First-Sound Attenuation and Viscosity of Superfluid ^{3}He-*B*

G. Eska, K. Neumaier, W. Schoepe,[a] K. Uhlig, and W. Wiedemann
Zentralinstitut für Tieftemperaturforschung der Bayerischen Akademie der Wissenschaften, 8046 Garching, West Germany

and

P. Wölfle
Fachbereich Physik, Technische Universität München, 8046 Garching, West Germany
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The attenuation of first sound in normal and superfluid ^{3}He in a cylindrical resonator has been measured from 40 kHz to 300 kHz at 8 and 28 bars. In the *B* phase a decreasing viscosity down to the lowest temperature ($0.5T_c$) is observed. In the normal phase the viscosity agrees with previously reported values. The data analysis is based on a recent theory of sound propagation in superfluid ^{3}He in a resonator.

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Measurements of the attenuation of sound in superfluid ^{3}He have revealed a variety of collective modes in the collisionless regime (zero sound).[1] However, the hydrodynamic mode (first sound) which allows a determination of the viscosity of the liquid has not been studied so far. It is the purpose of this Letter to report on the first measurements of the attenuation of first sound in superfluid ^{3}He. Our work was motivated by the considerable interest which the viscosity has gained recently because of conflicting experimental and theoretical results.[2] Whereas theory[1] predicts a nearly temperature-independent viscosity below $T/T_c \approx 0.77$ the experiments of Main *et al.*[3] and Alvesalo *et al.*[4] indicate a monotonic decrease; in the case of Ref. 4 even down to $T/T_c = 0.4$. Those experiments were performed in a torsional oscillator[3,4] or with a vibrating wire,[4] both involving a rather restricted geometry. In our method a comparatively open geometry was employed but mean-free-path effects occurring at the walls of our cylindrical sound resonator were still important; however, they could be treated quantitatively. Our data on sound attenuation in ^{3}He-*B* yield a monotonically decreasing viscosity. In the normal phase our results are in good agreement with previously reported values.[5]

Because of the very low temperatures and hence the long quasiparticle collision times τ, hydrodynamic sound exists only at rather low frequencies $\omega/2\pi$ ($\omega\tau \ll 1$). This condition is even more stringent in the superfluid phases where τ is rising very rapidly as a result of the decreasing number of quasiparticles with decreasing temperature. For this reason the frequencies have to be well below 1 MHz where the use of conventional quartz transducers is inconvenient if not impossible. Instead, we employed capacitive sound transducers of the design used earlier at higher temperatures.[6] Our pulsed method of measuring the sound attenuation consisted of first exciting a standing sound wave in a cylindrical resonator filled with liquid ^{3}He and terminated at either end by the transducers. The length l of the resonator was 1.0 cm and its diameter $2R = 0.8$ cm. At a pressure of 28 bars the fundamental plane-wave resonance occurred at 19.78 kHz. The odd resonances from the third to the fifteenth harmonic, i.e., 60 kHz $\lesssim \omega/2\pi \lesssim 300$ kHz, were found to have acceptably high background Q values ($Q \gtrsim 1000$) at high temperatures (~ 300 mK) where the damping due to the low viscosity of the liquid is neglgible.[7] After the standing wave had reached a constant amplitude, typically after 5 msec, the driving pulse (≈ 1 V ac on top of a constant dc polarizing voltage of about 400 V) was turned off and the free decay of the standing wave was recorded with a transient recorder. Data were analyzed with use of a computer by fitting an exponential decay $\exp(-\beta t)$ to the digitized signal. From the decay rate (which was independent of the applied driving voltage) the sound attenuation α can be determined: $\alpha = \beta/c$, where $c = \omega l/\pi n$ is the velocity of sound and $n = 1, 2, 3...$ is the order of the harmonic.

The sample cell containing the resonator was filled with 5.5 cm^3 of ^{3}He (including the liquid in the heat exchangers of 18-m^2 surface area) which was cooled by means of adiabatic nuclear demagnetization of 0.25 mole of $PrNi_5$ from a starting temperature of about 16 mK and an initial magnetic field of 3 T. Pulsed NMR on ^{195}Pt was used for thermometry. The Pt powder was immersed in the liquid in a separate tower close to the resonator. The static magnetic field of 28 mT was applied along the direction of the sound propagation. The field homogeneity over the entire ^{3}He cell was better than 1%. Above 10 mK the susceptibility of the Pt was calibrated by means of τ_1 measurements using a Korringa constant of 31.95 msec K.[8] The superfluid transition temperature T_c (La Jolla scale[9]) served as a second calibration point. T_c was determined by the rapid change of the longitudinal relaxation time of ^{3}He at T_c. The sample could be cooled to a lowest temperature of 1.2 mK and stayed below T_c at 28 bars for more than three days. The warmup rate was 20 μK/h in the temperature range of interest.

Before presenting our results for the superfluid phase, we mention that above T_c we tested our method in the normal fluid where the viscosity is well known[5] and classical hydrodynamics can be applied at our pressures. Hence the attenuation of sound in a cylincrical resonator is given by[10]

$$\alpha = 2\omega^2\eta/3\rho c^3 + \omega\delta/2Rc, \qquad (1)$$

where ρ is the liquid density and $\delta = (2\eta/\rho\omega)^{1/2}$ is the viscous penetration depth. The first term is the attenuation in the bulk liquid, the second term is due to the viscous losses at the walls and dominates in our case. Because of $\eta \propto T^{-2}$ the second term increases at T^{-1}. At lower temperatures the bulk attenuation ($\propto T^{-2}$) starts to contribute. This behavior is clearly demonstrated by our normal-fluid data shown in Fig. 1. (A frequency-dependent background attenuation which was determined from an extrapolation of the data to $1/T = 0$ has been subtracted.) The solid lines are fits to Eq. (1) from which we obtain $\eta T^2 = 1.62$ (± 0.18) P mK^2 at 8 bars and 1.04 (± 0.03) P mK^2 at 28 bars which agree well with Ref. 5.

In the superfluid phase, the attenuation of first sound drops rapidly as the temperature is lowerd, see Fig. 1. This drop is related to the decrease of both the viscosity and the density ρ_n of the normal-fluid component. However, extending Eq. (1) below T_c simply by replacing δ by $\delta_n\rho_n/\rho$ [where $\delta_n = (2\eta/\rho_n\omega)^{1/2}$] is a crude approximation even at our frequencies. Instead, the boundary conditions at the wall have to be reconsidered. Assuming diffuse scattering of the quasiparticles at the walls, Jensen *et al.*[11] find that with increasing frequency the velocity of the normal component starts to develop a slip at the wall.

A full description of sound propagation in this regime requires the solution of a boundary-value problem for the quasiparticle kinetic equation, which is prohibitively complicated. Instead, Na-

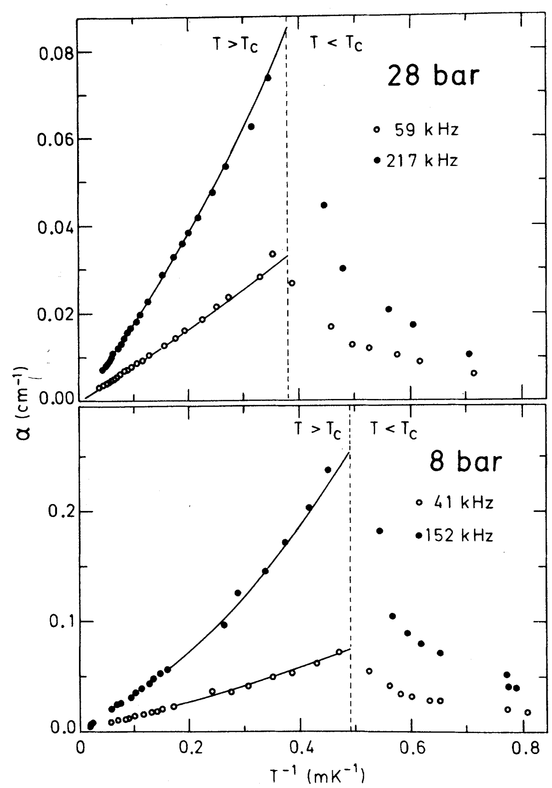

FIG. 1. Attenuation of first sound in normal and superfluid ^{3}He at different frequencies and pressures. The solid lines above T_c are fits to Eq. (1). For the sake of clarity only part of our data is plotted.

gai and Wölfle,[12] employing a viscoelastic-model description of the Fermi liquid which works in the hydrodynamic as well as the collisionless regime, solved a set of generalized hydrodynamic equations amended by a slip boundary condition. The full details of this theory turn out to be not essential in the present case, because of the fact that the viscous penetration depth δ_n is much less than the dimensions of the resonator as well as the wavelength of sound. The attenuation of sound by friction at the walls is therefore in good approximation given by the surface impedance $Z(\omega)$ (Ref. 12),

$$\alpha_{\text{wall}} = (1/Rc\rho)\,\mathrm{Re}Z(\omega). \qquad (2)$$

Corrections to this expression from the more detailed theory are[12]

$$\Delta\alpha_{\text{wall}} = \tfrac{3}{8}(\rho_n/\rho)\alpha_{\text{bulk}}[1 + O((kR)^{-2})]. \qquad (3)$$

The surface impedance has been calculated using a variational approach,[11]

$$Z=\tfrac{3}{8}\rho\frac{m^*}{m}v_f Y_1\left[1+\left(1+\frac{45}{64}\,\frac{l+\frac{1}{3}(1-i\omega\tau)Y_0F_1{}^s}{-i\omega\tau}\,\frac{Y_1{}^2}{Y_0Y_2}\right)^{1/2}\left(1-\lambda_2\frac{Y_2/Y_0}{l-i\omega\tau}\right)^{1/2}\right]^{-1}. \tag{4}$$

Here

$$Y_n(T)=-\int_{-\infty}^{\infty}d\xi\,|\xi/E|^n\,df(E)/dE\ ,$$

with $f(E)$ the Fermi function, $E=(\xi^2+\Delta^2)^{1/2}$, and λ_2 is a parameter characterizing the collision integral, typically 0.6–0.7. The quasiparticle lifetime τ may be related to the viscosity coefficient η by[11]

$$\eta=\tfrac{1}{5}\rho(m^*/m)v_f{}^2Y_2\tau_\eta, \tag{5}$$

where $\tau_\eta=\tau/(1-\lambda_2Y_2/Y_0)$ is the viscous relaxation time. The attenuation in the bulk liquid is given in the visco-elastic approximation by[12]

$$\alpha_{\text{bulk}}=(2\omega^2\eta/3c^3\rho)[1/1+(\omega\tau_\eta)^2]. \tag{6}$$

The total attenuation is given by the sum of Eqs. (2), (3), and (6).

With this theory we have calculated the viscosity from our data by expressing the theoretical attenuation in terms of the single variable $\omega\tau$ and solving numerically for $\omega\tau$ which yields η from Eq. (5). Fermi-liquid parameters were taken from Wheatley.[13] The Yosida functions were calculated by assuming $\Delta(0)=1.8T_c$ at 8 bars and $\Delta(0)=2.0T_c$ at 28 bars.[14] Changing the input parameters did not change the results very much: a 30% decrease of m^* gave a 10% higher η at low T/T_c (2% at high T/T_c); varying λ_2 by 20% or the $Y_n(T)$ by 10% changed η by less than 1 or 5%, respectively. The relative error of η is given mostly by twice the error of the measured α, being the largest at low α as a result of background subtraction and at high α as a result of poor signal quality.

The result of this analysis is shown in Fig. 2. We notice that all of our frequencies, which vary by a factor of 5 and reach well into the onset of zero sound ($\omega\tau$ ranging up to 0.3), yield the same result. It is interesting that the viscosity does not become temperature independent. Instead it decreases to a fraction of its value at T_c which is smaller than we had expected. If we had used the hydrodynamic limit instead, the viscosities would have been even lower by up to 15%. At higher T/T_c, however, our data are above those of Ref. 4 and theory.[15,16] Setting $\omega\tau=0$ in this regime has only little effect on the data and does not remove the discrepancy; the reason for this remains unclear to us. The deviation at low temperatures found at 28 bars is probably less serious. The result of Ref. 4 might well be affected by mean-free-path effects at low T/T_c and the theoretical curve is likely to be shifted to lower values if one allows for more structure in the quasiparticle scattering amplitude. Further experimental work at lower temperature is needed to determine where and at which value the viscosity finally will pass through a shallow minimum. Also we will extend our method by measuring accurately the small changes of the velocity of sound due to the temperature dependence of $Z(\omega)$.[17] From both the attenuation and the velocity of sound ρ_n/ρ and η can be determined simultaneously. An extension of our work to the more complicated A phase is being planned.

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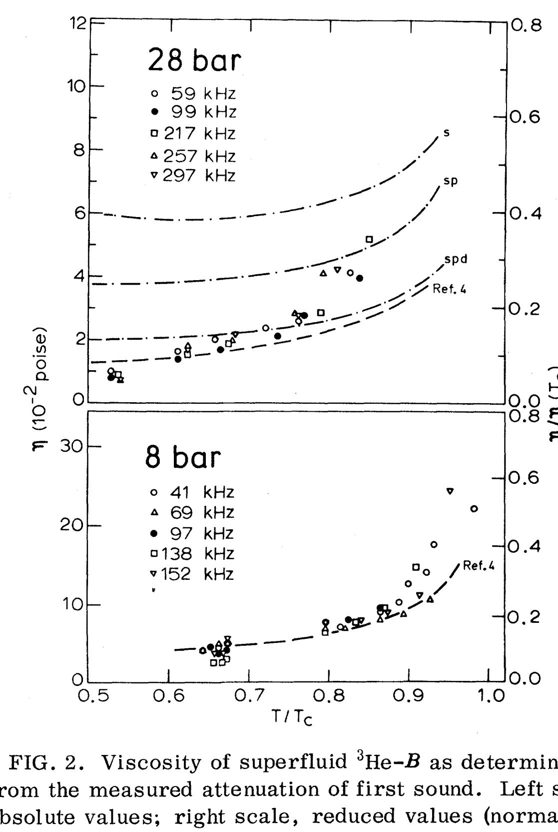

FIG. 2. Viscosity of superfluid ^{3}He-B as determined from the measured attenuation of first sound. Left scale, absolute values; right scale, reduced values (normalized at T_c). Dashed line, smoothed data of Ref. 4 (10 and 29 bars); dash-dotted lines, theory (s, sp, and spd approximation) from Ref. 16.

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[(a)]On leave from Regensburg University, Regensburg, West Germany.

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[7]In the middle between the transducers the resonator had a hole for filling. Therefore, the odd resonances were less influenced than the even ones which have a pressure maximum there.

[8]The Pt powder and the information on its Korringa constant were kindly provided to us by D. D. Osheroff.

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[10]See, e.g., L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1959).

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[17]Those measurements require a pressure stability of about 1 part in 10^5 which is an order of magnitude better than we had in our present work.

Shallow-Deep Instabilities of Donor Impurity Levels and Excitons in Many-Valley Semiconductors

L. Resca[(a)]
Department of Physics, Purdue University, West Lafayette, Indiana 47907

and

R. Resta
Laboratoire de Physique Appliquée, École Politechnique Fédérale, CH-1007 Lausanne, Switzerland

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Two recent theories are compared quantitatively in order to describe the intervalley mixing effect on donor impurity levels and excitons in many-valley semiconductors. Deep levels are predicted for a screened point-charge potential, regardless of the position at which the potential is centered. Altarelli-Hsu predict that the same potential has a deep level only if centered at an interstitial site. It is shown that this basic discrepancy is a direct consequence of a key Altarelli-Hsu approximation.

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Impurity levels and excitons play a central role in semiconductor physics. A vast experimental information has been gathered in the past in connection with shallow donor levels.[1] However, it has been experimentally determined that interstitial hydrogen and muonium exhibit deep donor levels in silicon and germanium.[2] Valence excitons are shallow levels in semiconductors. However, there is experimental evidence that that the corresponding core excitons may be deep.[3]

The most common semiconductors, such as Si, Ge, GaP, GaAs, etc., exhibit several equivalent minima. It is believed that this feature plays a key role for the shallow or deep nature of those

