Nonlinear magnetic field dependence of the emission frequency and bandwidth of a far-infrared p-type Ge cyclotron laser

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Using a high-resolution Fabry-Perot interferometer, we have studied the emission of a far-infrared p-type germanium cyclotron laser in a frequency range of strong emission (82–87 cm^{-1}). A nonlinear tuning of the emission frequency with magnetic field, a magnetic field dependent width, and a splitting of the emission line (near 85.8 cm^{-1}) were found. We present an analysis showing that the emission behavior can be understood by recognizing that the laser level separation depends not only on the magnetic-field strength, but also on the ratio of the electric and magnetic-field strengths. We further relate the observed line splitting to radiation absorption due to transitions between Zeeman-split states of shallow acceptors.

In earlier studies of the far-infrared p-type germanium cyclotron laser (FCL) a linear dependence of the emission frequency ν_L on the magnetic-field strength B has been reported.[1-3] The relation is $\nu_L = eB(2\pi m_d)^{-1}$ where m_d is the dynamic mass of light holes which is slightly larger than the classical cyclotron mass of $0.042m_e$,[4] m_e being the free-electron mass. In this paper we report on a high-resolution investigation of the tuning curve and of the linewidth of a FCL. The experimental results, together with a band-structure analysis, give evidence that the dynamic mass depends on both the electric- and the magnetic-field strengths.

Our measurements were made using a p-type germanium crystal with a boron doping concentration of 5×10^{13} cm^{-3}. The rectangular bar-shaped crystal had dimensions $37\times4\times6.5$ mm^3 and was cut with the longest dimension parallel to the [110] axis. Two opposite long sides of the sample were covered with aluminum forming Ohmic contacts. The laser crystal was placed in a superconducting solenoid which in turn was immersed in liquid helium. The magnetic field was applied parallel to the long side of the crystal and the electric field was parallel to another [110] axis. The experimental arrangement [Fig. 1(a)] is similar to that described recently,[5,6] i.e., using a $SrTiO_3$ crystal as a high reflectivity mirror. However we used a hybrid metal mesh mirror with nearly constant optical properties in the investigated spectral range as an outcoupling mirror. This mirror was made from a chess-board pattern of molybdenum squares (of 16 μm period) deposited on a sapphire substrate. A 10-μm Teflon film was placed between the metal film and the sample for insulation. The laser radiation was guided out of the cryostat by means of a stainless-steel pipe (20-mm diameter). The emission spectrum of the laser has been analyzed by use of a high-resolution Fabry-Perot interferometer consisting of two parallel 750 lines per inch copper meshes. The device was placed in a vacuum chamber to avoid water vapor absorption and had a finesse of 20 at 90 cm^{-1}. The interferometer was operated in 50th order yielding a resolution of 0.1 cm^{-1}. The radiation transmitted through the interferometer was detected by a photoconductive p-type germanium detector.

We found FCL oscillations in a frequency range of 60–100 cm^{-1}. For a detailed investigation the interval of 82–87 cm^{-1} was chosen, where laser oscillation was observed over a large range of the applied electric field. Figure 1(b) (upper curve) shows a typical frequency distribution of the laser output. The emission bandwidth was about 0.3 cm^{-1}. For a field near 4.4 T two emission lines were found [Fig. 1(b), lower curves] with a separation of ~0.5 cm^{-1}. In Fig. 1(c) we have drawn a tuning curve of laser emission for an electric-field strength of 3 kV/cm applied to the sample. The shaded band [Fig. 1(c)] shows the frequency range in which emission was

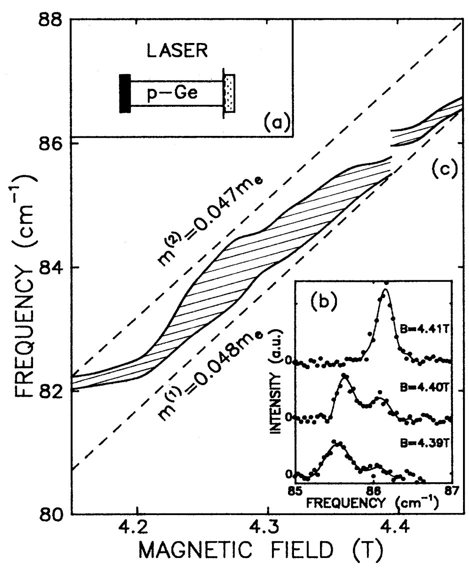

FIG. 1. (a) Sketch of the experimental arrangement; (b) laser emission for different magnetic fields; (c) emission frequency spectrum of the far-infrared p-type germanium cyclotron laser as a function of magnetic field for an applied electric field of 3 kV/cm.

found as a function of the magnetic field. The borders of the band indicate the frequencies where the intensity fell below one-half of its maximum value at constant magnetic field. The emission spectrum is narrow in the low magnetic-field region, then widens in the intermediate range but narrows again at high magnetic-field strengths. The break of the band at $B \simeq 4.4$ T reflects a splitting of the FCL line. The experimental tuning curve lies between straight lines corresponding to constant effective masses $m^{(1)}=0.048m_e$ and $m^{(2)}=0.047m_e$ [dashed lines in Fig. 1(c)].

For an analysis of the experimental results we have calculated the frequencies for the transitions between the lowest three Landau levels for light holes in crossed electric and magnetic fields. As a basis set of wave functions we have chosen the eigenfunctions of light holes in a magnetic field of the extended Luttinger Hamiltonian[7] for the three valence subbands.[8] Light holes with vanishing wave vectors parallel to the magnetic field ($k_B=0$) are most important due to the peak in the density of states. Therefore we limit our calculation to this case. For **B**$\|[110]$ one finds that the whole set of states splits into a and b sets, which do not interact with each other.[8] In the notation of Ref. 8, the basis eigenfunctions for the a and b sets can be, respectively, expressed as

$$|na+\rangle = a_{1,n}^{+}\varphi_{n-2}(\mathbf{r})u_{10}+a_{2,n}^{+}\varphi_n(\mathbf{r})u_{20}+a_{3,n}^{+}\varphi_n(\mathbf{r})u_{30}\ ,$$

$$|nb+\rangle = b_{1,n}^{+}\varphi_{n-2}(\mathbf{r})u_{10}+b_{2,n}^{+}\varphi_n(\mathbf{r})u_{20}+b_{3,n}^{+}\varphi_{n-2}(\mathbf{r})u_{30}\ ,$$

where $+$ indicates any one of three valence subbands, n is the number of Landau levels, φ_n is the harmonic-oscillator wave function of nth order, u_{10}, u_{20}, and u_{30} are the Bloch functions for the valence subbands, and $a_{i,n}^{+}$ and $b_{i,n}^{+}$ are numerical coefficients corresponding to the different levels.

In order to calculate the Stark shifts of light-hole levels, we only consider their wave functions and levels. In this approximation the Stark shift in crossed electric and magnetic fields can be determined with good accuracy,[9] as the influence of the heavy-hole and splitoff subband levels is quite small. We note that under our experimental conditions the ratio $eEa_L/(\hbar\omega_c^h)\approx 0.3$, where E is the total electric-field strength, a_L is the lattice constant of germanium, and ω_c^h is the cyclotron frequency of the heavy holes. Therefore, the heavy holes cannot be treated by the effective-mass approximation, since the criterion $eEa_L/(\hbar\omega_c^b)\ll 1$ (Ref. 10) is not fulfilled. The Stark shifts of the frequency are calculated using second-order perturbation theory; there are no first-order Stark shifts due to the inversion symmetry of germanium. Finally, two infinite matrices are obtained corresponding to light holes from the a set (H_a) and the b set (H_b). Only three diagonal lines of the matrices are different from zero.[9] The elements in the main diagonal are the energies of light-hole Landau levels without an applied electric field. We assume **E**$\|Y$ and **B**$\|Z$ where Y and Z are Cartesian coordinate axes. Then the two other diagonals, adjacent to the main one, consist of elements of the type $\langle n,a+|eEy|n-1,a+\rangle$ and $\langle n+1,a+|eEy|n,a+\rangle$. From this point on we consider only the a set, as all relations are the same for the b set. One can show that

$$\langle n,a|+eEy|n-1,a+\rangle = eEL_M\{a_{1,n}a_{1,n-1}[(n-2)/2]^{1/2}+a_{2,n}a_{2,n-1}(n/2)^{1/2}+a_{3,n}a_{3,n-1}(n/2)^{1/2}\}\ ,$$

$$\langle n+1,a+|eEy|n,a+\rangle = eEL_M\{a_{1,n+1}a_{1,n}[(n-1)/2]^{1/2}+a_{2,n+1}a_{2,n}[(n+1)/2]^{1/2}+a_{3,n+1}a_{3,n}[(n+1)/2]^{1/2}\}\ ,$$

where $L_M=(\hbar/eB)^{1/2}$ is the radius of the $n=0$ cyclotron orbit. We follow then the method used in Ref. 11 to diagonalize the infinite matrix. First a unitary transformation $H_a'=S^{-1}H_aS$ is applied, which diagonalizes the matrix exactly in the case of a simple parabolic band. Such a transformation strongly reduces the off-diagonal elements and it becomes possible to truncate the matrix H_a'. Our numerical calculations show that one can limit the matrix H_a' to eight rows and eight columns to obtain, after second diagonalization, the accurate values of the transition frequencies between the lowest three Landau levels. In Fig. 2 the frequencies of the transitions $2\rightarrow 1$ and $1\rightarrow 0$ in both the a and b sets are shown as a function of the dimensionless parameter $a_v^2=E^2(m_l^{\rm cl})^2/(e\hbar B^3)$. The frequencies were normalized to the classical cyclotron frequency with a light-hole mass $m_l^{\rm cl}=m_e/[\gamma_1+(\gamma'^2+3\gamma''^2)]^{1/2}$, where $\gamma_1(=13.38)$, $\gamma'(=5.26)$, and $\gamma''(=5.15)$ are the valence-band parameters[8] taken from Ref. 4 for **B**$\|[110]$. A comparison with the experimental frequency range (dashed in Fig. 2) shows that $1\rightarrow 0$ transitions in the b set yield reasonable values for a_v^2 and are

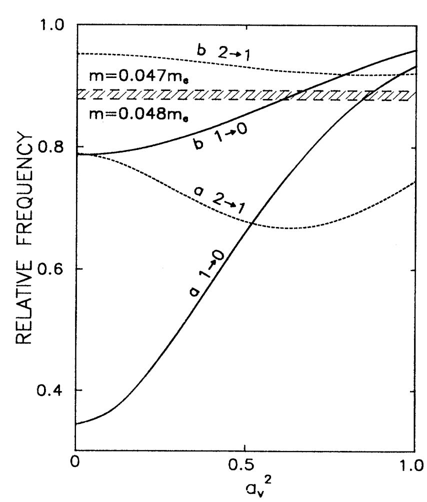

FIG. 2. The normalized frequencies of transitions in the a and b sets of Landau levels as a function of the dimensionless parameter a_v^2 and the experimental frequency range of laser emission (dashed).

therefore most likely responsible for the laser oscillation. We found a range $a_v^2=0.61 \div 0.67$ that corresponds to an electric-field strength of about 7.5 kV/cm. This value is higher than a value estimated recently for the same conditions.[5] The $1 \rightarrow 0$ transition in the a set would deliver still higher electric-field values.

We now discuss the magnetic-field dependence of the FCL emission. Coming back to Fig. 1(c) we see that the interval of the emission frequencies moves with increasing magnetic-field strength to heavier dynamic masses and becomes narrower at both ends of the magnetic-field tuning range. Figure 2 shows that the lighter masses correspond to higher values of a_v^2 and that, for a given electric field, different values of a_v^2 are possible. However, as any a_v^2 yields only one transition frequency, the different values of a_v^2 cannot explain the relatively broad emission line. We attribute the large width of the emission to the inhomogeneity of the electric field within the *p*-type germanium crystal. For a total field of 7.5 kV/cm and an applied field of 3 kV/cm we can estimate a Hall angle of $\sim 66°$. The combination of applied and Hall fields for the rectangular sample gives an inhomogeneous field that has, according to Ref. 12, values from 6.7 kV/cm up to 8.4 kV/cm around a circle of diameter 0.4 cm in the center of our 0.4-cm-thick germanium crystal. Thus we can estimate an amplification range of the FCL for the given applied electric field. This range (Fig. 3) is an overlap of the band $0.61 < a_v^2 < 0.67$ and a band of actual electric fields inside the crystal. The width of the electric-field band (Fig. 3) was chosen corresponding to a 3% inhomogeneity assuming that amplification does not take place in the whole volume of the sample. The total increase of this band with rising magnetic field is due to higher values of the Hall field for higher magnetic fields.[5] Thus we get the same behavior of the amplification band as has been observed in the experiment: it is broader in the middle of the obtained magnetic-field range and becomes narrower tending to either lower values or higher values of a_v^2 (corresponding to heavier and lighter masses) at lower and higher magnetic-field strengths, respectively.

We note that the present model cannot explain all details of the FCL line behavior because we did not take into account the intrinsic widths of the laser levels. These widths are mainly determined by broadening due to the hole distribution in the projection of the hole momentum parallel to the magnetic field ($k_B \neq 0$), by interaction of the light holes with the heavy-hole subband, and by scattering. These processes may explain, for example, a shift of the FCL line to lower frequencies if the applied electric field increases. This effect was observed only for high magnetic fields. Additional investigations would be necessary to make a conclusion about the nature of this shift.

We finally discuss the splitting of the FCL line near 85.8 cm^{-1} (Fig. 1). We propose that in this case the Zeeman-split states of acceptors are important. Figure 4 shows the positions of the FCL line and some absorption lines of boron impurities, reported in Ref. 13, as a function of magnetic field. The C_4 line and G group of lines correspond to transitions between shallow acceptor levels of different symmetry.[14] Solid and dashed parts of the straight line of Fig. 4 show the regions where the FCL emission appears and disappears, respectively.[1–3,5,15] One can see that the C_4 line may be responsible for the splitting we observed. We suggest that the frequency gap of the laser emission occurs because of absorption of radiation by transitions between the impurity levels. The experiment gives a slightly higher value of the frequency of the singularity (85.8 cm^{-1}) than we estimate (Fig. 4) for C_4(~ 82 cm^{-1}) at the same magnetic field. The discrepancy may be due to the fact that our estimation was performed with data[13] obtained at zero electric field. Another interesting point concerns the range of the magnetic field near 2.7 T where the FCL line crosses the group of G lines: just near this field a gap was observed for the FCL emission.[3,15]

In conclusion, we have observed a deviation of the

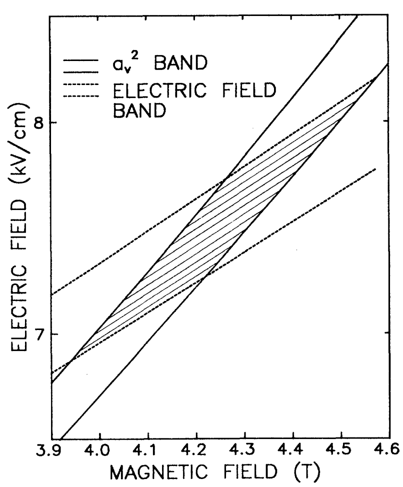

FIG. 3. Calculated range of the laser amplification (cross-hatched) for $0.61 < a_v^2 < 0.67$, with a total electric-field strength on the crystal axis of 7.5 kV/cm.

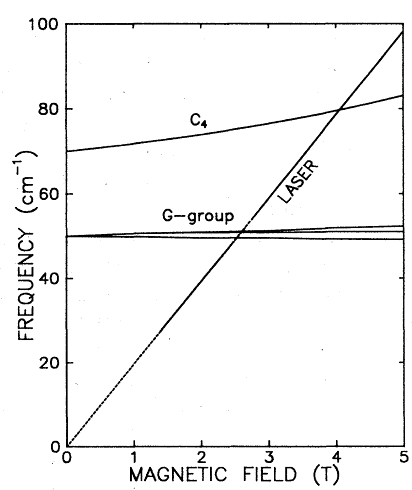

FIG. 4. Magnetic field dependences of the laser line and boron absorption line frequencies.

FCL line frequency from linearity in applied magnetic field. We present an analysis of this deviation and of the dependence of the linewidth on the magnetic-field strength. Gaps of laser emission are attributed to absorption of radiation by impurity levels.

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