Frequency shift of spin waves in tunnel-junction spin-transfer nano-oscillators

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The excitations of microwave spin waves in magnetic tunnel junctions are theoretically investigated. An analytical approach which describes the dependence of the microwave precession frequency on the applied voltage is developed. It is shown that the spin-wave frequency is directly related to both the in-plane and perpendicular spin-transfer torques. In the low field regime the perpendicular torque can induce changes in the slope of the oscillation frequency versus applied voltage (df/dv) from negative (redshift) to positive (blueshift) values.

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Since its first theoretical description,[1,2] the phenomenon of spin-transfer torque (STT) has been intensively investigated in metallic spin valves[3,6] (SVs) and in magnetic tunnel junctions[7,9] (MTJs) as well. The origin of the STT lies on the direct transfer of angular momentum from the conduction electrons that are spin polarized by the magnetization of the pinned layer to the magnetization of the free layer. This results in a torque that acts on the magnetization opposing the process of relaxation that may produce magnetization reversal or steady-state precession with frequencies in the microwave range. Since the STT is proportional to the current density this effect becomes important when the lateral dimensions of the contacts are on the order of a few hundred nanometers. The precession frequency can be adjusted by changing the current or the applied magnetic field,[3–6,10–13] opening a wide door of possibilities for a new class of spintronic devices, such as current-tunable microwave sources, often called spin-transfer-nano-oscillators (STNOs).

Extensive measurements and detailed analyses with macrospin models have shown that in metallic SVs the STNOs driving properties are very well described by Slonczewski's in-plane (IP) torque.[14,15] However, recent experiments in MTJs have provided evidence of the existence of an additional perpendicular or out-of-plane (OP) torque,[7,10] with magnitude that can reach 25% of the in-plane STT term.[7,8] The spin-wave (SW) theory previously presented[11–13] for the microwave generation considering only the in-plane STT explains the now well established[4,6] downward frequency shifts (redshifts) with increasing current when the magnetic field is applied in the film plane and upward frequency shifts (blueshifts) when the field is perpendicular to the film observed in metallic SVs. In this paper we investigate the influence of the OP spin transfer torque on the steady-state magnetization precession in magnetic tunnel junctions. An analytical approach which describes the dependence of the microwave precession frequency on the applied voltage shows that the OP torque can have a significant effect on the frequency shift.

Let us consider a multilayer structure with two ferromagnetic layers separated by a nonmagnetic spacer, metallic, or insulating, which is the active part of a spin valve or a magnetic tunnel junction (see Fig. 1). The magnetization of one of the ferromagnetic layers is fixed (reference layer), whereas the magnetization of the other ferromagnetic layer is free to rotate (free layer). The external magnetic field $\mathbf{H}$ is applied along the z direction with the y axis perpendicular to the plane of the structure. The dynamics of the magnetization $\mathbf{M}$ of the free layer is assumed to follow the Landau-Lifschitz-Gilbert equation including the spin-transfer terms[16]

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_s}\mathbf{M} \times \frac{d\mathbf{M}}{dt} - \mathbf{T}, \quad (1)$$

where γ is the gyromagnetic ratio, α is the Gilbert damping constant, and $\mathbf{H}_{eff}=-\delta U(M)/\delta\mathbf{M}$ is the effective magnetic field acting on the magnetization, where $U(M)$ is the magnetic energy. We shall consider the simplest case for the free layer with an in-plane magnetization and will neglect exchange interaction and anisotropy. Then, the effective magnetic field $\mathbf{H}_{eff}=\hat{\mathbf{z}}H-\hat{\mathbf{y}}4\pi M_y$, is the sum of the applied field H and the surface demagnetizing field.

The spin-transfer torque in Eq. (1) can be phenomenologically written as the following expression:[17,18]

$$\mathbf{T} = \gamma a_{\parallel}\mathbf{M} \times (\hat{\mathbf{z}} \times \hat{\mathbf{m}}) + \gamma b_{\perp}\hat{\mathbf{z}} \times \mathbf{M}, \quad (2)$$

where $\hat{\mathbf{z}}$ define the polarization of the reference layer, $a_{\parallel}$ is the coefficient of the Slonczewski in-plane STT, and the $b_{\perp}$ term represents the OP torque that acts as a supplementary field.[17,18] Note that both torque terms vary with the angle-dependent current density through the free layer $J(\theta)$, where

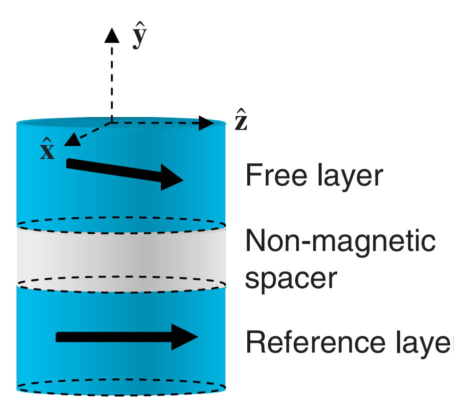

FIG. 1. (Color online) Schematic of the structure geometry.

θ is the angle between the reference and free layers. As is well established the IP torque coefficient is proportional to the current density, so we write $a_{\parallel}(\theta)=a_0 J(\theta)$. However detailed analysis[19] of the OP torque in MTJ STNOs has shown that it is proportional to $VJ(\theta)$, where V is the applied voltage across the junction. Thus we write $b_{\perp}(\theta)=b_0 V a_{\parallel}(\theta) = b_0 a_0 V J(\theta)$ so that $b_0 V = b_{\perp}/a_{\parallel}$ controls the relative amplitude between the OP and IP spin-torque terms.

We describe the angle dependence of the MTJ conductance by $G(\theta)=G_P+1/2(G_{AP}-G_P)(1-\cos\theta)$ and the size of the tunneling magnetoresistance (TMR) is characterized, as usual, by the quantity $\mathrm{TMR}=(G_P-G_{AP})/G_{AP}$. Then, the current density becomes

$$J(\theta) = \frac{I}{A_c} = \frac{VG_P}{A_c}\left\{1 - \frac{\mathrm{TMR}[1-\cos(\theta)]}{2(1+\mathrm{TMR})}\right\}, \qquad (3)$$

where A_c is the area of the current cross section, assumed to be spatially uniform. To describe the nonlinear dynamics of the system, Eq. (1) is transformed into a set of equations of motion for the scalar complex Fourier component SW amplitudes[12,17] c_k and c_k^*. The Holstein-Primakoff[20] transformation provides a way to achieve such a goal. First, the spin components $S_{x,y,z}$ ($\mathbf{S}=S\mathbf{M}/M_s$) are expressed in terms of the complex variable $a(\mathbf{r},t)$ according to

$$a(\mathbf{r},t) = \frac{S^+}{(S+S_z)^{1/2}} \qquad (4)$$

and

$$S_z = S - a^*(\mathbf{r},t)a(\mathbf{r},t), \qquad (5)$$

where, $S^{\pm}=S_x \pm iS_y$ and S is the spin. With the use of Eqs. (4) and (5), Eq. (1) can be transformed in an equation of motion for the complex variable $a(\mathbf{r},t)$ in Hamiltonian form

$$\frac{da}{dt} = -i\frac{\gamma S}{M}\frac{\delta U}{\delta a^*} - \frac{\delta a}{\delta \mathbf{S}}\cdot \mathbf{T}, \qquad (6)$$

where the torque term $\mathbf{T}$ from Eqs. (1)–(3) has the form

$$\mathbf{T} = \gamma a_0 J(0)\left[1 - \frac{\mathrm{TMR}(1-S_z/S)}{2(1+\mathrm{TMR})}\right] \times\left[\frac{1}{S}\mathbf{S}\times(\hat{\mathbf{z}}\times\mathbf{S}) + b_0 V\hat{\mathbf{z}}\times\mathbf{S}\right]. \qquad (7)$$

We assume that precession angles are not large so that the right side of Eq. (6) can be expanded in a power series in the operators a and a^*, written as $da/dt=(da/dt)^{(1)}+(da/dt)^{(3)}+\cdots$. The superscripts indicate the degree in powers of the different a, a^*, a^*aa, a^*a^*a, and aaa products. At this point, following the formalism described in Ref. 11 the conjugate variables $a(\mathbf{r},t)$ and $a^*(\mathbf{r},t)$ are expanded in a Fourier series according to

$$a(\mathbf{r},t) = \sum_k \phi_k a_k(t), \qquad (8)$$

where the basis functions are standing plane-wave functions, $\phi_k = 1/(2N)^{1/2}\cos\mathbf{k}\cdot\mathbf{r}$ appropriate for nanopillar structures. With this transformation Eq. (6) and its conjugate become coupled equations of motion for a given pair of a_k and a_k^* amplitudes. The decoupling can be achieved through the linear transformation[20]

$$a_k = u_k c_k + v_k c_k^*. \qquad (9)$$

Finally, the full equation of motion for c_k becomes

$$\frac{dc_k}{dt} = -i(\omega_k - \gamma T_k\beta n_k)c_k - \left[\eta_k - \gamma a_0 J(0) \times\left[1 - \frac{3}{4NS}\left(\frac{1+2\mathrm{TMR}}{1+\mathrm{TMR}}\right)(u_k^2+v_k^2)n_k\right]\right]c_k, \qquad (10)$$

where: $u_k=[(A_k+\omega_k)/2\omega_k]^{1/2}$, $v_k=[(A_k-\omega_k)/2\omega_k]^{1/2}$, $A_k = \gamma(H+2\pi M - b_0 a_0 VJ(0))$ and the frequency ω_k in the limit of $k \ll 1$ is given by

$$\omega_k = \gamma[H + 4\pi M - b_0 a_0 VJ(0)]^{1/2}[H - b_0 a_0 VJ(0)]^{1/2}. \qquad (11)$$

From Eq. (11) one can see that in first order the spin-wave frequency does not depend on the IP torque[11–13] but it does depend on the OP torque. This is expected because the OP torque acts as an additional magnetic field.[7,10] However, when nonlinear terms are incorporated an extra influence of the OP torque on the effective damping appears [see last term in Eq. (10)]. As a consequence, the voltage dependence of the spin-wave frequency may exhibits a nontrivial, nonmonotonic behavior (see discussion below).

Note that in Eq. (10) $\eta_k = \alpha\gamma(H+2\pi M)$ is the relaxation rate parameter and the other factors are

$$\beta = 4u_k^4 - 12u_k^3 v + 16u_k^2 v_k^2 - 12u_k v_k^3 + 4v_k^4 - \frac{2b_0 a_0 VJ(0)}{\pi M}\frac{\mathrm{TMR}}{2(1+\mathrm{TMR})}(u_k^4 + 4u_k^2 v_k^2 + v_k^4) \qquad (12)$$

and

$$T_k = \frac{3\pi M}{4NS}, \qquad (13)$$

where N is the number of spins S in the sample.

In order to interpret the roles of the nonlinear interactions, Eq. (10) was written using $n_k = c_k^* c_k$ for the number of magnons. This equation shows that when a voltage above the critical voltage $V_c = \eta_k A_c/\gamma a_0 G_p$ is applied across the MTJ the spin-wave amplitude initially grows exponentially while the number of magnons n_k is negligible ($n_k \ll NS$). However, as n_k increases the in-plane STT driving decreases and its effect is balanced by the relaxation so that the spin-wave amplitude saturates.

Further understanding of this process can be gained writing an equation for the magnon saturation number n_s. From Eqs. (10)–(12) introducing the driving parameter $r=V/V_c$ the equation for n_s can be shown to be

$$n_s = \frac{4NS}{3}\frac{(r-1)(1+\mathrm{TMR})}{r(1+2\mathrm{TMR})} \times\left\{1 - \left[\left(1+\frac{H}{2\pi M}\right)(1-\alpha b_0 Vr)\right]^{-2}\right\}^{1/2}. \qquad (14)$$

From Eqs. (10) and (14) we obtain the frequency shift

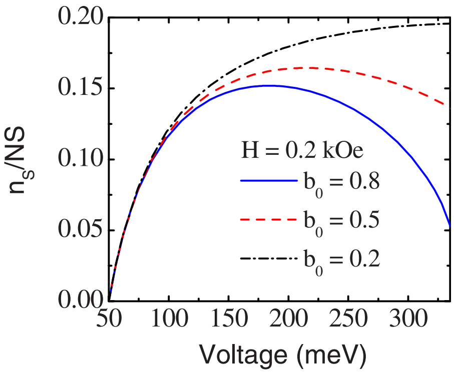

FIG. 2. (Color online) Magnon saturation number n_s vs voltage for three different values of b_0. The values of b_0 have been chosen such that the OP torque reaches 25% (b_0=0.8, solid line), 15% (b_0=0.5, dashed line), and 5% (b_0=0.2, dashed-dotted line) of IP torque for a voltage bias about 300 mV.

$$\delta\omega_k = -\gamma T_k \beta n_s = -\gamma\pi M\beta\frac{(r-1)(1+\text{TMR})}{r(1+2\text{TMR})} \times\left\{1-\left[\left(1+\frac{H}{2\pi M}\right)(1-\alpha b_0 Vr)\right]^{-2}\right\}^{1/2}, \quad (15)$$

where the coefficient β can be written in terms of the driving parameter r

$$\beta = (4u_k^4+16u_k^2v_k^2+4v_k^4)\left[1-\alpha b_0 Vr\left(1+\frac{H}{2\pi M}\right)\right] - 12u_k^3 v - 12u_k v_k^3. \quad (16)$$

Clearly the effect of the nonlinear term T_k arising from the surface dipolar energy (demagnetizing effect) is to shift the SW frequency downwards (redshift) with increasing voltage. However the presence of the OP torque introduces an additional dependence on the voltage through the term $b_0V = b_\perp/a_\|$.

At this point we are interested in the dependence of the frequency on the voltage in the nonlinear regime, i.e., for $V > V_c$. The parameters used in the calculation are, $H = 0.2$ kOe, $4\pi M = 10.0$ kG, $V_c = 50$ mV, $g = 2$, $\alpha = 0.02$, and TMR=100%. Here for simplicity we neglect the dependence of the TMR on the applied voltage. Figure 2 shows the dependence of the magnon saturation number n_s given by Eq. (14) as a function of the applied voltage for different values of b_0. The values of b_0 has been chosen such that for a voltage about 300 mV the OP torque reaches 25% (solid line), 15% (dashed line), 6% (dashed-dotted line) of the IP torque.[9]

As one can see, when the magnitude of the OP torque is small (solid line), for strong driving $V \gg V_c$ or $r \gg 1$ the number of magnons saturates approximately at $0.2NS$, corresponding [see Eq. (5)] to a cone angle of 36.9°, i.e., the spin-wave approach does provide an accurate description of the problem at such a level of excitation for the chosen parameters. On the other hand, as is clear from Eq. (14), when the relative amplitude between the OP and IP spin torques increases, and the applied voltage is sufficiently high, the number of magnons decreases as we show in Fig. 2 (solid line). For even larger voltages, a vanishing of the number of magnons is predicted by Eq. (14). However in such a regime (which is not shown in Fig. 2) higher order terms which were omitted in Eq. (10) may start to play a role and no conclusive statements can be made at the present level of approximation.

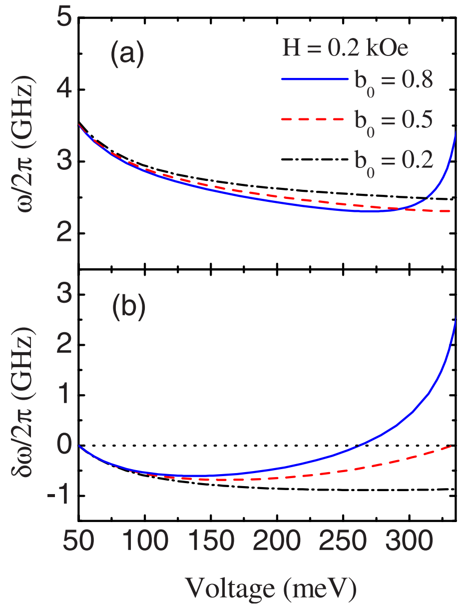

FIG. 3. (Color online) (a) Voltage dependence of the frequency for three different values of b_0 and H=0.2 kOe. (b) Voltage dependence of the corresponding frequency shift. The dotted line in (b) is a guide for the eye.

Figure 3 shows the calculated frequency vs voltage for the same parameters. As discussed above, the spin-wave frequency is given by $\omega_k(\theta=0)+\delta\omega_k$, where $\omega_k(\theta=0)$ and $\delta\omega_k$ are given by Eqs. (11) and (15), respectively. As we can see from the Eqs. (10) and (11), for $V < V_c$ the frequency shift is due only to the influence of the OP torque[7] because the magnon number has the thermal value. On the other hand, if $V > V_c$, the magnon number is large (see Fig. 2) and the shift can be appreciable and strongly dependent on the relative amplitude ($b_0V = b_\perp/a_\|$) between the OP and IP spin torques.

With the field in the plane of the film, when $b_\perp/a_\|$ is small as in the SVs structures, the frequency always decreases with increasing voltage (redshift) [dashed-dotted and dashed lines in Fig. 3(a)].[11–13] However, it follows from Eq. (15) that for large enough values of $b_\perp/a_\|$ and at a relatively high bias voltage, i.e., when $[(1+H/2\pi M)(1-\alpha b_0 Vr)]^{-2} > 1$, the frequency may shift upwards with increasing voltage (blueshift). This is shown in Fig. 3(b). This behavior will be more pronounced in the low field regime, $H \ll 4\pi M$ [Eq. (15)]. It is, therefore, expected that the blueshift cancels out the redshift under appropriate experimental conditions, reducing the nonlinearity of the oscillator. This could be very important in determining the influence of the out-of-plane spin-torque on the high-frequency magnetization oscillations in magnetic tunnel junctions, considering that up to now the experiments have been performed under the linear regime.

In summary, following the spin-wave theory[11–13] we have done an investigation of the influence of the OP spin transfer torque on the steady-state magnetization precession in magnetic tunnel junctions in the nonlinear regime. The oscillation frequency vs the applied voltage is directly related to both the in-plane and perpendicular spin-transfer torques. In the low field regime the perpendicular torque can induce changes in the slope of the oscillation frequency with respect to the voltage (df/dv) from negative (redshift) to positive (blueshift) values.

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