TUNNELING FROM ELECTRONIC BUBBLE STATES THROUGH THE LIQUID-VAPOR INTERFACE OF HeII *

G. W. RAYFIELD and W. SCHOEPE **
Physics Department, University of Oregon, Eugene, Oregon 97403, *USA*

Received 8 November 1971

A model is presented which shows that electrons escape through the free surface of HeII by tunneling from electronic bubble states. The results give a bubble radius of 12.9 Å and a binding energy of 0.72 eV for the electronic bubble.

Negative ions in HeII are trapped below the liquid surface by an image induced potential of the form $V(x) = A/x + e\mathcal{E}x$ [1,2]. The coefficient A depends on the dielectric constant of HeII and $\mathcal{E}$ is the electric field applied across the interface. In previous work [1,2] we have measured the lifetime of negative ions trapped in this potential as a function of temperature and electric field $\mathcal{E}$. We interpreted our results in terms of a model in which the negative ions diffuse through the liquid by Brownian motion up to a point near the surface and then escape to the vapor phase. A classical interpretation indicated an unusually large curvature of the top of the potential barrier and we proposed the possibility of tunneling to explain this result.

In this letter we present a detailed model in which the negative ions (electronic bubbles) move toward the surface by Brownian motion and then decay by electron tunneling into the vapor phase. A fit of our previous data to this theory gives the well depth and radius of the electronic bubble in HeII.

We first calculate the tunneling transition rate for an electron from the bubble state into the vapor. Let the bubble be a distance x from the surface as shown in fig.1. We will neglect surface distortions and bubble distortions in the present calculation. The barrier height seen by the electrons near the surface is $(E_1 - E_0)$, see fig. 1. E_1 is the energy of the untrapped electron [3,5] and E_0 is the zero point energy of the electron in the bubble state. We set $E_1 - E_0 = \hbar^2\alpha^2/2m$. The probability the electron tunnels a distance d through the liquid is $\exp(-2\alpha d)$. From fig.1: $d = (x/\cos\theta) - R$. The tunneling transition rate from the bubble at the solid angle $d\Omega$ is $\nu \exp[-2\alpha\{(x/\cos\theta) - R\}] d\Omega/4\pi$ where ν is the frequency at which the electron hits the walls of the bubble. The total transition rate is

$$\tfrac{1}{2}\nu \int_0^{\pi/2} \exp[-2\alpha\{(x/\cos\theta) - R\}] \sin\theta \, d\theta =$$

$$= \tfrac{1}{2}\nu \exp(2\alpha R)\, W(x)$$

where $W(x) \approx \exp(-2\alpha x)/2\alpha x$. If the number of particles between x and $x + dx$ is $n(x)dx$, the current through the surface from this region is $\frac{1}{2}n(x)\nu \exp(2\alpha R)\, W(x) dx$. The equation which determines n may be found from the continuity equation. The resulting equation is not easily solved but the rate of loss of particles from any given region is small compared to the divergence of the diffusion or electric field currents. We therefore take an equilibrium distribution (Maxwell-Boltzmann) in our regions of interest: $n(x) = C\exp(-V(x)/T)$. C is determined by the normalization condition $\int_0^\infty n(x)\,dx = N_0$ where N_0 is the total number of particles in the well at

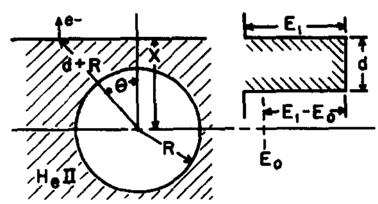

Fig.1. The spherical well in which the electron is trapped near the surface of HeII is shown. Also indicated in the figure is the barrier through which the trapped electron must tunnel. E_0 is the zero point energy of the electron in the bubble.

* Work supported by NSF Grant GP-29659.
** NATO post-doctoral fellow.

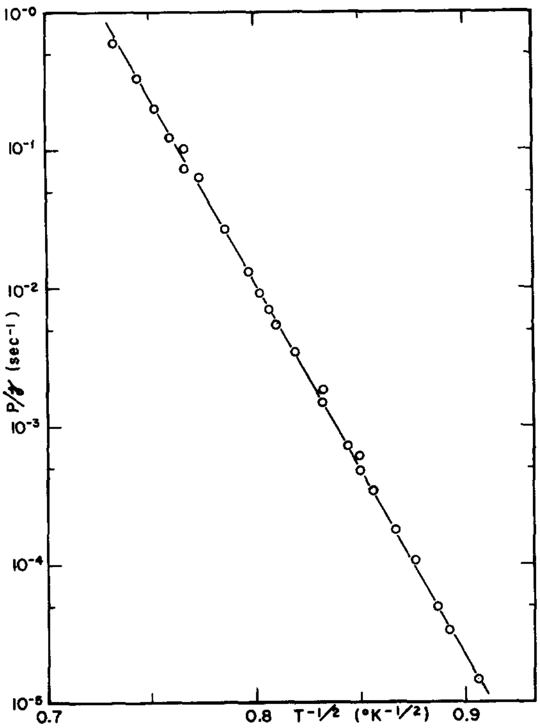

Fig.2. Reduced escape probability per second P/γ versus $1/\sqrt{T}$. Solid line is theory with $R = 12.9$ Å and $\alpha = 0.436$ Å^{-1}. Data are from refs. [1, 2].

some particular time *. Performing the integral over $n(x)$ we find $C = N_0/I_1$ with $I_1 = 2\sqrt{A/e\mathcal{E}}\,K_1(2\sqrt{Ae\mathcal{E}}/T)$ Å where K_1 is a modified Bessel function. In our units length is in Å, $e\mathcal{E}$ in $^{\circ}$K Å^{-1}, A in $^{\circ}$K Å^{-1} and T in $^{\circ}$K. The total current from the surface is:

$$j_S = \tfrac{1}{2}\nu \exp(2\alpha R) \int n(x)\, W(x)\, dx = $$

$$= \tfrac{1}{2}\nu \exp(2\alpha R)\, N_0\, I_2/I_1$$

where

$$I_2 = \int_R^{\infty} \frac{\exp[-\{(A/Tx) + (e\mathcal{E}x/T) + 2\alpha x\}]}{2\alpha x}\, dx .$$

* In general N_0 depends on temperature and electric field. See ref. [2] Section 4.

Most tunneling takes place from a region 20 Å to 40 Å below the liquid surface. Extending the lower limit of the integral I_2 to zero and performing the integration we find $I_2 = K_0(z)/\alpha$ Å where $z = \sqrt{(4A/T)(e\mathcal{E}/T + 2\alpha)}$ and K_0 is a modified Bessel function. Approximating the Bessel functions we find the probability per unit time a particle escapes from the surface:

$$P = j_S/N_0 \approx \tfrac{1}{2}\nu\gamma(\mathcal{E},T)\exp(2\alpha R) \times \exp - (8A\alpha/T)^{1/2}\ \mathrm{sec}^{-1} \qquad (1)$$

where

$$\gamma(\mathcal{E},T) \propto \frac{\mathcal{E}^{3/4}}{T^{1/4}} \frac{\exp[2\sqrt{Ae\mathcal{E}}/T - (e\mathcal{E}/\alpha T)(\alpha A/2T)^{1/2}]}{\{1 + 3T/16\sqrt{Ae\mathcal{E}}\}}$$

The trapping time $1/P$ varies exponentially with R and α.

Fig.2 shows the reduced escape probability per second P/γ versus $1/\sqrt{T}$. The experimental values for P are taken from our earlier work [1, 2]. A least squares fit yields $\alpha = 0.436$ Å^{-1} and $R = 12.9$ Å with a correlation coefficient 0.99955. Using our value for α we find $E_1 - E_0 = 0.72$ eV. Theoretical calculations by Fowler and Dexter [4] yield $E_1 - E_0 = 0.92$ eV and $R = 17.5$ Å. Energy levels of electronic bubble states have been studied in detail by other investigators, see ref. [5].

Experiments are presently underway to examine the emission of electrons from the surface of ^{3}He and ^{3}He-^{4}He mixtures.

References

[1] G.W.Rayfield and W.Schoepe, Phys.Letters 34A (1971) 133.
[2] W.Schoepe and G.W.Rayfield, Z.Naturforsch., to be published.
[3] W.T.Sommer, Phys.Rev.Lett. 12 (1964) 271; M.A.Woolf and G.W.Rayfield, Phys.Rev.Lett. 15 (1965) 235.
[4] W.B.Fowler and D.L.Dexter, Phys.Rev. 176 (1968) 337.
[5] C.Zipfel and T.M.Sanders Jr., Proc. 11th Int. Conf. on Low temperature physics, 1968, Vol. 1, p.296;
T.Miyakawa and D.L.Dexter, Phys.Rev. A1 (1970) 513.

* * * * *