Safety first portfolio choice based on financial and sustainability returns

Gregor Dorfleitner, Sebastian Utz*

Department of Finance, University of Regensburg, 93040 Regensburg, Germany

Abstract

The aim of this paper is to expand the methodological spectrum of socially responsible investing by introducing stochastic sustainability returns into safety first models for portfolio choice. We provide a foundation of the notion of sustainability in portfolio theory and establish a general model for generalized safety first portfolio management with probabilistic constraints and three specializations of it. Moreover, we prove theorems about conditions for unique optimal solutions and for the constraints of one model being more restrictive than those of another. In an empirical part, we calculate the costs of investing according to our approach in terms of less financial return.

Keywords: Finance, Socially Responsible Investing, Sustainability Value, Safety First Investor

1. Introduction

In recent years, investors' behavior has fundamentally changed. Although the financial return is still important for the investment decision, environmental, social, and governance (ESG) issues of investment opportunities are beginning to receive more and more attention. The amount of invested capital in sustainable funds has been rapidly increasing over the last few years. In Europe, the market for *socially responsible investments* (SRI) has amounted

*Corresponding author. Tel. +49 941 943 5669; fax +49 941 943 4608.

Email addresses: gregor.dorfleitner@wiwi.uni-r.de (Gregor Dorfleitner), sebastian.utz@wiwi.uni-r.de (Sebastian Utz)

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to 4,986 billion euros as of December 31, 2009 (see Eurosif, 2010). The growth from 2005 to 2009 in the euro amount of SRI is 338%.

Principally, we build on the ideas of SRI. However, we do not regard an investment's sustainability as an objective property like the financial return. On the contrary, an investment's sustainability depends on the individual preferences of every single investor. Every kind of investment can have non-financial impacts that are desirable for some investors. These impacts are summarized under the term sustainability.

Furthermore, the standard approach in SRI consists of screening methods which select the investable assets in a first step and optimize the portfolio conventionally in a second. This paper expands the methodological spectrum of SRI in four different ways. First, we provide a comprehensive foundation of sustainability in portfolio theory with axioms and definitions. Second, we introduce a general model for generalized safety first portfolio management with probabilistic constraints. Additionally we discuss three variants of this general model and establish conditions for unique optimal solutions. Our approach is not to represent investors' preferences with an inclusion or exclusion criterion for every single asset as screening does, but rather with a scalable quantity capturing the quality of sustainability of an investment as an additional stochastic objective variable. Third, we treat the case of deterministic sustainability returns and show that under this assumption two of the three models suggested are equivalent. Fourth, we show the practical applicability of our models and calculate financial costs of investing sustainably.

The remainder of the paper is organized as follows. After a concise literature review directly following, Section 3 covers sustainability ratings and quantifies the notion of an investment's sustainability return. We introduce three different models for downside risk portfolio choice in Section 4 and prove results on the solvability of the constructed models. Section 5 comprises an empirical application, while Section 6 concludes.

2. Literature Review

Renneboog et al. (2008) may serve as a comprehensive review of the developments and methods in SRI. Whereas Guerard (1997) studies the performance differences of portfolios with various screening criteria, Bello (2005) and Hamilton et al. (1993) compare the performance of sustainable and common funds. All of them show that there is no significant under- or overperformance of sustainable funds. Galema et al. (2008) consider the impact of SRI on stock returns and conclude that SRI has a significant impact on the stock returns. Benson & Humphrey (2008) find that SRI fund flow is less sensitive to returns than the fund flow of conventional fundss and that SRI investors are less concerned about returns than conventional investors. Bollen (2007) suggests measuring the utility of a portfolio with multi-attributive utility functions, but he still shapes the SRI optimization problem in a binary manner, using an indicator function for the fulfillment of SRI attitudes.

In contrast, Hallerbach et al. (2004) give a practical approach for portfolio selection utilizing multi-attributive preference functions. Ballestero et al. (2012) provide a financial-ethical bi-criteria model especially for SRI portfolio selection. Furthermore, studies like Abdelaziz et al. (2007) and Steuer et al. (2007) argue that portfolio selection is a multi-objective problem. While all of the above references shape the sustainability quantity as deterministic, Dorfleitner et al. (2011) introduce the idea of stochastic social returns and incorporate them into a Markowitz-like portfolio selection framework.

In recent years safety first approaches and variants of it as discussed in Haley & Whiteman (2008) or Huang (2008), all of which have been based on the pioneering work of Telser (1955) and Roy (1952), have increasingly gained attention due to the growing practical relevance of downside risk.

Instead of a two-step portfolio selection with sustainability screening first and financial optimization second, which all of the papers mentioned in the first paragraph use, we establish models with financial and sustainable real-valued objective variables. This idea is also considered by all papers cited in the second paragraph above. However, of these approaches only Dorfleitner et al. (2011) uses stochastic sustainability returns. We contribute to the literature of SRI by using this new concept in the context of safety first portfolio choice.

3. Modeling sustainability value and sustainability return

The basic idea is that an investment is characterized by different quantities, namely the initial wealth V^0 at the beginning, the final wealth[1] V^t and a value of sustainability at the end of the investment period $[0, t]$.

3.1. Sustainability ratings

The growing demand for sustainable investments introduces some associated developments. International committees pass standards for sustainability reporting like the AccountAbility 1000 AccountAbility Principles[2]. Sustainable rating agencies develop rankings for the sustainability of companies according to these reports and additional information. Most of these rankings are based on positive and negative indicators. Rating agencies score non-monetary values of these different positive indicators for each investment and condense indicators into factors. The scores of factors are aggregated to a number describing the grade of sustainability inherent in an investment. This number can be positive or negative and is often transformed into a relative quantity to ensure a comparison of companies of different size and branches. Therefore, it is appropriate to view this quantity as a 'sustainability return'. Negative indicators represent the set of exclusion criteria used for negative screenings. Some of the agencies only provide ordinal rankings, while others compute real returns, representing a cardinal order. The sustainability ratings are based on historical data and upcoming projects (see

[1] The final wealth consists of two components, the initial wealth V^0 and the financial profit $V^t - V^0$.

[2] Beckett & Jonker (2002) provide a comprehensive description concerning the AccountAbility 1000 standard.

for example Forrest et al., 2006). Taking the future actions into account, it is natural to consider the sustainability of a company as a random number.

3.2. Measuring sustainability value and return

In our approach we commence with determining the objective sustainability return of every single investment with respect to a set $\mathcal{F}$ of factors, taken from an existing sustainability rating. In a second step an individual investor aggregates these objective values according to her preferences.

The **objective sustainability return** $OSR_j^{[s,t]}(F, \omega)$ of factor F and state ω in investment period $[s,t]$ for investment j is directly given by a sustainability rating. In an ex ante view the objective sustainability returns are clearly random variables. Knowing the invested initial wealth V_j^s in investment j, the objective sustainability return can be transformed into an objective sustainability value.

Definition 1 (Objective sustainability value)
The **objective sustainability value** $OSV_j^{[s,t]} : \mathcal{F} \times \Omega \times \mathbb{R} \to \mathbb{R}$ of a factor $F \in \mathcal{F}$ is a real random number with sample space Ω representing the objective non-monetary value that is generated by factor F of an investment j at maturity t. The objective sustainability value depends on V_j^s and is defined as

$$OSV_j^{[s,t]}(F, \omega, V_j^s) := V_j^s \cdot OSR_j^{[s,t]}(F, \omega).$$

To shape the investor's preferences, let $\delta \in \mathbb{R}$ be a real number and $F \in \mathcal{F}$ a factor of sustainable interest such as the environment. Then $\delta(F, \pi)$ denotes the strength of sustainable impact of factor F on investor π. When factor F has a positive impact on the investment decision of investor π, then $\delta(F, \pi) > 0$ holds. If an investor is indifferent with respect to factor F, we set $\delta(F, \pi) = 0$. An investor, who rejects the common interpretation of objective sustainability of factor F, has $\delta(F, \pi) < 0$. Using the notation of Definition 1 we can define the sustainability value of an investment j for investor π.

Definition 2 (Sustainability value)
The **sustainability value** $SV_j^{[s,t]} : \Omega \times \Pi \times \mathbb{R} \to \mathbb{R}$ of an investment j is a real random number with sample space Ω representing the non-monetary value of an investment the investor π receives at maturity t. Sustainability value

$$SV_j^{[s,t]}(\omega, \pi, V_j^s) := \sum_{F \in \mathcal{F}} \delta(F, \pi) OSV_j^{[s,t]}(F, \omega, V_j^s)$$

depends on the state ω, the preference π and initial wealth V^s.

Analogously to the objective sustainability return, a preference-dependent sustainability return exists.

Definition 3 (Sustainability return)
The **sustainability return** $SR_j^{[s,t]} : \Omega \times \Pi \to \mathbb{R}$ of investment j to investor π in period $[s, t]$ with sample space Ω and preference space Π is defined by

$$SR_j^{[s,t]}(\omega, \pi) := \frac{SV_j^{[s,t]}(\omega, \pi, V_j^s)}{V_j^s}.$$

An implication of the definitions from above is that the sustainability return can be expressed as a weighted sum over all factors F of the objective sustainability returns with weights $\delta(F, \pi)$. Furthermore, sustainability returns evidently fulfill the property of portfolio additivity.

Lemma 1 (Portfolio Additivity)
Let $w_1, \ldots, w_N$ the weights of N assets with sustainability returns $SR_1^{[s,t]}, \ldots,$ $SR_N^{[s,t]}$. If the sustainability value of each factor is additive over different assets held then we have

$$SR_P^{[s,t]} = \sum_{j=1}^{N} w_j SR_j^{[s,t]}.$$

Proof. Lemma 1 follows straight from Definition 2 and Definition 3. $\square$

4. Model variants: Some results and discussion

Next, we present and discuss three models for generalized safety first investors. All considerations below are based on one single period; hence, we drop time and interval indices as well as parameters for the state and the investor's preferences. Let $N \geq 2$ be the number of all assets the portfolio is supposed to comprise. Note that not all of these assets require a risky financial return. The weight of asset i is w_i and a well-defined portfolio satisfies $\sum_{i=1}^{N} w_i = 1$. In general, we permit short sales, which are characterized by negative w_i. There must be at least one risky asset with $w_i \neq 0$ to prevent computations from singularities[3].

4.1. Approaches to safety first portfolio choice with stochastic sustainability returns

Even if it is not the standard view in SRI to model sustainability of an investment as a random number with a finite expected value and a variance, we regard this assumption as most realistic since ex ante one can never predict to what extent the good intentions the management of a company has will become reality. Let $R : \Omega \to \mathbb{R}^N$ and $SR : \Omega \to \mathbb{R}^N$ denote the random vectors depicting the financial and sustainability returns of all available assets. The models presented below utilize returns instead of absolute quantities. However, this comes without loss of generality since it still might be the case that the risk aversion depends on initial wealth, which will be represented by the restrictions of the optimization problems. More precisely the thresholds for R and SR introduced below can be considered generally dependent on the initial wealth V^0. The vector SR is calculated for a fixed but arbitrary investor's preference. The covariance matrices of R and SR are denoted by Σ_R and Σ_{SR}. If asset i is riskless, the ith row and the ith column of both Σ_R and Σ_{SR} are zero vectors.

[3] Contrary to classical portfolio choice, the investor here may face two or more riskless assets with possibly different interest rates and different sustainability returns.

4.1.1. General model

This subsection introduces the general structure of a portfolio problem of a generalized safety first investor with financial and sustainable interests. Let A_i denote a random $J \times 2N$ matrix whose elements are multiples of R and SR. Let be $\mathbb{1}_2 = (1,1)' \in \mathbb{R}^2$, then the Kronecker product $\mathbb{1}_2 \otimes w$ denotes a column vector which is w strung together twice. Let

$$K_i(w) := \mathbf{P}\left(\bigcap_{j=1}^{J_i}\left(A_i^{(j)}(\mathbb{1}_2 \otimes w) \geq c_i^{(j)}\right)\right), \quad i = 1, \ldots, I$$

be a set of probabilities ($K(w) : \mathbb{R}^{2N} \to \mathbb{R}^I$) that depends on portfolio weights w, R and SR with vectors of thresholds $c_i \in \mathbb{R}^{J_i}$ and with $I \in \mathbb{N}$. Some appropriate specifications of $A_i^{(j)}$ and $c_i^{(j)}$ are listed in the following.

Definition 4 (Generalized safety first investor)
An investor with financial and sustainable investment interests is called **generalized safety first investor** (GSFI), if she accepts a portfolio w which fulfills every probability condition $K_i(w) \geq 1 - \alpha_i$ for $i = 1, \ldots, I$ with a default probability $\alpha_i \in \mathbb{R}$.

In general, a safety first investor maximizes a preference-dependent functional $\Psi(w'\mathbf{E}[R], w'\mathbf{E}[SR])$ with $\partial\Psi/\partial w'\mathbf{E}[R] \geq 0$, $\partial\Psi/\partial w'\mathbf{E}[SR] \geq 0$ and $\Psi : \mathbb{R}^2 \to \mathbb{R}$ at the constraints given by Definition 4. Thus, the maximization problem is

$$\max_{w \in \mathcal{W}} \quad \Psi(w'\mathbf{E}[R], w'\mathbf{E}[SR]) \tag{1a}$$

$$\text{s. t.} \quad K_i(w) \geq 1 - \alpha_i \qquad i = 1, \ldots, I \tag{1b}$$

with a convex set $\mathcal{W} \in \mathbb{R}^N$ with $\mathcal{W} := \{w | l \leq w \leq u, Bw \propto b, \sum_{i=1}^N w_i - 1 = 0\}$, where $\propto$ stands for $=$, $\leq$ or $\geq$, $B \in \mathbb{R}^{k \times N}$ and $b \in \mathbb{R}^k$. The set $\mathcal{W}$ contains all linear constraints on w like lower and upper bounds ($l, u \in \mathbb{R}^N$) for portfolio weights and other budget constraints, where $k \in \mathbb{N}$ is the number of all constraints. The risk preferences of every single investor are conveyed by the thresholds c_i and the default probabilities α_i.

In general, problem (1) is a chance-constrained programming problem with random coefficient matrices A_i, which does not necessarily contain independent rows and is thus a non-convex programming problem. Kall & Mayer (2005, p. 142-143) display the conditions under which problem (1) is an easy-to-solve convex programming problem. The intersection of two convex sets is convex. Thus, it is accurate to consider the probability constraints (1b) in this context only. It is also a well-known fact that every local optimal solution of a convex optimization problem is a global optimal solution. Below we present specifications of the general model representing three different approaches to deal with the tradeoff between R and SR in a safety first context. As the objective function we use the straightforward specification $\Psi(x, y) = (1 - \gamma)x + \gamma y$, where the scale parameter is $\gamma \in (0, 1)$ with $\gamma = 0$ representing the case where only the financial return enters the objective function. We can derive the marginal rate of substitution between expected financial (x) and sustainability (y) return following

$$\frac{\partial\Psi(x, y)/\partial x}{\partial\Psi(x, y)/\partial y} = \frac{1 - \gamma}{\gamma}.$$

This quotient indicates how many units increase in the expected sustainability return one demands for a loss of one unit of the expected financial return while the objective function remains the same. Note that the sustainability return in general has one degree of freedom since the individual factors $\delta(\omega, F)$ could be multiplied by an arbitrary constant. However, when determining γ according to the preferences, one has to take the general level of SR into consideration and loses the degree of freedom again. Therefore γ can be interpreted most easily if R and SR have approximately the same range. A value of $\gamma = 1/2$ would then imply that both returns are equally weighted in the objective function. If one then chooses to replace SR by, for instance, $10 \cdot SR$, then γ would have to change from 1/2 to 1/11 in order to express the same preferences as before.

4.1.2. Joint distribution model (JDM)

The first interpretation of a GSFI utilizes the joint probability distribution of financial and sustainability portfolio returns. A joint distribution type GSFI is defined as follows.

Definition 5 (Joint distribution type GSFI)
An investor with preference functional Ψ is called **joint distribution type GSFI**, if there are thresholds c_R and c_{SR} for financial and sustainability portfolio returns which she allows to underperform with joint probability α.

In this case the parameters of the general problem (1) are $I = 1$, $J = 2$,

$$A = \begin{pmatrix} R_1 & \cdots & R_N & 0 & \cdots & 0 \\ 0 & \cdots & 0 & SR_1 & \cdots & SR_N \end{pmatrix}, c = \begin{pmatrix} c_R \\ c_{SR} \end{pmatrix}$$

and $\alpha \in \mathbb{R}$. For a fixed portfolio P, we can determine the joint probability density function and the joint cumulative distribution function (cdf). The preference functional Ψ is the convex combination of expected returns, where the relation between financial and sustainable quantities is reflected by γ. Hence, our first model for generalized portfolio choice is

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{2a}$$

$$\text{s. t.} \quad \mathbf{P}(w'R \geq c_R, w'SR \geq c_{SR}) \geq 1 - \alpha, \tag{2b}$$

where c_R and c_{SR} are the thresholds and the financial and sustainability portfolio returns $w'R$ and $w'SR$ are random numbers. The following theorem deals with the uniqueness of the solution if financial and sustainability returns are assumed to be multivariately normally distributed.

Theorem 1 *The deterministic equivalent of chance-constrained programming problem* (2) *has a unique solution, if R and SR are multivariately normally* *distributed and the feasible set of the deterministic equivalent is not empty.*

Proof. Solving problem (2) we issue a deterministic equivalent problem. In the spirit of Watanabe & Ellis (1994) we consider a problem with multivariate normally distributed coefficients of matrix A. Therefore, as the deterministic equivalent of problem (2) we obtain

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR] \right] \tag{3a}$$

$$\text{s. t.} \quad \int_{l_{SR}}^{\infty} \int_{l_R}^{\infty} \phi_Z(z) dz_1 dz_2 \geq 1 - \alpha, \tag{3b}$$

where

$$l_R(w) = \frac{c_R - \mu_R' w}{\sqrt{w' \Sigma_R w}} \quad \text{and} \quad l_{SR}(w) = \frac{c_{SR} - \mu_{SR}' w}{\sqrt{w' \Sigma_{SR} w}},$$

$$z_1(w) = \frac{r - \mu_R' w}{\sqrt{w' \Sigma_R w}} \quad \text{and} \quad z_2(w) = \frac{s - \mu_{SR}' w}{\sqrt{w' \Sigma_{SR} w}},$$

$$z = (z_1, z_2)', \quad \varrho(w) = \begin{pmatrix} 1 & \rho(w'R, w'SR) \\ \rho(w'R, w'SR) & 1 \end{pmatrix} \quad \text{and}$$

$$\phi_Z(z) = \frac{1}{2\pi \sqrt{\det\left(\varrho(w)\right)}} \exp\left\{ -\frac{1}{2} z' \varrho(w)^{-1} z \right\}.$$

The correlation coefficient of financial and sustainability portfolio returns is denoted by $\rho(w'R, w'SR)$ and the probability density function of the bivariate normal distribution with mean $\mu_Z = (0,0)'$ and covariance matrix $\varrho(w)$ by ϕ_Z. Constraint (3b) yields to a convex set because $K(w)$ is quasi-concave due to the attitudes of the bivariate normal distribution. The objective function (3a) is not constant, so it has a local optimum on the bounded feasible set, if this it not empty. Hence this solution is also a global optimal solution. □

4.1.3. Convolution model (CM)

The second approach is probably the most obvious one: We generate a convex combination of financial and sustainability portfolio returns.

Definition 6 (Convolution type GSFI)
An investor with preference functional Ψ is called **convolution type GSFI**, if there is a threshold c for the convex combination of financial and sustainability portfolio returns that he allows to underperform with probability α.

In this context, the problem is to maximize the combination of expected financial and sustainability portfolio returns under the constraint that their convex combination underperforms a threshold c with probability less than α. The parameters are $I = 1$, $J = 1$, $A = ((1-\gamma)R_1, \ldots, (1-\gamma)R_N, \gamma SR_1, \ldots, \gamma SR_N)$, $c \in \mathbb{R}$ and $\alpha \in \mathbb{R}$. The formal notation is given by

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{4a}$$

$$\text{s. t.} \quad \mathbf{P}((1-\gamma)w'R + \gamma w'SR \geq c) \geq 1 - \alpha. \tag{4b}$$

If $\gamma = 0$ holds, an investor is not interested in sustainability; we obtain the standard safety first portfolio optimization problem in accordance with Telser (1955) without sustainable interests. On the contrary, an investor who is only interested in sustainability has $\gamma = 1$. For each investor with $0 < \gamma < 1$ and a fixed threshold c, the constraint (4b) contains the tradeoff between financial and sustainable quantities. Let $R_P^\gamma := (1-\gamma)w'R + \gamma w'SR$ be the convex combination of financial and sustainability portfolio returns with $\gamma \in (0, 1)$.

Theorem 2 *For multivariately normally distributed financial and sustainability portfolio returns the deterministic equivalent of problem (4) has a unique solution, if the feasible set is not empty.*

Proof. A deterministic equivalent of problem (4) can be derived because due to the normally distributed coefficients of matrix A the sum of $2N$ normally distributed random numbers R_P^γ is normally distributed with parameters

$$\mu_{R_P^\gamma} = w'((1-\gamma)\mu_R + \gamma\mu_{SR})$$
$$\sigma^2_{R_P^\gamma} = (D(\mathbb{1}_2 \otimes w))'\Sigma D(\mathbb{1}_2 \otimes w),$$

where

$$\Sigma = \begin{pmatrix} \Sigma_R & \Sigma_{R,SR} \\ \Sigma'_{R,SR} & \Sigma_{SR} \end{pmatrix},$$

and $D \in \mathbb{R}^{2N\times 2N}$ is a diagonal matrix with $D = \mathrm{diag}(1-\gamma, \ldots, 1-\gamma, \gamma, \ldots, \gamma)$, while $\Sigma_{R,SR} \in \mathbb{R}^{N\times N}$ is the estimated (not necessarily symmetric) covariance matrix between R and SR. For $\alpha \in (0, 0.5)$ we get as the deterministic equivalent of problem (4) the convex programming problem

$$\max_{w\in\mathcal{W}} \quad w'\left[(1-\gamma)\mathbf{E}[R] + \gamma\mathbf{E}[SR]\right] \tag{5a}$$

$$\text{s. t.} \quad \Phi^{-1}(\alpha)\sigma_{R_P^\gamma} + \mu_{R_P^\gamma} \geq c, \tag{5b}$$

where $\Phi(\cdot)$ denotes the univariate standard normal cdf. The argumentation for a unique global optimal solution is the same as in proof of Theorem 1. $\square$

4.1.4. Marginal distributions model (MDM)

The third model we describe in this paper differs from problems (2) and (4) in its constraints. Again, the objective function is the maximization of a convex combination of expected financial and sustainability portfolio returns. The risk constraints build on the marginal distributions of financial and sustainability portfolio returns.

Definition 7 (Marginal type GSFI)

An investor with preference functional Ψ is called **marginal type GSFI**, if there is a threshold c_R for the portfolio return which she allows to underperform with probability α_R and a threshold c_{SR} for the portfolio sustainability return which she allows to underperform with probability α_{SR}.

The parameters are $I = 2$, $J_1 = J_2 = 1$, $A_1 = \begin{pmatrix} R_1 & \cdots & R_N & 0 & \cdots & 0 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & \cdots & 0 & SR_1 & \cdots & SR_N \end{pmatrix}$, $(c_1, c_2) = (c_R, c_{SR})$ and $(\alpha_1, \alpha_2) =$

(α_R, α_{SR}). The formal notation of the third model is

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{6a}$$
$$\text{s. t.} \quad \mathbf{P}(w'R \geq c_R) \geq 1 - \alpha_R \tag{6b}$$
$$\mathbf{P}(w'SR \geq c_{SR}) \geq 1 - \alpha_{SR}. \tag{6c}$$

Model (6) does not make use of any correlation between R and SR. The lower bounds c_R and c_{SR} are fixed. Model (6) excludes all portfolios that underperform in one or both dimensions with a higher probability than α_R and α_{SR}.

Theorem 3 *If the marginal distributions of financial and sustainability portfolio returns are marginally normally distributed and the feasible set of the deterministic equivalent is not empty, a unique optimal solution exists.*

Proof. The deterministic equivalent of problem (6) under normality assumptions is the convex programming problem (7)

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma \mathbf{E}[SR]\right] \tag{7a}$$
$$\text{s. t.} \quad \Phi^{-1}(\alpha_R)\sqrt{w'\Sigma_R w} + w'\mu_R \geq c_R \tag{7b}$$
$$\Phi^{-1}(\alpha_{SR})\sqrt{w'\Sigma_{SR} w} + w'\mu_{SR} \geq c_{SR}. \tag{7c}$$

with a local, thus global, optimal solution if the feasible set is not empty. □

4.2. Joint versus marginal distributions model

In this section we explore the connection between the feasible sets of the JDM and the MDM and proof a corresponding result. Both models differ from each other solely in terms of probability constraints. In contrast to the CM, where the probability constraint imitates the convex combination of the objective function and hence the feasible set depends on γ, the feasible sets of the JDM and the MDM do not depend on that part of the investor's preferences displayed by γ. This implies a more flexible shaping. An investor can choose between financial and sustainable quantities in the objective function and the thresholds for both quantities in the probability constraints independently. By choosing $\gamma = 0$ both models are appropriate in shaping the preferences of an investor, who only maximizes expected financial portfolio return under the constraint that the sustainability portfolio return exceeds a threshold with a certain probability. Moreover, we find that the JDM and the MDM have both an efficient frontier in the $\mathbf{E}[w'R]$-$\mathbf{E}[w'SR]$-space for given parameters c_R, c_{SR}, α or α_R, α_{SR}, respectively. Hence, given these parameters, the optimal portfolio depends solely on the choice of γ, which is graphically displayed in the Appendix B in the supplementary material.

The MDM is very simple to implement, because the user does not have to estimate correlations between financial and sustainable quantities and no computation of any joint probability functions is necessary. Thereby, the feasible set is that of the intersection of the sets corresponding to the probability constraints. However, this is a very restrictive way to handle the downside risk, but it is the only model which guarantees underperformance of each single threshold by less than the given default probability. In Theorem 4 we show a general result regarding the relationship between objective values of both models for positive quadrant dependence.

Theorem 4 *Let the financial and sustainability portfolio returns of the marginal distributions model's optimal solution (6) be distributed according to a bivariate distribution with positive quadrant dependence. Then the MDM becomes more restrictive than the JDM if* $(1-\alpha_R)\cdot(1-\alpha_{SR}) \geq 1-\alpha$.

Proof. Let $\hat{w}$ be the optimal solution of problem (7), i.e. $\hat{w}$ satisfies the constraints (6b) and (6c). Obviously, $\hat{w} \in \mathcal{W}$ is satisfied for any feasible w in both models. Confirming that problem (6) is more restrictive than problem (2), we have to show that an optimal solution of problem (6) is also feasible for problem (2). Hence, if $\hat{w}$ is the optimal solution of problem (7), we obtain

$$\mathbf{P}(\hat{w}'R \geq c_R)\cdot\mathbf{P}(\hat{w}'SR \geq c_{SR}) \geq (1-\alpha_R)\cdot(1-\alpha_{SR}). \tag{8}$$

A joint probability distribution has the attitude of positive quadrant dependence if $\mathbf{P}(X \geq x, Y \geq y) \geq \mathbf{P}(X \geq x) \cdot \mathbf{P}(Y \geq y)$ is satisfied (see Lehmann, 1966). Therefore, inequality

$$\mathbf{P}(\hat{w}'R \geq c_R, \hat{w}'SR \geq c_{SR}) \geq \mathbf{P}(\hat{w}'R \geq c_R) \cdot \mathbf{P}(\hat{w}'SR \geq c_{SR}) \tag{9}$$

holds. Due to the probability condition $1 - \alpha \leq (1 - \alpha_R)(1 - \alpha_{SR})$ and (8), constraint (2b) is satisfied and the result is proven. $\square$

Corollary 1 *Let the financial and sustainability portfolio returns of the marginal distributions model's optimal solution* (6) *be bivariately normally distributed with correlation $\rho(\hat{w}'R, \hat{w}'SR) \geq 0$. Then the MDM becomes more restrictive than the JDM if $(1 - \alpha_R) \cdot (1 - \alpha_{SR}) \geq 1 - \alpha$.*

Proof. Lehmann (1966) shows that every bivariate normal distribution with a positive correlation coefficient is positively quadrant dependent. In this case the proof follows straight from Theorem 4. $\square$

We display this observation in Appendix C in the supplementary material.

4.3. Safety first portfolio choice with deterministic sustainability returns

Since in state-of-the-art SRI practice a deterministic sustainability return is used, we now consider the GSFI approach under the assumption of deterministic sustainability returns. Let $F_P(\cdot)$ be the appropriate cdf of the financial portfolio return $w'R$. We consider a setup with a fixed investment period and a confidence level $1-\alpha > 0.5$. There are two different approaches derived from the general models in Section 4. The first one is the counterpart of the CM with a deterministic sustainability return, i.e.

$$\max_{w \in \mathcal{W}} \quad w' \left[(1 - \gamma)\mathbf{E}[R] + \gamma SR\right] \tag{10a}$$

$$\text{s. t.} \quad 0 \geq c - \gamma w'SR - (1 - \gamma)F_P^{-1}(\alpha). \tag{10b}$$

We interpret this model as a sustainable shifted quantile model, which means that the α-quantile of the financial portfolio return distribution is shifted by the sustainability return of the portfolio. The second model with deterministic sustainability return is deduced from the MDM and can be regarded as an improved screening portfolio selection. Hence, this model is given by

$$\max_{w \in \mathcal{W}} \quad w' \left[(1-\gamma)\mathbf{E}[R] + \gamma SR \right] \tag{11a}$$

$$\text{s. t.} \quad 0 \geq c_R - F_P^{-1}(\alpha_R) \tag{11b}$$

$$0 \geq c_{SR} - w' SR. \tag{11c}$$

Probability constraint (6c) from the general MDM is merged into a deterministic linear constraint (11c) and thereby unchanged in the deterministic equivalent. Therefore, a portfolio is feasible for this model if its sustainability return is higher than a given threshold and it satisfies the probability constraint for financial return.

It is easy to show that the JDM is equivalent to (11) in this setting because without variability in the sustainability return the joint distribution essentially degenerates into the marginal distribution of the financial return with a deterministic value in the sustainability return which lies above the required level c_{SR} with probability 0 or 1.

5. Empirical application and model comparison

To show the relevance of our theoretical models we next discuss several aspects of an empirical implementation. Here we also discuss the economic consequences of our approach.

5.1. Computational complexity

First, we analyze the computational effort of finding the optimal portfolio for 3, 50, 100 and 630 assets.[4] The results are displayed in Table 1 in relation to the minimal computational time. Obviously, the JDM is the one with

Table 1: Computational time analysis. The minimum time is normalized to 1.0. The absolute time for computing the optimal solution with the interior point algorithm for $\gamma = 0.2$ of the CM with 3 assets is 0.0658 seconds on a Pentium(R) Dual-Core E5300 2.60 GHz, 3.21 GB RAM. The estimation of the input parameters is not considered.

Number of assets	3	50	100	630
Joint distribution model	7.3	303.4	820.6	31417.0
Convolution model	1.0	25.4	69.0	11913.0
Marginal distributions model	1.0	27.7	73.1	9615.4

the highest computational effort. But the optimal solution of the portfolio problem with 630 risky assets from our sample lasts 34.5 minutes. In contrast, the CM and MDM have very low computational costs because there are no multivariate probabilities to deal with.

5.2. Data and empirical methodology

To apply our methodology, the expected values, variances and covariances of the financial and sustainability returns need to be estimated using real world data. To compute the sustainability returns, we use annual ESG scores from the sustainability rating agency Inrate for the years dating from 2005 to 2009 for all companies in their rating universe. These ESG (Environment, Social, Governance) scores consist out of a high number of indicators, which score the efforts of each company in several fields of environmental protection, social issues and corporate governance issues. Corresponding to the shaping of objective and subjective sustainability returns, the scores of

[4]This computation exercise was conducted under the assumption of normally distributed returns and with realistic values of all expected values, variances and covariances.

these indicators, that can be seen as the objective sustainability returns, are aggregated by special coefficients to a subjective sustainability return. In our case, we take the given coefficients from Inrate as the considered investors' preference and use the provided ESG score as the subjective sustainability return for each asset. Basically, we suggest that not only ratings like the ones by Inrate can be used for the aggregation of subjective sustainability returns. Moreover, every investor gathers benefits from non-financial factors, which also can comprise being a home-biased investor or a sport supporter and several further issues. Every investor is required to score these factors to his own preference. Ratings provided by independent rating agencies are useful proxies for a wide set of factors but can not cover all conceivable preferences. As we wish to be as objective as possible, we do not take into account further preferences except the ones incorporated into the Inrate ESG scores.

The range of the score is $[0, 100]$. The company with the best score in 2009 had an ESG score of 90, and the one with the lowest had an ESG score of 38. As we are required to estimate returns and variance, we drop all companies that do not have annual ESG scores and obtain a sample with 735 stocks, most of which are also included in the MSCI World Index. We export monthly financial stock prices and market capitalization in USD from Thomson Reuters Datastream and estimate the monthly financial quantities using an expected weighted moving average model with decay factor 0.97 for a period dating from January 1, 1990 to December 1, 2009. We use annualized expected financial returns and variances in our portfolio choice.

Following the sector-based screening analysis in the SRI literature (Guerard, 1997), we use ten market-capitalization weighted subportfolios, each consisting of ten randomly chosen companies belonging to the same industrial sector. These ten subportfolios are the investment instruments, which an investor uses to develop her portfolio. To achieve sustainability returns which are in a comparable range to the financial returns we linearly transform the ESG scores S_j by $SR_j = (S_j - 50) \cdot \mathbf{E}[R_j]/(\mathbf{E}[S_j] - 50)$ for all subportfolios

$j \in \{1, \ldots, 10\}$. The ESG score of 50 is the minimum score a sustainably acceptable company should reach. Thus, values below 50 should correspond to a negative SR.

The covariance matrices of the sustainability returns are estimated using an expected weighted moving average model with decay factor 0.97 for the years between 2005 and 2009, while the expected sustainability returns are shown by the current scores. The expected financial and sustainability returns of each subportfolio are displayed in Table A.1, the estimated covariance matrices Σ_R, Σ_{SR} and $\Sigma_{R,SR}$ in Appendix A in the supplementary material. Both financial and sustainability returns are assumed to be multivariately normally distributed[5].

5.3. Results

Now we calculate the optimal portfolios for the three models and different values of γ and the thresholds[6]. As a first model consistency check we compare the three models with respect to the optimal portfolios' probabilities of exceeding the thresholds[7]. We find that the CM may generate outcomes that violate the maximum probabilities very clearly. This is a phenomenon, which is implied by the effect that both return dimensions compensate for each other in this model.

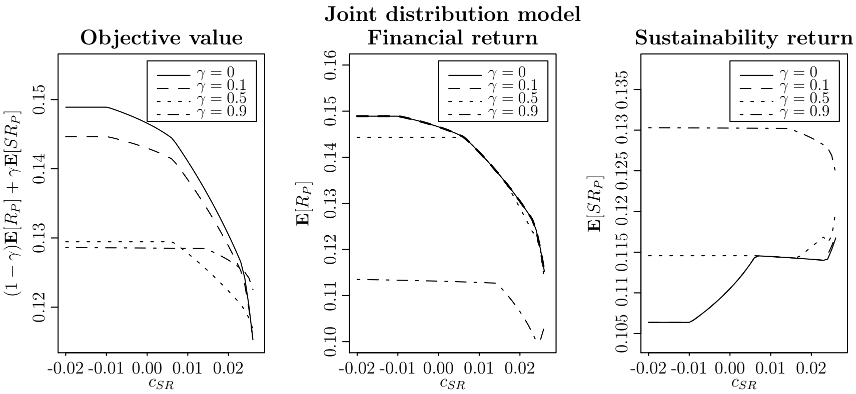
Figure 1 shows the optimal value of the objective function and the expected financial and sustainability portfolio returns of the JDM for varying threshold c_{SR} and γ. An increasing threshold c_{SR} implies that the portfolio is required to satisfy a higher sustainability level. The objective function value decreases with increasing c_{SR} because the feasible set becomes smaller.

[5]We check this by applying Royston's test (Royston, 1982) to our financial and sustainability returns and find that our assumption of multivariately normally distributed returns cannot be rejected at any reasonable significance level.

[6]We restrict each portfolio weight to the interval $[0, 0.25]$. The values for α are 0.13 in the JDM, 0.067 in the CM and $\alpha_R = 0.09375$, $\alpha_{SR} = 0.04$ in the MDM.

[7]Cf. Figure D.3 in the supplementary material.

Figure 1: Variations of optimal objective function value, expected financial and expected sustainability portfolio returns of the JDM dependent on c_{SR} and γ. Calculations are based on the parameter specification used throughout this section.



5.3.1. Portfolio weights variation

When considering the objective value, the expected financial and the expected sustainability return of the optimal portfolios for $\gamma = 0.9$ dependent on c_{SR}, the functions reveal segments with constant levels. This is due to the weight constraints. Table 2 displays how the portfolio weights change when increasing the sustainability threshold for $\gamma = 0.5$ in the JDM[8]. Furthermore, we provide the expected financial and sustainability portfolio returns as well as the financial and sustainability portfolio variance of the optimal solutions in each case. Obviously, by increasing the threshold c_{SR} as displayed here, riskier portfolios with weights on the upper and lower bounds are changed into more diversified portfolios. The result of this observation is a decrease in expected financial portfolio return, a decrease in financial portfolio standard deviation and an increase in expected sustainability portfolio return as well as a decrease in sustainability portfolio standard deviation, because a higher sustainable threshold c_{SR} implies a higher sustainability risk aversion and

[8]We provide further tables with portfolio weights for several model and parameter combinations in Appendix E in the supplementary material.

Table 2: Portfolio weights of the optimal solutions of the JDM with $\gamma = 0.5$ in percent. It is due to rounding errors that the weights do not amount to 100 in all cases.

c_{SR}	-0.020	-0.013	-0.006	0.001	0.008	0.015	0.022	0.026
w_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_2	25.0	25.0	25.0	25.0	25.0	25.0	25.0	24.8
w_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.5
w_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_5	25.0	25.0	25.0	25.0	25.0	25.0	21.3	13.7
w_6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_8	25.0	25.0	25.0	25.0	25.0	25.0	25.0	25.0
w_9	0.0	0.0	0.0	0.0	2.0	11.1	22.5	25.0
w_{10}	25.0	25.0	25.0	25.0	23.0	14.0	6.2	0.0
μ_R	0.144	0.144	0.144	0.144	0.143	0.136	0.126	0.114
σ_R	0.229	0.229	0.229	0.229	0.226	0.213	0.200	0.189
μ_{SR}	0.115	0.115	0.115	0.115	0.115	0.114	0.116	0.120
σ_{SR}	0.029	0.029	0.029	0.029	0.029	0.028	0.028	0.028

simultaneously a higher demand for expected sustainability return.

5.3.2. Financial cost analysis

In the previous subsection one could see that the impact of a variation of c_{SR} is higher for investors with small γ since these investors mainly maximize the expected financial portfolio return. In Figure 1, the line for an investor with $\gamma = 0$ shows how strongly the expected financial return decreases if the investor increases her sustainable threshold c_{SR}. Hence, the loss in the expected financial return can be viewed as the financial costs of a sustainable investment. We investigate this issue more precisely. As a benchmark, we model an investor with no sustainable interest, i.e. $\gamma = 0$ (and $c_{SR} = -\infty$ in the JDM and MDM). Basically, this kind of investor is only interested in financial risk and return[9]. Next, we increase the sustainability threshold to

[9]With our GSFI models applied to $\gamma = 0$ (and $c_{SR} = -\infty$ in the JDM and MDM) the optimal portfolios are even μ-σ efficient in the sense of Markowitz, at least for normally distributed financial returns (Arzac & Bawa, 1977), which is the assumption we apply.

$c_{SR} = 0$, which reflects a sustainability return equivalent to an ESG score of 50, and optimize portfolios for $\gamma \in \{0, 0.1, 0.5, 0.9\}$. Note that for $\gamma = 0$ the investor is not interested in maximizing the sustainability return, but takes sustainability into account by the constraints in the JDM and the MDM.

In the JDM and the MDM, for fixed model specific default probabilities, a fixed sustainable threshold c_{SR} and a fixed γ, the only free parameter with effect on the feasible set is the financial threshold c_R. An increase in c_R means that the riskiness of the portfolio declines. Therefore, we also run the optimization for varying c_R. In line with Adler & Kritzman (2008) we regard the difference between the expected financial returns of a conventional and a sustainable investment as the cost of socially responsible investing. The expected return a sustainable investor achieves is compared to the optimal return at a threshold c_R, which is used as our term of risk in the following. We calculate the cost of sustainable interests by determining the difference between the optimal expected financial returns, i.e. without considering sustainability, and the expected financial return of the model and the parameter setting under consideration. Table 3 and 4 display the results for the JDM.

Table 3: Expected financial returns and costs of investing sustainably in the JDM for varying c_R, c_{SR} and γ. Line 2 displays the expected financial returns of the optimal portfolio without considering sustainability. Lines 3 to 6 display the differences between the optimal portfolios' expected financial returns (according to the JDM) and line 2. The last column displays the average difference for $c_R \in [-0.2, -0.108]$.

c_R	-0.200	-0.177	-0.154	-0.131	-0.108	average diff.
$c_{SR} = -\infty$, $\gamma = 0$	0.149	0.149	0.149	0.149	0.146	–
$c_{SR} = 0$, $\gamma = 0$	0.000	0.000	-0.002	-0.009	-0.027	-0.005
$c_{SR} = 0$, $\gamma = 0.1$	-0.000	-0.000	-0.002	-0.009	-0.027	-0.005
$c_{SR} = 0$, $\gamma = 0.5$	-0.005	-0.005	-0.005	-0.009	-0.027	-0.008
$c_{SR} = 0$, $\gamma = 0.9$	-0.034	-0.035	-0.036	-0.043	-0.028	-0.036

We receive similar results for the MDM, which are shown in Table 5 and 6. Note that the feasible interval for c_R is smaller for the MDM as compared to the JDM, an observation which is due to the fact that the MDM here is more restrictive than the JDM (cf. Theorem 4.)

Table 4: Expected sustainability returns and benefits of investing sustainably in the JDM for varying c_R, c_{SR} and γ. Line 2 displays the expected sustainability returns of the optimal portfolio without considering sustainability. Lines 3 to 6 display the differences between the optimal portfolios' expected sustainability returns (according to the JDM) and line 2. The last column displays the average difference for $c_R \in [-0.2, -0.108]$.

c_R	-0.200	-0.177	-0.154	-0.131	-0.108	average diff.
$c_{SR} = -\infty$, $\gamma = 0$	0.106	0.106	0.106	0.106	0.104	–
$c_{SR} = 0$, $\gamma = 0$	0.000	0.000	0.003	0.008	0.007	0.004
$c_{SR} = 0$, $\gamma = 0.1$	0.000	0.000	0.003	0.008	0.007	0.004
$c_{SR} = 0$, $\gamma = 0.5$	0.008	0.008	0.008	0.008	0.007	0.008
$c_{SR} = 0$, $\gamma = 0.9$	0.024	0.024	0.024	0.022	0.007	0.022

Table 5: Expected financial returns and costs of investing sustainably in the MDM for varying c_R, c_{SR} and γ. Line 2 displays the expected financial returns of the optimal portfolio without considering sustainability. Lines 3 to 6 display the differences between the optimal portfolios' expected financial returns (according to the MDM) and line 2. The last column displays the average difference for $c_R \in [-0.2, -0.116]$.

c_R	-0.200	-0.179	-0.158	-0.137	-0.116	average diff.
$c_{SR} = -\infty$, $\gamma = 0$	0.149	0.149	0.148	0.140	0.110	–
$c_{SR} = 0$, $\gamma = 0$	0.000	0.000	-0.000	0.000	0.000	-0.000
$c_{SR} = 0$, $\gamma = 0.1$	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
$c_{SR} = 0$, $\gamma = 0.5$	-0.005	-0.005	-0.003	-0.002	-0.001	-0.003
$c_{SR} = 0$, $\gamma = 0.9$	-0.035	-0.036	-0.035	-0.021	-0.003	-0.027

The methodology for the financial cost comparison for the CM is different to that of the JDM and the MDM. Because the common threshold depends on the financial and sustainability thresholds as well as on γ, we would not obtain comparable results for different c_{SR} and different γ. Therefore, we repeatedly use the combined threshold as the term of risk and calculate the optimal portfolio $\tilde{w}$ for a specific parameter set first. Subsequently, we take an inverse optimization step to calculate which threshold $\hat{c}$ is suitable for receiving the calculated portfolio $\tilde{w}$ as the optimal portfolio of the financial investor optimization problem ($\gamma = 0$). In a final step we optimize with the threshold $\hat{c}$ and $\gamma = 0$. The optimal portfolio possesses the same risk in

Table 6: Expected sustainability returns and benefits of investing sustainably in the MDM for varying c_R, c_{SR} and γ. Line 2 displays the expected sustainability returns of the optimal portfolio without considering sustainability. Lines 3 to 6 display the differences between the optimal portfolios' expected sustainability returns (according to the MDM) and line 2. The last column displays the average difference for $c_R \in [-0.2, -0.116]$.

c_R	-0.200	-0.179	-0.158	-0.137	-0.116	average diff.
$c_{SR} = -\infty$, $\gamma = 0$	0.106	0.106	0.103	0.108	0.097	–
$c_{SR} = 0$, $\gamma = 0$	0.000	0.000	0.002	0.000	0.000	0.000
$c_{SR} = 0$, $\gamma = 0.1$	0.000	0.000	0.006	0.001	0.001	0.002
$c_{SR} = 0$, $\gamma = 0.5$	0.008	0.008	0.012	0.006	0.003	0.008
$c_{SR} = 0$, $\gamma = 0.9$	0.024	0.024	0.025	0.012	0.003	0.019

terms of the CM and only maximizes financial quantities. Hence, we use this portfolio as the financial benchmark model and calculate the differences to the sustainable investor's portfolio. The results are shown in Table 7 and 8.

The results show that JDM, CM and MDM investors have to give up expected financial return if they account for sustainability. For instance, a sustainable investor with $\gamma = 0.5$ (and $c_{SR} = 0$) loses, on average, 0.8% in the JDM and 0.3% in the MDM, both with regard to the expected financial return. In reverse, she increases her expected sustainability return by an absolute value, on average, by 0.8% for the JDM and by 0.8% for the MDM. In this setting, the CM choses all weights equal to the lower or upper bound and thus does not reveal any variation. Finally, Figure 2 visualizes the variation of the expected financial return in all three models for varying parameters c_R, c_{SR} and γ.

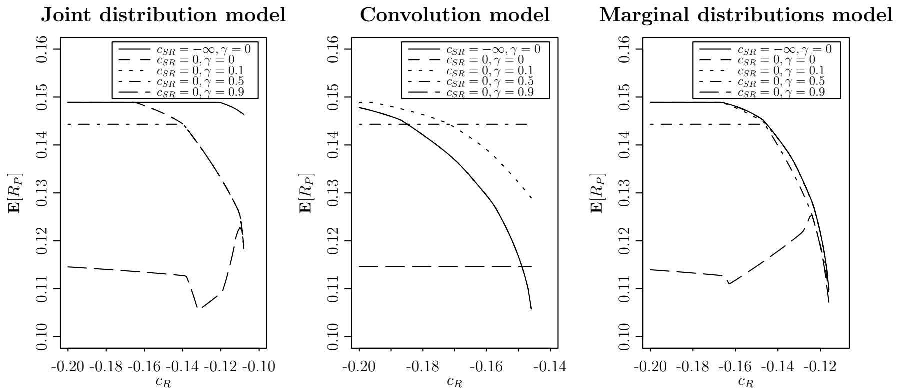
6. Conclusion

We present a mathematical framework for modeling the sustainability return as a computable quantity. Based on sustainability ratings, one can derive objective sustainability returns of every sustainability dimension of an investment asset. These objective sustainability returns are then aggregated linearly according to the investor's preferences. Based on these considerations

Table 7: Expected financial returns and costs of investing sustainably in the CM for varying c_R, c_{SR} and γ. Line 2 displays the expected financial returns of the optimal portfolio without considering sustainability. Lines 4, 6 and 8 display the optimal portfolios' expected financial returns. Lines 3, 5, 7 and 9 contain the differences between the optimal portfolios' expected financial returns (according to the CM in the appriopriate value of risk) and the corresponding line above. The last column displays the average difference for $c_R \in [-0.2, -0.148]$.

c_R	-0.200	-0.187	-0.174	-0.161	-0.148	average diff.
$c_{SR} = 0$, $\gamma = 0$	0.148	0.145	0.139	0.130	0.113	–
financial costs	-0.000	0.000	0.000	0.000	0.000	-0.000
$c_{SR} = 0$, $\gamma = 0.1$	0.149	0.148	0.145	0.140	0.131	–
financial costs	0.000	-0.001	-0.004	-0.009	-0.016	-0.009
$c_{SR} = 0$, $\gamma = 0.5$	0.144	0.144	0.144	0.144	0.144	–
financial costs	0.000	0.000	0.000	0.000	0.000	0.000
$c_{SR} = 0$, $\gamma = 0.9$	0.115	0.115	0.115	0.115	0.115	–
financial costs	0.000	0.000	0.000	0.000	0.000	0.000

Figure 2: Expected financial return dependent on c_R, c_{SR} and γ for all three models.



we present a general model for safety first portfolio selection with stochastic financial and sustainability returns and introduce the notion of a generalized safety first investor. Whereas the objective function is fixed as a convex combination of the expected financial and sustainability return, the conditions determining the feasible set may vary. We specify this general model in three different forms, namely the convolution type, the marginal distri-

Table 8: Expected sustainability returns and benefits of investing sustainably in the CM for varying c_R, c_{SR} and γ. Line 2 displays the expected sustainability returns of the optimal portfolio without considering sustainability. Lines 4, 6 and 8 display the optimal portfolios' expected sustainability returns. Lines 3, 5, 7 and 9 contain the differences between the optimal portfolios' expected sustainability returns (according to the CM in the appriopriate value of risk) and the corresponding line above. The last column displays the average difference for $c_R \in [-0.2, -0.148]$.

c_R	-0.200	-0.187	-0.174	-0.161	-0.148	average diff.
$c_{SR} = 0$, $\gamma = 0$	0.103	0.107	0.108	0.104	0.098	–
sustainable benefits	0.000	-0.000	0.000	0.000	0.000	0.000
$c_{SR} = 0$, $\gamma = 0.1$	0.106	0.109	0.113	0.109	0.108	–
sustainable benefits	0.000	0.002	0.007	0.002	-0.003	0.000
$c_{SR} = 0$, $\gamma = 0.5$	0.106	0.109	0.113	0.109	0.108	–
sustainable benefits	0.000	0.000	0.000	0.000	0.000	0.000
$c_{SR} = 0$, $\gamma = 0.9$	0.130	0.130	0.130	0.130	0.130	–
sustainable benefits	0.000	0.000	0.000	0.000	0.000	0.000

butions type and the joint distribution type generalized safety first investor, and show that each model has an optimal solution under the assumption of normally distributed returns. The implementation of the models follows the constructive proofs. We demonstrate the practical implementability and calculate the costs of optimizing portfolios according to our approach in terms of financial return by using real world data.

Summarizing, we provide the following recommendations. The convolution type model is the most straightforward one. However, this model may not be favourable to those investors who really care about the shortfall in each of both dimensions. The marginal and joint distribution type models are generally recommendable; the first for less information and computation time available, the latter for good informational and computational resources. However, it could happen that a substantial amount of objective value is lost by implementing the marginal model. The deterministic setup is also easily implementable, but naturally does not account for risk in the sustainability dimension. Hence, it may lead to unfavourable outcomes for a safety first investor.

The models presented are suggestions for the real world investors. Our results only concern theoretical properties and implementational details of these models. It is up to the investors to find those variants best describing their views and needs.

Supplementary material. We provide further tables, figures and texts on the data of our empirical study, the efficient frontier in the $\mathbf{E}[w'R]$-$\mathbf{E}[w'SR]$ space, our Theorem 4, a model comparison with respect to the default probabilities in each dimension and the optimal portfolio weights with varying parameter values in the appendices contained in the supplementary material.

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