variables $\{\Psi, z\}$. So lets assume that these variables can be expressed as functions of the economywide state variables, i.e. we have prices, $p_h = P_h(\Psi, z)$, $p_d = P_d(\Psi, z)$, $p_l = P_l(\Psi, z)$, $r = R(\Psi, z)$, $w = W(\Psi, z)$, transfers, $\upsilon = \Upsilon(\Psi, z)$, and control variables, $c = C(\Psi, z)$, $n_c = N_c(\Psi, z)$, $n_d = N_d(\Psi, z)$, $s_J = S_J(\Psi, z)$, and suppose that k_c, k_d, and h evolve in equilibrium according to the laws of motion $k_c' = K_c(\Psi, z)$, $k_d' = K_d(\Psi, z)$, $h' = H(\Psi, z)$, and z evolves according to equations (3.7) and (3.8).

3.2.1 The Household Sector

Each period the representative household chooses its consumption level, $\hat{c}$, capital stocks $\hat{k}_c', \hat{k}_d'$, housing investment, $\hat{s}_J$, and time allocated to work, $\hat{n}_c$,and $\hat{n}_d$, so as to solve the following dynamic programming problem:

$$V(\psi, \Psi, z) = \max_{\hat{\phi}}\{U(\hat{c}, h, (1 - \hat{n}_c - \hat{n}_d)) + \beta E[V(\psi', \Psi', z')]\} \tag{3.27}$$

[8]As allready stated, the price of the consumption good is normalized to 1.

subject to

$$
\begin{aligned}
& \hat{c} + \eta \hat{k}_c' + \eta \hat{k}_d' + P_h(\Psi, z) \left(\sum_{j=1}^{J-1} \omega_j s_j + \omega_J \hat{s}_J \right) + \eta P_h(\Psi, z) h' \\
\leq \; & [1 + (1 - \tau_k)(R(\Psi, z) - \delta_k)](\hat{k}_c + \hat{k}_d) + (1 - \tau_n) W(\Psi, z)(\hat{n}_c + \hat{n}_d) \\
& + P_l(\Psi, z) x_l + P_h(\Psi, z)(s_{1t} + (1 - \delta_h) h_t) + \Upsilon(\Psi, z) \qquad (3.28)
\end{aligned}
$$

and to the equations for technological progress, (3.7) and (3.8), and $k_c' = K_c(\Psi, z)$, $k_d' = K_d(\Psi, z), h' = H(\Psi, z)$.[9]

3.2.2 The Production Sectors

Each sector consists of a large number of identical firms. Since I assume constant returns to scale technologies for all firms the scale of each firm operating is indeterminate, and the production sector can be represented by three price-taking firms. Two of these representative firms (the CI good producer and the RESI good producer) hire labor,$\widetilde{n}_i$, and capital, $\widetilde{k}_i$, to maximize their profits $\Pi_i, i \in \{c, d\}$

$$\Pi_c = \max_{\widetilde{k}_c, \widetilde{n}_c} \{ y_c - R(\Psi, z) \widetilde{k}_c - W(\Psi, z) \widetilde{n}_c \} \qquad (3.29)$$

$$\Pi_d = \max_{\widetilde{k}_d, \widetilde{n}_d} \{ P_d(\Psi, z) y_d - R(\Psi, z) \widetilde{k}_d - W(\Psi, z) \widetilde{n}_d \} \qquad (3.30)$$

subject to their production technologies:

$$y_{ct} = F^c(\widetilde{k}_c, \widetilde{n}_c, z) \qquad (3.31)$$

$$y_{dt} = F^d(\widetilde{k}_d, \widetilde{n}_d, z) \qquad (3.32)$$

[9]The first order necessary conditions of the households dynamic programming problem for the calibrated model are given in Appendix B. The steady state equations can be found there as well.

and the following constraints:

$$\begin{aligned} \widetilde{k}_c + \widetilde{k}_d &\leq \widetilde{k} \\ \widetilde{n}_c + \widetilde{n}_d &\leq \widetilde{n} \\ \{\widetilde{k}_i, \widetilde{n}_i\}_{i \in \{c,d\}} &\geq 0 \end{aligned}$$

The representative real estate developer is buying the RESI good, x_d, and is combining it with land, x_l, to produce new houses, y_h, so as to maximize profits Π_h

$$\Pi_h = \max_{x_d, x_l} \{P_h(\Psi, z) y_h - P_d(\Psi, z) x_d - P_l(\Psi, z) x_l\} \tag{3.33}$$

subject to the production technology:

$$y_h = F^h(x_d, x_l) \tag{3.34}$$

Due to the underlying assumptions, these optimization problems yields zero profits for all firms and the following factor prices

$$r = F_1^c(\widetilde{k}_c, \widetilde{n}_c, z) = p_d F_1^d(\widetilde{k}_d, \widetilde{n}_d, z) \tag{3.35}$$

$$w = F_2^c(\widetilde{k}_c, \widetilde{n}_c, z) = p_d F_2^d(\widetilde{k}_d, \widetilde{n}_d, z) \tag{3.36}$$

$$p_d = p_h F_1^h(x_d, x_l) \tag{3.37}$$

$$p_l = p_h F_2^h(x_d, x_l) \tag{3.38}$$

where F_i stand for the partial derivative with respect to the $i-th$ argument of the function F. Furthermore, we need the condition that government transfers are a function of economy wide states

$$\upsilon = \Upsilon(\Psi, z) \tag{3.39}$$

3.2.3 Definition

A competitive recursive equilibrium for this economy is a set of allocation rules, $c = C(\Psi, z)$, $n_c = N_c(\Psi, z)$, $n_d = N_d(\Psi, z)$, $k'_c = K_c(\Psi, z)$, $k'_d = K_d(\Psi, z)$, $s_J = S_J(\Psi, z)$, and pricing and transfer functions, $p_h = P_h(\Psi, z)$, $p_d = P_d(\Psi, z)$, $p_l = P_l(\Psi, z)$, $r = R(\Psi, z)$, $w = W(\Psi, z)$, $\upsilon = \Upsilon(\Psi, z)$, such that:

i) households solve problem (3.27), taking as given the aggregate state in the economy $\{\Psi, z\}$, and the form of the functions $P_h(\cdot)$, $P_d(\cdot)$, $P_l(\cdot)$, $R(\cdot)$, $W(\cdot)$, $\Upsilon(\cdot)$, $K_c(\cdot)$, $K_d(\cdot)$, $H(\cdot)$, with the equilibrium solution of the problem satisfying $\hat{c} = C(\Psi, z)$, $\hat{n}_c = N_c(\Psi, z)$, $\hat{n}_d = N_d(\Psi, z)$, $\hat{k}'_c = K_c(\Psi, z)$, $\hat{k}'_d = K_d(\Psi, z)$, $\hat{s}_J = S_J(\Psi, z)$;

ii) final good producers solve problems (3.29) and (3.30), and satisfy conditions (3.31) and (3.32), taking as given $\{\Psi, z\}$, and the functions, $R(\cdot)$, $W(\cdot)$, $P_d(\cdot)$, with the equilibrium solution of the problem satisfying $\widetilde{k}_c = k_c$, $\widetilde{k}_d = k_d$, $\widetilde{n}_c = N_c(\Psi, z)$, $\widetilde{n}_d = N_d(\Psi, z)$;

iii) real estate developers solve problem (3.33), and satisfy condition (3.34) , taking as given $\{\Psi, z\}$, and the functions $P_h(\cdot)$, $P_d(\cdot)$, $P_l(\cdot)$.

iiii) the goods markets for the consumption/investment good, for housing, for residental structures, and for land clear each period, implying that

$$c + i_c + i_d + p_h \left(\sum_{j=1}^{J} \omega_j s_j \right) = y_c + p_h y_h \quad (3.40)$$

$$\eta h' = s_1 + (1 - \delta_h) h \quad (3.41)$$

$$y_h = \sum_{j=1}^{J} \omega_j s_j \quad (3.42)$$

$$x_d = y_d \quad (3.43)$$

$$x_l = 1 \quad (3.44)$$

where

$$i_c = k_c' - (1 - \delta_k) k_c$$

$$i_d = k_d' - (1 - \delta_k) k_d$$

3.3 Balanced Growth and Solution Method

For most industrialized countries it is well known that in the long-run the majority of economic quantities is not stable. For example, output per capita, capital per worker, and productivity is growing over time.

However, long-run growth of various economic variables occurs at rates that are roughly constant within economies. These observations, also known as the "stylized facts" of economic growth became the point of departure for applied business cycle research.[10] The theoretical counterpart to these empirical observations is the concept of the balanced-growth path, a situation in which quantities in the model grow at constant - but possible different

[10]Labeld this way by Nicholas Kaldor (1957)

- non-negative rates.[11] Since preferences and production functions all have Cobb-Douglas functional forms we know that a balanced growth path exists for our model economy.[12] Denoting one plus the growth rate of a variable X as g_x (i.e. X_{t+1}/X_t) it is shown in Appendix C that we have the following gross growth rates for the variables of the model:

$$
\begin{aligned}
g_z &= g_c = g_{k_c} = g_{k_d} = g_k = \\
g_{y_c} &= g_{p_d y_d} = g_{p_h y_h} = g_{i_c} = \\
g_{i_d} &= g_i = g_w = g_\Upsilon \\
g_{n_c} &= g_{n_d} = g_n = r = 1 \\
g_{x_l} &= \eta^{-1} \\
g_{y_h} &= g_h = g_{s_1} = g_{s_2} = g_z^\theta g_{x_l}^{1-\theta}
\end{aligned}
$$

These gross rates state that the trend growth of the variables $c, k_c, k_d, k, y_c, p_d y_d, p_h y_h, i_c, i_d, i, w, \Upsilon$ are all equal to g_z, the trend growth of technology. On the other hand, for variables for which land is a relevant factor of production, the growth rate is a weighted product of productivity growth and trend growth of land. These rates reflect the fact that land is in fixed supply and thus, if more new structures are produced they have to be combined with a constant quantity of land.

In order to get a stationary model economy the next step is to use these trend growth rates to transform all variables in the economy such that the transformed variables exhibit no trends. This transformation involves dividing all variables in the system by their

[11] Another terminology for the same concept is steady state growth or dynamic euqilibrium (see also Barro and Sala-i-Martin (2004), chapter 1).

[12] As noted allready above, the restriction of one sector which is more labor intensive is assumed to be satisfied in the model. For a general reference on restriction on preferences and production in one-sector models consistent with balanced growth see also King et al. (1988).

respective growth rates, i.e. for a generic old variable x_t, with gross growth rate g_x, the stationary transformation $\bar{x}_t$ is given by

$$\bar{x}_t = \frac{x_t}{g_x^t} \tag{3.45}$$

Having made the system stationary, the next step is to find steady state values of the model without technology shocks. Given these steady state values, the first-order conditions of the system are then linearized near the steady state. A popular linearization procedure also used here is the Taylor series expansion. The associated system of linear difference equations can then be solved numerically using standard dynamic-programming methods. In fact, I use a generalized Schur factorization to get the policy functions for the system. Technical details can be found in Appendix D.

3.4 Calibration and Data

In order to calculate steady state values and simulated data I have to assign values to various parameters of the model. To this end, parameter values are assigned according to economic theory, a priory information and estimates for economic aggregates and prices. Data are taken basically from the National Income and Product Accounts (NIPA) tables and Fixed Asset tables, both published by the U.S. Department of Commerce.

The model period is one quarter of a year. The population growth rate is set to 0.59% per quarter, the average rate of growth of hours worked in the private sector between 1970 and 2007, which is the sample period.

3.4.1 Consumption

Consumption is taken from the NIPA tables 1.5.5 "Gross Domestic Product, Expanded Detail", the series personal consumption expenditure (including expenditure for housing services) and government consumption are added since in the model it is assumed that government consumption is zero. The series is transformed into real terms by converting it with the equivalent series from "Price Indexes for Gross Domestic Product, Expanded Detail", NIPA table 1.5.4.

3.4.2 House Prices

Data one house prices are taken from the Freddie Mac Conventional Mortgage Home Price Index[13]. To calculate the relative price of houses, this price index is divided by the NIPA price index for Personal Consumption Expenditure.

3.4.3 Total Fixed Investment

Total fixed investment is divided into residential fixed investment, and business fixed investment (also nonresidential fixed investment in the text) from NIPA tables 1.5.5. Business fixed investment is the additive of nonresidential fixed investment, non-defense government investment, state and local government investment and consumer durables. Residential investment is gross private domestic residental investment Both investment series are adjusted to real terms by the respective price indices from NIPA.

[13] This price index is constructed so as to account for changes in the quality of houses over time.

3.4.4 Aggregate Stocks and Depreciation of Capital and Housing

To calibrate the total "productive" capital stocks of the model, i.e. the empirical equivalence to capital stock, k, in the model, NIPA estimates of nonresidential fixed private capital stock are added with NIPA estimates of non-defense federal government stocks, state and local government stocks, and consumer durables. For the housing stock, h, NIPA estimates of residential fixed private capital are added with NIPA estimates of residential fixed government capital[14]. By employing the depreciation of fixed assets series I calculate the annual depreciation rates for capital, $\delta_k = 0.0703$, and for housing, $\delta_h = 0.0155$. From the adjusted NIPA data I get a housing stock which is 1.3 times GDP.

3.4.5 Productivity and Preference Parameters

Capital shares in both industries are derived as 1 minus the ratios from compensations in that industry over the nominal value added of that industry minus the nominal proprietors income in that industry[15]. The average capital shares I get for the period 1970 to 2007 is $\alpha = 0.317$, for the consumption good sector and $\gamma = 0.197$, for the RESI good sector[16]. These estimates imply that the RESI sector is more labor intensive then the consumption good sector. The calibration for the land shares follows Davis and Heathcote (2005) who estimate the land share in the production of new homes by 10.6% which gives $\theta = 0.896$.

A critical parameter which can not be determined from time series alone is the pa-

[14]From the "Fixed Assets Accounts Tables" of U.S. Department of Commerce.
[15]See also Cooley and Prescott (1995) or Davis and Heathcote (2005).
[16]For the RESI sector I computed industrie compensation as the additive of compensation in the real estate sector and the construction sector.

rameter of relative risk aversion σ. Microeconomic studies do not provide clear-cut evidence across heterogenous population characteristics in this respect. However, if homogeneity is assumed, Browning, Hansen and Heckman (1999) argue for a σ slightly above one. I decide to use $\sigma = 2$ which implies a smoother consumption profile of agents than with the widely used logarithmic preferences. The discount factor beta is set to a value of $\beta = (1/1.066)^{\frac{1}{4}}$, which indicates an annual interest rate of 6.6%, the average of the fed funds rate for the sample period.

The preference parameters μ_c and μ_h are set such that agents spend around 25% of their time endowment working[17], and such that the ratio of the housing stock to GDP is 1.3, the average ratio found in the data for our sample period. This yields values for $\mu_c = 0.35$, and $\mu_h = 0.051$.

3.4.6 Tax Rates

The constant tax-rate on capital income is set such that on the balanced growth path the ratio of the nonresidential capital stock to output is as close as possible to the average ratio found for the 1970 to 2007 data period. This ratio is 1.932, and the tax rate is set to $\tau_k = 0.15$. For the tax rate on labor income the value range in the literature is between 25% (Greenwood et al. 1995) and 40% (Lucas 1990). I decide to stay in between these extremes, and set it to $\tau_l = 0.33$.

[17]This is equivalent to a working time of 42 hours per week.

3.4.7 Solow Residual

As in Greenwood and Hercowitz (1991) I assume no sector specific shocks but one shock process for the overall economy. To calculate the multifactor productivity growth or Solow residual, it requires data on real output, the real capital stock and hours worked. With respect to the relative size of both sectors these calculations are carried out by using real GDP (net of housing), data on real fixed nonresidential private capital and total work hours employed by nonresidential factor inputs. Using these date the annual Solow residual is derived from:

$$\log(z_t) = \frac{1}{1-\alpha}\left[\log(y_{ct}) - \alpha \log(k_{ct}) - (1-\alpha)\log(n_{ct})\right]$$

Given my estimates for the share values, I calculate the time-series path of the annual logarithm of the technology shocks for the period 1970 to 2007. To estimate the rate of growth of the technology shock, I regress the log shocks on a constant and a time trend. From this procedure I obtain an annual growth in technology of 2.56% which gives us a quarterly gross growth rate of $g_z = 1.0064$.

Having calculated these growth rates, the logged detrended residuals of this estimate, which is equivalent to $\log(\widetilde{z})$ in the model, is regressed on its own past value to get the autocorrelation coefficient of the $\widetilde{z}$ series, this gives the annual AR coefficient. However, I am working with quarterly data. Unfortunately, OSL estimates of these AR coefficients are biased since NIPA annual estimates are arithmetic averages over quarterly estimates (expressed at annual rates). So, to determine quarterly AR coefficients out of annual data we are confronted with an endogeneity problem. Therefore I use Generalized Method of Moments (GMM) estimation and employ as instrument the $t-3$ value of the annual logged detrended residuals[18]. The annual estimate from GMM is found to be, $b_a = 0.532$. Taking residuals of this estimate gives us the empirical equivalence to the models $\varepsilon's$, which has a standard deviation of $\sigma_{\varepsilon,a} = 0.0303$. From these annual estimates we can construct quarterly estimates for the AR coefficient and the standard deviation of the residuals which are given by $b = 0.853$ and $\sigma_\varepsilon = 0.0226$[19].

3.4.8 Time-to-Build Technology

Concerning the time-to-build technology, the *Length of Time for New Residential Construction* series of the U.S. Census Bureau indicates average construction periods in between 6 months (single unit dwellings) and around 9 months (multiple unit dwellings) on average for the time span from 1971 - 2007. These numbers lead me to set the time-to-build period for housing projects to two quarters, i.e. $J = 2$, and each period an equal amount of resources has to be invested to an project so that $\omega_i = \frac{1}{2}$ for $j = 1, 2$.

For a summary of the chosen parameter values see Table 3.1.

[18]see also Davis and Heathcote (2000)

[19]A detailed description of this procedure is in the Data Appendix.

Table 3.1

Basline Parameters		
	Value	Definition
η	1.005925	Population growth
Preferences		
β	0.983	Discount factor
σ	2	Coefficient of relative risk aversion
μ_c	0.35	Consumption's share in utility
μ_h	0.051	Housing's share in utility
$(1- \mu_c - \mu_h)$	0.599	Leisure's share in utility
Production		
α	0.317	Capital's share in CI sector
γ	0.197	Capital's share in RESI sector
$(1- \theta)$	0.106	Land's share in new housing
δ_k	0.0176	Depreciation rate of capital
δ_h	0.00388	Depreciation rate of housing
g_z	1.0064	Productivity growth
Time to build		
J	2	Number of project periods
ω_j	1/J	Fraction of resources used at stage j
Government		
G	0	Government spending
τ_k	0.15	Tax rate on capital income
τ_n	0.33	Tax rate on labor income
Shocks		
ρ	0.853	Autocorrelation of innovations
σ_ε	0.0226	SD of innovations

Chapter 4

Results

Prior to stating results from the simulations of the calibrated model, I have to define some new variables in order to match with their empirical equivalences. NIPA private consumption includes an imputed value for rents from owner-occupied housing. Thus private consumption expenditures are given by

$$PC_t = c_t + q_t h_t$$

with q_t a rental rate on housing, defined such that households are indifferent to renting a marginal unit of housing, i.e.

$$q_t = \frac{U_h(c_t, h_t, (1 - n_t))}{U_c(c_t, h_t, (1 - n_t))}$$

Since the cost of raw land is not imputed in NIPA estimates of GDP the value of newly build houses is not included in the models overall output. To be consistent with the NIPA the value of residential investment is added to GDP instead. This gives us the following definition for the overall output[1]:

$$GDP_t = y_{ct} + p_{dt}y_{dt} + q_t h_t$$

4.1 First Moments

As a first attempt in evaluating the model accuracy a comparison of varies macroeconomic ratios along the balanced growth path is made. The real world equivalences are averages of ratios of variables as displayed in Table 4.1[2]. We observe that the ratios of the model match the U.S. ratios properly. In particular, the model reproduces the observed shares of consumption, residential investment, and nonresidential investment in GDP very well. So, the first moments indicate that the model starts out of a steady state well in relation to observed long-run patterns of macroeconomic aggregates in the U.S.

TABLE 4.1

Properties of steady state: Ratios to GDP %		
	Data 1970 - 2007	Model*
Capital stock (k)	193.2	160.7
Housing stock (h)	129.9	130.0
Private consumption (PC)	78.6	71.3
Non-residental investment (Non-RESI)	17.8	15.1
Residental investment (RESI)	4.3	4.1

* Since some data are availabe only annualy the model is adjusted to the same frequency here.

4.2 Second Moments

To determine whether the model economy is capable of accounting for some of the facts regarding the behavior of housing in the United States, simulations are run for two versions of the model; the benchmark model with no explicit modeling for the gestation lag,[3]

[1] see also Davis and Heathcote (2005)

[2] The description of the data is given in Table A.1 in Appendix A.

[3] In this version of the model w_1 is set to 1, and w_2 is set to 0.

and the model with a two quarter time-to-build period. A set of second moments measuring the business cycle properties of the simulated data is presented in Table 4.2. Both models can account for most of the facts we find in the data at business cycle frequencies. However, the model with time to build is doing much better in all respects concerning the real estate sector. In particular, in the benchmark model nonresidential investment is much more volatile then residential investment. In the time to build model, both investment volatilities are overestimated, but the relative volatilities of the series are similar to what we observe in the data, i.e. residential investment is twice as volatile as nonresidential investment.[4]

Concerning house prices, in the data we observe a disproportionally high house price volatility. The benchmark model is producing a standard deviation (SD) of house prices lower then the standard deviation of GDP, but the time to build model is able to approximate this house price feature quite properly. The correlations of GDP with consumption and house prices are reproduced by both models. The correlation of consumption to nonresidential investment is the only measure for which the benchmark model is producing more accurate results then the time to build version. However, the correlations of residential investment to both, consumption and house prices are replicated with the wrong sign from the benchmark model, but the time to build version is doing well here. The correlation of business investment and output is reproduced similary well by both model versions.

[4]Both investment series are very sensitive to the low depriciation rates for captial and housing. For example, if these rates are doubled, the volatilites of both series decrease by about 40 percent.

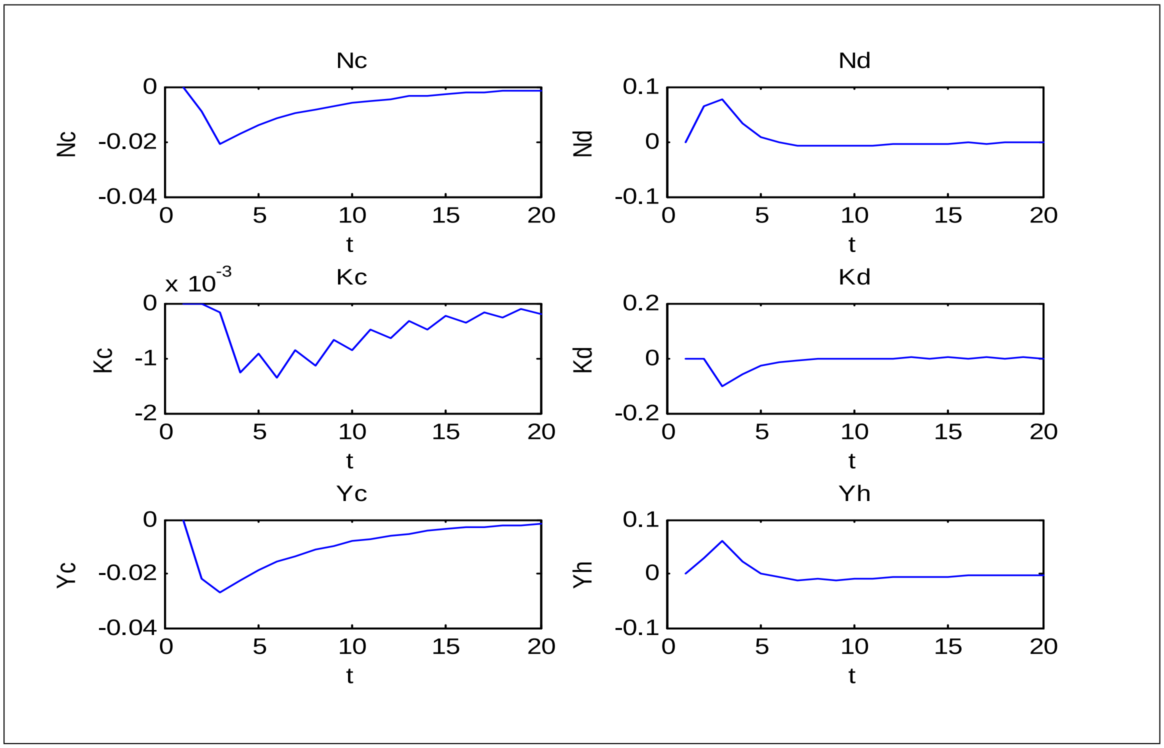
Table 4.2

Business Cycle Properties									
	Data (1970 - 2007)			Benchmark Model			Time-to-Build Model		
% SD (relative to GDP)									
GDP		1.52			1.72			1.61	
PC		0.72			0.51			0.66	
Labor (n)		0.96			0.91			0.98	
Non-RESI		2.73			6.64			7.47	
RESI		6.88			1.34			12.74	
House prices (Ph)		1.30			0.89			1.71	
Correlations									
PC, GDP		0.86			0.62			0.55	
Ph, GDP		0.58			0.57			0.68	
PC, Non-RESI		0.58			0.30			0.05	
PC, RESI		0.79			-0.06			0.36	
Ph, RESI		0.45			-0.03			0.33	
Lead-lag Correlations	i =2	i = 0	i = -2	i =2	i = 0	i = -2	i =2	i = 0	i = -2
Non-RESI(t-i), GDP(t)	0.37	0.80	0.84	0.34	0.77	0.49	0.31	0.71	0.43
RESI(t-i), GDP(t)	0.80	0.70	0.27	-0.21	-0.34	-0.55	-0.12	-0.24	-0.30
Non-RESI(t-i), RESI(t)	-0.16	0.28	0.63	-0.45	-0.50	-0.20	-0.48	-0.54	-0.17

Statistics are averages over 600 simulations, each of length 152 periods, the length of the data sample. Prior to computing statistics, all variables are transformed from their stationary representations back into nonstationary representations incorporating trend growth. Finally, all variables are logged and Hodtrick-Prescott filtered.

Unfortunately, the model can not account for two striking features noted in the introduction, namely that residential investment co-moves with business investment, and that residential investment strongly leads the business cycles. My results indicate that non-residential investment and property investment show negative correlations, a result which is expected in this class of model, since profitable investment opportunities in one sector should lead to a decrease of investment in the other sector. Still, in the data we observe a co-movement. The second puzzle is the lead of residential investment to GDP and business investment. At this stage, however, I have to conclude that real estate construction lags alone can not account for the lead-lag pattern of sector specific investments.

Figure 4.1: Impulse Responses I

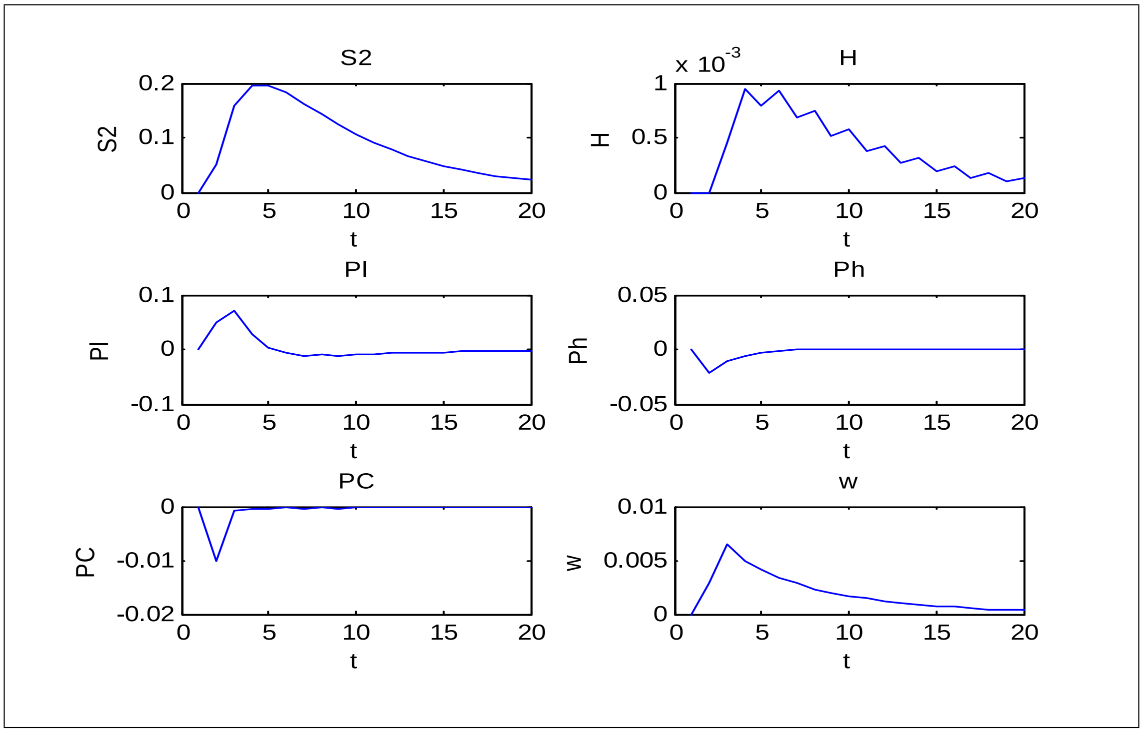


4.3 Impulse Responses

The behavior of the model economy as reaction to technology shocks is another interesting domain to consider. The standard procedure in this respect is to examine the impulse response functions of several key variables due to an one standard deviation technology shock (i.e. a shock of $\sigma_\varepsilon = 0.0226$).

For the two sector economy we observe in Figure 4.1 that a positive productivity shock to the system results in an increase of employment in the more labor intensive real estate sector. Since a positive shock to labor productivity is shifting the (downward sloping)

Figure 4.2: Impulse Responses II



labor demand schedule for both sectors outwards,[5] equilibrium in the labor market requires higher wages and, as a result, households supply more labor in the more labor intensive RESI sector. Capital on the other hand is decreasing in both sectors since the real rental rate of capital is decreasing as a reaction to the shock. Thus, the immediate impact of the shock is an increase of the real wage, and of both, output and the labor supply in the RESI sector. As a consequence, the output in the consumption good sector is decreasing on impact.

In Figure 4.2, we can observe that investment in housing, s_2, is increasing on impact. Potentially, the increase in real income raises the demand for housing. If consumers

[5]see euqation (10.14) in Appendix B.

like to move in a new property two periods from now, they have to decide for the investment today. The stock of new houses is raising with a time lag as described by the law of motion for housing. Firms observe the increase in the demand for housing and their demand for land increases, thus the price for land, p_l, starts to rise too. As a reaction to the increase of the housing stock, the price for real estate, p_h, is decreasing temporarily. In order to finance the new demand for housing, agents consume a little bit less initially, thus personal consumption expenditures, PC, are decreasing on impact. What we observe in general is, that a shock to productivity leads to short-term deviations from equilibrium. But, most of the variables are back to their balanced path after a few periods.

Chapter 5

Conclusions

This part of my thesis investigates the role of time-to-build for explaining aggregate fluctuations in general and patterns of residential investment in particular. To this end, I analyse a multisector real business cycle model with a consumption/capital investment (CI) good sector and a real estate sector. The CI sector is modeled as usual: the representative household decides how much to consume and how much to invest in physical capital to be productive in the next period. In contrast, the housing sector is restricted by a time-to-build technology. That is, households have to decide for investments in housing today which can be occupied half a year later. This asymmetric modeling of the economic domains in my model is motivated be the inability of the housing sector to adjust capacities in the short run and has the adventage of adding one more piece of reality to the artificial economy.

To obtain qualitative and quantitative results, respectively, the model economy is calibrated to U.S. data. A linearized version of the model is solved via a generalized Schur decomposition. The policy functions obtained are used to perform Monte Carlo experiments to approximate business cycle moments. In addition, impulse response functions are considered to analyze the behavior of the model economy as reaction to transitory technology shocks.

First moments from the long-run equilibrium solution of the model indicate a reasonable point of departure of the model economy when contrasted with the ratios from real world equivalences. The comparison of second moments from U.S. data and the artificial economy is done for two different model versions, the two-sector model without time-to-build - the benchmark model - and the two-sector model with a gestation period. Both version can account for most of the facts found in the data. However, the model with a gestation lag is performing much better in reflecting the real estate sector. Namely, the model with time-to-build can account for the following stylized facts: i) residential investment is twice as volatility as nonresidential investment; ii) house price volatility is disproportionally high in comparison to GDP; iii) hours worked show a volatility nearly similar to the volatility of output, and consumption is less volatile then output; iv) house prices and residential investment comove positively; v) GDP and both, consumption and house prices show positive correlations; vi) personal consumption expenditure show positive comovement with residential investment and nonresidential investment, respectively; and vii) business investment and output are highly correlated.

Both versions, unfortunately can not account for the positive co-movement of residential investment and GDP, and of residential and nonresidential investment. Additionally, my model versions are not able to account for the lead-lag structure of investments in the housing sector and the business sector. The failure of my model to replicate these facts stems potentially from a missing link between capital in the real estate sector and business capital. There are several ways to establish such an interrelation, e.g. by introducing intermediate goods like in Davis and Heathcote (2005) or by modeling capital in real estate sector as a complementary input in market production, like in Fischer (2007).[1] However, I decided to keep the model as simple a possible in order to test if a time-to-build specification in the housing sector alone can account for the co-movement puzzle, the relative volatility puzzle and the lead-lag structure of residential and nonresidential investment, respectively.

[1]Fischer's (2007) model is a home production model with household capital a complementary input in market production.

Part II

German Commercial Property Prices and Bank Lending

Chapter 6

Introduction

Interest in modeling the relationship between the macroeconomy and the real estate economy has risen steadily in the last ten years with a proliferation of macroeconomic models that highlight the role of financial intermediaries in business cycle activity. With variations on this theme referred to as models of the credit channel, agency cost models, or financial accelerator models, the common element is that lending activity is characterized by asymmetric information between borrowers and lenders. As a consequence, interest rates may not move to clear lending markets (as in models with moral hazard and adverse selection elements) or firms' net worth may play a critical role as collateral in influencing lending activity (as in models with agency costs): borrowers' financial positions affect their external finance premiums and thus their overall cost of credit. While debate on the empirical support for these models continues, there is little doubt that, as a whole, they have improved our understanding of financial intermediation and broadened the scope of how monetary policy, through the impact of interest rates on firms' net worth, can influence macroeconomic performance.[1]

The objective of this chapter is to study the potential effects of macro-policy and bank shocks on the German real estate sector. In particular, I estimate a structural vector autoregression (SVAR) model using both aggregate German and two largest regional states of Bavaria and Nordrhein - Westfalen (NRW) data for commercial property, bank loan, and other macro-policy variables from 1975 to 2004. Numerous authors have examined the effects of bank loans and general price level shocks on both residential and commercial real estate variables with a bag of non-resolute results (e.g. Iacoviello (2005), Davis and Zhu (2004), Tsolacos and McGough (1999), Hoffman (2002)). A variety of techniques have also been used to examine these relationships. Econometric models and time series approaches have both been employed. Among the previous studies, it is well known that analysis that employs a single equation setup (Goodhard, 1995) potentially suffer from simultaneity problems. Consequently, I use a VAR approach to analyze the dynamic relationship between bank lending (CR), commercial property prices (PP), investment in construction (INV), and Gross Domestic Product (GDP) for Germany, Bavaria, and NRW.

The commonly used VAR model in most real estate economics applications uses an identification which implies the system being modeled has a recursive structure: variables affect each other in the order specified by the modeler. This recursive structure implies that the variable at the top of the ordering will have a contemporaneous effect on all the variables in the model, and that the variable at the bottom of the ordering will have a contemporaneous effect on itself only. Most evaluations of the importance of this assumption

[1] The credit channel literature is large and continues to expand. Some prominent contributions are: Williamson (1987), Bernanke and Gertler (1989, 1990), Bernanke, Gertler, and Gilchrist (1999), Kiyataki and Moore (1997), Carlstrom and Fuerst (1997), Cooper and Ejarque (2000), and Dorofeenko, Lee and Salyer (2007). Walsh (1998) presents an overview, both theoretical and empirical, of the literature.

have relied on changing the order in the recursive specification and examining the impact on the model. In this work, since the objective is to analyze the effect of bank loans for commercial real estate sector on property prices, the principal ordering of variables[2] is the gross domestic product (in aggregate as well as for regional level), aggregate investment, the amount of bank loans to commercial real estate sector, and property prices. This ordering makes economic sense, in that it can be thought of as corresponding to an ordering of "increasing endogeneity": Commercial real estate decision makers are more likely to have a large impact on commercial property prices than on the gross domestic product.[3]

The empirical results initial show that on the aggregate level, bank (credit) loan shocks have a small but positive effect on property prices up to about eight periods and a negative effect afterwards. Other macro (e.g. GDP and Investment) shocks have the expected effect on the prices: a positive investment shock decreases prices whereas a positive shock to income increases prices in the medium-run. These effects are reinforced in the variance decomposition analysis. We observe that nearly 70% of the variance in property prices comes from price shocks initially. However, after a while shocks in bank loans start to gain importance in explaining volatility in property prices. The share is increasing up to nearly 30%. Shocks to the GDP and investment have a weak or less meaningful impact on the forecast error variance of commercial property prices.

In regard to the regional level the empirical results indicate somewhat different effects. From the impulse response analysis of Bavaria, we observe a negative response of prices to a shock in credits. Moreover, from the variance decomposition analysis we also get

[2]For the Identification Restricion, the Cholesky factorization is used to orthogonalize the residuals.

[3]Since this particular ordering is assumed the estimated model is a structural VAR, described in more detail below.

a different picture at the regional level. First, the proportion of the variance in prices due to shocks in prices is much lower initially, and is decreasing sharply to around 20% after three periods. Second, shocks to income seems to have much more influence on the variance of prices then for Germany (GDP starting form 33%, reaching a peak of 65% after 3 periods and staying at a level of more then 50% after 10 periods). And lastly, credit volatility seems not so sensitive to price shocks as observed on the national level. In contrast, for NRW property price shocks seem to explain some of the variance in bank loans.

The next section provides a short description of the data set as well as the German commercial property development. Section seven introduces a linkage between the macroeconomy, bank lending and real estate with a brief review of previous studies. Section eight then outlines VAR models and the role of identification restrictions. The empirical analysis, results and some discussion of their implications are also reported in this section. The final section concludes part II.

Chapter 7

Data and Related Literature

7.1 Data Description

All variables are real using the consumer price index. I use annual series from 1975 to 2004 for Germany and from 1980 to 2004 for Bavaria and NRW. For the aggregate German data, commercial property prices is an index series form Bulwien AG, Germany. Bank lending data corresponds to credit in real estate construction form the Deutsche Bundesbank. For the measure of aggregate economic activity, I use investment in construction from the Federal Statistical Office of Germany, and gross domestic production obtained from the International Financial Statistical Office.

For Bavaria, commercial property prices are again from Bulwien AG. I take an average of the eight largest cities in the region. Bank lending is defined as the commercial credits in construction form, which is obtained from the regional bank "Landeszentralbank Bayern" (LZB). As a proxy for investment in construction, I use the order inflow in the construction sector from the LZB. Bavarian GDP is obtained from the State Offices for

Statistics of Baden-Würtenberg and Bayern (Statistischem Landesamt Baden-Würtenberg, and Bayrischen Landesamt für Statistik and Datenverarbeitung).

Commercial property prices for NRW are constructed as an average of prices of the twelve largest cities in the region, prices are from the Bulwien AG. For the remaining series the same proxies are used as for Bavaria and are taken from the State Office for Statistics of Nordrhein-Westfalen (Landesamt für Datenverarbeitung und Statistik Nordrhein-Westfalen).

7.2 German Commercial Property Development

Figure 7.1: Germany - Plot of Commercial Property Prices (PP) against Bank Lending (CR), Investment in Construction (INV), GDP

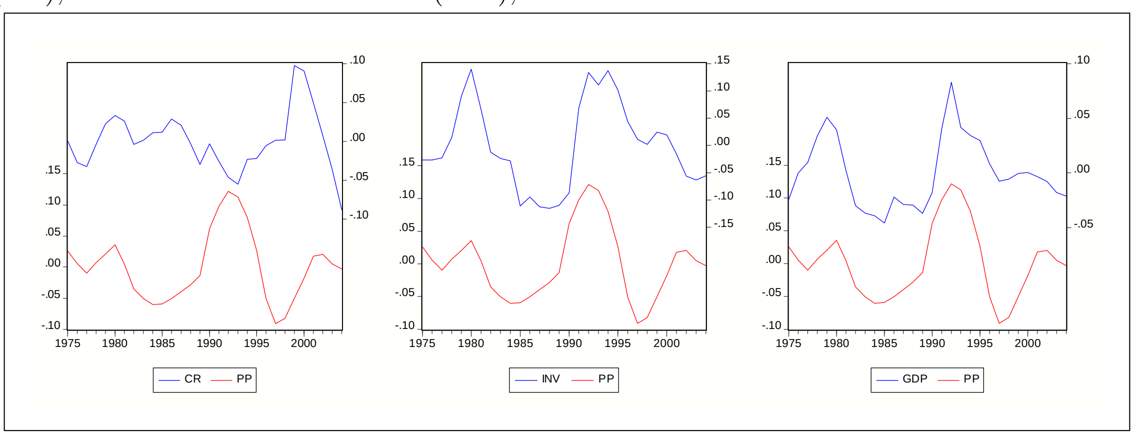


Figure 7.1 shows some of the data in annualized form to exhibit the cyclical patterns most clearly. The main series to be explained is the German commercial property price index in relation to the amount of bank loans.[1] The property prices peaked whereas the bank loan was at the trough in 1992 (two years after the German unification). The

[1] All time series are in logs and detrended using the Hodrick-Prescott filter.

bank loan peaked in 1999 and took a steep downturn. Until 1999, there is a clear negative correlation between the bank loans and property prices. The last panel in figure 7.1 shows the co-movement between the GDP (proxy for real income) and prices. Since investment in construction is a component of GDP and is known to be highly correlated with output the second panel shows a similar pattern[2].

Figure 7.2: Bavaria - Plot of Commercial Property Prices (PP) against Bank Lending (CR), Investment in Construction (INV), GDP.

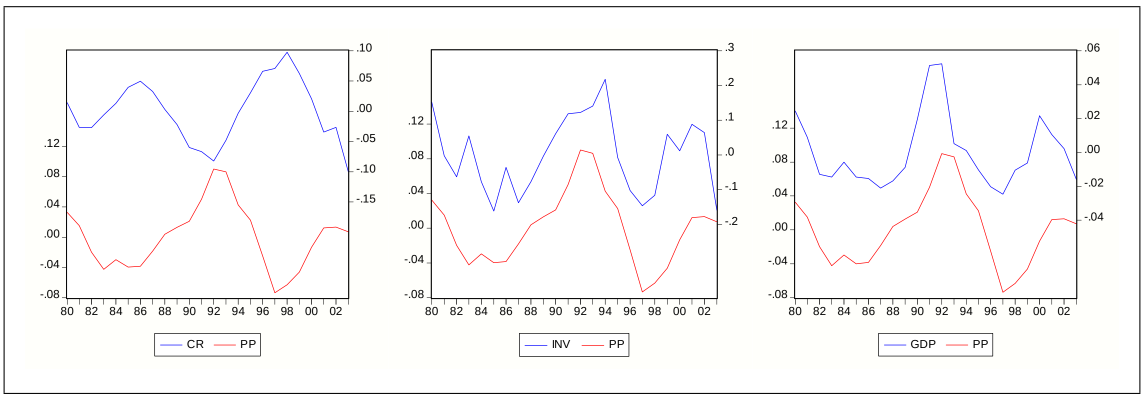
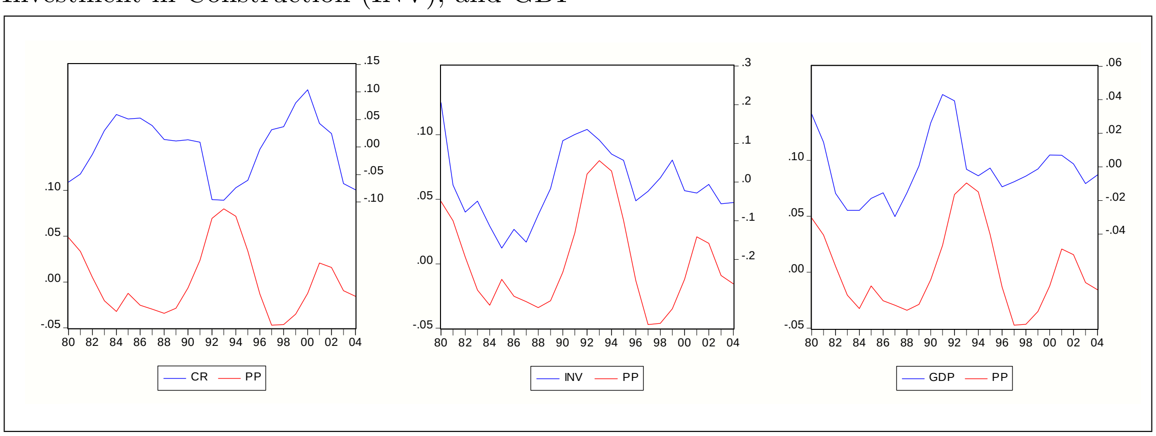


Figure 7.3: NRW - Plot of Commercial Property Prices (PP) against Bank Lending (CR), Investment in Construction (INV), and GDP



[2]The correlation for Germany is 86%.

Figure 7.2 shows the regional level time series for Bavaria, where we observe a similar picture to the aggregate level. Again, property prices peaked two years after the German unification and bank loans were at a through. For investment and output we observe the same co-movement with prices as at the national level.

For Nordrhein - Westfalen (NRW), in Figure 7.3 we observe similar swings for commercial property prices. The co-movement for prices, investment and income is very distinctive for this region.

Concerning the stability of the data in use, the Augmented Dicky-Fuller (ADF) unit-root test is applied. As a lag-length selection criterion the Schwarz Information Criterion (SIC) was chosen. The tests in Table 7.1 indicate that all variables are stationary for Germany, Bayern, and NRW.[3]

Table 7.1: ADF Unit Root Test

	Germany		Bavaria		NRW	
	Level	P-value	Level	P-value	Level	P-value
PP	-4.40	0.00	-3.26	0.00	-4.27	0.00
CR	-2.79	0.01	-3.70	0.00	-2.57	0.01
INV	-2.78	0.01	-2.80	0.01	-2.94	0.00
GDP	-3.11	0.00	-3.20	0.00	-3.58	0.00

PP = Commercial Property Prices, CR = Bank Lending in the RE-Sector, INV = Investment in Construction in the RE-Sector

7.3 Macroeconomics, Bank Lending and Real Estate Linkage

In recent years a number of theoretical models that highlights the role of a financial accelerator in propagating and amplifying macroeconomic shocks has further casted doubts

[3]Enders (1995), for example, shows that the power of the unit root tests is very limited when sample size is small at a lower frequency. Moreover, Sims (1980), for example, argues against diffenencing variables even if the variables contain a unit root, because the goal of VAR analysis is to determine the interrelationship among variables and not to determine the parameter estimates.

on aggregate technology shocks in the standard real business cycle model as the driving force in business activities.[4] This literature addresses the question "can credit constraints and (or) asymmetric information between borrows and lenders propagate and amplify business cycles?" Although the theoretical contributions have improved our understanding of the propagation mechanism, the lack of empirical support has led many to question the relevance of financial accelerator type models.[5]

Empirical support for or against credit channel effect in real estate literature is, however, less prominent. Among few is Davis and Zhu (2004), who study on the bank lending and property prices, both for the commercial and the residential sector. In their commercial real estate paper, they develop a reduced form theoretical model, based on the financial accelerator framework. The model suggests that bank lending is closely related to commercial property prices, and their interaction can develop cycles given plausible assumptions (e.g. lags of supply and property evaluation based on current prices). Their study of 17 different countries shows a strong link of commercial property prices to credit in commercial property in the countries that have experienced banking crises linked to property losses in 1985-95. They found a significant impact of prices on bank credit. However, the impact in the reverse direction is less clear. In addition, they found GDP to be an important factor for commercial property prices and bank credits. Peng, Cheung and Leung (2001) also report similar result for Hong Kong: they state that excessive bank lending was not the main reason for the boom and bust cycle of the Hong Kong residential property market.

[4] Financial accelerator models are usually clasified into two catergories: agency costs models and credit constraint models.

[5] See, for example, Kocherlakota (2000), Cooper and Ejarque (2000), and Cordoba and Ripoll (2004) for a negative stance on the role that financial sector plays in the actual economy. Carlstrom and Fuerst (1997) and Dorofeenko, Lee and Salyer (2007) are the only few that document the empirical relevance for financial acceleration.

Hoffman (2004) uses both time series and panel data estimation and distinguishes between long and short run causality. In 15 out of 20 cases, he finds that long-run causality goes from property prices to bank lending. He also states that property price bubbles are rather caused by property prices then by bank lending. Short-run causality is going in both directions, which supports the theoretical argument of self-reinforcing cycles in property markets. Gerlach and Peng (2004) find a similar result as in Peng, Cheung and Leung(2001) for residential properties in the Hong Kong market. They have large contemporaneous correlation between bank lending and property prices, but they argue their results (of an Vector Error Correction Model) suggest that the direction of influence goes from prices to credits rather than converse.

Chapter 8

Vector Autoregression (VAR) and Empirical Analysis

This section begins with a brief introduction to a recursive structural VAR that the empirical analysis is based on. Then some results and some of their implications are discussed.

8.1 VAR

As outlined briefly below, VAR models are a form of dynamic simultaneous equations model. As in any simultaneous equations exercise, the VAR model requires identification restrictions to interpret the model in a causal framework. The approach in VAR modeling is to develop a statistical model before the imposition of identification restrictions.

In VAR the dependent variables are, by definition, all endogenous variables and the independent variables are lagged observations of all variables in the system.[1] All variables affect each other through a system of lags. This allows the data to provide a representation of the changes in the system without "zero restrictions" (i.e. restricting the coefficient of some explanatory variables in an equation to zero) as required in traditional simultaneous equation techniques. While VAR models do not impose zero restrictions on the parameters in the traditional simultaneous equation fashion, the model does require identification restrictions to provide information on the response of system variables to shocks.

Traditional econometric modeling of a simultaneous system would require the construction of a structural model using theory and the placement of restrictions on this structural model in order to be able to identify the parameters of the structural model from the reduced form or statistically estimated model. Typically, the reduced form is based on a reduced parameter space and the identifying restrictions are used to derive the structural parameters (see for example Sims, 1980). The VAR approach uses the set of lags of all of the endogenous variables in each behavioral equation as the reduced form. The economic structure is identified using the covariance matrix of the residuals to place identifying restrictions on the matrix of contemporaneous coefficients. For example, the Cholesky decomposition of the covariance matrix results in orthogonal behavioral shocks and a contemporaneous coefficient structure that implies a recursive ordering between variables. A VAR imposing this error structure is also known as structural VAR (SVAR).[2]

While both the SVAR approach and traditional econometric approaches require identification restrictions, the nature of these restrictions are quite different. The traditional

[1] Where they are considered to be important, exogenous (or deterministic) variables may be included in the set of independent variables in the system.

[2] see e.g. Enders (1995) or Lütkepohl and Krätzig (2004).

approaches tend to place little emphasis on lags in equations while the SVAR approach emphasizes it. The traditional approach places strict interpretations on the parameters of each equation while the SVAR approach interprets the system as a whole and analyzes responses to the behavioral shocks. The traditional approach uses zero restrictions on parameters for identification while the SVAR approach uses the covariance matrix of the reduced form residuals and the assumption of orthogonal behavioral shocks to establish identification. In VAR models, the statistical model is developed first and then the structural model is identified. This approach is opposite to that followed in traditional econometrics and is favoured by some statistical theorists.

The SVAR model begins with a dynamic equation system of the form

$$\sum_{s=0}^{p} B(s)Y(t-s) = \eta(t) \tag{8.1}$$

where $Y(t)$ and $\eta(t)$ are $k \times 1$ vectors and $B(s)$ is a $k \times k$ matrix of coefficients for each time period (s) previous to current time (t) with $B(0)$ having only unity entries on its main diagonal.[3] Equation (8.1) relates the observable data $Y(t)$ to sources of variation in the economy $\eta(t)$. The shocks in $\eta(t)$ are assumed to represent behaviorally distinct sources of variation that drive the economy over time. The vector $\eta(t)$ has an expected value of zero and an assumed diagonal covariance matrix, D. The covariance matrix is assumed to be diagonal so that individual shocks $(\eta(t))$ apply to only one behavioral equation at a time. Thus we can evaluate the effect of shocks to each behavioral equation on each variable in the system.

[3]The constant is ignored in this exposition for notational convenience.

Equation (8.1) can be rewritten in autoregressive form as

$$A(L)Y(t) = \varepsilon(t) \tag{8.2}$$

The matrix $A(0) = I$,and $A(L), L \geq 1$ represents the lag. Equations (8.2) is the autoregressive equation system which is estimated given an assumption on the lag length. It is the reduced form model. The reduced form errors, $\varepsilon(t)$, are linear combinations of the structural errors, $\eta(t)$, and have covariance matrix Ω.

Since all the variables are related in the system, it is not possible to disentangle the effects of one variable on another using the reduced form representation. However, one could orthogonalize the shocks of a SVAR system as follows: Rewrite equation (2) as $QA(L)Y(t) = Q\varepsilon(t)$, where Q is a matrix, and call the new matrices $C(L)Y(t) = e(t)$ with $C(L) = QA(L)$ and $e(t) = Q\varepsilon(t)$. The new system will then have the properties $C(0) = Q$, $E(e(t)e(t)') = Q\Omega Q'$. The new system and the old one are observationally equivalent (so long as Q is not singular). In particular, a convenient choice for Q is a Q such that $Q^{-1}Q^{-1\prime} = \Omega$. With this Q, $E(e(t)e(t)') = Q\Omega Q' = I$. Thus, it makes the shocks orthogonal. One way to construct such a Q is via the Choleski decomposition. The Choleski decomposition produces a lower triangular matrix Q^{-1}, and thus a lower triangular Q. This implies that, for example, in a 2×2 equation the second variable is not included in the first equation, but the first variable is included in the second equation.

Given a stable SVAR, the autoregressive representation can be used to find the moving average representation which expresses the level of a particular variable as a function of the error process. Transforming the reduced from model (8.1) into a moving average representation results in:

$$Y(t) = \Psi(L)\eta(t) \tag{8.3}$$

with

$$\begin{aligned} \Psi(L) &= A(L)^{-1}B(0)^{-1} \\ &= \sum_{s=0}^{\infty} \Psi(s)L^s \end{aligned}$$

Moreover, this moving average representation is the impulse response function (IRF), which describes the effect of shocks to the behavioral relations on variables in the system. The IRF summarizes the dynamic multipliers as implied by our identification. A shock may be represented by the placement of the value unity in one element of the vector $\eta(t)$. The IRF provides the response of all variables in the system to this unit shock.

The moving average representation can also be used to decompose the forecast error variance of one of the variables in the system into portions attributable to each element in $Y(t)$. Using the autoregressive structure of the SVAR model, the conditional expectation of $Y(t+h)$ given $Y(t), Y(t-1), \cdots$, can be determined. These are the h-step ahead forecasts of the series $Y(t)$. The forecast error covariance can also be established since it depends only on information up to time t. Forecast error decompositions are derived from the result that the contribution of each variable to the forecast error is linear thus allowing the evaluation of each separate variable's impact on the forecast error. This linearity of forecast errors results from the orthogonalization procedure used in SVAR models explained above.

The forecast error variance decompositions provide a useful measure of the strength of explanation between variables at different forecast horizons. Interpreted together with IRFs, decompositions can provide valuable insight into the dynamics of variables under investigation.

8.2 Empirical Results and Implications

The SVARs are estimated for 1975 to 2004 for Germany and for 1980 to 2004 for Bavaria and NRW. As mentioned allready above, all time series are in logs and detrended by the Hodrick-Prescott filter. A system was selected with one lag for both the aggregate and regional models. For diagnostic purpose, we find that the SVAR(1)'s estimates are stable for Germany and Bavaria but SVAR(2)'s estimates are statistically more appropriate for NRW. Moreover, the residuals are checked for autocorrelation, conditional heteroscedasticity as well as for the deviations for the Gaussian assumption, as proposed by Johanson (1995).

For the Identification Restriction the Cholesky factorization is used to orthogonalize the residuals. The Cholesky Ordering is specified as follows: GDP, INV, CR, PP. To see what this ordering implies consider the structural errors $\eta(t)$ in relation to the reduced form errors $\varepsilon(t)$:

$$\varepsilon(t) = \Gamma\eta(t)$$

with $\Gamma = B(0)^{-1}$, and γ_{ij} the elements in Γ. Using this notation the ordering implies the following structure for the contemporaneous residuals:

$$\varepsilon_{GDP,t} = \eta_{GDP,t} \tag{8.4}$$

$$\varepsilon_{INV,t} = \gamma_{21}\eta_{GDP,t} + \eta_{INV,t} \tag{8.5}$$

$$\varepsilon_{CR,t} = \gamma_{31}\eta_{GDP,t} + \gamma_{32}\eta_{INV,t} + \eta_{CR,t} \tag{8.6}$$

$$\varepsilon_{PP,t} = \gamma_{41}\eta_{GDP,t} + \gamma_{42}\eta_{INV,t} + \gamma_{43}\eta_{CR,t} + \eta_{PP,t} \tag{8.7}$$

Hence, we can observe that a shock to GDP effects all other variables contemporaneously, a shock to investment in construction effects bank lending and property prices contemporaneously, and a shock to credits does the same for property prices. The economic reasoning for this ordering is the well known fact, that income can have immediate impacts on credits, investment and prices, whereas this is not so obvious in the other direction.[4]

8.2.1 Germany: Impulse Response Analyses and Variance Decomposition

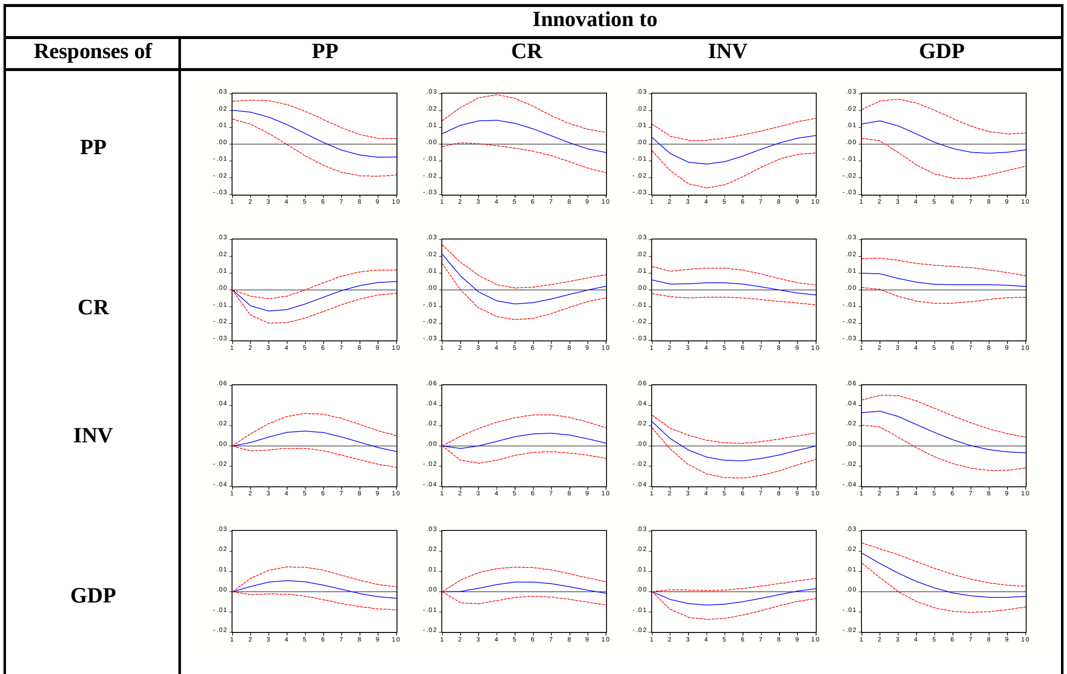
The impulse responses for Germany are shown over 10 periods in Figure 8.1. On the national level, we observe that a one-standard deviation shock to credits has a small positive effect on property prices up to about eight years, and afterwards the response gets negative. This could imply that in the first periods prices are increasing due to a weaker conditions in the credit market, but the result of persistence in high prices probably lead to an over-supply of properties and subsequently a downward pressure on prices.

However, we see a negative effect of credits due to an increase in prices. This result could imply that commercial properties are not an important component in the portfolio of German banks and so they do not react as expected to an increase in property prices. For Investment and GDP, we have the expected impulse responses: a positive shock to investment decreases prices, and a positive shock to income will increase prices in the medium-run.

The reaction of investment to a positive price shock is also as expected, it leads to

[4] Also the impulse responses are not very sensitive to the ordering. Additionally, I tried other orderings, but the pattern of the responses do not change.

Figure 8.1: Germany - Impulse Responses from SVAR(1)

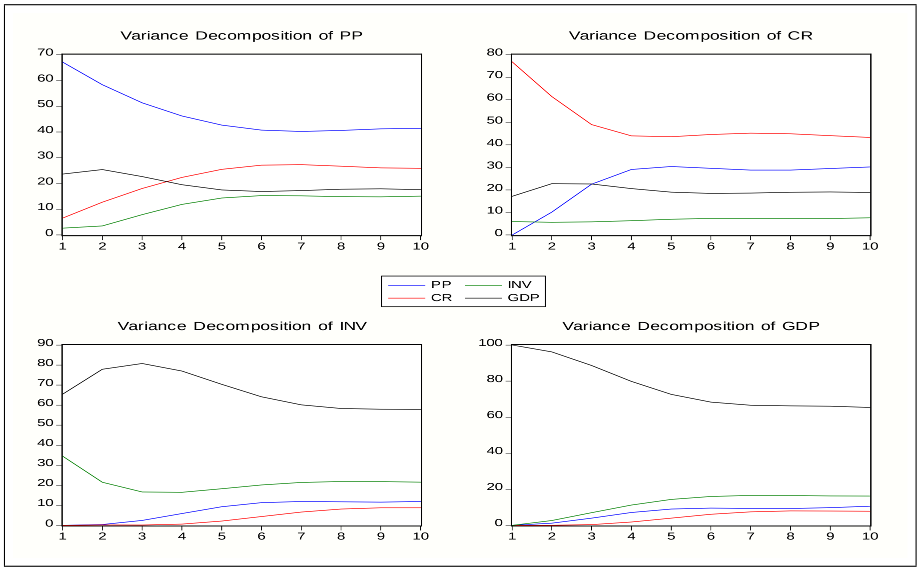


Variables: PP - Commercial Property Prices, CR - Bank Lending, INV - Investment in Construction.

an increase in commercial property investment for some periods.

From the forecast error variance decomposition in Figure 8.2, we observe that initially nearly 70% of the variance in property prices comes from a shock in this prices. However, after some periods shocks in credits for commercial properties start to gain importance in explaining volatility in commercial property prices, the share is increasing up to nearly 30%. For the variance decomposition of credits we can observe the opposite pattern. Within three to four periods price shocks help to explain nearly one third of credit volatility, a result which strongly supports the hypothetical interrelation of price and credits. For the investment volatility in the commercial property sector the variance decomposition indicates a minor role of price shocks and a dominant role of income shocks.

Figure 8.2: Germany - Variance Decompositon from SVAR(1)



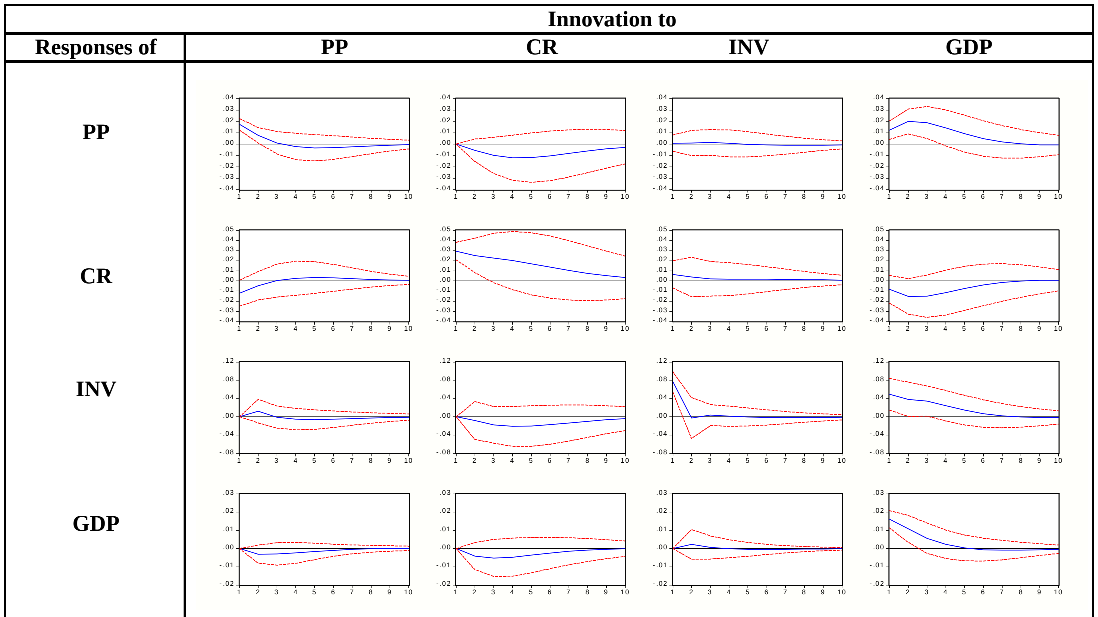
8.2.2 Bavaria: Impulse Response Analyses and Variance Decomposition

The Cholesky Ordering for Bavaria is the same as for Germany. I also tried different orderings and the pattern of the responses do not change (the correlations of the VAR residuals are here from -0.43 to 0.57)

For the impulse responses for Bavaria in Figure 8.3, we obtain different results than for the national level. Figure 8.3 shows a negative response of property prices to a shock in credits. Since the credit series represents credit outstanding we can not distinguish whether this effect is due demand or supply. The demand side argument is that credits will increase if property prices moves up and therefore demand will increase since new loans are available. Thus, prices will rise further. But here we see that a positive shock to credits has a negative effect on prices. Thus, a supply side effect also needs to be examined.

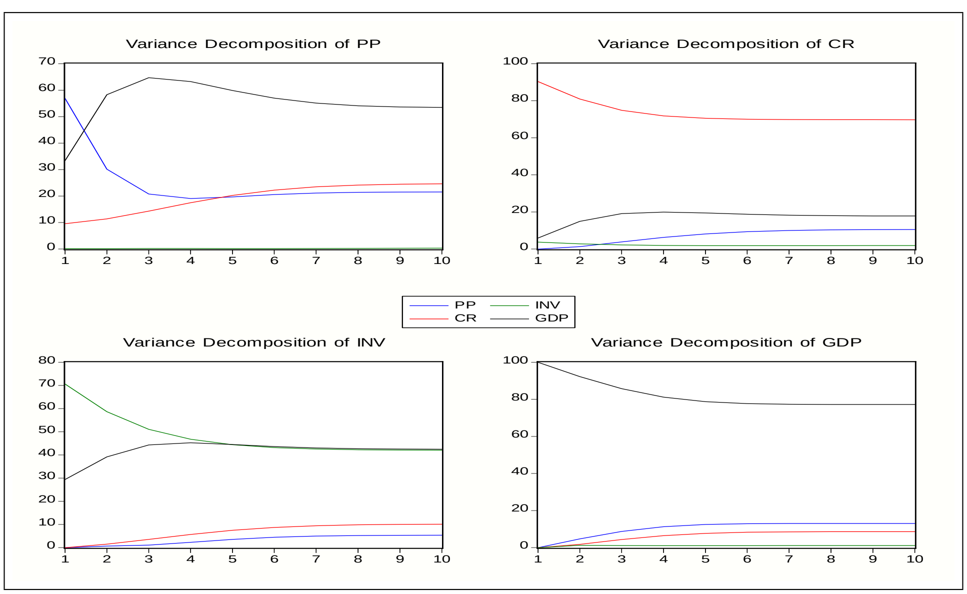
Here we see an immediate impact of prices (which is by definition possible due to

Figure 8.3: Bavaria - Impulse Responses from SVAR(1)



Variables: PP - Commercial Property Prices, CR - Bank Lending, INV - Investment in Construction.

Figure 8.4: Bavaria - Variance Decompositon from SVAR(1)



the Cholesky ordering we impose). Since we have this immediate impact we can argue that, if prices go down due to an increase in credits this comes from the supply side, because if more credits are available the supply of properties will increase due to increase in investment in commercial properties, therefore the prices will decline.This supply side argument is also supported by the positive response of credits due to a price shock. If prices move up the default risk of banks decreases and therefore credit supply is increased.

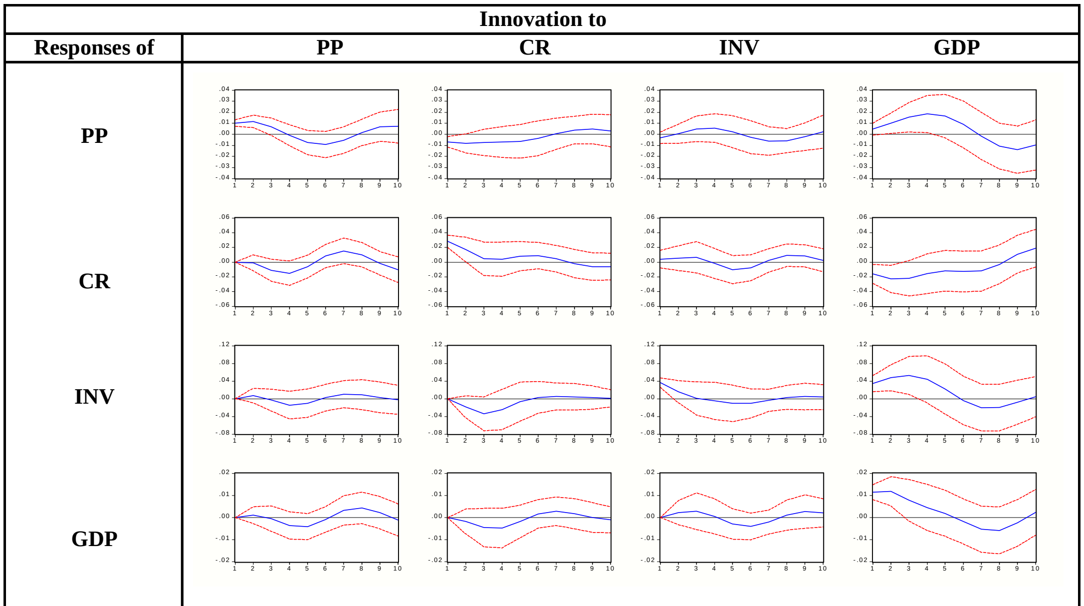
From the variance decomposition, in Figure 8.4, we also get a different picture on the regional level. First, the proportion of the variance in prices due to shocks in prices is much lower (only about 57% at the first period). Second, the forecast error variance is decreasing sharply to around 20% after three periods. Third, shocks to income seems to have much more influence on the variance of prices then for Germany.(GDP starting form 33%, reaching a peak of 65% after 3 periods and staying at a level of more then 50% after 10 periods). And lastly, credit volatility seems not so sensitive to price shocks as observed on the national level.

8.2.3 Nordrhein - Westfalen: Impulse Response Analyses and Variance Decomposition

For the last region under investigation, Nordrhein - Westfalen (NRW), we observe in the first column of Figure 8.5, that a price shock leads to an increase of property prices for two periods. After that, prices show small swings around the equilibrium level. For the response of credits to shocks in price, there is an immediate decrease.

This indicates that there is a contemporaneous effect of prices on credits (which is possible due to the ordering implied). For the same time span as for the price increase we

Figure 8.5: NRW - Impulse Responses from SVAR(2)



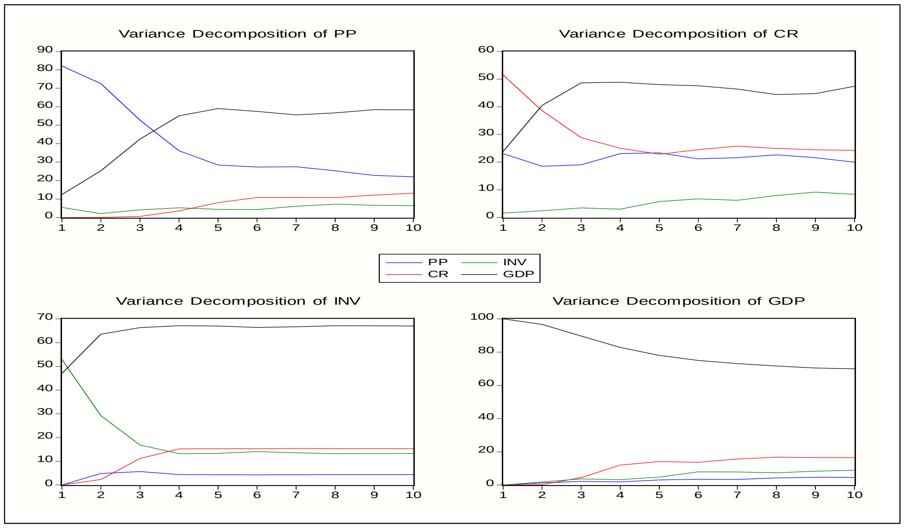
Variables: PP - Commercial Property Prices, CR - Bank Lending, INV - Investment in Construction.

observe an increase in credits as well. When prices start to fall, the bank lending decreases, but then it increases before property prices start to rise again. This is a puzzling reaction, since (with the construction lag in mind) it could indicate a causal direction form credits to prices. For Investment and GDP we observe similar patterns as above.

In column two, a credit shock leads to a decrease of property prices again. The difference here is the reaction of bank lending to a credit shock. For NRW we observe an increase of credits after four periods. Property prices, on the other hand, increase with one lag. Again, this is a reaction which was not expected since this could tell us that prices increase due to an increase in credits.

The variance decomposition for NRW in Figure 8.6 indicates that the regional GDP is the most important variable for property prices and bank lending in this country.

Figure 8.6: NRW - Varaince Decomposition from SVAR(2)



In contrast to Bavaria, shocks to property prices seem to explain some of the variance in bank loans.

Chapter 9

Conclusion

This part of my thesis investigates the importance of some macroeconomic factors, in particular bank shocks, on the dynamics of German commercial real estate prices for both, national and regional levels. The results can be summarized as follows. On the aggregate level, bank (credit) loan shocks have a small but positive effect on property prices up to eight periods and negative effect afterwards. Other macro (e.g. GDP and Investment) shocks have the expected effect on the prices: a positive investment shock decreases prices whereas a positive shock to income increases prices. These effects are reinforced in the variance decomposition analysis. We observe that most of the variance in property prices comes from a shock to itself. Thus, the shocks to other variables have a weak or less meaningful impact on the forecast error variance of commercial property prices. In regards to the regional level, the empirical results indicate somewhat different effects. From the impulse response analysis, one can observe a negative response of prices to a shock in credits. Moreover, from the variance decomposition analysis we also get a different picture at the regional level.

As much as I would like to draw a resolute conclusion and lesson from this study, one has to acknowledge that the empirical results are highly dependent on the availability of the commercial real estate data in Germany. One could, however, draw a plausible conclusion that there is a long-lasting link (either positive or negative) between bank loans and property prices in Germany. The feedback from property prices to credit growth is strongest in places with a greater prevalence of variable rate mortgages and more market-based property valuation practices for loan accounting.

Chapter 10

Appendix Part I

10.1 Appendix A

10.1.1 Data

Table A.1

Data Sources Table

Variable*	Source**	Computation
GDP	NIPA	Chain-weighted GDP (Tables 1.1.3 line 1.1.5., lines 1, respectively)
PC	NIPA	Chain-weighted aggregate of personal consumption expenditures (Tables 1.5.4 and 1.5.5, lines 2, respectively) and government consumption expenditures (same Tables, lines 50 plus lines 53, respectively)
Non-RESI	NIPA	Chain-weighted aggregate of nonresidential fixed investment (Tables 1.5.4 and 1.5.5, lines 23, respectively), non-defense government investment (same Tables lines 51, respectively), state durables and local government investment (same Tables, lines 54, respectively) and consumption expenditures on consumer durables (same Tables, lines 3, respectively)
RESI	NIPA	Chain-weighted residential fixed investment (Table 1.5.4 and 1.5.5, lines 33, respectively)
Total labor	BLS	Aggregate hours worked in private industries
House prices	CMHPI, NIPA	Conventional Mortgage Home Price Index (CMHPI, USA) divided by the price index for personal consumption expenditures (NIPA, Table 2.3.4, line 1)
Capital Non-RESI	FAT	Chain-weighted aggregate of nonresidential fixed private capital stocks (Tables 1.1 and 1.2 , lines 4, respectively), non-defense federal government stocks (Tables 7.1.A&B, 7.2.A&B, lines 30, respectively) state and local government stocks (same Tables, lines 41, respectively) and stocks of consumer durables (Tables 1.1 and 1.2, lines 13, respectively)
Capital RESI	FAT	Chain-weighted aggregate of residential fixed private capital stocks (Tables 1.1 and 1.2, lines 7, respectively) and residential fixed government capital (same Tables, lines 12, respectively)

* With the exception of the labor series and house prices, all variables are real chain-weighted variables with the base year 2000.

**NIPA = National Income and Product Accounts Tables, Bureau of Economic Analysis; BLS = Bureau of Labor Statistics, U.S. Department of Labor; CMHPI = Conventional Mortgage Home Price Index, Freddie Mac; FAT = Fixed Asset Tables, Bureau of Economic Analysis.

10.1.2 Solow Residual

To get the real output data net of housing, the quantity indices of GDP and housing (NIPA Table 1.5.4, line 1 and 13) are multiplied by the 2000 nominal values respectively (NIPA Table 1.5.5, lines 1 and 13). The real housing series is subtracted from GDP yielding real output net of housing in chain-weighted 2000 U.S. Dollars. The real nonresidential (Non-RESI) capital stock is constructed by the respective quantity indices multiplied by 2000 nominal values from the Fixed Asset Tables (see the Data Sources Table) . The labor supply series is taken from NIPA Table 6.9B "Hours Worked by Full-Time and Part-Time Employees by Industry" and is calculated as annual hours worked in domestic industry minus hours in finance, insurance and real estate. Using these date the annual Solow residual is derived from:

$$\log(z_t) = \frac{1}{1-\alpha}\left[\log(y_{ct}) - \alpha \log(k_{ct}) - (1-\alpha)\log(n_{ct})\right]$$

Given my estimates for the share values, I calculate the time-series path of the annual logarithm of the technology shocks for the period 1970 to 2007. To estimate the rate of growth of the technology shock, I regress the log shocks on a constant and a time trend. From this procedure I get an annual growth in technology of 2.56% which gives us a quarterly gross growth rate of $g_z = 1.0064$

Having calculated these growth rates, the logged detrended residuals of this estimate, which is equivalent to $\log(\widetilde{z})$ in the model, is regressed on its own past value to get the autocorrelation coefficient of the $\widetilde{z}$ series, this gives the annual AR coefficient. However, I am working with quarterly data. Unfortunately, OSL estimates of these AR coefficients are biased since NIPA annual estimates are arithmetic averages over quarterly estimates (expressed at annual rates). So, to determine quarterly AR coefficients out of annual data we are confronted with an endogeneity problem. Therefore I use GMM estimation and employ as instrument the $t-3$ value of the annual logged detrended residuals. The annual estimate from GMM is found to be, $b_a = 0.532$. Taking residuals of this estimate gives us the empirical equivalence to the models $\varepsilon' s$, which has a standard deviation of, $\sigma_{\varepsilon,a} = 0.0303$. These GMM estimates (and also the biased OLS estimates) can then be converted into quarterly terms by the relationship[1]

$$b = b_a^{1/4}$$

with b_a the annual AR coefficient and b the quarterly analog. So we get a quarterly coefficient of $b = 0.853$. To derive the corresponding standard deviations for innovations to quarterly logged detrended residuals note that

$$\varepsilon_t = \frac{1}{4}\sum_{q=1}^{4} \hat{e}_{t,q} = \frac{1}{4}[\iota_4 B \widetilde{e}_t] \tag{10.1}$$

where q stand for the quarter in period t, ι_4 is a 1×4 vector of the element 1, $\widetilde{e}'_t = [e_{t-1,2}, e_{t-1,3}, e_{t-1,4}, e_{t,1}, e_{t,2}, e_{t,3}, e_{t,4}]$, the elements in the $\widetilde{e}'_t$ vector are unobserved quarterly errors, and

$$B = \begin{bmatrix} b^3 & b^2 & b & 1 & 0 & 0 & 0 \\ 0 & b^3 & b^2 & b & 1 & 0 & 0 \\ 0 & 0 & b^3 & b^2 & b & 1 & 0 \\ 0 & 0 & 0 & b^3 & b^2 & b & 1 \end{bmatrix} \tag{10.2}$$

If we assume that the innovations to the quarterly log detrended residuals are independently distributed over time, then the variance of unobserved quarterly errors can then computed

[1] see also Davis and Heathcote (2000) for this and the following.

via the relationship[2]

$$E(\varepsilon_t^2) = \sigma_{\varepsilon,a}^2 = \frac{1}{16}[\iota_4 BB'\iota_4']E(e_{t,q}^2) \tag{10.3}$$

so if we plug in our numbers we get

$$\begin{aligned}\sigma_{\varepsilon,a}^2 &= 1.7744\sigma_\varepsilon^2 \Rightarrow \\ \sigma_\varepsilon &= 0.0226\end{aligned}$$

with $\sigma_\varepsilon^2 = E(e_{t,q}^2)$.

[2] I assume $E(e_{t,q}) = 0 \ \forall \ t, q$

10.2 Appendix B

10.2.1 First Order Conditions

Goods Production

$$
\begin{aligned}
y_{ct} &= k_{ct}^{\alpha}(z_t n_{ct})^{(1-\alpha)} && (10.4)\\
y_{dt} &= k_{dt}^{\gamma}(z_t n_{dt})^{(1-\gamma)} && (10.5)\\
y_{ht} &= y_{dt}^{\theta} x_{lt}^{1-\theta} && (10.6)
\end{aligned}
$$

The price of the consumption good is normalized to one, each price p_{it} is then the price of good i in units of the final consumption good. Let w and r denote the rental rates of labor and capital, respectively, in the same units. Since there is no link of successive periods on the production side, maximization of the firm's present value is equivalent to maximize one-period profits, for period t they are given by

$$
\Pi = \max_{\{k_{it}, n_{it}\}_{i\in\{c,d\}}, x_{lt}} \{y_{ct} + p_{ht}y_{ht} - r_t k_t - w_t n_t - p_{lt}x_{lt}\} \tag{10.7}
$$

subject to equations 10.4 and 10.6 and to the constraints

$$
\begin{aligned}
k_{ct} + k_{dt} &\leq k_t && (10.8)\\
n_{ct} + n_{dt} &\leq n_t && (10.9)\\
\{k_{it}, n_{it}\}_{i\in\{c,d\}}, x_{lt} &\geq 0 && (10.10)
\end{aligned}
$$

From 10.7 we get the following first order necessary conditions (FOC's)

$$\frac{\partial \Pi}{\partial k_{ct}} \quad : \quad \alpha \frac{y_{ct}}{k_{ct}} = r_t \tag{10.11}$$

$$\frac{\partial \Pi}{\partial n_{ct}} \quad : \quad (1-\alpha)\frac{y_{ct}}{n_{ct}} = w_t \tag{10.12}$$

$$\frac{\partial \Pi}{\partial k_{dt}} \quad : \quad p_{ht}\gamma\theta \frac{y_{ht}}{k_{dt}} = r_t \tag{10.13}$$

$$\frac{\partial \Pi}{\partial n_{dt}} \quad : \quad p_{ht}(1-\gamma)\theta \frac{y_{ht}}{n_{dt}} = w_t \tag{10.14}$$

$$\frac{\partial \Pi}{\partial x_{lt}} \quad : \quad p_{ht}(1-\theta)\frac{y_{ht}}{x_{lt}} = p_{lt} \tag{10.15}$$

and we get the following relation for prices:

$$p_d = \theta p_h \frac{y_h}{y_d} \tag{10.16}$$

In addition, the full employment conditions are:

$$\begin{aligned} n_{ct} + n_{dt} &= n_t \\ k_{ct} + k_{dt} &= k_t \\ x_{lt} &= 1 \end{aligned}$$

Households

To get the FOC's the value function formulation for the model with two periods time to build can be written as

$$\begin{aligned} V(\psi_t, z_t) &= \max_{\phi_t}\{U(c_t, h_t, (1-n_{ct}-n_{dt})) + \beta E_t[V(\psi_{t+1}, z_{t+1})] \\ &\quad +\lambda_t[(1+(1-\tau_k)(r_t-\delta_k))(k_{ct}+k_{dt}) + (1-\tau_n)w_t(n_{ct}+n_{dt}) + p_{lt}x_{lt} \\ &\quad +p_{ht}(s_{1t}+(1-\delta_h)h_t) + \upsilon_t - p_{ht}(\omega_1 s_{1t} + \omega_2 s_{2t}) \\ &\quad -c_t - \eta k_{c,t+1} - \eta k_{d,t+1} - \eta p_{ht}h_{t+1}]\} \end{aligned}$$

with the vectors of state variables (ψ_t) and control variables (ϕ_t) defined as:

$$\begin{aligned} \psi_t &\equiv \{k_{ct}, k_{dt}, s_{1t}, h_t, x_{lt}\} \\ \phi_t &\equiv \{c_t, n_{ct}, n_{dt}, s_{2t}, k_{c,t+1}, k_{d,t+1}\} \end{aligned}$$

and z_t the state variable of the shock process. For our convenience I rewrite the utility function here:

$$U(c_t, h_t, (1-n_t)) = \frac{(\bar{u}_t)^{1-\sigma}}{1-\sigma}$$

with

$$\bar{u}_t \equiv (c_t^{\mu_c} h_t^{\mu_H} (1-n_t)^{(1-\mu_c-\mu_h)})$$

Having defined the variables we get the following first order necessary conditions for the consumers problem

$$\begin{aligned} \frac{\partial V}{\partial c_t} &: \frac{\mu_c}{c_t}(\bar{u}_t)^{1-\sigma} = \lambda_t && (10.17) \\ \frac{\partial V}{\partial s_{2t}} &: \beta p_{h,t+1}\lambda_{t+1} = \beta\lambda_{t+1}p_{h,t+1}\omega_1 + \lambda_t p_{ht}\omega_2 \\ \frac{\partial V}{\partial h_{t+2}} &: \beta p_{h,t+1}\lambda_{t+1} = \eta^{-1}\beta^2\left(\frac{\mu_h}{h_{t+2}}(\bar{u}_{t+2})^{1-\sigma} + \lambda_{t+2}p_{h,t+2}(1-\delta_h)\right) \end{aligned}$$

taken these two FOC's together we get:

$$\frac{\mu_h}{h_{t+2}}(\bar{u}_{t+2})^{1-\sigma} = \frac{\eta}{\beta^2}\lambda_t p_{ht}\omega_2 + \frac{\eta}{\beta}\lambda_{t+1}p_{h,t+1}\omega_1 - \lambda_{t+2}p_{h,t+2}(1-\delta_h) \quad (10.18)$$

$$\begin{aligned} \frac{\partial V}{\partial n_{ct}} &: \frac{(1-\mu_c-\mu_h)}{(1-n_{ct}-n_{dt})}(\bar{u}_t)^{1-\sigma} = \lambda_t(1-\tau_n)w_t && (10.19) \\ \frac{\partial V}{\partial n_{dt}} &: \frac{(1-\mu_c-\mu_h)}{(1-n_{ct}-n_{dt})}(\bar{u}_t)^{1-\sigma} = \lambda_t(1-\tau_n)w_t && (10.20) \\ \frac{\partial V}{\partial k_{c,t+1}} &: \beta E_t[\lambda_{t+1}(1+(1-\tau_k)(r_{t+1}-\delta_k))] = \eta\lambda_t && (10.21) \\ \frac{\partial V}{\partial k_{d,t+1}} &: \beta E[\lambda_{t+1}(1+(1-\tau_k)(r_{t+1}-\delta_k))] = \eta\lambda_t && (10.22) \end{aligned}$$

$$
\begin{aligned}
\frac{\partial V}{\partial \lambda_t} \quad &: \quad c_t + \eta k_{c,t+1} + \eta k_{d,t+1} + p_{ht}(\omega_1 s_{1t} + \omega_2 s_{2t}) + \eta p_{ht} h_{t+1} = \\
&(1 + (1-\tau_k)(r_t - \delta_k))(k_{ct} + k_{dt}) + (1-\tau_n) w_t (n_{ct} + n_{dt}) + p_{lt} x_{lt} \\
&+ p_{ht}(s_{1t} + (1-\delta_h) h_t) + \upsilon_t \qquad (10.23) \\
\frac{\partial V}{\partial p_{ht}} \quad &: \quad \eta h_{t+1} = k_{dt}^{\gamma\theta} (z_{dt} n_{dt})^{(1-\gamma)\theta} x_{lt}^{(1-\theta)} + s_{1t} + \\
&(1-\delta_h) h_t - (\omega_1 s_{1t} + \omega_2 s_{2t}) \qquad (10.24)
\end{aligned}
$$

10.2.2 FOC with Efficiency Conditions from Goods Production

Using the efficiency conditions from the goods production we get the following expressions for 10.19 to 10.22

$$
\begin{aligned}
\frac{\partial V}{\partial n_{ct}} \quad &: \quad \frac{(1-\mu_c-\mu_h)}{(1-n_{ct}-n_{dt})} (\bar{u}_t)^{1-\sigma} = \lambda_t (1-\tau_n)(1-\alpha) \frac{y_{ct}}{n_{ct}} \qquad (10.25) \\
\frac{\partial V}{\partial n_{dt}} \quad &: \quad \frac{(1-\mu_c-\mu_h)}{(1-n_{ct}-n_{dt})} (\bar{u}_t)^{1-\sigma} = \lambda_t p_{ht} (1-\tau_n) \theta (1-\gamma) \frac{y_{ht}}{n_{dt}} \qquad (10.26) \\
\frac{\partial V}{\partial k_{c,t+1}} \quad &: \quad \beta E_t [\lambda_{t+1} [(1-\tau_k) \alpha \frac{y_{c,t+1}}{k_{c,t+1}} + (1-\delta_k) + \tau_k \delta_k]] = \eta \lambda_t \qquad (10.27) \\
\frac{\partial V}{\partial k_{d,t+1}} \quad &: \quad \beta E [\lambda_{t+1} [\gamma \theta (1-\tau_k) p_{h,t+1} \frac{y_{h,t+1}}{k_{d,t+1}} + (1-\delta_k) + \tau_k \delta_k]] = \eta \lambda_t \qquad (10.28)
\end{aligned}
$$

Efficiency condition 10.23 can also rewritten as

$$
\begin{aligned}
&c_t + \eta k_{c,t+1} + \eta k_{d,t+1} + p_{ht}(\omega_1 s_{1t} + \omega_2 s_{2t}) + \eta p_{ht} h_{t+1} \\
= \quad &(1-\tau_k)[\alpha y_{ct} + p_{ht} \gamma \theta y_{ht}] + (1-\tau_n)[(1-\alpha) y_{ct} + (1-\gamma) \theta p_{ht} y_{ht}] \\
&+ ((1-\delta_k) + \tau_k \delta_k)[k_{ct} + k_{dt}] \\
&+ (1-\theta) p_{ht} y_{ht} + p_{ht}(s_{1t} + (1-\delta_h) h_t) + \upsilon_t \qquad (10.29)
\end{aligned}
$$

10.2.3 Steady State Equations

From the FOC's we get the following equilibrium balanced growth path equations, where $t's$ subscripts for time are omitted here:

$$\begin{aligned}
\frac{\mu_c}{c}(\bar{u})^{1-\sigma} &= \lambda && (10.30)\\
\frac{\mu_h}{h}(\bar{u})^{1-\sigma} &= \lambda p_h \eta[\beta^{-2}\omega_2 + \beta^{-1}\omega_1 - \frac{1}{\eta}(1-\delta_h)] && (10.31)\\
\frac{(1-\mu_c-\mu_h)}{(1-n_c-n_d)}(\bar{u})^{1-\sigma} &= \lambda(1-\tau_n)(1-\alpha)\frac{y_c}{n_c} && (10.32)\\
\frac{(1-\mu_c-\mu_h)}{(1-n_c-n_d)}(\bar{u})^{1-\sigma} &= \lambda(1-\tau_n)(1-\gamma)\theta p_h \frac{y_h}{n_d} && (10.33)\\
(1-\tau_k)\alpha\frac{y_c}{k_c} &= \frac{\eta}{\beta} - (1-\delta_k) - \tau_k\delta_k && (10.34)\\
(1-\tau_k)p_h\gamma\theta\frac{y_h}{k_d} &= \frac{\eta}{\beta} - (1-\delta_k) - \tau_k\delta_k && (10.35)\\
y_c + p_h y_h &= c + [\eta - (1-\delta_k)](k_c+k_d) + (\eta-(1-\delta_h))p_h h && (10.36)\\
(\eta+\delta_h-1)h &= y_h && (10.37)\\
(\eta+\delta_h-1)h &= s_2 && (10.38)\\
p_l &= p_h(1-\theta)y_h && (10.39)\\
p_d &= \theta p_h \frac{y_h}{y_d} && (10.40)\\
\upsilon &= \tau_n((1-\alpha)y_c + p_h(1-\gamma)\theta y_h) + \\
& \quad \tau_k(\alpha y_c + p_h\gamma\theta y_h) - \delta_k\tau_k(k_c+k_d) && (10.41)\\
y_c &= k_c^{\alpha} n_c^{1-\alpha} && (10.42)\\
y_d &= k_d^{\gamma} n_d^{(1-\gamma)} && (10.43)\\
y_h &= y_d^{\theta} && (10.44)
\end{aligned}$$

10.3 Appendix C

10.3.1 Growth Rates

From (10.17) we get:

$$\begin{aligned}
\frac{\mu_c}{c_t}(\bar{u}_t)^{1-\sigma} &= \lambda_t \\
\frac{\mu_c}{c_{t+1}}(\bar{u}_{t+1})^{1-\sigma} &= \lambda_{t+1} \Rightarrow \\
\frac{\lambda_{t+1}}{\lambda_t} &= \frac{c_t}{c_{t+1}}(\frac{\bar{u}_t}{\bar{u}_{t+1}})^{1-\sigma} \Leftrightarrow \\
g_\lambda &= \frac{(g_{\bar{u}})^{1-\sigma}}{g_c} \qquad (10.45)
\end{aligned}$$

with $\frac{X_{t+1}}{X_t} = g_x$, the gross growth rate of variable X.

Since the amount of time devoted to work has to be between zero and one, the only feasible constant growth rate for labor n is zero, that is the gross growth rate is $g_n = 1$. But this does also imply that the only admissible constant growth rate for both, n_c and n_d has to be zero. Otherwise, if labor in one sector would grow then it would have to shrink in the other sector until no more labor is used in that sector. Thus we can state that

$$g_{n_c} = g_{n_d} = g_n = 1 \qquad (10.46)$$

If labor is growing at a rate of zero then leisure is growing at the same rate as well, i.e. $g_{(1-n)} = 1$.

From the commodity resource constraint of the consumption good sector (net of taxes, since they can be substituted out),

$$c_t + \eta k_{c,t+1} = y_{ct} + (1 - \delta_k)k_{ct}$$

dividing this equation by k_{ct} we obtain

$$\frac{c_t}{k_{ct}} + \eta\frac{k_{c,t+1}}{k_{ct}} = \frac{y_{ct}}{k_{ct}} + (1-\delta_k) \tag{10.47}$$

from the FOC for $k_{c,t+1}$ we know that (expectation operator omitted)

$$\frac{1}{(1-\tau_k)\alpha}\left[\frac{\lambda_t}{\lambda_{t+1}}\frac{\eta}{\beta} - (1-\delta_k) - \tau_k\delta_k\right] = \frac{y_{c,t+1}}{k_{c,t+1}}$$

since $\frac{\lambda_t}{\lambda_{t+1}} = \frac{g_c}{(g_{\bar{u}})^{\sigma-1}}$, and we assume that all grow rates are constant we know the right hand side (r.h.s.) of this equation is constant, i.e.

$$\frac{1}{(1-\tau_k)\alpha}\left[\frac{g_c}{(g_{\bar{u}})^{\sigma-1}}\frac{\eta}{\beta} - (1-\delta_k) - \tau_k\delta_k\right] \equiv C_1 = \frac{y_{c,t+1}}{k_{c,t+1}} \tag{10.48}$$

where C_1 stands for constant which is of course the same in period t. If we plug that in (10.47) we obtain

$$C_1 + (1-\delta_k) - \eta g_{k_c} = \frac{c_t}{k_{ct}} \tag{10.49}$$

since we know that the left hand side (l.h.s) of (10.49) is constant we can conclude that the r.h.s. must be constant as well. Thus consumption, c_t, and capital in the consumption good sector, k_{ct}, must grow at the same rate, i.e.

$$g_c = g_{k_c}$$

From (10.48) we know that since $g_{y_c} = g_{k_c}$ it is also true that $g_{y_c} = g_c$. So it follows immediately that the growth rate of overall investment i_t is also the same since

$$\begin{aligned} c_t + i_t &= y_{ct} \Leftrightarrow \\ \frac{i_t}{c_t} &= \frac{y_{ct}}{c_t} - 1 \Rightarrow \\ g_i &= g_c \end{aligned}$$

From (10.25) we know that

$$\frac{(1-\mu_c-\mu_h)(\bar{u}_t)^{1-\sigma}n_{ct}}{(1-\tau_n)(1-\alpha)(1-n_t)} = \lambda_t y_{ct}$$

and the update for one period is given by

$$\frac{(1-\mu_c-\mu_h)(\bar{u}_{t+1})^{1-\sigma}n_{c,t+1}}{(1-\tau_n)(1-\alpha)(1-n_{t+1})} = \lambda_{t+1} y_{c,t+1}$$

dividing the later equation by the former gives

$$\begin{aligned}
\frac{(1-n_t)}{(1-n_{t+1})}\frac{n_{c,t+1}}{n_{ct}}\left(\frac{\bar{u}_{t+1}}{\bar{u}_t}\right)^{1-\sigma} &= \frac{\lambda_{t+1}}{\lambda_t}\frac{y_{c,t+1}}{y_{ct}} \Leftrightarrow \\
\frac{g_{n_c}(g_{\bar{u}})^{1-\sigma}}{g_{(1-n)}} &= g_\lambda g_{y_c} \Leftrightarrow \\
\frac{(g_{\bar{u}})^{1-\sigma}}{g_\lambda} &= g_c = g_{y_c}
\end{aligned}$$

A similar growth rate can be obtained for $p_{ht}y_{ht}$ since from (10.26) and the one period updated equation we obtain

$$\frac{(g_{\bar{u}})^{1-\sigma}}{g_\lambda} = g_c = g_{p_h y_h}$$

thus $p_{ht}y_{ht}$ is growing at the same rate as c_t.

Using the production function of the consumption good sector (equ. 10.11) we obtain

$$g_{y_c} = \alpha g_{k_c} + (1-\alpha)g_z$$

since $g_{n_c} = 1$. And since $g_{y_c} = g_{k_c}$ we can conclude that

$$g_z = g_{y_c} = g_{k_c}$$

From (10.22) we know that

$$\frac{\beta\eta\frac{g_c}{(g_{\bar{u}})^{1-\sigma}} - (1-\delta_k) - \tau_k\delta_k}{\gamma\theta(1-\tau_k)} = C_2 = \frac{p_{h,t+1}y_{h,t+1}}{k_{d,t+1}}$$

with C_2 stands for a constant. From that we can conclude that

$$g_{k_d} = g_{p_h y_h}.$$

To show the growth rates of investment in the different sectors I use the law of motion for the capital stocks in sector $j = c, d$

$$\begin{aligned} k_{j,t+1} &= i_{jt} + (1-\delta_k)k_{jt} \Leftrightarrow \\ \frac{k_{j,t+1}}{k_{jt}} &= \frac{i_{jt}}{k_{jt}} + (1-\delta_k) \Leftrightarrow \\ g_{k_j} - (1-\delta_k) &= \frac{i_{jt}}{k_{jt}} \end{aligned}$$

since the l.h.s. is constant at the steady state we conclude that

$$g_{i_c} = g_{k_c} = g_{i_d} = g_{k_d} = g_z.$$

If we consider efficiency condition (10.11)

$$\begin{aligned} \alpha\frac{y_{ct}}{k_{ct}} &= r_t \Rightarrow \\ g_r &= 1 \end{aligned}$$

since y_c and k_c grow at the same rate. From condition (10.13)

$$\gamma\frac{p_{dt}y_{dt}}{k_{dt}} = r_t$$

we can then conclude that

$$g_{p_d y_d} = g_{k_d}.$$

From condition (10.12)

$$(1-\alpha)\frac{y_{ct}}{n_{ct}} = w_t$$

we obtain that $g_w = g_{y_c}$, since $g_{n_c} = 1$.

For the growth rate of new houses h we get:

$$\begin{aligned}
\eta h_{t+1} &= s_{1t} + (1-\delta_h)h_t \Rightarrow \\
\frac{h_{t+1}}{h_t} &= \frac{s_{1t}}{h_t} + (1-\delta_h) \Leftrightarrow \\
g_h - (1-\delta_h) &= \frac{s_{1t}}{h_t} \Rightarrow \\
g_h &= g_{s_1}
\end{aligned}$$

since the l.h.s. is constant by definition. If we update the last equation one period we also get $g_{s_2} = g_h$. From the resource constraint

$$y_{ht} = \omega_1 s_{1t} + \omega_2 s_{2t}$$

we can see that it must hold that

$$g_{y_h} = g_h.$$

From efficiency condition (10.15) we know that

$$p_{ht}(1-\theta)y_{ht} = p_{lt}x_{lt}$$

so, we see that

$$g_{p_h y_h} = g_{p_l x_l}.$$

From the production function for y_h (equ. 10.6) we obtain

$$g_{y_h} = g_{y_d}^{\theta} g_{x_l}^{1-\theta}$$

The growth rate of y_d can be obtained from the production function of the RESI good

$$\begin{aligned} y_{dt} &= k_{dt}^{\gamma}(z_t n_{dt})^{(1-\gamma)} \Rightarrow \\ g_{y_d} &= g_{k_d}^{\gamma} g_z^{(1-\gamma)} \Leftrightarrow \\ g_{y_d} &= g_z \end{aligned}$$

since k_d and z grow at the same rate, and labor is growing at gross rate of 1.

Since land is in fixed supply and the population is growing by the rate η, land per capita is shrinking over time by the rate

$$g_{x_l} = \eta^{-1}.$$

Concerning government spending, since it is assumed that $G_t = 0$, for all t we are interested in the growth rate of the transfer Υ_t:

$$\begin{aligned} \Upsilon_t &= \tau_n w_t n_t + \tau_k r_t k_t - \delta_k \tau_k k_t \Leftrightarrow \\ \frac{\Upsilon_t}{k_t} &= \frac{\tau_n w_t n_t}{k_t} + \tau_k (r_t - \delta_k) \end{aligned}$$

Since we know that the growth rate of r and n is zero and w and k are growing at the same rate, it must be that Υ is growing at the same rate as k. So, we obtain

$$g_{\Upsilon} = g_z.$$

To conclude, we have the following gross growth rates:

$$\begin{aligned} g_z &= g_c = g_{k_c} = g_{k_d} = g_k = g_{y_c} = g_{p_d y_d} = g_{p_h y_h} = \\ g_{i_c} &= g_{i_d} = g_i = g_{s_1} = g_{s_2} = g_w = g_{\Upsilon} \end{aligned}$$

$$
\begin{aligned}
g_{n_c} &= g_{n_d} = g_n = r = 1 \\
g_{x_l} &= \eta^{-1} \\
g_{y_h} &= g_h = g_{s_1} = g_{s_2} = g_z^{\theta} g_{x_l}^{1-\theta} \\
g_{p_h} &= \left(\frac{g_z}{g_{x_l}}\right)^{(1-\theta)} \\
g_{p_l} &= \frac{g_z}{g_{x_l}}
\end{aligned}
$$

10.4 Appendix D

10.4.1 Solution Procedure

The first-order conditions derived above constitutes a system of stochastic difference equations. Since the Schur factorization used to solve the system is designed for first-order systems I have to define new variables to transform equation 10.18. To that end I define

$$
\begin{aligned}
h_{1t} &= h_t \\
h_{2t} &= h_{1,t+1} \Rightarrow \\
h_{2,t+1} &= h_{t+2} \\
x_{1t} &= \lambda_t p_{ht} \\
x_{2t} &= x_{1,t+1} \Rightarrow \\
x_{2,t+1} &= \lambda_{t+2} p_{h,t+2} \\
\bar{u}_{1t} &= \bar{u}_t \\
\bar{u}_{2t} &= \bar{u}_{1,t+1} \Rightarrow \\
\bar{u}_{2,t+1} &= \bar{u}_{t+2}
\end{aligned}
$$

Using these new variables and substituting out for λ_t by the relation $\lambda_t = \frac{X_{1t}}{p_{ht}}$ we obtain the following system of first-order stochastic difference equations (written in the ordering as used to solve the system in Matlab, and expectation operators omitted)

$$\frac{\mu_c}{c_t}(\bar{u}_t)^{1-\sigma} = \frac{x_{1t}}{p_{ht}} \tag{10.50}$$

$$\frac{(1-\mu_c-\mu_h)}{(1-n_{ct}-n_{dt})}(\bar{u}_t)^{1-\sigma} = \frac{x_{1t}}{p_{ht}}(1-\tau_n)(1-\alpha)\frac{y_{ct}}{n_{ct}} \tag{10.51}$$

$$\frac{(1-\mu_c-\mu_h)}{(1-n_{ct}-n_{dt})}(\bar{u}_t)^{1-\sigma} = x_{1t}(1-\tau_n)\theta(1-\gamma)\frac{y_{ht}}{n_{dt}} \tag{10.52}$$

$$p_{lt} = p_{ht}(1-\theta)\frac{y_{ht}}{x_{lt}} \tag{10.53}$$

$$p_{dt} = \theta p_{ht}\frac{y_{ht}}{y_{dt}} \tag{10.54}$$

$$x_{lt} = 1 \tag{10.55}$$

$$y_{ct} = k_{ct}^{\alpha}(z_t n_{ct})^{(1-\alpha)} \tag{10.56}$$

$$y_{dt} = k_{dt}^{\gamma}(z_t n_{dt})^{(1-\gamma)} \tag{10.57}$$

$$y_{ht} = y_{dt}^{\theta}x_{lt}^{(1-\theta)} \tag{10.58}$$

$$y_{ht} = (\omega_1 s_{1t} + \omega_2 s_{2t}) \tag{10.59}$$

$$\eta h_{2t} = s_{1t} + (1-\delta_h)h_{1t} \tag{10.60}$$

$$\bar{u}_{1t} = c_t^{\mu_c}h_t^{\mu_h}(1-n_{ct}-n_{dt})^{(1-\mu_c-\mu_h)} \tag{10.61}$$

$$\begin{aligned} \upsilon_t &= \tau_n[(1-\alpha)y_{ct} + (1-\gamma)\theta p_{ht}y_{ht}] \\ &\quad +\tau_k[\alpha y_{ct} + \gamma\theta p_{ht}y_{ht}] - \tau_k\delta_k[k_{ct}+k_{dt}] \end{aligned} \tag{10.62}$$

$$\eta\frac{x_{1t}}{p_{ht}} = \beta[\frac{x_{1,t+1}}{p_{h,t+1}}[(1-\tau_k)\alpha\frac{y_{c,t+1}}{k_{c,t+1}} + (1-\delta_k) + \tau_k\delta_k]] \tag{10.63}$$

$$\eta\frac{x_{1t}}{p_{ht}} = \beta[\frac{x_{1,t+1}}{p_{h,t+1}}[\gamma\theta(1-\tau_k)p_{h,t+1}\frac{y_{h,t+1}}{k_{d,t+1}} + (1-\delta_k) + \tau_k\delta_k]] \tag{10.64}$$

$$h_{2t} = h_{1,t+1} \tag{10.65}$$

$$s_{2t} = s_{1,t+1} \tag{10.66}$$

$$\begin{aligned} & c_t + \eta k_{c,t+1} + \eta k_{d,t+1} + p_{ht}(\omega_1 s_{1t} + \omega_2 s_{2t}) + \eta p_{ht} h_{2t} \\ = \; & y_{ct} + p_{ht} y_{ht} + (1-\delta_k)[k_{ct} + k_{dt}] + p_{ht}(s_{1t} + (1-\delta_h) h_{1t}) \end{aligned} \tag{10.67}$$

$$x_{2t} = x_{1,t+1} \tag{10.68}$$

$$\bar{u}_{2t} = \bar{u}_{1,t+1} \tag{10.69}$$

$$\frac{\mu_h}{h_{2,t+1}} (\bar{u}_{2,t+1})^{1-\sigma} = \frac{\eta}{\beta^2} x_{1t}\omega_2 + \frac{\eta}{\beta} x_{2t}\omega_1 - x_{2,t+1}(1-\delta_h) \tag{10.70}$$

$$z_t = z_{t-1}^{\rho} \tag{10.71}$$

This system of equations is log-linearized around the steady state such that we obtain a system of linear stochastic difference equations which can be stated as the following system:[3]

$$C_u \mathbf{u}_t = C_{x\pi} \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\pi}_t \end{bmatrix} + C_z z_t \tag{10.72}$$

$$\begin{aligned} D_{x\pi} E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \boldsymbol{\pi}_{t+1} \end{bmatrix} + F_{x\pi} \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\pi}_t \end{bmatrix} = \; & D_u E_t \mathbf{u}_{t+1} + F_u \mathbf{u}_t \\ & + D_z E_t z_{t+1} + F_z z_t \end{aligned} \tag{10.73}$$

with $\mathbf{u}_t$ the vector containing the variables determined within period t as linear functions of the model's state variables, this is the vector $\{c_t,\ n_{ct},\ n_{dt},\ p_{lt},\ p_{dt},\ x_{lt},\ y_{ct},\ y_{dt},\ y_{ht},\ s_{2t},\ h_{2t},\ v_t,\ \bar{u}_{1t}\}$. Then we distinguish between three kind of state variables: those with given initial conditions build the vector $\mathbf{x}_t = \{k_{ct},\ k_{dt}, h_{1t},\ s_{1t}\}$, those variables whose initial values may be chosen freely, the vector $\pi_t = \{x_{1t},\ x_{2t},\ \bar{u}_{2t},\ p_{ht}\}$, and the purely exogenous stochastic variable z_t.

[3] There are several ways to state and solve such a system. Classical references in this context are e.g. Blanchard and Kahn (1980), Farmer (1993), Uhlig (1999) and King and Watson (2002). However, I follow here closely the presentation of Heer and Maussner (2005), Chapter 2.

10.4.2 The Reduced System

The system (10.72)-(10.73)can be reduced and rewritten as:

$$E_t \begin{bmatrix} \mathbf{x}_{t+1} \\ \boldsymbol{\pi}_{t+1} \end{bmatrix} = W \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\pi}_t \end{bmatrix} + Rz_t \tag{10.74}$$

with:

$$\begin{aligned} W &= -(D_{x\boldsymbol{\pi}} - D_u C_u^{-1} C_{x\boldsymbol{\pi}})^{-1}(F_{x\boldsymbol{\pi}} - F_u C_u^{-1} C_{x\boldsymbol{\pi}}) & (10.75) \\ R &= (D_{x\boldsymbol{\pi}} - D_u C_u^{-1} C_{x\boldsymbol{\pi}})^{-1} \\ &\quad \times [(D_z + D_u C_u^{-1} C_z)\Pi + (F_z + F_u C_u^{-1} C_z)] & (10.76) \end{aligned}$$

$$\text{and } E_t z_{t+1} = b z_t \tag{10.77}$$

with b the AR coefficient for the shock process. This system is solved via Schur factorization written as:

$$S = T^{-1} W T \tag{10.78}$$

with S an upper triangular matrix. The absolute values of the eigenvalues of W are assumed to appear in ascending order on the main diagonal of S. To obtain a unique solution, $n(x)$ eigenvalues must have modulus smaller then unity and $n(\pi)$ eigenvalues must have modulus greater than one, with $n(y)$ denoting the dimension of vector y. The transformation matrix T is known to have the following properties:

$$\begin{aligned} T' &= T^{-1} \\ TT' &= TT^{-1} = I \end{aligned}$$

so S is the given by:

$$
\begin{aligned}
S &= \begin{bmatrix} S_{xx} & S_{x\boldsymbol{\pi}} \\ 0 & S_{\lambda\boldsymbol{\pi}} \end{bmatrix} \\
&= \begin{bmatrix} T^{xx} & T^{x\boldsymbol{\pi}} \\ T^{\boldsymbol{\pi}x} & T^{\boldsymbol{\pi\pi}} \end{bmatrix} \begin{bmatrix} W_{xx} & W_{x\boldsymbol{\pi}} \\ W_{\boldsymbol{\pi}x} & W_{\boldsymbol{\pi\pi}} \end{bmatrix} \begin{bmatrix} T_{xx} & T_{x\boldsymbol{\pi}} \\ T_{\boldsymbol{\pi}x} & T_{\boldsymbol{\pi\pi}} \end{bmatrix} \qquad (10.79)
\end{aligned}
$$

We can define new variables as:

$$
\begin{bmatrix} \widetilde{\mathbf{x}}_t \\ \widetilde{\boldsymbol{\pi}}_t \end{bmatrix} = \begin{bmatrix} T^{xx} & T^{x\boldsymbol{\pi}} \\ T^{\boldsymbol{\pi}x} & T^{\boldsymbol{\pi\pi}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\pi}_t \end{bmatrix} \qquad (10.80)
$$

so we write the dynamic system (10.74) as:

$$
E_t \begin{bmatrix} \widetilde{\mathbf{x}}_{t+1} \\ \widetilde{\boldsymbol{\pi}}_{t+1} \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{x\boldsymbol{\pi}} \\ 0 & S_{\boldsymbol{\pi\pi}} \end{bmatrix} \begin{bmatrix} \widetilde{\mathbf{x}}_t \\ \widetilde{\boldsymbol{\pi}}_t \end{bmatrix} + \begin{bmatrix} Q_x \\ Q_\pi \end{bmatrix} z_t \qquad (10.81)
$$

with

$$
Q = [Q_x, Q_\pi]' = T^{-1}R \qquad (10.82)
$$

To solve the system (10.81) we consider the second line first:

$$
E_t \widetilde{\boldsymbol{\pi}}_{t+1} = S_{\pi\pi}\widetilde{\boldsymbol{\pi}}_t + Q_{\boldsymbol{\pi}} z_t \qquad (10.83)
$$

Since this is a system of $\widetilde{\boldsymbol{\pi}}$ alone the solution is given by:

$$
\widetilde{\boldsymbol{\pi}}_t = \Phi z_t \qquad (10.84)
$$

The rows of the matrix Φ are computed in the following steps: we take the matrix $S_{\boldsymbol{\pi\pi}}$, which is upper triangular with its eigenvalues, μ_i, on the diagonal being larger then one in modulus. In our case we have $\boldsymbol{\pi}_t = (x_{1t}, x_{2t}, \bar{u}_{2t}, p_{ht})'$, so we have a matrix $S_{\boldsymbol{\pi}\boldsymbol{\pi}}$ of the form:

$$S_{\boldsymbol{\pi}\boldsymbol{\pi}} = \begin{bmatrix} \mu_1 & s_{12} & s_{13} & s_{14} \\ 0 & \mu_2 & s_{23} & s_{24} \\ 0 & 0 & \mu_3 & s_{34} \\ 0 & 0 & 0 & \mu_4 \end{bmatrix} \tag{10.85}$$

Since we have the conditions for eigenvalues fulfilled, we know that the last line of (10.83) is a stochastic difference equation in the single variable p_{ht} :

$$E_t \widetilde{p}_{h,t+1} = \mu_4 \widetilde{p}_{h,t} + q_4 z_t \tag{10.86}$$

where q_4 is the fourth element in Q_π. Given the sequence

$$\{E_t \widetilde{p}_{h,t+\tau}\}_{\tau=0}^{\infty}$$

is bounded, we know that the sequence

$$\left\{\frac{1}{\mu^\tau} E_t \widetilde{p}_{h,t+\tau}\right\}_{\tau=0}^{\infty}$$

will converge to zero. Since all variables are percentage deviations from the steady state, we know that this price sequence is bounded and thus, the solution to equation (10.86) is given by

$$\widetilde{p}_{ht} = \phi_4 z_t$$

with ϕ_4 the fourth element in Φ. To figure out this element we substitute this solution into equation (10.86):

$$\begin{aligned} E_t(\phi_4 z_{t+1}) &= \mu_4 \phi_4 z_t + q_4 z_t \Leftrightarrow \\ (\phi_4 b - \phi_4 \mu_4) z_t &= q_4 z_t \Leftrightarrow \\ \phi_4 (b - \mu_4) z_t &= q_4 z_t \end{aligned}$$

because of (10.77). This then gives us the solution to the unknown element ϕ_4 :

$$\phi_4 = q_4(b-\mu_4)^{-1}$$

since $b < 1$ by assumption. Thus, we have the solution to the last row in (10.83) given by :

$$\widetilde{p}_{ht} = q_4(b-\mu_4)^{-1} z_t \tag{10.87}$$

This result is used for the solution of the third row of (10.83):

$$\begin{aligned} E_t \widetilde{\widetilde{u}}_{t+1} &= \mu_3 \widetilde{\widetilde{u}}_t + s_{34} \widetilde{p}_{ht} + q_3 z_t \\ &= \mu_3 \widetilde{\widetilde{u}}_t + s_{34} q_4 (b-\mu_4)^{-1} z_t + q_3 z_t \end{aligned}$$

again we substitute the solution, $\widetilde{\widetilde{u}}_t = \phi_3 z_t$, updated one period on the l.h.s. to get:

$$\begin{aligned} E_t(\phi_3 z_{t+1}) &= \mu_3 \phi_3 z_t + s_{34} q_4 (b-\mu_4)^{-1} z_t + q_3 z_t \Leftrightarrow \\ \phi_3 b z_t &= \mu_3 \phi_3 z_t + s_{34} q_4 (b-\mu_4)^{-1} z_t + q_3 z_t \Rightarrow \\ \phi_3 &= (s_{34} q_4 (b-\mu_4)^{-1} + q_3)(b-\mu_3)^{-1} \end{aligned}$$

so we get the solution for $\widetilde{\widetilde{u}}_t$ as:

$$\widetilde{\widetilde{u}}_t = (s_{34} q_4 (b-\mu_4)^{-1} + q_3)(b-\mu_3)^{-1} z_t \tag{10.88}$$

Repeating the steps from above we find the solutions for the remaining variables of system (10.83).

10.4.3 Policy Functions for $\boldsymbol{\pi}_t$

Given the solution for $\widetilde{\boldsymbol{\pi}}_t$ we can use system (10.80) to find the solutions for $\boldsymbol{\pi}_t$ in terms of $\mathbf{x}_t$ and z_t. The second part of (10.80) is given by:

$$\widetilde{\pi}_t = T^{\pi x} \mathbf{x}_t + T^{\pi\pi} \boldsymbol{\pi}_t \tag{10.89}$$

together with (10.84) this gives:

$$\begin{aligned}
\Phi z_t &= T^{\pi x}\mathbf{x}_t + T^{\pi\pi}\boldsymbol{\pi}_t \Rightarrow \\
\boldsymbol{\pi}_t &= \underbrace{-(T^{\pi\pi})^{-1}T^{\pi x}}_{J_{\pi x}}\mathbf{x}_t + \underbrace{(T^{\pi\pi})^{-1}\Phi}_{J_{\pi z}} z_t \Leftrightarrow \\
\boldsymbol{\pi}_t &= J_{\pi x}\mathbf{x}_t + J_{\pi z} z_t.
\end{aligned} \tag{10.90}$$

10.4.4 Policy Functions for $\mathbf{x}_{t+1}$

The first part of of (10.74) can be written as:

$$\mathbf{x}_{t+1} = W_{xx}\mathbf{x}_t + W_{x\pi}\boldsymbol{\pi}_t + R_x z_t$$

with (10.90) this gives:

$$\begin{aligned}
\mathbf{x}_{t+1} &= W_{xx}\mathbf{x}_t + W_{x\pi}((T^{\pi\pi})^{-1}\Phi z_t - (T^{\pi\pi})^{-1}T^{\pi x}\mathbf{x}_t) + R_x z_t \\
&= \underbrace{(W_{xx} - W_{x\pi}(T^{\pi\pi})^{-1}T^{\pi x})}_{J_{xx}}\mathbf{x}_t + \underbrace{(W_{x\pi}(T^{\pi\pi})^{-1}\Phi + R_x)}_{J_{xz}} z_t.
\end{aligned} \tag{10.91}$$

10.4.5 Policy Functions for $\mathbf{u}_t$

It is assumed that equation (10.72) can be solved for $\mathbf{u}_t$, i.e.

$$\mathbf{u}_t = C_u^{-1}C_{x\pi}\begin{bmatrix} \mathbf{x}_t \\ \boldsymbol{\pi}_t \end{bmatrix} + C_u^{-1}C_z\mathbf{z}_t. \tag{10.92}$$

Using the solutions for $\mathbf{x}_t$ and $\boldsymbol{\pi}_t$ we get from (10.92) the following policy function for the vector $\mathbf{u}_t$:

$$\begin{aligned}
\mathbf{u}_t &= C_u^{-1} C_{x\pi} \begin{bmatrix} \mathbf{x}_t \\ J_{\pi x}\mathbf{x}_t + J_{\pi z} z_t \end{bmatrix} + C_u^{-1} C_z z_t \Leftrightarrow \\
&= \underbrace{C_u^{-1} C_{x\pi} \begin{bmatrix} \mathbf{I}_{n(x)} \\ J_{\pi x} \end{bmatrix}}_{J_{ux}} \mathbf{x}_t + \underbrace{\left(C_u^{-1} C_{x\pi} \begin{matrix} \mathbf{0}_{n(x)\times n(z)} \\ J_{\pi z} \end{matrix} + C_u^{-1} C_z \right)}_{J_{uz}} z_t \qquad (10.93)
\end{aligned}$$

with $I_{n(x)}$ the identity matrix of dimension $n(x)$ and $\mathbf{0}_{n(x)\times n(z)}$ the null matrix of dimension $n(x) \times n(z)$.

10.5 Appendix E

10.5.1 Matlab Code for the DSGE Model

Two-sector RBC model with a 2-period time to build in the RE sector

```
clear all;

(1)Parameter Description——————-%

tic;

alfa = 0.317;

gama = 0.197;

teta = 0.894;

mc = 0.35;

mh = 0.052017;

beta = (1/1.066)^0.25;

dk = 0.0703/4;

dh = 0.0155/4;

%annual population growth is 2.36%

eta = 1 + (0.005925);

tk = 0.15;

tn = 0.33;

w1 = 1/2; w2 = w1;

b = 1-mc-mh;

roh = 0.853;

sigma = 0.0226;
```

```
nue = 2;

gz = 1.0064;

gzz = gz + eta -1;

%——-SS values calculated by the function SSgov_2_per_ces

x0 = [0.27; 1.1; 0.73; 0.78; 0.15; 0.02; 0.34; 0.02;0.03; 0.5];

options=optimset('LargeScale', 'on', 'MaxFunEvals',10000);

[x,fval] = fsolve(@SSgov_2_per_ces,x0,options);

Cs = real(x(1));

Ls = real(x(2));

Hs = real(x(3));

Phs = real(x(4));

Ncs = real(x(5));

Nds = real(x(6));

Kcs = real(x(7));

Kds = real(x(8));

S2s = real(x(9));

Us = real(x(10));


U1s = Us; U2s = Us; Ycs = (Kcs^alfa)*(Ncs^(1-alfa)); Yds = (Kds^gama)*(Nds^(1-gama));

Yhs = Yds^teta; Pds = teta*Phs*(Yhs/Yds); Pls = Phs*(1-teta)*Yhs;

Ups = tn*((1-alfa)*Ycs + Phs*(1-gama)*teta*Yhs) + tk*(alfa*Ycs + Phs*teta*gama*Yhs)

    - dk*tk*(Kcs + Kds);
```

```
qs = (Cs/Hs)*(mh/mc); Ys = Ycs + Phs*Yhs; Xls = 1; Ics = (eta -(1-dk))*Kcs; Ids = (eta -(1-dk))*Kds;

Is = Ics + Ids; Ks = Kcs + Kds; PCs = Cs+qs*Hs; Ns = Ncs + Nds; GDPs = Ycs + Pds*Yds + qs*Hs;

Pros = GDPs/Ncs; RESIs = Phs*S2s; Ihs = RESIs; Xs = Ls*Phs; X1s = Xs; X2s = Xs; H1s = Hs;

H2s = Hs; S1s = S2s ; Zs = 1; Zds = Zs; Zcs = Zs;

%—————-System FOC's————-

%definition of variables, X is at time t, X1 at t+1, X2 = t+2, etc.

syms U11 U21 C1 Nc1 Nd1 L1 Pl1 Ph1 Yh1 Yc1 Yd1 Xl1 H11 H21 Pd1 Kc1 Kd1 S11 S21 X11
    X21 Z1 Up1 ;

syms U1 U2 C Nc Nd Pl Xl H1 Kc Yh Yc Yd Kd H2 S1 S2 Ph X1 Pd X2 L Z r w Up;

Q1 = (mc/C)*(U1^(1-nue)) - (X1/Ph);

Q2 = (b/(1-Nc-Nd))*(U1^(1-nue)) - (X1/Ph)*(1-tn)*(1-alfa)*(Yc/Nc);

Q3 = (b/(1-Nc-Nd))*(U1^(1-nue))- X1*(1-tn)*(1-gama)*teta*(Yh/Nd);

Q4 = Pl - Ph*(1-teta)*(Yh/Xl);

Q5 = Pd - Ph*teta*(Yh/Yd);

Q6 = Xl - 1;

Q7 = Yc - (Kc^alfa)*((Nc*Z)^(1-alfa));

Q8 = Yd - (Kd^gama)*((Z*Nd)^(1-gama));

Q9 = Yh - (Yd^teta)*(Xl^(1-teta));

Q10 = Yh - w1*S1 - w2*S2;

Q11 = eta*H2 - S1 - (1-dh)*H1;

Q12 = Up - tn*((1-alfa)*Yc + Ph*(1-gama)*teta*Yh)- tk*(alfa*Yc + Ph*gama*teta*Yh)
    + dk*tk*(Kc + Kd);
```

```
Q13 = U2 - (C1^mc)*(H2^mh)*((1-Nc1-Nd1)^b);

Q14 = beta*((X2/Ph1)*((1-tk)*alfa*(Yc1/Kc1) + (1-dk) + tk*dk)) - (eta*(X1/Ph));

Q15 = beta*((X2/Ph1)*((1-tk)*Ph1*gama*teta*(Yh1/Kd1) + (1-dk) + tk*dk)) - (eta*(X1/Ph));

Q16 = H2 - H11;

Q17 = S2 - S11;

Q18 = ((1-dk) + tk*dk)*(Kc + Kd) + (1-tk)*(alfa*Yc + Ph*gama*teta*Yh) + (1-tn)*((1-alfa)*Yc +

      Ph*(1-gama)*teta*Yh) + Ph*(1-teta)*Yh + Up - C - eta*Kc1 - eta*Kd1 - Ph*(w1*S1 + w2*S2)

      + Ph*(S1 +(1-dh)*H1)- eta*Ph*H2;

Q19 = X2 - X11;

Q20 = (mh/H21)*(U21^(1-nue)) - eta*(beta^(-2))*X1*w2 - eta*(beta^(-1))*X2*w1 + X21*(1-dh) ;

Q21 = U2 - U11; %U1

Q22 = log(Z1) - roh*log(Z);

control_eq = [Q1; Q2; Q3; Q4; Q5; Q6; Q7; Q8 ; Q9; Q10; Q11; Q12;Q13];

state_eq = [Q14; Q15; Q16;Q17];

prices = [Q18; Q19;Q20;Q21];

shocks = Q22;

eq = [control_eq; state_eq; prices; shocks];

% ———————— [3] Linearization proc ——————————%

xx = [C1 Nc1 Nd1 Pl1 Pd1 Xl1 Yc1 Yd1 Yh1 S21 H21 Up1 U21 Kc1 Kd1 H11 S11 Ph1 X11 X21 U11

     Z1 C Nc Nd

 Pl Pd Xl Yc Yd Yh S2 H2 Up U2 Kc Kd H1 S1 Ph X1 X2 U1 Z];

jopt = jacobian(eq,xx);
```

```
%————Evaluate each derivative at the SS

% time t:

C = Cs; Nc = Ncs; Nd = Nds; Pl = Pls; Xl = Xls; S1 = S1s; H1 = H1s; H2 = H2s; Yh = Yhs;

Kc = Kcs; Kd = Kds; S2 = S2s; Pd = Pds; Up=Ups; Yc = Ycs; Yd = Yds;

L = Ls; Ph = Phs; Z = Zs; X1 = X1s; X2 = X2s; U1 = Us; U2 = Us;

% time t+1:

C1 = Cs; Nc1 = Ncs; Nd1 = Nds; Pl1 = Pls; Xl1 = Xls; S11 = S1s; H11 = H1s; H21 = H2s;

Up1 = Ups; Kc1 = Kcs; Kd1 = Kds; S21 = S2s;Yc1 = Ycs; Yd1 = Yds;

Ph1 = Phs; Z1 = Zs; X11 = X1s; X21 = X2s; Pd1 = Pds;

Yh1 = Yhs; U11 = Us; U21 = Us;L1 = Ls;

coef = eval(jopt);

vo = [Cs Ncs Nds Pls Pds Xls Ycs Yds Yhs S2s H2s Ups U2s Kcs Kds H1s S1s Phs X1s X2s U1s Zs] ;

TW = [vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo; vo;vo; vo];

nu = 13;   %# of controls = (C, Nc, Nd, Pl, Pd, Xl, Yc, Yd, Yh, S2, H2, Up U2)

nx = 4; %# of predetermined states = ( Kc, Kd, H1, S1)

nL = 4; %# of non-pred. states = (X1, X2, Ph, U2)

nz = 1; %# of shocks = z

nxL = nx + nL; nCxL = nu + nxL; nCz = nCxL+nz;

na = nu+nx+nL+nz;

A1 = (-coef(:,1:na)).*TW;          %Matrix of derivatives at ss times ss in t+1

A0 = (coef(:,na+1:2*na)).*TW;    %Matrix of derivatives at ss times ss in t

Cu = A0(1:nu,1:nu);
```

```
CxL = A0(1:nu,nu+1:nCxL);

Cz = A0(1:nu,na);

Fu = A0(nu+1:nCxL,1:nu);

FxL = A0(nu+1:nCxL,nu+1:nCxL);

Fz = A0(nu+1:nCxL,na);

Du = A1(nu+1:nCxL,1:nu);

DxL = A1(nu+1:nCxL,nu+1:nCxL);

Dz = A1(nu+1:nCxL,na);

ut1 = inv(Cu)*CxL;

ut2 = inv(Cu)*Cz;

D1 = (DxL - (Du*(inv(Cu))*CxL));

F1 = (FxL - (Fu*(inv(Cu))*CxL));

D2 = (Dz + (Du*(inv(Cu))*Cz));

F2 = (Fz + (Fu*(inv(Cu))*Cz));

Pi = roh;

W = -(inv(D1)*F1);

R = (inv(D1))*((D2*Pi)+F2);

kappa = condeig(W);

[T S M]= cschur(W,6);

lambda = diag(S);  %eigenvalues of W

lambda = abs(lambda);

Ti = (T');
```

```
TLxi = (Ti(nx+1:size(Ti,1), 1:nx));

TLLi = (Ti(nx+1:size(Ti,1), nx+1:size(Ti,2)));

TLLii = inv(TLLi);

temp = Ti*R;

q1 = (temp(1:nx,:));

q2 = (temp(nx+1:size(temp,1),:));

nz = size(Cz,2);

phi = zeros(n1,nz);

sLL = (S(nx+1:size(S,1), nx+1:size(S,2)));

i = n1;

while i > 0

 j=i+1;

 temp = zeros(1,nz);

while j <= n1

 temp = temp + sLL(i,j)*phi(j,:);

 j = j+1;

 end

 phi(i,:) = (q2(i,:)+temp)*inv(roh-sLL(i,i)*eye(nz));

 i = i-1;

end

%Policy functions for lamda(t)(L(t)), as functions of x(t) and z(t)

JLx = -TLLii*TLxi;
```

```
JLz = TLLii*phi;

JLx = real(JLx);

JLz = real(JLz);

%Policy functions of x(t+1) as functions of x(t), z(t) and the solutions for L(t)

Jxx = T(1:nx,1:nx)*S(1:nx,1:nx)*inv(T(1:nx,1:nx));

Jxz = W(1:nx,nx+1:size(W,2))*JLz + R(1:nx,:);

Jxx = real(Jxx);

Jxz = real(Jxz);

%Policy functions for u(t) as functions of x(t),z(t) and the solutions for L(t)

Jux = ut1*vertcat(eye(nx),JLx);

Juz = ut1*vertcat(zeros(nx,nz),JLz)+ ut2;

Jux = real(Jux);

Juz = real(Juz);

n = 500;

ns =600

eps = sigma.*randn(n,ns);

z = zeros(n,ns);

H1 = zeros(n+1,ns); H2 = H1; Kc = zeros(n+1,ns); S1 = H1; S2 = H1;

Kd = zeros(n+1,ns); C = z; Nc = z; Nd = z; Ph = z; Pl = z; Xl = z; Up = z;

X1 = z; X2 = z; Yc = z; Yd = z; Ic = z; Ih = z; I = z; r = z; w = z;

 Yh = z; Pd = z; GDP = z; PC = z; RESI = z; q = z; Y = z; K = z; Id = z; z(1) = 0; N =z;

H2(1)= 0; H3(1)=0; S1(1)=0; S2(1)=0; S3(1)=0; H1(1)= 0; Kc(1) = 0; Kd(1) = 0; Pro = z;
```

```
for it = 1:ns

for t = 1:n

for j = 2:n

z(j,it) = roh*(z(j-1,it))+(eps(j,it));

end

 %policy functions for prices (Ph, X1, X2,U1) and states (Kc, Kd, H1,S1)

Ph(t,it)= JLx(1,1)*Kc(t) + JLx(1,2)*Kd(t) + JLx(1,3)*H1(t) + JLx(1,4)*S1(t)+ JLz(1,1)*z(t,it);

X1(t,it)= JLx(2,1)*Kc(t) + JLx(2,2)*Kd(t) + JLx(2,3)*H1(t) + JLx(2,4)*S1(t)+ JLz(2,1)*z(t,it);

X2(t,it)= JLx(3,1)*Kc(t) + JLx(3,2)*Kd(t) + JLx(3,3)*H1(t) + JLx(3,4)*S1(t)+ JLz(3,1)*z(t,it);

U1(t,it)= JLx(4,1)*Kc(t) + JLx(4,2)*Kd(t) + JLx(4,3)*H1(t) + JLx(4,4)*S1(t)+ JLz(4,1)*z(t,it);

Kc(t+1,it)= Jxx(1,1)*Kc(t)+ Jxx(1,2)*Kd(t)+ Jxx(1,3)*H1(t) + Jxx(1,4)*S1(t) + Jxz(1,1)*z(t,it);

Kd(t+1,it)= Jxx(2,1)*Kc(t) + Jxx(2,2)*Kd(t)+ Jxx(2,3)*H1(t) + Jxx(2,4)*S1(t) + Jxz(2,1)*z(t,it);

H1(t+1,it)= Jxx(3,1)*Kc(t) + Jxx(3,2)*Kd(t)+ Jxx(3,3)*H1(t) + Jxx(3,4)*S1(t) + Jxz(3,1)*z(t,it);

S1(t+1,it)= Jxx(4,1)*Kc(t) + Jxx(4,2)*Kd(t)+ Jxx(4,3)*H1(t) + Jxx(4,4)*S1(t) + Jxz(4,1)*z(t,it);

%policy functions for variables determined within one period C, Nc, Nd, Pl, Pd, Xl,

    Yc,Yd Yh S2, H2, Up, U2

C(t,it)= Jux(1,1)*Kc(t) + Jux(1,2)*Kd(t)+ Jux(1,3)*H1(t) + Jux(1,4)*S1(t) + Juz(1,1)*z(t,it);

Nc(t,it)= Jux(2,1)*Kc(t) + Jux(2,2)*Kd(t)+ Jux(2,3)*H1(t) + Jux(2,4)*S1(t) + Juz(2,1)*z(t,it);

Nd(t,it)= Jux(3,1)*Kc(t) + Jux(3,2)*Kd(t)+ Jux(3,3)*H1(t) + Jux(3,4)*S1(t) + Juz(3,1)*z(t,it);

Pl(t,it)= Jux(4,1)*Kc(t) + Jux(4,2)*Kd(t)+ Jux(4,3)*H1(t) + Jux(4,4)*S1(t) + Juz(4,1)*z(t,it);

Pd(t,it)= Jux(5,1)*Kc(t) + Jux(5,2)*Kd(t)+ Jux(5,3)*H1(t) + Jux(5,4)*S1(t) + Juz(5,1)*z(t,it);

Xl(t,it)= Jux(6,1)*Kc(t) + Jux(6,2)*Kd(t)+ Jux(6,3)*H1(t) + Jux(6,4)*S1(t) + Juz(6,1)*z(t,it);
```

```
Yc(t,it)= Jux(7,1)*Kc(t) + Jux(7,2)*Kd(t)+ Jux(7,3)*H1(t) + Jux(7,4)*S1(t) + Juz(7,1)*z(t,it);

Yd(t,it)= Jux(8,1)*Kc(t) + Jux(8,2)*Kd(t)+ Jux(8,3)*H1(t) + Jux(8,4)*S1(t) + Juz(8,1)*z(t,it);

Yh(t,it)= Jux(9,1)*Kc(t) + Jux(9,2)*Kd(t)+ Jux(9,3)*H1(t) + Jux(9,4)*S1(t) + Juz(9,1)*z(t,it);

S2(t,it)= Jux(10,1)*Kc(t) + Jux(10,2)*Kd(t)+ Jux(10,3)*H1(t) + Jux(10,4)*S1(t) + Juz(10,1)*z(t,it);

H2(t,it)= Jux(11,1)*Kc(t) + Jux(11,2)*Kd(t)+ Jux(11,3)*H1(t) + Jux(11,4)*S1(t) + Juz(11,1)*z(t,it);

Up(t,it)= Jux(12,1)*Kc(t) + Jux(12,2)*Kd(t)+ Jux(12,3)*H1(t) + Jux(12,4)*S1(t) + Juz(12,1)*z(t,it);

U2(t,it)= Jux(13,1)*Kc(t) + Jux(13,2)*Kd(t)+ Jux(13,3)*H1(t) + Jux(13,4)*S1(t) + Juz(13,1)*z(t,it);

%Variables determined residually

q(t,it) = C(t,it)- H1(t,it);

RESI(t,it) = Ph(t,it) + w1*S1(t,it) + w2*S2(t,it);

Ih(t,it) = RESI(t,it);

Id(t,it) = eta*(Kds/Ids)*Kd(t+1,it)- (1-dk)*(Kds/Ids)*Kd(t,it);

Ic(t,it) = eta*(Kcs/Ics)*Kc(t+1,it) - (Kcs/Ics)*(1-dk)*Kc(t,it);

Y(t,it) = (Ycs/Ys)*Yc(t,it)+ (Phs*Yhs/Ys)*(Ph(t,it) + Yh(t,it));

GDP(t,it) = (Ycs/GDPs)*Yc(t,it) + ((Pds*Yds)/GDPs)*(Pd(t,it)+ Yd(t,it))+((qs*Hs)/GDPs)*(q(t,it)+H1(t,it));

PC(t,it) = (Cs/PCs)*C(t,it) + ((qs*Hs)/PCs)*q(t,it)+ ((qs*Hs)/PCs)*H1(t,it);

K(t,it) = (Kcs/Ks)*Kc(t,it) + (Kds/Ks)*Kd(t,it);

I(t,it) = (Ycs/Is)*Yc(t,it)-(Cs/Is)*C(t,it);

N(t,it) = (Ncs/Ns).*Nc(t,it) + (Nds/Ns).*Nd(t,it);

r(t,it) = Yc(t,it) - Kc(t,it);

w(t,it) = Yc(t,it) - Nc(t,it);

L(t,it) = X1(t,it) - Ph(t,it);
```

```
end
    end
%All series are transformed into trending series by adding the trend growth of productivity
 and population, taking logs and then removing the trend by the HP filter
o2 = n - 152;
%I like to have here that 152 periods are left for the quarters from 1970 to 2007 = 4x38 = 152
o1 = n - o2;
r = r(o2:n-1,:); w = w(o2:n-1,:); L = L(o2:n-1,:); Ph = Ph(o2:n-1,:);H1 = H1(o2:n-1,:);H2 = H2(o2:n-1,:);
Kc = Kc(o2:n-1,:); Kd = Kd(o2:n-1,:);K = K(o2:n-1,:); C = C(o2:n-1,:);Nc = Nc(o2:n-1,:);
Nd = Nd(o2:n-1,:); N = N(o2:n-1,:); Pl = Pl(o2:n-1,:);Xl = Xl(o2:n-1,:);X2 = X2(o2:n-1,:);
S1 = S1(o2:n-1,:);S2 = S2(o2:n-1,:); Ic = Ic(o2:n-1,:); Ih = Ih(o2:n-1,:);I = I(o2:n-1,:);
Y = Y(o2:n-1,:); Yc = Yc(o2:n-1,:);Yh = Yh(o2:n-1,:); Up =Up(o2:n-1,:); Yd = Yd(o2:n-1,:);
GDP = GDP(o2:n-1,:);PC = PC(o2:n-1,:); q = q(o2:n-1,:);Pd = Pd(o2:n-1,:); Id = Id(o2:n-1,:);
nss = ns;
nn = o1;
for itt = 1:nss
for tt = 1:nn
PC(tt,itt) = (log(PC(tt,itt)+ gz^tt));
C(tt,itt) = (log(C(tt,itt)+ gz^tt));
GDP(tt,itt) = (log(GDP(tt,itt)+ gz^tt));
c(tt,itt) = (log(Ic(tt,itt)+ gz^tt));
Id(tt,itt) = (log(Id(tt,itt)+ gz^tt));
```

```
Ih(tt,itt) = (log(Ih(tt,itt)+ gz^tt));

I(tt,itt) = (log(I(tt,itt)+ gz^tt));

Yc(tt,itt) = (log(Yc(tt,itt)+ gz^tt));

Yd(tt,itt) = (log(Yd(tt,itt)+ gz^tt));

Y(tt,itt) = (log(Y(tt,itt)+ gz^tt));

Kc(tt,itt) = (log(Kc(tt,itt)+ gz^tt));

Kd(tt,itt) = (log(Kd(tt,itt)+ gz^tt));

K(tt,itt) = (log(K(tt,itt)+ gz^tt));

L(tt,itt) = (log(L(tt,itt)+ gz^tt));

w(tt,itt) = (log(w(tt,itt)+ gz^tt));

q(tt,itt) = (log(q(tt,itt)+ gz^tt));

K(tt,itt) = (log(K(tt,itt)+ gz^tt));

Pl(tt,itt) = (log(Pl(tt,itt)+ gz*eta));

Ph(tt,itt) = (log(Ph(tt,itt)+ ((gz*eta)^(1-teta))^tt));

Pd(tt,itt) = (log(Pd(tt,itt)+ 1));

Nc(tt,itt) = log(Nc(tt,itt)+ 1);

Nd(tt,itt) = log(Nd(tt,itt)+ 1);

N(tt,itt) = log(N(tt,itt)+1);

r(tt,itt) = log(r(tt,itt)+1);

Yh(tt,itt) = (log(Yh(tt,itt)+ ((gz^teta)*((1/eta)^(1-teta)))^tt));

S1(tt,itt) = (log(S1(tt,itt)+ ((gz^teta)*((1/eta)^(1-teta)))^tt));

S2(tt,itt) = (log(S2(tt,itt)+ ((gz^teta)*((1/eta)^(1-teta)))^tt));
```

```
H1(tt,itt) = (log(H1(tt,itt)+ ((gz^teta)*((1/eta)^(1-teta)))^tt));

H2(tt,itt) = (log(H2(tt,itt)+((gz^teta)*((1/eta)^(1-teta)))^tt));

end

end

C = real(C) - real(hpfilter(C,1600)); PC = real(PC) - real(hpfilter(PC,1600));

GDP = real(GDP) - real(hpfilter(GDP,1600)); Ic = real(Ic) - real(hpfilter(Ic,1600));

Id = real(Id) - real(hpfilter(Id,1600)); Ih = real(Ih) - real(hpfilter(Ih,1600));

I = real(I) - real(hpfilter(I,1600)); Ph = real(Ph) - real(hpfilter(Ph,1600));

Pl = real(Pl) - real(hpfilter(Pl,1600)); Pd = real(Pd) - real(hpfilter(Pd,1600));

Yc = real(Yc) - real(hpfilter(Yc,1600)); Yh = real(Yh) - real(hpfilter(Yh,1600));

Yd = real(Yd) - real(hpfilter(Yd,1600)); Y = real(Y) - real(hpfilter(Y,1600));

Kc = real(Kc) - real(hpfilter(Kc,1600)); Kd = real(Kd) - real(hpfilter(Kd,1600));

K = real(K) - real(hpfilter(K,1600)); Nc = real(Nc) - real(hpfilter(Nc,1600));

Nd = real(Nd) - real(hpfilter(Nd,1600)); N = real(N) - real(hpfilter(N,1600));

S1 = real(S1) - real(hpfilter(S1,1600)); S2 = real(S2) - real(hpfilter(S2,1600));

H1 = real(H1) - real(hpfilter(H1,1600)); H2 = real(H2) - real(hpfilter(H2,1600));

L = real(L) - real(hpfilter(L,1600)); N = real(N) - real(hpfilter(N,1600));

q = real(q) - real(hpfilter(q,1600)); K = real(K) - real(hpfilter(K,1600));

r = real(r) - real(hpfilter(r,1600)); w = real(w) - real(hpfilter(w,1600));

%this commands calculate the means and SD of each collum of the matrixes

La = mean(L); Lb = std(L); Pha = mean(Ph); Phb = std(Ph); H1a = mean(H1); H1b = std(H1);

H2a = mean(H2); H2b = std(H2); Kca = mean(Kc); Kcb = std(Kc); Kda = mean(Kd); Kdb = std(Kd);
```

```
Ka = mean(K); Kb = std(K); S1a = mean(S1); S1b = std(S1); S2a = mean(S2); S2b = std(S2);
Ca = mean(C); Cb = std(C); Nca = mean(Nc); Ncb = std(Nc); Nda = mean(Nd); Ndb = std(Nd);
Na = mean(N); Nb = std(N); X1a = mean(X1); X1b = std(X1); X2a = mean(X2); X2b = std(X2);
Pla = mean(Pl); Plb = std(Pl); Xla = mean(Xl); Xlb = std(Xl); Ica = mean(Ic); Icb = std(Ic);
Iha = mean(Ih); Ihb = std(Ih); Yca = mean(Yc); Ycb = std(Yc); Yha = mean(Yh); Yhb = std(Yh);
ra = mean(r); rb = std(r); wa = mean(w); wb = std(w); Ya = mean(Y); Yb = std(Y);
Ia = mean(I); Ib = std(I); Ida = mean(Id); Idb = std(Id); Upa = mean(Up); Upb = std(Up);
Yda = mean(Yd); Ydb = std(Yd); GDPa = mean(GDP); GDPb = std(GDP); Pda = mean(Pd);
Pdb = std(Pd); qa = mean(q); qb = std(q); PCa = mean(PC); PCb = std(PC);
%The following function "Mean_Std_f" gets the means and std's of all series, the function is in
a separeta file named Mean_Std_f
[C_bar, PC_bar, Yc_bar, Yd_bar, Yh_bar, Y_bar, GDP_bar, Ic_bar, Id_bar, Ih_bar, I_bar,
Kc_bar, Kd_bar, K_bar, S1_bar, S2_bar, H1_bar, H2_bar, Pd_bar, Ph_bar, Pl_bar,
Nc_bar, Nd_bar, L_bar, q_bar, w_bar, r_bar, N_bar Pro_bar, C_std, PC_std, Yc_std,
Yd_std, Yh_std, Y_std, GDP_std, Ic_std, Id_std, Ih_std, I_std, Kc_std, Kd_std, K_std,
S1_std, S2_std, H1_std, H2_std, Pd_std, Ph_std, Pl_std, Nc_std, Nd_std, L_std, q_std,
w_std, r_std, N_std, Pro_std ]
= Mean_Std_2per(Ca, PCa, Yca, Yda, Yha, Ya, GDPa, Ica, Ida, Iha, Ia, Kca, Kda, Ka, S1a,
S2a, H1a, H2a, Pda, Pha, Pla, Nca, Nda, La, qa, wa, ra, Na, Proa, Cb, PCb, Ycb, Ydb, Yhb, Yb,
GDPb, Icb, Idb, Ihb, Ib, Kcb, Kdb, Kb, S1b, S2b, H1b, H2b, Pdb, Phb, Plb, Ncb, Ndb, Lb, qb, wb,rb,
    Nb, Prob );
YY = [C_bar PC_bar Yc_bar Yd_bar Yh_bar Y_bar GDP_bar Ic_bar Id_bar Ih_bar I_bar Kc_bar
```

```
Kd_bar K_bar S1_bar S2_bar H1_bar H2_bar Pd_bar Ph_bar Pl_bar Nc_bar Nd_bar L_bar q_bar
w_bar r_bar N_bar Pro_bar; C_std PC_std Yc_std Yd_std Yh_std Y_std GDP_std Ic_std Id_std
Ih_std I_std Kc_std Kd_std K_std S1_std S2_std H1_std H2_std Pd_std Ph_std Pl_std Nc_std
Nd_std L_std q_std w_std r_std N_std Pro_std ]';
YYY = [(YY(:,1)./GDP_bar), (YY(:,2)./GDP_std)];
toc
```

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