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Vibrations of a magnetic microsphere levitated above a superconductor: a high-Q oscillator for studies of vorticity in superconductors and superfluids

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The oscillations of a small magnetic sphere (radius 100 μm) levitated above a superconductor are investigated. Resonance frequencies between 300 Hz and 600 Hz are observed. At low amplitudes (≤ 1 μm) the oscillator has Q values of about 10^6. At larger amplitudes both the resonance frequency and the damping become amplitude dependent. Nonlinear and hysteretic friction is attributed to vortex motion in the superconducting environment. Application of this oscillator for investigation of vorticity in superfluids is discussed.

1. Introduction

The friction a moving particle experiences in superfluid helium is determined by its interaction with the elementary excitations. At sufficiently low temperatures it may be possible to accelerate a particle up to a critical velocity where vorticity is produced and kinetic energy is lost to the liquid. Among the various probes which have been used so far are ions (diameter approx 3 nm), vibrating wires, membranes with small holes, and torsional oscillators of macroscopic size. Recently, we have presented a new method [1, 2] based on magnetic levitation of a permanent magnet above a superconductor. The spherical ferromagnetic particle has a radius of about 10^{-4} m, a mass of 10^{-8} kg, a magnetic dipole moment of 10^{-6} A m^{-2}, and is given an electric charge of 10^{-12} C. The particle vibrates due to electrical excitation inside a capacitor. The main advantages of this oscillator for liquid helium studies are the following:

(1) Low mass and high Q value (10^6 in vacuum) allow a high energy resolution $\Delta E = E/Q \sim 10^{-19}$ J.

(2) Spherical shape and macroscopic radius permit simple hydrodynamic description. The critical velocity for vortex production is lower than the pair breaking velocity in superfluid ^{3}He.

The magnetic field of up to 0.4 T at the surface of the particle will, however, influence the superfluid state of ^{3}He. Depending on the particular experiment, this may introduce some complications.

A completely different aspect of this oscillator refers to its properties in vacuum, when the only interaction of the particle is with its superconducting environment. The dynamics of a levitated permanent magnet above a superconducting surface has recently attracted considerable interest, in particular since high-T_c superconductors have made levitation experiments both simpler and more spectacular [3]. A quantitative interpretation of the observed phenomena has not yet emerged, even the spatial variation of the levitation force is unclear. This is caused by flux pinning and flux motion which gives rise to hysteretic forces and amplitude dependent stiffness and friction [4–8]. An understanding of the dynamics of our oscillator in vacuum, therefore, will shed more light on the physics of magnetic

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levitation and the properties of flux motion in superconductors, both of which are of considerable practical significance.

2. Experimental

A piece of a $SmCo_5$ permanent magnet is ground to almost spherical shape. Visual inspection under an optical microscope yields a diameter of 200 μm which varies less than 10% when the particle is moved into different positions. From the density and the saturation magnetization of the material we conclude that the particle has a mass m of 2×10^{-8} kg and a magnetic dipole moment of 2×10^{-6} A m^2 (by assuming a homogeneous magnetization). The particle is placed inside a niobium capacitor with a spacing of 1 mm. The upper plate consists of a collector connected to an electrometer amplifier and a ground guard ring (fig. 1). Before cooling the capacitor below $T_c = 9$ K, a DC voltage of several hundred volts is applied to the bottom plate. Therefore, the particle acquires a charge and starts moving up and down between the capacitor plates until the temperature has dropped below T_c. Shielding currents on the superconducting surfaces then repel the magnet and it comes to rest at an equilibrium position where magnetic forces cancel gravitation and electrostatic forces. Owing to the type II behaviour of Nb the shielding of the magnetic field will not be perfect (fig. 2). Some flux lines are expected to exist in the superconductor. Therefore, the usual image technique does not describe the magnetic field distribution properly. Furthermore, the flux lines are subject to pinning forces thus contributing to vertical stiffness (and providing lateral stability). Electrostatic forces and gravitation are much weaker than the magnetic forces. This, however, does not mean that the equilibrium position is necessarily in the middle of the gap, because of hysteresis during descent and ascent [8]. In fact, we have evidence that the equilibrium position is indeed a very asymmetric one (see below).

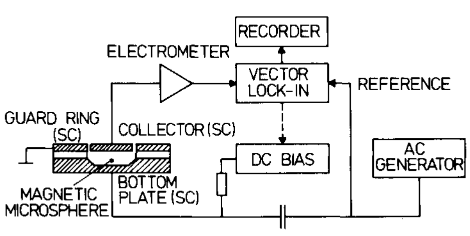

Fig. 1. Experimental set-up. The measuring cell is made of superconducting niobium cooled to 4.2 K.

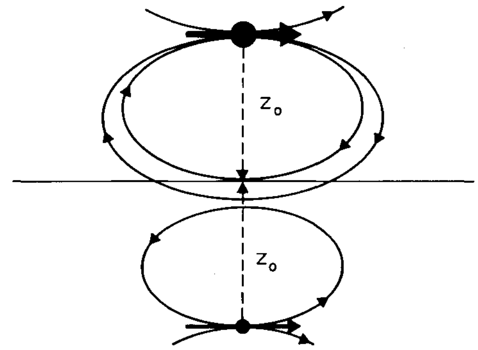

Fig. 2. Qualitative picture of a magnetic dipole at a distance z_0 above the surface of a type II superconductor. The image dipole below cannot completely expel the magnetic field if the critical field is exceeded at some part of the surface.

The resonance frequency of the vertical oscillations of the particle is found by sweeping the AC excitation frequency at a given amplitude of a few millivolts and monitoring the electrometer output voltage with a two-phase lock-in amplifier. The current I induced on the collector by the charge q of the particle is given by the velocity v, i.e.

$$I = qv/d = q\omega A/d \,, \tag{1}$$

where $d = 1$ mm is the gap between the plates and A is the amplitude of the oscillation at frequency ω. In the present experiment the particle has a charge $q \approx 1$ pC, thus a current of 1 pA corresponds to $v \approx 1$ mm/s and $A \approx 0.5$ μm because $\omega/2\pi$ is about 350 Hz. At resonance there is a 90° phase shift between the signal and capacitively coupled pick-up, off resonance the pick-up is eliminated numerically by complex analysis. All data shown below were taken with an evacuated cell at a temperature of 4.2 K. A new equilibrium position with a new resonance frequency $\omega/2\pi$ ranging from 300 to 600 Hz can be produced within a few minutes by briefly heating the niobium parts above T_c and letting the cell then cool back to 4.2 K. Because of the high Q values (up to 10^6) and the rather low frequencies, long time constants have to be tolerated when studying the properties of these oscillators. In the following section we present the typical results obtained with four very similar oscillators which were studied in detailed in the present measuring cell.

3. Results

In fig. 3 the frequency dependence of the amplitude is depicted at a fixed drive. Nonlinear return forces lead to a typical inclination of the resonance curve and therefore to a huge hysteresis. Large amplitudes can be excited only if the frequency is lowered from values above the resonance, i.e. moving along the upper part of the resonance curve. After the maximum value is reached, the amplitude drops discontinuously to almost zero. By sweeping the frequency up again, a comparatively small jump occurs where the resonance curve becomes single-valued (see fig. 4).

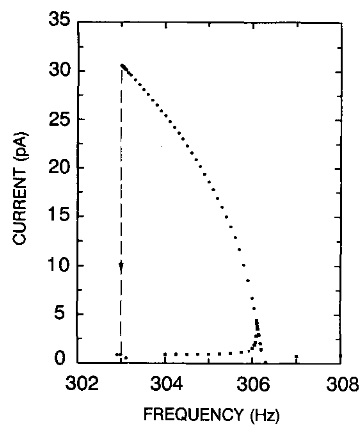

Fig. 3. Frequency dependence of the signal near resonance with an AC excitation of 50 mV. Because of the triple-valuedness of the resonance curve, the uppermost branch is accessible only by frequency sweeps from above and the lowest branch by sweeps from below. The missing third branch is unstable.

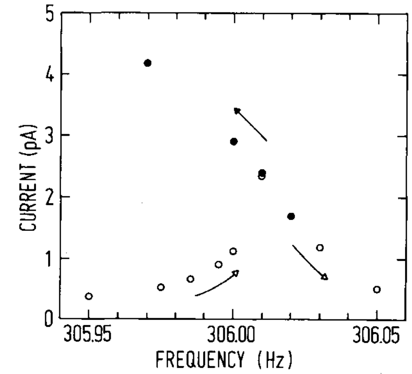

Fig. 4. Close-up of the resonance curve taken at a drive of 7.3 mV. Open circles indicate frequency sweep upwards, full circles downward sweep.

In order to characterize the oscillator in more detail, the following data were taken: the maximum current I_{max} and the corresponding frequency shift $\Delta f = f_{max} - f_0$ were measured as a function of the AC excitation, where f_0 is the resonance frequency at very low drive (i.e. without hysteresis). From this we obtain the following plots. In fig. 5, the relative frequency shift is shown as a function of I_{max}. We clearly find a quadratic dependence:

$$\frac{\Delta f}{f_0} = -aI^2_{max} , \tag{2}$$

where $a = 5.6 \times 10^{18}$ A^{-2}. Inserting eq. (1) gives

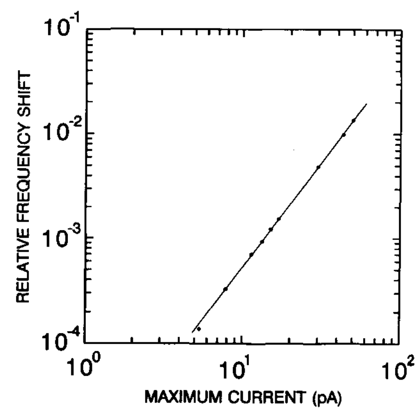

Fig. 5. Relative shift of the resonance frequency $-\Delta f/f_0$ as a function of the current amplitude. The straight line indicates a quadratic dependence.

the typical square-law dependence of the frequency shift on the amplitude of a nonlinear oscillator:

$$\frac{\Delta f}{f_0} = -\frac{aq^2\omega^2}{d^2} A^2 . \tag{3}$$

In fig. 6 the current amplitude as a function of the AC excitation amplitude is depicted. Here we find a square-root dependence at large drive:

$$I_{\max} = b\sqrt{U}_{AC} \tag{4}$$

with $b = 2.9 \times 10^{-10}\ \mathrm{AV}^{-1/2}$. Because the driving force is $F_d = qU_{AC}/d$, eq. (4) can be written as

$$A = \left(\frac{b^2 d^3}{\omega^2 q^3} F_d\right)^{1/2} . \tag{5}$$

Such a square-root behaviour is not due to nonlinear return forces but instead is typical for a nonlinear friction (see below).

It is interesting to follow the amplitude to a very low drive level (see fig. 7). At very small amplitudes (≤ 2 pA) we find a linear relation, indicating that the damping force has become linear in velocity. Most peculiar, though, is the plateau at 5 pA (and probably another one at 7.3 pA). It is quite obvious that the damping mechanism is amplitude dependent (except for the lowest amplitudes) and that there are sudden changes of the frictional force at certain values of amplitudes.

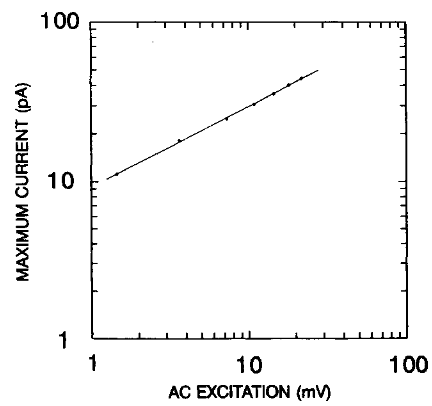

Fig. 6. Amplitude at large driving force. The straight line is a square-root dependence. Resonance frequency is 555.75 Hz.

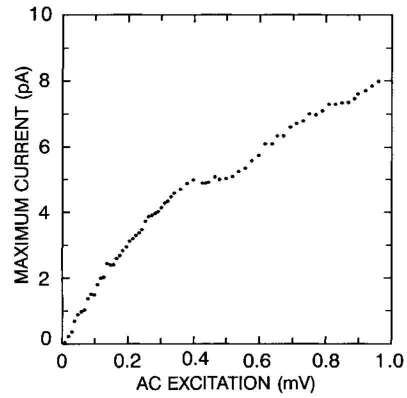

Fig. 7. Amplitude at low drive (same oscillator as in fig. 6.)

The same behaviour is evident in the free decay of the oscillations (see fig. 8). There are two sections of exponential decays with different time constants τ. The Q value is given by $Q = \omega\tau/2$. From fig. 8 we find, for example $Q_1 = 2.6 \times 10^5$ and $Q_2 = 4.7 \times 10^5$. The transition occurs at a current amplitude of 5 pA, which is close to the plateau in fig. 7. Initially, that is at larger amplitudes, the Q values are always smaller than later on. Because of its amplitude dependence, the frequency is changing during the free decay; this has prevented us so far from recording free decays from larger levels.

Finally, we have applied a DC voltage to the cell thereby changing the equilibrium position of the particle. This results in a frequency shift, which in all observed cases is perfectly linear

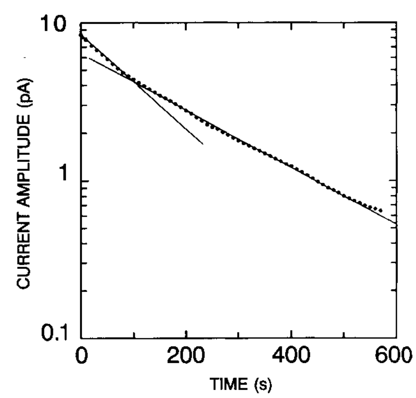

Fig. 8. Free decay of the oscillations. Note the different decay constants $\tau_1 = 151$ s and $\tau_2 = 272$ s (same oscillator as in fig. 6).

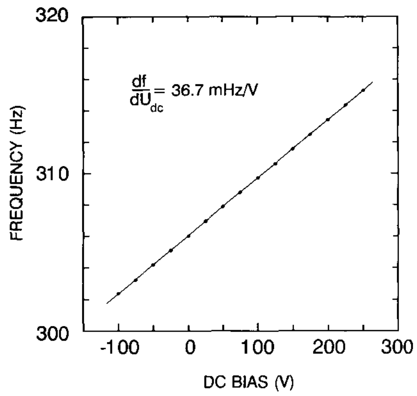

Fig. 9. Change of resonance frequency with DC voltage applied to the capacitor.

with the voltage and independent of the AC drive (see fig. 9). From the slope we find $\mathrm{d}f/\mathrm{d}U_{\mathrm{DC}} = 36.7\,\mathrm{mHz/V}$, which is a typical value. For the oscillator at $f_0 = 556\,\mathrm{Hz}$ we get $27.9\,\mathrm{mHz/V}$. The voltage dependence may be used to keep the oscillator at a fixed resonance frequency by means of a feedback loop which nulls the imaginary signal channel of the vector lock-in. This method is indispensible when working at resonance at low drive levels because of the high Q values of the oscillator.

4. Discussion

We first discuss the nonlinear characteristics of the oscillator which can be attributed to nonlinear return forces. It is well known [9] that corrections up to cubic terms, that is

$$\frac{F}{m} = -\omega_0^2 x - \alpha x^2 - \beta x^3 , \tag{6}$$

lead to an amplitude dependent frequency, namely,

$$\frac{\Delta f}{f_0} = \left(\frac{3\beta}{8\omega_0^2} - \frac{5\alpha^2}{12\omega_0^4}\right) A^2 . \tag{7}$$

This has to be compared with the experimental result (see eqs. (2) and (3)). Because there is no quantitative description of the magnetic force available as yet, the nonlinear coefficients α and β are unknown. We therefore proceed with our analysis as follows. We use the linear dependence of the frequency on the applied DC voltage (see fig. 9) to make the following Ansatz:

$$\omega(\mathscr{E}) = \omega_{00} + c(mg \pm q\mathscr{E}) , \tag{8}$$

where ω_{00} is the resonance frequency without the external forces mg (gravitation) and $q\mathscr{E}$ (electrostatic charge times external field) and c is a constant. At the equilibrium position the unknown magnetic force F cancels the external forces:

$$F = mg \pm q\mathscr{E} . \tag{9}$$

Next, we assume that the dynamical stiffness is given by the gradient of F at the equilibrium position which depends on the electric field $\mathscr{E}$:

$$-\frac{\mathrm{d}F}{\mathrm{d}x} = m\omega^2 = m(\omega_{00} + cF)^2 . \tag{10}$$

Integration of eq. (10) gives

$$F(x) = \frac{1}{c}\left(\frac{1}{cmx} - \omega_{00}\right) . \tag{11}$$

The integration constant is set to zero, that is we choose $x = 0$ as the position of the divergence. This position will not be right at the superconducting surface but probably close. We will discuss this point further below.

The equilibrium position x_0 follows from eq. (9):

$$x_0 = (cm\omega)^{-1} . \tag{12}$$

If the charge q is known, the unknown constant c can be determined experimentally from

$$cq = \frac{\mathrm{d}\omega}{\mathrm{d}\mathscr{E}} = 2\pi d\,\frac{\mathrm{d}f}{\mathrm{d}U} . \tag{13}$$

Alternatively, an expansion of $F(x)$ at x_0 gives the nonlinear coefficients α and β (see eq. (6)). Inserting these into eq. (7) yields

$$\frac{\Delta f}{f_0} = -\frac{1}{24x_0^2} A^2 . \tag{14}$$

Comparing with eq. (3) and using (13) we obtain the charge q and the constant c separately. Thus the magnetic force and the equilibrium position x_0 (with respect to $x = 0$) are known. For all of our oscillators we find that x_0 is about 100 μm and that q varies from about 0.5 pC to the theoretical maximum value of 1.1 pC (at our $\mathscr{E} = 600$ kV/m). The constant c amounts to several 10^8 Hz/N. This implies that the frequency is always quite close to ω_{00}, gravitational and electrostatic forces (we are neglecting image forces) have only little effect on the equilibrium position. The external voltages of several hundred volts change the frequency and hence x_0 only by a few percent, that is by a few micrometers. The position $x = 0$ where F diverges is probably the return point of the sphere after approaching the first superconducting surface (see e.g. [7, 8]). During the approach, most of the rather large kinetic energy ($\sim 10^{-10}$ J) is stored or dissipated in the superconductor, but some will be left for a rebound. Because the cooling below T_c occurs with the particle bouncing back and forth, we have the so-called "field-cooled" levitation (or suspension) force [7]. The particle then sits in a potential well whose curvature is given by ω. The repulsive part of the force varies as $1/x$ (from some unknown position $x = 0$ on) and the attractive force is constant. We should emphasize, though, that this is valid only over our very limited interval of equilibrium positions.

The assumption that the dynamical stiffness is given by the gradient of the static force (see eq. (10)) has to be discussed. While this is certainly wrong for the hysteresis curves during descent and ascent [4–8], they seem to agree better when only small displacements of 0.5 mm are performed [8]. In our case the displacements are even two orders of magnitude smaller, therefore the above assumption appears to be reasonable.

Finally, the frictional force has to be analysed. The square-root dependence of the amplitude as a function of drive (see eq. (5)) implies a frictional force F_f which is quadratic in velocity, namely,

$$F_f = -m\gamma |v| v \,. \tag{15}$$

At resonance this force leads to [10]

$$A = \left(\frac{3\pi}{8\gamma m\omega^2} F_d \right)^{1/2} , \tag{16}$$

which when compared with (5) gives

$$\gamma = \frac{3\pi q^3}{8mb^2d^3} \,. \tag{17}$$

Our typical values are about 0.5 m^{-1}.

Towards lower amplitudes, the frictional force obviously changes to a linear behaviour, namely,

$$F_f = -m\lambda v \,. \tag{18}$$

We observe stepwise changes in λ as the drive is reduced (a fixed value of λ corresponds to a straight line in fig. 7, see also [2]). As a qualitative interpretation we suggest that the amplitude- and time-dependent magnetic field at the surface of the superconductor leads to flux motion which causes amplitude-dependent and hysteretic damping. Similar effects have been observed in levitation experiments with bigger magnets [6] and vibrating superconducting reeds [11]. The stepwise changes, however, have not been observed before, probably because of too low energy resolution.

While the linear friction force corresponds to an exponential decay of the free oscillation with a well defined Q value, the nonlinear force has a different decay law [10], namely A^{-1} becoming linear in time. We have no clear confirmation of this law in our experiment as yet due to the above mentioned frequency shifts. However, at lower amplitudes, the decay in fig. 8 is seen to consist of two different time constants from which the two different Q values, $Q_1 = 2.6 \times 10^5$ and $Q_2 = 4.7 \times 10^5$ can be obtained. Calculating the Q values for the data of fig. 7 we use [9]

$$A = \frac{F_d}{m\omega\lambda} \tag{19}$$

and eq. (1). This gives

$$Q = \frac{\omega\tau}{2} = \frac{\omega}{\lambda} = \frac{m\omega d^2}{q^2} \frac{I}{U_{AC}} \,. \tag{20}$$

From fig. 7 we obtain from the data above the plateau $Q_1 = 6.8 \times 10^5$ and for low amplitudes $Q_2 = 1.2 \times 10^6$. (These Q values are larger than those obtained from the free decay (fig. 8) because of a somewhat different conversion factor from current to amplitude in eq. (20). The ratio of the Q values is the same.) The transition from linear friction in eq. (19) to nonlinear friction is considered to occur in a series of discrete steps by which on the average, the friction coefficient becomes a linear function of the velocity: $\lambda(v) =$ constant for low velocities and $\lambda(v) = \gamma v$ for large velocities. At large velocities the steps cannot be resolved anymore (see also [2]) and a smooth square-root law (5) results.

5. Conclusion

Although we have made some encouraging progress in understanding the dynamical behaviour of the levitated sphere, we are still far from a complete quantitative description. A physical model for the spatial variation of the levitation force is lacking as well as a calculation of the friction coefficients. Further experiments with different superconductors and at different temperatures are under way. Of course, high T_c superconductors will also be studied. Our method is certainly promising enough for future work on the physics of magnetic levitation.

Returning to our original goal to use this oscillator for studies of vorticity in superfluid ^{3}He, we note that the low residual damping is particularly useful for low viscosity applications. Of course, simple hydrodynamic friction can be studied in detail because of the spherical shape of the particle. However, even more interesting appears to be an experiment involving supercritical velocities which because of the large radius can easily be reached. The question is: Can we detect the nucleation of vorticity? What is the nucleation rate then? And finally: What happens when the oscillating sphere gets attached to a vortex in rotating superfluid helium? We are looking forward to attempting to answer these questions experimentally.

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