Low-temperature electronic transport measurements on a gated δ -doped GaAs sample: magnetoresistance, quantum Hall effect and conductivity fluctuations

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Abstract. We present magnetotransport measurements (up to 7 T) performed at very low temperatures (down to 20 mK) on a GaAs sample containing two parallel δ -doped layers whose carrier concentration can be varied by means of a gate electrode. With increasing negative gate voltage the resistance becomes more strongly temperature-dependent, indicating a more localized electron system. The magnetoresistance is found to be strongly anisotropic. When the field is parallel to the layers we find a large positive magnetoresistance which we attribute to orbital shrinking of the strongly localized donor wavefunction. In contrast, in the perpendicular orientation, we observe a strong negative magnetoresistance at low fields whose origin remains unclear, and the quantum Hall effect at larger fields. At low gate voltages both δ -layers are in the quantum Hall state whereas at larger negative voltages the layer adjacent to the gate becomes insulating. In the case of strong depletion the high-ohmic sample shows reproducible conductivity fluctuations as a function of either the gate voltage or the magnetic field. The fluctuations diminish at higher temperatures and larger measuring currents.

1. Introduction

Magnetotransport in disordered two-dimensional electron systems depends on whether the electron wavefunctions are extended or localized. Phenomena resulting from weak localization or the quantum Hall effect (QHE) are observed in the former case, whereas in strongly localized systems the effect of the magnetic field is based on a reduction of the overlap of the localized electron wavefunction. This orbital shrinking (OS) describes well the huge and anisotropic positive magnetoresistance (MR) in large fields which is found in these systems [1]. In low fields, however, various experiments have shown that a strong negative MR exists in localized systems, especially for the perpendicular orientation of the magnetic field with respect to the conductive layer [2-6]. The origin of this enhancement of the electron transport is currently under discussion. Two mechanisms have been proposed. One is based on interference phenomena affecting the hopping probability from an occupied site to an empty site [7, 8]; the second deals with an increase in the density of states near the Fermi level due to the reduced overlap [9]. While the interference mechanism applies only for a perpendicular field, the effect on the density of states may also exist in a parallel field although it should be weaker than in the perpendicular orientation where OS is stronger. These models are strictly applicable only in the strongly localized variable-range hopping (VRH) regime. In the intermediate regime near the metal-insulator transition, the effect of the magnetic field on the conductivity, that is on the transition itself, is even less clear.

We have made an extensive experimental study of magnetotransport on a sample of GaAs containing two parallel δ -doped layers whose carrier concentration can be varied by means of a gate voltage. The gate allows us to tune the conductivity of the layers from an almost metallic behaviour deeper into the insulating regime, depending, in addition to gate voltage, on the strength and orientation of the magnetic field and, in general, on

temperature.

Our results demonstrate the existence of a large negative MR in perpendicular fields, which merges into the QHE at larger fields. Under the same conditions, we find a large positive MR in parallel fields, which can be described by the OS model for strongly localized systems. This implies that in a perpendicular field there is a delocalizing effect that is stronger than the localizing one. This effect is much weaker in the parallel orientation, and OS dominates at higher fields.

By sweeping the gate voltage we can fully deplete the adjacent δ -layer while the other one remains conductive. Therefore we can observe the transition from a state with both layers in the QH state to that with only one layer in the OH state.

Finally, at large gate voltages, we find reproducible resistance fluctuations as a function of either the magnetic field or the gate voltage. The size of the fluctuations depends on temperature and measuring current. These fluctuations occur in the strongly localized regime and are to be distinguished from the universal fluctuations in the metallic regime.

Experimental details are given in the following section 2. Results on the longitudinal resistivity are discussed in 3.1. Observations of the QHE are presented in section 3.2, and section 3.3 is devoted to the resistivity fluctuations. Section 4 concludes the paper.

2. Experimental set-up

The MBE-grown sample investigated in this work consists of GaAs with two δ -layers at depths of 270 nm and 470 nm respectively: see figure 1. The donor atoms are Si with a concentration of 7×10^{11} cm⁻². Due to background compensation the effective electron concentration is estimated to be half of the donor concentration. The surface of the sample is covered by a gate electrode made of Al. A negative gate voltage depletes the carrier concentration in the layers. The sample geometry is the usual Hall bar structure, so that measurements of either the longitudinal resistance R_{xx} or the Hall resistance R_{xy} are possible. The contacts connect both layers and are made by diffusing Ge into the sample. At negative gate voltages of up to -1.0 V, only the upper layer is depleted, while a further increase

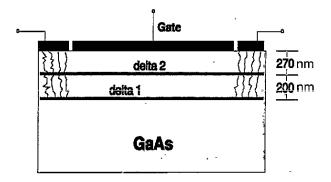


Figure 1. Schematic set-up of the GaAs sample. On the sample surface are shown two of the contacts and the gate.

of the negative gate potential diminishes the carrier concentration in the lower layer too.

The sample resistance was measured in a fourpoint geometry by using a constant alternating current between 10 pA and 10 nA with a frequency of either 2.5 Hz or 12.5 Hz in order to avoid heating effects and current leakage to the gate or to ground via the cable capacitances. The voltage signal was recorded by a highimpedance AC voltmeter. The sample was immersed in the dilute phase of the mixing chamber of a ³He-⁴He dilution refrigerator having a final temperature of 15 mK. A RuO2 thick-film resistor was placed in the immediate vicinity of the sample as a low-temperature thermometer. A superconducting magnet allowed the application of fields up to 7 T. By changing the orientation of the sample, the field can be either perpendicular or parallel to the δ -layers. In the parallel case, the measuring current was perpendicular to the field.

3. Results and discussion

3.1. Dependence of the longitudinal resistance on temperature and magnetic field

Figure 2 shows the temperature dependence of the longitudinal sample resistance in zero magnetic field at various constant gate voltages. The reason for plotting the logarithm of the resistance versus $T^{-1/3}$ is to compare the data with Mott's law for VRH in two dimensions at a constant density of states:

$$\ln(R/R_0) = (T_0/T)^{1/3} \tag{1}$$

where T_0 is determined by the localization length and the density of states. At larger negative gate voltages the resistance does increase strongly, but the data do not show the straight-line behaviour of equation (1). Also, other versions of Mott's law such as $T^{-1/2}$ (twodimensional hopping with a Coulomb gap) do not fit the results. We are left with a strongly increasing resistance, in particular for large gate voltages where the sample is in the insulating regime, but we cannot describe its divergence quantitatively. We believe that this behaviour is the result of inhomogeneities of the carrier concentration leading to local variations of T_0 and R_0 of equation (1). (Further effects due to inhomogeneities in the layers will be discussed in section 3.3.) At low gate voltages the weaker temperature dependence suggests an increase of the localization length although the resistance still rises by more than a factor of three between 500 mK and 20 mK. In order to characterize the sample in more detail, it is more informative to investigate its magnetoresistance for both perpendicular and parallel geometries.

Figure 3 shows an example of the dependence of the sample resistance R_{xx} on the magnetic field for the case of either parallel or perpendicular orientation of the field with respect to the layers. The MR is strongly anisotropic. In fields up to 7 T the resistance for the parallel orientation is larger than for the perpendicular

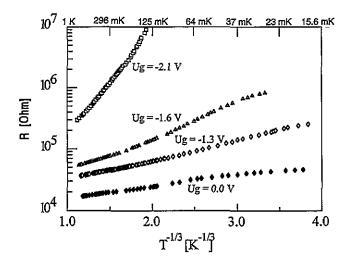


Figure 2. Longitudinal sample resistance versus temperature at various constant gate voltages.

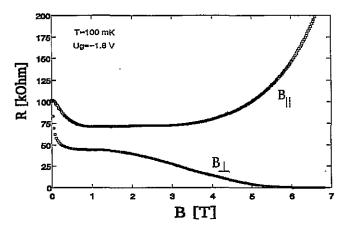


Figure 3. Longitudinal sample resistance as a function of the magnetic field at T=100 mK and gate voltage = -1.8 V for parallel and perpendicular orientations of the field.

one. Before entering into a discussion of these results we should like to present further experimental results on the MR.

The MR for parallel orientation at various constant gate voltages is shown in figure 4. The common feature is a small negative MR at fields up to 4 T, which changes into a strong and exponentially growing positive MR at higher fields. The positive MR is observed even at zero gate voltage, and it increases when the sample is rendered more insulating by larger gate voltages.

Figure 5 shows the MR measured under the same conditions when the field orientation is changed to the perpendicular direction. In contrast to the parallel case, a strong and very steep negative MR is observed at fields below 0.2 T. At intermediate fields (between 1 T and 2.5 T) we see a non-monotonic structure of the negative MR, which vanishes when the negative gate voltage is raised up to -1.6 V. Then the negative MR shows only a saturation. When the field is increased above 2.5 T we find a strong decrease of the resistance, which finally vanishes at a magnetic field of 6 T. This behaviour is due to the QHE, as discussed in the following section.

The negative MR, in the perpendicular orientation, at different constant gate voltages is shown in more detail

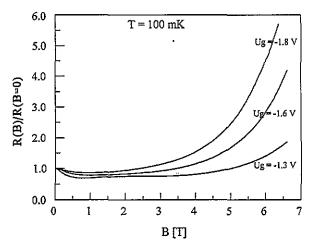


Figure 4. Longitudinal sample resistance as a function of the parallel magnetic field at T=100 mK and various constant gate voltages.

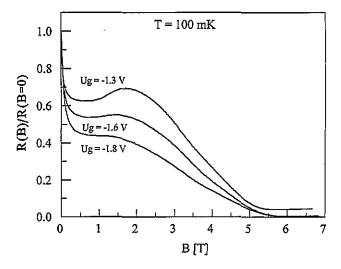


Figure 5. Longitudinal sample resistance as a function of the perpendicular magnetic field at $T=100\,\mathrm{mK}$ and various constant gate voltages.

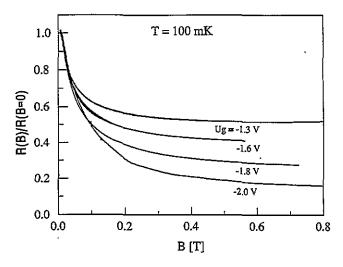


Figure 6. Longitudinal sample resistance versus magnetic field at T = 100 mK and various constant gate voltages for perpendicular sample orientation and small fields.

in figure 6. We find that the gate voltage seems to have only a negligible influence on the negative MR below 50 mT. The saturation value of the negative

MR, however, is altered considerably when the carrier concentration is changed by the gate voltage: a lower carrier density leads to a stronger negative MR.

To commence a discussion of the above results we first notice that the positive MR in the parallel orientation of the sample can be attributed to the effect of orbital shrinking. This effect is well known in the regime of strong localization and can be described in general [1] by the dependence

$$R(B) = R(0) \exp(B^2/B_0^2).$$
 (2)

In the case of VRH the parameter B_0^2 is temperaturedependent, and for two-dimensional systems with a constant density of states it is given by

$$B_0^2 = \alpha (\hbar^2/e^2a^4)(T/T_0)$$

where a is the localization length and α is a numerical factor: $\alpha = 720$ for parallel orientation and $\alpha = 360$ for the perpendicular case (see [5]).

Analysing our data for $U_g = -1.6$ V and large parallel fields, we find that a fit of equation (2) yields $B_0^2(T) \propto T^{0.5}$ instead of a linear dependence. This again indicates that a quantitative description of our results in terms of the VRH mechanism is not possible, although a decrease of B_0^2 with temperature is consistent with it.

When considering the anisotropy, however, we note that equation (2) predicts a larger MR for the perpendicular geometry. Experimentally, the opposite is found to be true, a situation that has been termed an 'inverted relation' [5]. Obviously, there is another mechanism that compensates this effect of orbital shrinking, at least for fields that are not too strong. Furthermore, this mechanism must be stronger in a perpendicular field (but it does not completely vanish in the parallel case) and ultimately delocalizes the wavefunctions to such an extent as to make the system undergo a QHE, whereas in a parallel field orbital shrinking is observed.

There are at least two different theoretical models to describe the anisotropic negative MR in the VRH regime and the 'inverted relation' as its consequence. One is based on the interference of the partial waves caused by the effect of impurity scattering [7, 8]; the other considers the effect of orbital shrinking on the density of states [9] and is called the 'incoherent mechanism'. For a short review of both models see chapter IV of reference [5]. -Two important differences between the results of these calculations have to be mentioned. Firstly, the interference mechanism requires a field perpendicular to the layer whereas the incoherent mechanism does not (although it is anisotropic). Secondly, the interference mechanism leads to a relative reduction of the resistance in a perpendicular field, which is expected to saturate at a factor of 1/2†. In contrast, the incoherent mechanisms give non-monotonic MR of either sign, depending on how strongly the density of states is affected by orbital shrinking.

We face several problems in attempting to discuss our results within these theories. Firstly, there is the large size of the negative MR: the resistance is reduced by a factor of $R(B)/R(0) \sim 0.15$ at fields of only about 0.5 T (see figure 6), the effect being too large to be accounted for by the interference mechanism or the field being too small for the concept of orbital shrinking and therefore also for invoking the incoherent mechanism. Furthermore, the negative MR in the parallel orientation is strong enough to compensate orbital shrinking at low fields (see figure 4). In the perpendicular orientation the increase of the effect for more negative gate voltages is remarkable and may be expected from both But the temperature sweeps at a constant field of 0.5 T (not shown here) yield a greatly reduced temperature dependence of the resistance, indicating a more delocalized sample. In summary, we are unable to describe our results on the low-field MR in terms of the above models quantitatively [11]. Similar results on the large negative MR in a different two-dimensional sample were reported in reference [6]. In that work, which contains no data for the parallel orientation, a definite conclusion as to the origin of this effect could also not be reached.

For larger parallel fields we find the typical MR of strongly localized systems, i.e. orbital shrinking, whereas in the perpendicular case the MR shows nonmonotonic behaviour before it completely vanishes because of the QHE (see below). Clearly, there is a delocalizing mechanism that is based on the existence of a perpendicular field. One might speculate that the impurity band contains a narrow strip of extended states around the maximum of the density of states [12]. Even small shifts of the Fermi level with respect to this strip could result in large changes in the conductivity similar to those observed in our experiments, thereby transforming the system from a strongly localized regime to a delocalized one exhibiting a QHE. This picture might apply also for the parallel orientation of the field if we assume the shifts to be smaller than in the perpendicular orientation so that OS dominates already at smaller fields.

Finally, we should mention that further experiments on a similar sample confirm our results on the perpendicular MR [13].

3.2. Hall resistance

The QHE is shown in figure 7 for a gate voltage of -1.3 V and a temperature of 100 mK. The longitudinal resistance vanishes while the transverse resistance increases and saturates at $12.90 \text{ k}\Omega$, indicating a filling factor $\nu=2$. No other filling factors could be observed even when other gate voltages or temperatures were investigated. The effect of the temperature is shown in figure 8 for a gate voltage of -1.6 V. The slope is slightly steeper and the plateau for $\nu=2$ is reached somewhat earlier at lower temperatures. This implies that the carrier concentration has only a weak

[†] Theoretical predictions might be different when orbital shrinking is properly included [10].

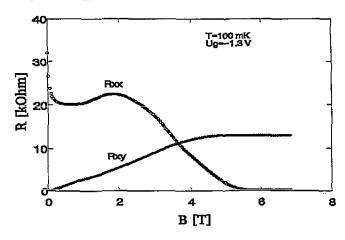


Figure 7. Longitudinal and Hall resistance versus magnetic field at T=100 mK and a gate voltage of -1.3 V. Note the OHE at filling factor $\nu=2$.

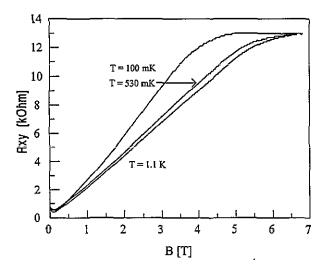


Figure 8. Hall resistance versus magnetic field at a gate voltage of ~1.6 V and different constant temperatures.

temperature dependence. More pronounced is the effect of the gate voltage at constant temperature: see figure 9. While both layers are conductive at zero voltage and the plateau at $6.45~\mathrm{k}\Omega$ indicates $\nu=2$ for both of them, only one layer remains in the QHE at $\nu=2$ when the voltages are increased to $-1.3~\mathrm{V}$ and more. This behaviour can be seen more clearly when sweeping the gate potential in a field of 6 T: see figure 10. Between $-0.1~\mathrm{V}$ and $-0.3~\mathrm{V}$ we observe the transition from a double-layer QHE to a single-layer QHE at $\nu=2$. In the transition regime transverse currents between the layers cause the peak of the longitudinal resistance (see below). This behaviour could be seen at all temperatures between 1.1 K and 50 mK.

The absence of higher filling factors may indicate that on the corresponding length scales $\lambda = (\hbar/eB)^{1/2}$ there is too much disorder in the layers. Only for $B \gtrsim 5$ T, i.e. $\lambda \lesssim 10$ nm, is a QH state detected. The question of whether a further increase in the field would lead to a QHE at $\nu = 1$ or to orbital shrinking could not be investigated because our maximum field was 7 T. The data of figure 10 can be described by a model that assumes a non-vanishing transverse current flowing through both δ -layers while the QHE in the

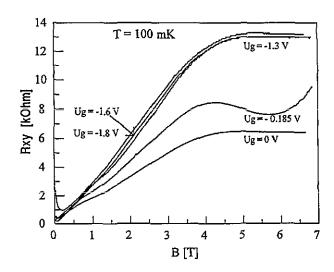


Figure 9. Hall resistance versus magnetic field for various constant gate voltages at T=100 mK. At zero gate potential both layers exhibit the OHE at v=2, while at -1.3 V only one layer remains in the OH state while the layer close to the gate is insulating.

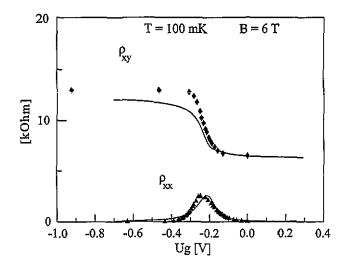


Figure 10. Hall and longitudinal resistance as functions of gate voltage at a constant perpendicular field of 6 T and $\mathcal{T}=100$ mK. The symbols show the measured data; the curves are obtained by calculating the fitting functions (see equation (3)).

upper layer breaks down due to a rising negative gate potential. When the negative gate voltage is very low, the upper layer is also in the QH regime and therefore the Hall voltages of the two layers are equal and transverse currents are zero. If the gate has a high negative potential the upper δ -layer is depleted very strongly and the Hall voltage vanishes. The upper layer then forms a high ohmic load connected to the Hall voltage of the lower layer. In this case the transverse current is negligibly small. If the gate voltage is fixed to a value at which the QHE in the upper layer is just breaking down, we have two layers with unequal Hall voltages, resulting in a transverse current. To describe this effect qualitatively, we calculate the conductivity of the connected layers using the conductivity tensors and assuming the additivity of the two components. In general, this assumption is not correct because the transverse currents through the layer are probably

inhomogeneous and concentrated near the contact region, while our assumption is based on a conductivity tensor being constant in space. The data in figure 10 suggest the following ansatz for the gate-voltage-dependent conductivities of the upper layer (units of $2e^2/h$):

$$\sigma_{xx}^{u} = \frac{a_{1}}{1 + (U - U_{0})^{2}/\delta^{2}}$$

$$\sigma_{xy}^{u} = -\frac{1}{a_{2}\pi} \arctan\left(\frac{U - U_{0}}{\delta}\right) + 0.5$$
(3)

where a_1 , a_2 , U_0 and δ are fitting parameters and U is the gate voltage. The lower layer is assumed to remain in the QH state at any gate voltage:

$$\sigma_{xx}^l = 0, \qquad \sigma_{xy}^l = 1.$$

By adding the two tensors and solving for the inverse we obtain the full curves in figure 10 in qualitative agreement with the data for $a_1 = a_2 = 0.5$, $\delta = 3$ V and $U_0 = -0.21$ V. The maximum of R_{xx} is reproduced rather well, but the transition between the two quantized values of R_{xy} is not steep enough. A more rapidly varying function for σ^u_{xy} would improve the agreement.

3.3. Conductivity fluctuations

When the longitudinal sample resistance R_{xx} becomes very high (at least several hundred $k\Omega$), at high negative gate voltages or at low temperatures, very stable and reproducible fluctuations of the sample resistance can be observed by sweeping either the gate voltage or the magnetic field. Such fluctuations have already been observed in mesoscopic systems on both sides of the metal-insulator transition (MIT). On the metallic side of the MIT these fluctuations have their origin in a weakly localized carrier transport being governed by a large phase coherence length [14]. In the insulating regime it is the hopping length that sets the length scale for observing mesoscopic effects [15].

In our measurements the appearance of such fluctuations was restricted to a temperature range below 1 K. The fluctuations could be observed in both the longitudinal resistance R_{xx} and the transverse To understand the occurrence of resistance R_{xy} . resistance fluctuations in our large-area samples we have to consider inhomogeneities in the two-dimensional electron system which are created either during doping or by the gate. It has to be pointed out that the sample dimensions are many orders of magnitude larger than any reasonable estimate of the mesoscopic length scale†. Therefore, we would expect an 'averaging out' of mesoscopic effects [16]. Thus, we may only speculate that, especially in the contact region or at the edges of the sample, inhomogeneities lead to a distribution of a small number of conducting paths, which in addition are

† From the data on the MR and the temperature dependence of the resistance we estimate mesoscopic effects to exist in our sample only of the order of 100,nm on either side of the MRT.

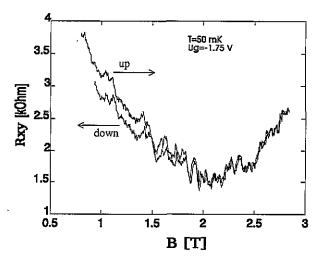


Figure 11. Transverse sample resistance R_{xy} versus magnetic field at T=50 mK and a gate voltage of -1.75 V. The two curves show the fluctuating resistance during up and down sweeps.

sufficiently restricted in space to make these fluctuations observable.

Fluctuations of R_{xy} are shown in figure 11 as a function of magnetic field. We see that sweeping the magnetic field up and down shows nearly the same structure. This structure has a long-time stability, i.e. it can be observed a few days later with little change when the sample is kept below 4 K.

Figure 12 shows fluctuations, produced by sweeping the gate voltage at various constant perpendicular magnetic fields. We see that small magnetic fields have little influence on the structure, but the amplitude of the resistance fluctuations decreases with increasing magnetic field. Further analysis shows, however, that

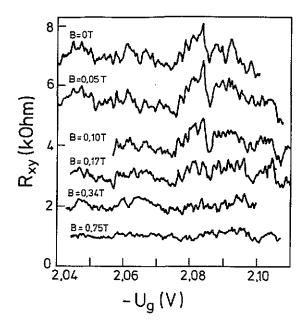


Figure 12. Fluctuations of the transverse sample resistance at a temperature of 300 mK when sweeping the gate voltage at different constant magnetic fields. Note the reduction of the amplitudes of the fluctuations with increasing field. For better comparison of the structures the zero point of the resistance curves is suppressed.

the relative size of the fluctuations, i.e. $\Delta R/R$, is rather independent of the field below 1 T. No fluctuations could be observed when sweeping a magnetic field parallel to the layers. Alternatively, when sweeping the gate voltage the fluctuations were not affected by a parallel field of 0.5 T (not shown here).

Fluctuations as a function of gate voltage are shown at different constant sample currents in figure 13. It can be easily recognized that the overall structure is not changed by an increase in sample current, but the variations become much smoother. In figure 14 we can see fluctuations in R_{xy} as a function of gate voltage at various constant temperatures. The amplitude of the fluctuations decreases when the temperature is increased. The appearance of these fluctuations is an indicator that the electronic transport of our macroscopic sample is governed by only a few conducting paths. Because the number of these paths is small, electronic transport is determined by quantum interference effects since no averaging out takes place. A change in the external parameters, like gate voltage or magnetic field, leads to a preference of single paths over other ones. This results in a change of quantum interference and therefore in a change of resistance, showing a fluctuating These effects can only be observed at very low temperatures where the number of electrons in an energy interval kT near the Fermi energy is sufficiently small. At higher temperatures the energy interval kT increases and so does the number of electrons contributing to electronic transport. Then an averaging out takes place and the fluctuations vanish. This can be seen when the sample temperature is increased to values higher than 1 K. Two effects can be observed with increasing temperature: a diminishing of the amplitude of the fluctuations and a change in their frequency spectrum. The two same effects occur when the current is increased. This is probably due to Joule heating of the current paths. Because these heating effects are locally restricted they do not appear as an overall decrease of the sample resistance. In order to characterize the temperature dependence of the fluctuations we evaluate the temperature dependences of their root mean square (RMS) values. The result of our measurements is shown in figure 15. We see that the RMS fluctuation in R_{xy} increases with temperature by more than an order of magnitude as $\exp(T_0/T)^{1/3}$.

In the strongly localized transport regime theory predicts a temperature dependence of the RMS of the resistance fluctuations [17]:

$$RMS(\Delta R(T)) \propto \exp\left[m(T_0/T)^{1/(d+1)}\right]$$
 (4)

where m can have values between 3.5 and 5, and d is the dimensionality of the sample. Obviously, our data in figure 15 are correctly described by equation (4) for d=2. Hence, the size of the fluctuations is in quantitative agreement with the predictions for the VRH mechanism. The values of T_0 depend on the choice of m and are in the interval 2-7 K. Therefore, in our case the fluctuations clearly have their origin in the strongly localized regime. This is consistent with our observation

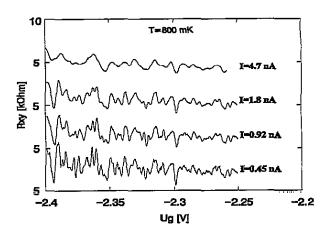


Figure 13. Transverse sample resistance versus gate voltage at T=800 mK, measured with different constant sample currents. Note the gradual disappearance of the 'high frequency' structure with increasing current.

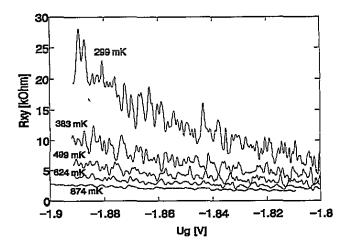


Figure 14. Transverse sample resistance R_{xy} versus gate voltage at various constant temperatures.

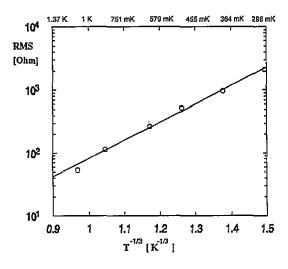


Figure 15. Root mean square of the fluctuations shown in figure 14 versus temperature. The straight line behaviour is described by equation (4).

that these resistance fluctuations occur only when the δ -layers are so far on the insulating side of the MIT that even the delocalizing effect of the perpendicular field is no longer operative.

4. Conclusions

Our experiments demonstrate that the gated δ -layers provide us with a 2D electron system whose localization length can be varied not only by means of the gate voltage but also by a magnetic field.

Typical effects of strong localization are observable at large negative gate voltages or large parallel fields: a diverging resistance as the temperature drops to zero, an exponentially growing positive magnetoresistance due to orbital shrinking and reproducible conductance fluctuations as a function of the gate voltage. Although an analysis is only qualitatively in agreement with the variable-range hopping mechanism, probably because of sample inhomogeneities, these effects are understood in principle.

The more interesting results, however, are those that demonstrate the delocalizing effects of a perpendicular magnetic field. Firstly, there is the giant negative magnetoresistance in small fields, for which we have no adequate theoretical model. Secondly, we observe a QHE at a field of about 5 T, corresponding to a filling factor of 2, whereas in the parallel orientation we have orbital shrinking of the strongly localized wavefunction. Obviously, the 2D electron system becomes completely delocalized in the perpendicular orientation although the orbital shrinking mechanism should be stronger in this geometry. And finally, the delocalizing mechanism can be observed in the rather weak temperature dependence of the resistance in a field of about 0.5 T.

Theoretical models for the negative magnetoresistance in the strongly localized regime probably have to be extended to more delocalized situations before they can be applied to our results quantitatively. Until then, the origin of the delocalizing effect of the magnetic field remains unclear.

Acknowledgments

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