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Abstract: This paper analyzes the role of uncertainty in a multi-sector housing model with financial frictions. We include time varying uncertainty (i.e. risk shocks) in the technology shocks that affect housing production. The analysis demonstrates that risk shocks to the housing production sector are a quantitatively important impulse mechanism for the business cycle. Also, we demonstrate that bankruptcy costs act as an endogenous markup factor in housing prices; as a consequence, the volatility of housing prices is greater than that of output, as observed in the data. The model can also account for the observed countercyclical behavior of risk premia on loans to the housing sector.

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Dear Editors:

Please consider for publication in the *Review of Economic Dynamics* the attached manuscript, "Risk Shocks and Housing Supply: A Quantitative Analysis," co-authored with Gabriel Lee and Victor Dorofeenko. As directed, the manuscript has been submitted via the online submission web site.

I certainly appreciate your editorial efforts and look forward to hearing from you in the near future.

Sincerely,

Kevin Salyer

Research highlights "Risk Shocks and Housing Supply: A Quantitative Analysis"

- We study housing in a multi-sector DSGE model that includes a lending channel.
- We introduce risk shocks (time varying uncertainty) in the production of housing.
- Bankruptcy costs affect housing prices and house price volatility.
- With risk shocks, the model matches the observed volatility of housing prices.

Risk Shocks and Housing Supply: A Quantitative Analysis*

Abstract

This paper analyzes the role of uncertainty in a multi-sector housing model with financial frictions. We include time varying uncertainty (i.e. risk shocks) in the technology shocks that affect housing production. The analysis demonstrates that risk shocks to the housing production sector are a quantitatively important impulse mechanism for the business cycle. Also, we demonstrate that bankruptcy costs act as an endogenous markup factor in housing prices; as a consequence, the volatility of housing prices is greater than that of output, as observed in the data. The model can also account for the observed countercyclical behavior of risk premia on loans to the housing sector.

- *JEL Classification: E4, E5, E2, R2, R3*
- *Keywords: agency costs, credit channel, time-varying uncertainty, residential investment, housing production, calibration*

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1 Introduction

Given the recent macroeconomic experience of most developed countries, few students of the economy would argue with the following three observations: 1. Financial intermediation plays an important role in the economy. 2. The housing sector is a critical component for aggregate economic behavior and 3. Uncertainty, and, in particular, increased uncertainty is a quantitatively important source of business cycle activity. However, while an extensive research literature is associated with each of these ideas individually, there are none that we know of which studies their joint influences and interactions.¹ The research presented here attempts to fill this void; in particular, we analyze the role of time varying uncertainty (i.e. risk shocks) in a multi-sector real business cycle model that includes housing production (developed by Davis and Heathcote, (2005)) and a financial sector with lending under asymmetric information (e.g. Carlstrom and Fuerst, (1997), (1998); Dorofeenko, Lee, and Salyer, (2008)). We model risk shocks as a mean preserving spread in the distribution of the technology shocks affecting house production and explore quantitatively how changes in uncertainty affect equilibrium characteristics.

Our aim in examining this environment is twofold. First, we want to develop a framework that can capture one of the main components of the recent financial crises, namely, changes in the risk associated with the housing sector. In our analysis, we focus entirely on the variations in risk associated with the production of housing and the consequences that this has for lending and economic activity. Hence our analysis is very much a fundamental-based approach so that we sidestep the delicate issue of modeling housing bubbles and departures from rational expectations. The results, as discussed below, suggest (to us) that this conservative approach is warranted.² Second,

¹ Some of the recent works which also examine housing and credit are: Iacoviello and Minetti (2008) and Iacoviello and Neri (2008) in which a new-Keynesian DGSE two sector model is used in their empirical analysis; Iacoviello (2005) analyzes the role that real estate collateral has for monetary policy; and Aoki, Proudman and Vlieghe (2004) analyse house price amplification effects in consumption and housing investment over the business cycle. None of these analyses use risk shocks as an impulse mechanism. Some recent papers that have examined the effects of uncertainty in a DSGE framework include Bloom et al. (2008), Fernandez-Villaverde et al. (2009), and Christiano et al. (2008). The last paper is most closely related to the analysis presented here in that it also uses a credit channel model.

² In a closely related analysis, Kahn (2008) also uses a multi-sector framework in order to analyze time variation in the growth rate of productivity in a key sector (consumption goods). He demonstrates that a change in regime growth, combined with a learning mechanism, can account for some of the observed movements in housing prices.

we want to cast the analysis of risk shocks in a model that is broadly consistent with some of the important stylized facts of the housing sector such as: (i) residential investment is about twice as volatile as non-residential investment and (ii) residential investment and non-residential investment are highly procyclical.³ Hence, we view our analysis as more of a quantitative rather than qualitative exercise.

With this in mind, we employ the Davis and Heathcote (2005) housing model which, when calibrated to the U.S. data, can replicate the high volatility observed in residential investment despite the absence of any frictions in the economy. The recent analysis in Christiano et al. (2008), however, provides compelling evidence that financial frictions do play an important role in business cycles and, given the recent financial events, it seems reasonable to investigate this role when combined with a housing sector.⁴ Consequently, we modify the Davis and Heathcote (2005) analysis by adding a financial sector in the economy and require that housing producers must finance their inputs via loans from the banking sector. While this modification improves the model upon some dimensions, a notable discrepancy between model output and the data is the volatility of housing prices; this inconsistency was also prominent in the original Davis and Heathcote (2005) analysis. However, when risk shocks are added to the production of housing, the model is capable of producing house price volatility consistent with observation. But this comes at the cost of excess volatility in several real variables such as residential investment. We find that adding adjustment costs to housing production with a quite reasonable value for the adjustment cost parameter eliminates this excess volatility in the real side while still matching the standard deviation of housing prices. We demonstrate that housing prices in our model are affected by expected bankruptcies and the associated agency costs; these serve as an endogenous, time-varying markup factor affecting the price of housing. When risk shocks are added to the

³ One other often mentioned stylized fact is that housing prices are persistent and mean reverting (e.g. Glaeser and Gyourko (2006)). See Figure 1 and Table 4 for these cyclical and statistical features during the period of 1975 until the second quarter of 2007.

⁴ Christiano et al. (2008) use a New Keynesian model to analyze the relative importance of shocks arising in the labor and goods markets, monetary policy, and financial sector. They find that time-varying second moments, i.e. risk shocks, are quantitatively important relative to the other impulse mechanisms.

model, volatility in this markup translates into increased volatility in housing prices. Moreover, the model implies that this endogenous markup to housing as well as the risk premium associated with loans to the housing sector should be countercyclical; both of these features are seen in the data.⁵

Our analysis also finds that plausible calibrations of the model with time varying uncertainty produce a quantitatively meaningful role for uncertainty over the housing and business cycles. For instance, we compare the impulse response functions for aggregate variables (such as output, consumption expenditure, and investment) due to a 1% increase in technology shocks to the construction sector to a 1% increase in uncertainty to shocks affecting housing production. We find that, quantitatively, the impact of risk shocks is almost as great as that from technology shocks. This comparison carries over to housing market variables such as the price of housing, the risk premium on loans, and the bankruptcy rate of housing producers. The model is not wholly satisfactory in that it can not account for the lead-lag structure of residential and non-residential investment but this is not surprising given that the analysis focuses entirely on the supply of housing. Still, we think the approach presented here provides a useful start in studying the effects of time-varying uncertainty on housing, housing finance and business cycles.

2 Model Description

As stated above, our model builds on two separate strands of literature: Davis and Heathcote's (2005) multi-sector growth model with housing, and Dorofeenko, Lee and Salyer's (2008) credit channel model with time-varying uncertainty. For expositional clarity, we first briefly outline our variant of the Davis and Heathcote model and then introduce the credit channel model.

⁵ In addition to these cyclical features, a marked feature of the housing sector has been the growth in residential and commercial real estate lending over the last decade. As shown in Figure 2, residential real estate loans (excluding revolving home equity loans) account for approximately 50% of total lending by domestically chartered commercial banks in the United States over the period October 1996 to July 2007. Figure 3 shows the strong co-movement between the amount of real estate loans and house prices.

2.1 Production

2.1.1 Firms

The economy consists of two agents, a consumer and an entrepreneur, and four sectors: an intermediate goods sector, a final goods sector, a housing goods sector and a banking sector. The intermediate sector is comprised of three perfectly competitive industries: a building/construction sector, a manufacturing sector and a service sector. The output from these sectors are then combined to produce a residential investment good and a consumption good which can be consumed or used as capital investment; these sectors are also perfectly competitive. Entrepreneurs combine residential investment with a fixed factor (land) to produce housing; this sector is where the lending channel and financial intermediation play a role.

Turning first to the intermediate goods sector, the representative firm in each sector is characterized by the following Cobb-Douglas production function:

$$x_{it} = k_{it}^{\theta_i} (n_{it} \exp^{z_{it}})^{1-\theta_i} \quad (1)$$

where $i = b, m, s$ (building/construction, manufacture, service), k_{it} , n_{it} , and z_{it} are capital, household labor, and a labor augmenting productivity shock respectively for each sector, with θ_i being the share of capital for sector i .⁶ In our calibration we set $\theta_b < \theta_m$ reflecting the fact that the manufacturing sector is more capital intensive (or less labor intensive) than the construction sector.

Productivity in each sector exhibits stochastic growth as given by:

$$z_{it} = t \log(g_{z,i}) + \tilde{z}_{it} \quad (2)$$

⁶ Real estate developers, i.e. entrepreneurs, also provide labor to the intermediate goods sectors. This is a technical consideration so that the net worth of entrepreneurs, including those that go bankrupt, is positive. Labor's share for entrepreneurs is set to a trivial number and has no effect on output dynamics. Hence, for expositional purposes, we ignore this factor in the presentation.

where $g_{z,i}$ is the trend growth rate in sector i .

The vector of technology shocks, $\tilde{\mathbf{z}} = (\tilde{z}_b, \tilde{z}_m, \tilde{z}_s)$, follows an $AR(1)$ process:

$$\tilde{\mathbf{z}}_{t+1} = B \cdot \tilde{\mathbf{z}}_t + \vec{\varepsilon}_{t+1} \quad (3)$$

The innovation vector $\vec{\varepsilon}$ is distributed normally with a given covariance matrix Σ_ε .⁷

These intermediate firms maximize a conventional static profit function every period. That is, at time t , the objective function is:

$$\max_{\{k_{it}, n_{it}\}} \left\{ \sum_i p_{it} x_{it} - r_t k_{it} - w_t n_{it} \right\} \quad (4)$$

which results in the usual first order conditions for factor demand:

$$r_t k_{it} = \theta_i p_{it} x_{it}, \quad w_t n_{it} = (1 - \theta_i) p_{it} x_{it} \quad (5)$$

where r_t , w_t , and p_{it} are the capital rental, wage, and output prices (with the consumption good as numeraire).

The intermediate goods are then used as inputs to produce two final goods, y_j , where $j = c, d$ (consumption/capital investment and residential investment respectively). This technology is also assumed to be Cobb-Douglas with constant returns to scale:

$$y_{jt} = \prod_{i=b,m,s} x_{ijt}^{\rho_{ij}}, \quad j = c, d. \quad (6)$$

⁷ In their analysis, Davis and Heathcote (2005) introduced a government sector characterized by non-stochastic tax rates and government expenditures and a balanced budget in every period. We abstract from these features in order to focus on time varying uncertainty and the credit channel. Our original model included these elements but it was determined that they did not have much influence on the policy functions that characterize equilibrium (although they clearly influence steady-state values).

Note that there are no aggregate technology shocks in the model. The input matrix is defined by

$$\mathbf{x1} = \begin{pmatrix} b_c & b_d \\ m_c & m_d \\ s_c & s_d \end{pmatrix}, \quad (7)$$

where, for example, m_j denotes the quantity of manufacturing output used as an input into sector j . The shares of construction, manufactures and services for sector j are defined by the matrix

$$\rho = \begin{pmatrix} B_c & B_d \\ M_c & M_d \\ S_c & S_d \end{pmatrix}. \quad (8)$$

The relative shares of the three intermediate inputs differ in producing the two final goods. For example, in the calibration of the model, we set $B_c < B_d$ to represent the fact that residential investment is more construction input intensive relative to the consumption good sector. The first degree homogeneity of the production processes implies $\sum_i \rho_{ij} = 1$; $j = c, d$ while market clearing in the intermediate goods markets requires $x_{it} = \sum_j x1_{ijt}$; $i = b, m, s$.

With intermediate goods as inputs, the final goods' firms solve the following static profit maximization problem at t (as stated earlier, the price of consumption good, p_{ct} , is normalized to 1):

$$\max_{x_{ijt}} \left\{ y_{ct} + p_{dt}y_{dt} - \sum_j \sum_i p_{it}x1_{ijt} \right\} \quad (9)$$

subject to the production functions (eq.(6)) and non-negativity of inputs.

The first order conditions associated with profit maximization are given by the typical marginal conditions

$$p_{it}x1_{ijt} = \rho_{ij}p_{jt}y_{jt}; \quad i = b, m, s; \quad j = c, d \quad (10)$$

Constant returns to scale implies zero profits in both sectors so we have the following relationships:

$$\sum_j p_{jt} y_{jt} = \sum_i p_{it} x_{it} = r_t k_t + w_t n_t \quad (11)$$

Finally, new housing structures, y_{ht} , are produced by entrepreneurs (i.e. real estate developers) using the residential investment good, y_{dt} , and land, x_{lt} , as inputs. This sector is discussed below following the description of the household and financial sectors.

2.1.2 Households

The representative household derives utility each period from consumption, c_t , housing, h_t , and leisure, $1 - n_t$; all of these are measured in per-capita terms. Instantaneous utility for each member of the household is defined by the Cobb-Douglas functional form of

$$U(c_t, h_t, 1 - n_t) = \frac{\left(c_t^{\mu_c} h_t^{\mu_h} (1 - n_t)^{1 - \mu_c - \mu_h} \right)^{1 - \sigma}}{1 - \sigma} \quad (12)$$

where μ_c and μ_h are the weights for consumption and housing in utility, and σ represents the coefficient of relative risk aversion. It is assumed that population grows at the (gross) rate of η so that the household's objective function is written as:

$$E_0 \sum_{t=0}^{\infty} (\beta \eta)^t U(c_t, h_t, 1 - n_t) \quad (13)$$

Each period agents combine labor income with income from capital and land and use these to purchase consumption, new housing and investment. In the purchase of housing (addition to the existing housing stock), agents interact with the financial intermediary which offers one unit of housing for the price of p_{ht} units of consumption. As described below, the financial intermediary lends these resources to risky entrepreneurs who use them to buy the inputs into the housing production. For the household, these choices are represented by the per-capita budget constraint

and the laws of motion for per-capita capital and housing:

$$w_t n_t + r_t k_t + p_{lt} x_{lt} = c_t + i_{kt} + p_{ht} i_{ht} \quad (14)$$

$$\eta k_{t+1} = k_t (1 - \delta_k) + G(i_{kt}, i_{kt-1}) \quad (15)$$

$$\eta h_{t+1} = h_t (1 - \delta_h) + G(i_{ht}, i_{ht-1}) \quad (16)$$

where i_{kt} is capital investment, i_{ht} is housing investment, δ_k and δ_h are the capital and house depreciation rates respectively.⁸ The function $G(\bullet)$ is used to introduce adjustment costs into both capital and housing accumulation. For both stocks, we use the same functional form:

$$G(i_{zt}, i_{zt-1}) = i_{zt} \left(1 - S \left(\frac{i_{zt}}{i_{zt-1}} \right) \right); z = (h, k) \quad (17)$$

It is assumed that $S(1) = S'(1) = 0$ and $S''(1) = \kappa_z > 0; z = (h, k)$; this is sufficient structure on the function given that we log-linearize the economy when solving for equilibrium. As shown by Christiano, Eichenbaum and Evans (2005), the parameter κ_z is the inverse of the elasticity of housing (capital) investment with respect to a temporary change in the shadow price of the housing (capital) stock, denoted q_{ht} (q_{kt}). For more on the properties of this form of adjustment costs (which, importantly, affect the second derivative of the law of motion of capital rather than the usual first derivative), the reader is directed to Christiano, Eichenbaum, and Evans (2005).

It is also important to note that we follow Davis and Heathcote (2005) in that h_t denotes effective housing units. Specifically, they exploit the geometric depreciation structure of housing structures in order to derive h_t . Furthermore, they derive the law of motion for effective housing units (in a model that does not include agency costs) and demonstrate that the depreciation rate δ_h is related to the depreciation rate of structures. As discussed in their analysis, it is not

⁸ Note that lower case variables for capital, labor and consumption represent per-capita quantities while upper case denote will denote aggregate quantities.

necessary to keep track of the stock of housing structures as an additional state variable; the amount of effective housing units, h_t , is a sufficient statistic.

The optimization problem leads to the following necessary conditions which represent, respectively, the Euler conditions associated with capital, housing, and the intra-temporal labor-leisure decision:

$$U_{1t} = U_{1t}q_{kt}G_1(i_{kt}, i_{kt-1}) + \beta\eta E_t [U_{1t+1}q_{kt+1}G_2(i_{kt+1}, i_{kt})] \quad (18)$$

$$U_{1t}q_{kt} = \beta\eta E_t [U_{1t+1}(r_{t+1} + q_{kt+1}(1 - \delta))] \quad (19)$$

$$U_{1t}p_{ht} = U_{1t}q_{ht}G_1(i_{ht}, i_{ht-1}) + \beta\eta E_t [U_{1t+1}q_{ht+1}G_2(i_{ht+1}, i_{ht})] \quad (20)$$

$$U_{1t}q_{ht} = \beta\eta E_t [U_{1t+1}(1 - \delta_h)q_{ht+1} + U_{2t+1}] \quad (21)$$

$$w_t = \frac{U_3}{U_1}. \quad (22)$$

As mentioned above, the terms q_{kt} and q_{ht} denote the shadow prices of capital and housing, respectively. Note that in the absence of adjustment costs (so that $G_1(i_{kt}, i_{kt-1}) = G_1(i_{ht}, i_{ht-1}) = 1$ and $G_2(i_{kt+1}, i_{kt}) = G_2(i_{ht+1}, i_{ht}) = 0$), these shadow prices are 1 and p_{ht} as expected. These Euler equations have the standard marginal cost = marginal benefit interpretations. For instance, the left-hand side of eq.(21) is the marginal cost of purchasing additional housing; at an optimum this must equal the expected marginal utility benefit of housing which comes from the capital value of undepreciated housing and the direct utility that additional housing provides. The shadow price, q_{ht} , used in this calculus is, in turn, related to the price of housing and the utility costs associated with housing adjustment costs as reflected in eq. (20). The equations associated with capital have an analogous interpretation.

2.2 The Credit Channel

2.2.1 Housing Entrepreneurial Contract

The economy described above is identical to that studied in Davis and Heathcote (2005) except for the addition of productivity shocks affecting housing production.⁹ We describe in more detail the nature of this sector and the role of the banking sector. It is assumed that a continuum of housing producing firms with unit mass are owned by risk-neutral entrepreneurs (developers). The costs of producing housing are financed via loans from risk-neutral intermediaries. Given the realization of the idiosyncratic shock to housing production, some real estate developers will not be able to satisfy their loan payments and will go bankrupt. The banks take over operations of these bankrupt firms but must pay an agency fee. These agency fees, therefore, affect the aggregate production of housing and, as shown below, imply an endogenous markup to housing prices. That is, since some housing output is lost to agency costs, the price of housing must be increased in order to cover factor costs.

The timing of events is critical:

1. The exogenous state vector of technology shocks and uncertainty shocks, denoted $(z_{i,t}, \sigma_{\omega,t})$, is realized.
2. Firms hire inputs of labor and capital from households and entrepreneurs and produce intermediate output via Cobb-Douglas production functions. These intermediate goods are then used to produce the two final outputs.
3. Households make their labor, consumption, housing, and investment decisions.
4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of the loan (fp_{at}) and a cutoff level of productivity for the entrepreneurs' technology

⁹ Also, as noted above, we abstract from taxes and government expenditures.

shock, $\bar{\omega}_t$.

5. Entrepreneurs use their net worth and loans from the banking sector in order to purchase the factors for housing production. The quantity of factors (residential investment and land) is determined and paid for before the idiosyncratic technology shock is known.
6. The idiosyncratic technology shock of each entrepreneur is realized. If $\omega_{at} \geq \bar{\omega}_t$ the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional (but time varying) to total factor payments.
7. Solvent entrepreneur's sell their remaining housing output to the bank sector and use this income to purchase current consumption and capital. The latter will in part determine their net worth in the following period.
8. Note that the total amount of housing output available to the households is due to three sources: (1) The repayment of loans by solvent entrepreneurs, (2) The housing output net of agency costs by insolvent firms, and (3) the sale of housing output by solvent entrepreneurs used to finance the purchase of consumption and capital.

A schematic of the implied flows is presented in Figure 5.

For entrepreneur a , the housing production function is denoted $F(x_{alt}, y_{adt})$ and is assumed to exhibit constant returns to scale. Specifically, we assume:

$$y_{aht} = \omega_{at} F(x_{alt}, y_{adt}) = \omega_{at} x_{alt}^{\zeta} y_{adt}^{1-\zeta} \quad (23)$$

where, ζ denotes the share of land. It is assumed that the aggregate quantity of land is fixed and equal to 1. The technology shock, ω_{at} , is an idiosyncratic shock affecting real estate developers. The technology shock is assumed to have a unitary mean and standard deviation of $\sigma_{\omega,t}$. The

standard deviation, $\sigma_{\omega,t}$, follows an $AR(1)$ process:

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^\chi \exp^{\varepsilon_{\sigma,t+1}} \quad (24)$$

with the steady-state value σ_0 , $\chi \in (0, 1)$ and $\varepsilon_{\sigma,t+1}$ is a white noise innovation.¹⁰

Each period, entrepreneurs enter the period with net worth given by nw_{at} . Developers use this net worth and loans from the banking sector in order to purchase inputs. Letting fp_{at} denote the factor payments associated with developer a , we have:

$$fp_{at} = p_{dt}y_{adt} + p_{lt}x_{alt} \quad (25)$$

Hence, the size of the loan is $(fp_{at} - nw_{at})$. The realization of ω_{at} is privately observed by each entrepreneur; banks can observe the realization at a cost that is proportional to the total input bill.

It is convenient to express these agency costs in terms of the price of housing. Note that agency costs combined with constant returns to scale in housing production (see eq. (23)) implies that the aggregate value of housing output must be greater than the value of inputs; i.e. housing must sell at a markup over the input costs, the factor payments. Denote this markup as \bar{s}_t (which is treated as parametric by both lenders and borrowers) which satisfies:

$$p_{ht}y_{ht} = \bar{s}_t fp_t \quad (26)$$

Also, since $E(\omega_t) = 1$ and all firms face the same factor prices, this implies that, at the individual

¹⁰ This autoregressive process is used so that, when the model is log-linearized, $\hat{\sigma}_{\omega,t}$ (defined as the percentage deviations from σ_0) follows a standard, mean-zero $AR(1)$ process.

level, we have¹¹

$$p_{ht}F(x_{alt}, y_{adt}) = \bar{s}_t f p_{at} \quad (27)$$

Given these relationships, we define agency costs for loans to an individual entrepreneur in terms of foregone housing production as $\mu \bar{s}_t f p_{at}$.

With a positive net worth, the entrepreneur borrows $(f p_{at} - n w_{at})$ consumption goods and agrees to pay back $(1 + r_t^L)(f p_{at} - n w_{at})$ to the lender, where r_t^L is the interest rate on loans. The cutoff value of productivity, $\bar{\omega}_t$, that determines solvency (i.e. $\omega_{at} \geq \bar{\omega}_t$) or bankruptcy (i.e. $\omega_{at} < \bar{\omega}_t$) is defined by $(1 + r_t^L)(f p_{at} - n w_{at}) = p_{ht} \bar{\omega}_t F(\cdot)$ (where $F(\cdot) = F(x_{alt}, y_{adt})$). Denoting the *c.d.f.* and *p.d.f.* of ω_t as $\Phi(\omega_t; \sigma_{\omega,t})$ and $\phi(\omega_t; \sigma_{\omega,t})$, the expected returns to a housing producer is therefore given by:¹²

$$\int_{\bar{\omega}_t}^{\infty} [p_{ht} \omega F(\cdot) - (1 + r_t^L)(f p_{at} - n w_{at})] \phi(\omega; \sigma_{\omega,t}) d\omega \quad (28)$$

Using the definition of $\bar{\omega}_t$ and eq. (27), this can be written as:

$$\bar{s}_t f p_{at} f(\bar{\omega}_t; \sigma_{\omega,t}) \quad (29)$$

where $f(\bar{\omega}_t; \sigma_{\omega,t})$ is defined as:

$$f(\bar{\omega}_t; \sigma_{\omega,t}) = \int_{\bar{\omega}_t}^{\infty} \omega \phi(\omega; \sigma_{\omega,t}) d\omega - [1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})] \bar{\omega}_t \quad (30)$$

Similarly, the expected returns to lenders is given by:

$$\int_0^{\bar{\omega}_t} p_{ht} \omega F(\cdot) \phi(\omega; \sigma_{\omega,t}) d\omega + [1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})] (1 + r_t^L)(f p_{at} - n w_{at}) - \Phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu \bar{s}_t f p_{at} \quad (31)$$

¹¹ The implication is that, at the individual level, the product of the markup (\bar{s}_t) and factor payments is equal to the expected value of housing production since housing output is unknown at the time of the contract. Since there is no aggregate risk in housing production, we also have $p_{ht} y_{ht} = \bar{s}_t f p_{at}$.

¹² The notation $\Phi(\omega; \sigma_{\omega,t})$ is used to denote that the distribution function is time-varying as determined by the realization of the random variable, $\sigma_{\omega,t}$.

Again, using the definition of $\bar{\omega}_t$ and eq. (27), this can be expressed as:

$$\bar{s}_t f p_{at} g(\bar{\omega}_t; \sigma_{\omega,t}) \quad (32)$$

where $g(\bar{\omega}_t; \sigma_{\omega,t})$ is defined as:

$$g(\bar{\omega}_t; \sigma_{\omega,t}) = \int_0^{\bar{\omega}_t} \omega \phi(\omega; \sigma_{\omega,t}) d\omega + [1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})] \bar{\omega}_t - \Phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu \quad (33)$$

Note that these two functions sum to:

$$f(\bar{\omega}_t; \sigma_{\omega,t}) + g(\bar{\omega}_t; \sigma_{\omega,t}) = 1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu \quad (34)$$

Hence, the term $\Phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu$ captures the loss of housing due to the agency costs associated with bankruptcy. With the expected returns to lender and borrower expressed in terms of the size of the loan, $f p_{at}$, and the cutoff value of productivity, $\bar{\omega}_t$, it is possible to define the optimal borrowing contract by the pair $(f p_{at}, \bar{\omega}_t)$ that maximizes the entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur). That is, the optimal contract is determined by the solution to:

$$\max_{\bar{\omega}_t, f p_{at}} \bar{s}_t f p_{at} f(\bar{\omega}_t; \sigma_{\omega,t}) \text{ subject to } \bar{s}_t f p_{at} g(\bar{\omega}_t; \sigma_{\omega,t}) \geq f p_{at} - n w_{at} \quad (35)$$

A necessary condition for the optimal contract problem is given by:

$$\frac{\partial(\cdot)}{\partial \bar{\omega}_t} : \bar{s}_t f p_{at} \frac{\partial f(\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t} = -\lambda_t \bar{s}_t f p_{at} \frac{\partial g(\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t} \quad (36)$$

where λ_t is the shadow price of the lender's resources. Using the definitions of $f(\bar{\omega}_t; \sigma_{\omega,t})$ and

$g(\bar{\omega}_t; \sigma_{\omega,t})$, this can be rewritten as:¹³

$$1 - \frac{1}{\lambda_t} = \frac{\phi(\bar{\omega}_t; \sigma_{\omega,t})}{1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})} \mu \quad (37)$$

As shown by eq.(37), the shadow price of the resources used in lending is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of housing production, i.e. $\lambda_t = 1$.

The second necessary condition is:

$$\frac{\partial(\cdot)}{\partial f p_{at}} : \bar{s}_t f(\bar{\omega}_t; \sigma_{\omega,t}) = \lambda_t [1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})] \quad (38)$$

These first-order conditions imply that, in general equilibrium, the markup factor, \bar{s}_t , will be endogenously determined and related to the probability of bankruptcy. Specifically, using the first order conditions, we have that the markup, \bar{s}_t , must satisfy:

$$\begin{aligned} \bar{s}_t^{-1} &= \left[(f(\bar{\omega}_t; \sigma_{\omega,t}) + g(\bar{\omega}_t; \sigma_{\omega,t})) + \frac{\phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu f(\bar{\omega}_t; \sigma_{\omega,t})}{\frac{\partial f(\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t}} \right] \\ &= \left[1 - \underbrace{\Phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu}_A - \underbrace{\frac{\phi(\bar{\omega}_t; \sigma_{\omega,t})}{1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})} \mu f(\bar{\omega}_t; \sigma_{\omega,t})}_B \right] \end{aligned} \quad (39)$$

First note that the markup factor depends only on economy-wide variables so that the aggregate markup factor is well defined. Also, the two terms, A and B , demonstrate that the markup factor is affected by both the total agency costs (term A) and the marginal effect that bankruptcy has on the entrepreneur's expected return. That is, term B reflects the loss of housing output, μ , weighted by the expected share that would go to entrepreneur's, $f(\bar{\omega}_t; \sigma_{\omega,t})$, and the conditional probability of

¹³ Note that we have used the fact that $\frac{\partial f(\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t} = \Phi(\bar{\omega}_t; \sigma_{\omega,t}) - 1 < 0$

bankruptcy (the Inverse Mill's ratio). Finally, note that, in the absence of credit market frictions, there is no markup so that $\bar{s}_t = 1$. In the partial equilibrium setting, it is straightforward to show that equation (39) defines an implicit function $\bar{\omega}(\bar{s}_t, \sigma_{\omega,t})$ that is increasing in \bar{s}_t .

The incentive compatibility constraint implies

$$fp_{at} = \frac{1}{(1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}))} nw_{at} \quad (40)$$

Equation (40) implies that the size of the loan is linear in entrepreneur's net worth so that aggregate lending is well-defined and a function of aggregate net worth.

The effect of an increase in uncertainty on lending can be understood in a partial equilibrium setting where \bar{s}_t and nw_{at} are treated as parameters. As shown by eq. (39), the assumption that the markup factor is unchanged implies that the costs of default, represented by the terms A and B , must be constant. With a mean-preserving spread in the distribution for ω_t , this means that $\bar{\omega}_t$ will fall (this is driven primarily by the term A). Through an approximation analysis, it can be shown that $\bar{\omega}_t \approx g(\bar{\omega}_t; \sigma_{\omega,t})$ (see the Appendix in Dorofeenko, Lee, and Salyer (2008)). That is, the increase in uncertainty will reduce lenders' expected return ($g(\bar{\omega}_t; \sigma_{\omega,t})$). Rewriting the binding incentive compatibility constraint (eq. (40)) yields:

$$\bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}) = 1 - \frac{nw_{at}}{fp_{at}} \quad (41)$$

the fall in the left-hand side induces a fall in fp_{at} . Hence, greater uncertainty results in a fall in housing production. This partial equilibrium result carries over to the general equilibrium setting.

The existence of the markup factor implies that inputs will be paid less than their marginal products. In particular, profit maximization in the housing development sector implies the following necessary conditions:

$$\frac{p_{lt}}{p_{ht}} = \frac{F_{xl}(x_{lt}, y_{dt})}{\bar{s}_t} \quad (42)$$

$$\frac{p_{dt}}{p_{ht}} = \frac{F_{yd}(x_{lt}, y_{dt})}{\bar{s}_t} \quad (43)$$

These expressions demonstrate that, in equilibrium, the endogenous markup (determined by the agency costs) will be a determinant of housing prices.

The production of new housing is determined by a Cobb-Douglas production with residential investment and land (fixed in equilibrium) as inputs. Denoting housing output, net of agency costs, as y_{ht} , this is given by:

$$y_{ht} = x_{lt}^\zeta y_{dt}^{1-\zeta} [1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu] \quad (44)$$

In equilibrium, we require that $i_{ht} = y_{ht}$; i.e. household's housing investment is equal to housing output. Recall that the law of motion for housing is given by eq. (16).

2.2.2 Entrepreneurial Consumption and House Prices

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the household. This is represented by following expected utility function:

$$E_0 \sum_{t=0}^{\infty} (\beta \eta \gamma)^t c_t^e \quad (45)$$

where c_t^e denotes entrepreneur's per-capita consumption at date t , and $\gamma \in (0, 1)$. This new parameter, γ , will be chosen so that it offsets the steady-state internal rate of return due to housing production.

Each period, entrepreneur's net worth, nw_t is determined by the value of capital income and the remaining capital stock.¹⁴ That is, entrepreneurs use capital to transfer wealth over time

¹⁴ As stated in footnote 6, net worth is also a function of current labor income so that net worth is bounded above zero in the case of bankruptcy. However, since entrepreneur's labor share is set to a very small number, we ignore this component of net worth in the exposition of the model.

(recall that the housing stock is owned by households). Denoting entrepreneur's capital as k_t^e , this implies:¹⁵

$$nw_t = k_t^e [r_t + 1 - \delta_\kappa] \quad (46)$$

The law of motion for entrepreneurial capital stock is determined in two steps. First, new capital is financed by the entrepreneurs' value of housing output after subtracting consumption:

$$\eta k_{t+1}^e = p_{ht} y_{aht} f(\bar{\omega}_t; \sigma_{\omega,t}) - c_t^e = \bar{s}_t f p_{at} f(\bar{\omega}_t; \sigma_{\omega,t}) - c_t^e \quad (47)$$

Note we have used the equilibrium condition that $p_{ht} y_{aht} = \bar{s}_t f p_{at}$ to introduce the markup, \bar{s}_t , into the expression. Then, using the incentive compatibility constraint, eq. (40), and the definition of net worth, the law of motion for capital is given by:

$$\eta k_{t+1}^e = k_t^e (r_t + 1 - \delta_\kappa) \frac{\bar{s}_t f(\bar{\omega}_t; \sigma_{\omega,t})}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})} - c_t^e \quad (48)$$

The term $\bar{s}_t f(\bar{\omega}_t; \sigma_{\omega,t}) / (1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}))$ represents the entrepreneur's internal rate of return due to housing production; alternatively, it reflects the leverage enjoyed by the entrepreneur since

$$\frac{\bar{s}_t f(\bar{\omega}_t; \sigma_{\omega,t})}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})} = \frac{\bar{s}_t f p_{at} f(\bar{\omega}_t; \sigma_{\omega,t})}{nw_t} \quad (49)$$

That is, entrepreneurs use their net worth to finance factor inputs of value $f p_{at}$, this produces housing which sells at the markup \bar{s}_t with entrepreneur's retaining fraction $f(\bar{\omega}_t; \sigma_{\omega,t})$ of the value of housing output.

Given this setting, the optimal path of entrepreneurial consumption implies the following Euler equation:

$$1 = \beta \eta \gamma E_t \left[(r_{t+1} + 1 - \delta_\kappa) \frac{\bar{s}_{t+1} f(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})}{1 - \bar{s}_{t+1} g(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})} \right] \quad (50)$$

¹⁵ For expositional purposes, in this section we drop the subscript a denoting the individual entrepreneur.

Finally, we can derive an explicit relationship between entrepreneur's capital and the value of the housing stock using the incentive compatibility constraint and the fact that housing sells at a markup over the value of factor inputs. That is, since $p_{ht}F(x_{alt}, y_{adt}) = \bar{s}_t f p_t$, the incentive compatibility constraint implies:

$$p_{ht} \left(x_{lt}^\zeta y_{dt}^{1-\zeta} \right) = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})} \bar{s}_t \quad (51)$$

Again, it is important to note that the markup parameter plays a key role in determining housing prices and output.

2.2.3 Financial Intermediaries

The banks in the model act as risk-neutral financial intermediaries that, in equilibrium, earn zero profits. There is a clear role for banks in this economy since, through pooling, all aggregate uncertainty of housing production can be eliminated. The banking sector receives deposits from households and, in return, agents receive a housing for a certain (i.e. risk-free) price. Hence, in this model, financial intermediaries act more like an aggregate housing cooperative rather than a typical bank.

3 Equilibrium

Prior to solving for equilibrium, it is necessary to express the growing economy in stationary form. Given that preferences and technologies are Cobb-Douglas, the economy will have a balanced growth path. Hence, it is possible to transform all variables by the appropriate growth factor. As discussed in Davis and Heathcote (2005), the output value of all markets (e.g. $p_d y_d, y_c, p_i x_i$ for $i = (b, m, s)$) are growing at the same rate as capital and consumption, g_k . This growth rate, in turn, is a geometric average of the growth rates in the intermediate sectors: $g_k = g_{zb}^{B_c} g_{zm}^{M_c} g_{zs}^{S_c}$. It is also the case that factor prices display the normal behavior along a balanced growth path: interest

Table 1: Growth Rates on the Balanced Growth Path

n_b, n_m, n_s, n, r	1
$k_b, k_m, k_s, k, c, y_c, w$	$g_k = \left[g_{zb}^{B_c(1-\theta_b)} g_{zm}^{M_c(1-\theta_m)} g_{zs}^{S_c(1-\theta_s)} \right]^{(1/(1-B_c\theta_b-M_c\theta_m-S_c\theta_s))}$
b_c, b_d, x_b	$g_b = g_k^{\theta_b} g_{zb}^{1-\theta_b}$
m_c, m_d, x_m	$g_m = g_k^{\theta_m} g_{zm}^{1-\theta_m}$
s_c, s_d, x_s	$g_s = g_k^{\theta_s} g_{zs}^{1-\theta_s}$
y_d	$g_d = g_b^{B_h} g_m^{M_h} g_s^{S_h}$
x_l	$g_l = \eta^{-1}$
y_h, h	$g_h = g_l^\zeta g_d^{1-\zeta}$
$p_h y_h, p_d x_d, p_l x_l, p_b x_b, p_m x_m, p_s x_s$	g_k

rates are stationary while the wage in all sectors is growing at the same rate. The growth rates for the various factors are presented in Table 1 (again see Davis and Heathcote (2005) for details). These growth factors were used to construct a stationary economy; all subsequent discussion is in terms of this transformed economy.

Equilibrium in the economy is described by the vector of factor prices (w_t, r_t) , the vector of intermediate goods prices, (p_{bt}, p_{mt}, p_{st}) , the price of residential investment (p_{dt}) , the price of land (p_{lt}) , the price of housing (p_{ht}) , the shadow prices associated with housing and capital (due to adjustment costs) (q_{ht}, q_{kt}) , and the markup factor (\bar{s}_t) . In total, therefore, there are eleven equilibrium prices. In addition, the following quantities are determined in equilibrium: the vector of intermediate goods (x_{mt}, x_{bt}, x_{st}) , the vector of labor inputs (n_{mt}, n_{bt}, n_{st}) , the total amount of labor supplied, (n_t) , the vector of inputs into the final goods sectors $(b_{ct}, b_{dt}, m_{ct}, m_{dt}, s_{ct}, s_{dt})$, the vector of capital inputs (k_{mt}, k_{bt}, k_{st}) , entrepreneurial capital (k_t^e) , household investment (k_{t+1}) , the vector of final goods output (y_{ct}, y_{dt}) , the technology cutoff level $(\bar{\omega}_t)$, the effective housing stock (h_{t+1}) , and the consumption of households and entrepreneurs (c_t, c_t^e) . In total, there are 24 quantities to be determined; adding the eleven prices, the system is defined by 35 unknowns.

These are determined by the following conditions:

Factor demand optimality in the intermediate goods markets

$$r_t = \theta_i \frac{p_{it} x_{it}}{k_{it}} \quad (3 \text{ equations}) \quad (52)$$

$$w_t = (1 - \theta_i) \frac{p_{it} x_{it}}{n_{it}} \quad (3 \text{ equations}) \quad (53)$$

Factor demand optimality in the final goods sector:

$$p_{ct} y_{ct} = \frac{p_{bt} b_{ct}}{B_c} = \frac{p_{mt} m_{ct}}{M_c} = \frac{p_{st} s_{ct}}{S_c} \quad (3 \text{ equations}) \quad (54)$$

$$p_{dt} y_{dt} = \frac{p_{bt} b_{dt}}{B_d} = \frac{p_{mt} m_{dt}}{M_d} = \frac{p_{st} s_{dt}}{S_d} \quad (3 \text{ equations}) \quad (55)$$

Factor demand in the housing sector (using the fact that, in equilibrium $x_{it} = 1$) produces two more equations:

$$\frac{p_{it}}{p_{ht}} = \frac{\zeta y_{dt}^{1-\zeta}}{\bar{s}_t} \quad (56)$$

$$\frac{p_{dt}}{p_{ht}} = \frac{(1 - \zeta) y_{dt}^{-\zeta}}{\bar{s}_t} \quad (57)$$

The household's necessary conditions provide 5 more equations:

$$1 = q_{kt} G_1(i_{kt}, i_{kt-1}) + \beta \eta E_t \left[\frac{U_{1t+1}}{U_{1t}} q_{kt+1} G_2(i_{kt+1}, i_{kt}) \right] \quad (58)$$

$$q_{kt} = \beta \eta E_t \left[\frac{U_{1t+1}}{U_{1t}} (r_{t+1} + q_{kt+1} (1 - \delta)) \right]$$

$$p_{ht} = q_{ht} G'_1(i_{ht}, i_{ht-1}) + \beta \eta E_t \left[q_{ht+1} G'_2(i_{ht+1}, i_{ht}) \frac{U'_{1t+1}}{U'_{1t}} \right] \quad (59)$$

$$q_{ht} = \beta \eta E_t \left[(1 - \delta_h) q_{ht+1} \frac{U'_{1t+1}}{U'_{1t}} + \frac{U'_{2t+1}}{U'_{1t}} \right] \quad (60)$$

$$w_t = \frac{U_3(c_t, h_t, 1 - n_t)}{U_1(c_t, h_t, 1 - n_t)}. \quad (61)$$

The financial contract provides the condition for the markup and the incentive compatibility constraint:

$$\bar{s}_t^{-1} = \left[(f(\bar{\omega}_t; \sigma_{\omega,t}) + g(\bar{\omega}_t; \sigma_{\omega,t})) + \frac{\phi(\bar{\omega}_t; \sigma_{\omega,t}) \mu f(\bar{\omega}_t; \sigma_{\omega,t})}{\frac{\partial f(\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t}} \right] \quad (62)$$

$$p_{ht} y_{dt}^{1-\zeta} = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})} \bar{s}_t \quad (63)$$

The entrepreneur's maximization problem provides the following Euler equation:

$$1 = \beta \eta \gamma E_t \left[(r_{t+1} + 1 - \delta_\kappa) \frac{\bar{s}_{t+1} f(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})}{1 - \bar{s}_{t+1} g(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})} \right] \quad (64)$$

To these optimality conditions, we have the following market clearing conditions:

Labor market clearing:

$$n_t = \sum_i n_{it}, \quad i = b, m, s \quad (65)$$

Market clearing for capital:

$$k_t = \sum_i k_{it}, \quad i = b, m, s \quad (66)$$

Market clearing for intermediate goods:

$$x_{bt} = b_{ct} + b_{dt}, \quad x_{mt} = m_{ct} + m_{dt}, \quad x_{st} = s_{ct} + s_{dt}. \quad (67)$$

The aggregate resource constraint for the consumption final goods sector (i.e. the law of motion for capital)

$$\eta k_{t+1} = (1 - \delta_\kappa) k_t + y_{ct} - c_t - c_t^e \quad (68)$$

The law of motion for the effective housing units:

$$\eta h_{t+1} = (1 - \delta_h) h_t + y_{dt}^{1-\zeta} (1 - \Phi(\bar{\omega}_t) \mu) \quad (69)$$

The law of motion for entrepreneur's capital stock:

$$\eta k_{t+1}^e = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})} \bar{s}_t f(\bar{\omega}_t; \sigma_{\omega,t}) - c_t^e \quad (70)$$

Finally, we have the production functions. Specifically, for the intermediate goods markets:

$$x_{it} = k_{it}^{\theta_i} (n_{it} \exp^{z_{it}})^{1-\theta_i}; \quad i = b, m, s \quad (71)$$

For the final goods sectors, we have:

$$y_{ct} = b_{ct}^{B_c} m_{ct}^{M_c} s_{ct}^{S_c} \quad (72)$$

$$y_{dt} = b_{dt}^{B_d} m_{dt}^{M_d} s_{dt}^{S_d} \quad (73)$$

These provide the required 35 equations to solve for equilibrium. In addition there are the laws of motion for the technology shocks and the uncertainty shocks.

$$\bar{\mathbf{z}}_{t+1} = B \cdot \bar{\mathbf{z}}_t + \bar{\boldsymbol{\varepsilon}}_{t+1} \quad (74)$$

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^\chi \exp^{\varepsilon_{\sigma,t+1}} \quad (75)$$

To solve the model, we log linearize around the steady-state. The solution is defined by 35 equations in which the endogenous variables are expressed as linear functions of the vector of state variables $(z_{bt}, z_{mt}, z_{st}, \sigma_{\omega,t}, k_t, k_t^e, h_t)$.

Table 2: Key Preference and Production Parameters

Depreciation rate for capital: δ_κ	0.056
Depreciation rate for effective housing (h): δ_h	0.014
Land's share in new housing: ζ	0.106
Population growth rate: η	1.017
Discount factor: β	0.951
Risk aversion: σ	2.00
Consumption's share in utility: μ_c	0.314
Housing's share in utility: μ_h	0.044
Leisure's share in utility: $1-\mu_c - \mu_h$	0.642

Table 3: Intermediate Production Technology Parameters

	B	M	S
Input shares for consumption/investment good (B_c, M_c, S_c)	0.031	0.270	0.700
Input shares for residential investment (B_d, M_d, S_d)	0.470	0.238	0.292
Capital's share in each sector ($\theta_b, \theta_m, \theta_s$)	0.132	0.309	0.237
Sectoral trend productivity growth (%) (g_{zb}, g_{zm}, g_{zs})	-0.27	2.85	1.65

4 Calibration and Data

A strong motivation for using the Davis and Heathcote (2005) model is that the theoretical constructs have empirical counterparts. Hence, the model parameters can be calibrated to the data. We use directly the parameter values chosen by the previous authors; readers are directed to their paper for an explanation of their calibration methodology. Parameter values for preferences, depreciation rates, population growth and land's share are presented in Table 2. In addition, the parameters for the intermediate production technologies are presented in Table 3.¹⁶

As in Davis and Heathcote (2005), the exogenous shocks to productivity in the three sectors are assumed to follow an autoregressive process as given in eq. (3). The parameters for the vector autoregression are the same as used in Davis and Heathcote (2005) (see their Table 4, p. 766 for details). In particular, we use the following values (recall that the rows of the B matrix correspond

¹⁶ Davis and Heathcote (2005) determine the input shares into the consumption and residential investment good by analyzing the two sub-tables contained in the "Use" table of the 1992 Benchmark NIPA Input-Output tables. Again, the interested reader is directed to their paper for further clarification.

to the building, manufacturing, and services sectors, respectively):

$$B = \begin{pmatrix} 0.707 & 0.010 & -0.093 \\ -0.006 & 0.871 & -0.150 \\ 0.003 & 0.028 & 0.919 \end{pmatrix}$$

Note this implies that productivity shocks have modest dynamic effects across sectors. The contemporaneous correlations of the innovations to the shock are given by the correlation matrix:

$$\Sigma = \begin{pmatrix} \text{Corr}(\varepsilon_b, \varepsilon_b) & \text{Corr}(\varepsilon_b, \varepsilon_m) & \text{Corr}(\varepsilon_b, \varepsilon_s) \\ & \text{Corr}(\varepsilon_m, \varepsilon_m) & \text{Corr}(\varepsilon_m, \varepsilon_s) \\ & & \text{Corr}(\varepsilon_s, \varepsilon_s) \end{pmatrix} = \begin{pmatrix} 1 & 0.089 & 0.306 \\ & 1 & 0.578 \\ & & 1 \end{pmatrix}$$

The standard deviations for the innovations were assumed to be: $(\sigma_{bb}, \sigma_{mm}, \sigma_{ss}) = (0.041, 0.036, 0.018)$.

For the financial sector, we use the same loan and bankruptcy rates as in Carlstrom and Fuerst (1997) in order to calibrate the steady-state value of $\bar{\omega}_t$, denoted ϖ , and the steady-state standard deviation of the entrepreneur's technology shock, σ_0 . The average spread between the prime and commercial paper rates is used to define the average risk premium (rp) associated with loans to entrepreneurs as defined in Carlstrom and Fuerst (1997); this average spread is 1.87% (expressed as an annual yield). The steady-state bankruptcy rate (br) is given by $\Phi(\varpi, \sigma_0)$ and Carlstrom and Fuerst (1997) used the value of 3.9% (again, expressed as an annual rate). This yields two equations which determine (ϖ, σ_0) :¹⁷

$$\begin{aligned} \Phi(\varpi, \sigma_0) &= 3.90 \\ \frac{\varpi}{g(\varpi, \sigma_0)} - 1 &= 1.87 \end{aligned} \tag{76}$$

¹⁷ Note that the risk premium can be derived from the markup share of the realized output and the amount of payment on borrowing: $\bar{s}_t \bar{\omega}_t f p_t = (1 + rp)(f p_t - n w_t)$. And using the optimal factor payment (project investment), $f p_t$, in equation (40), we arrive at the risk premium in equation (76).

yielding $\varpi \approx 0.65$, $\sigma_0 \approx 0.23$.¹⁸

The entrepreneurial discount factor γ can be recovered by the condition that the steady-state internal rate of return to the entrepreneur is offset by their additional discount factor:

$$\gamma \left[\frac{\bar{s}f(\varpi, \sigma_0)}{1 - \bar{s}g(\varpi, \sigma_0)} \right] = 1$$

and using the mark-up equation for \bar{s} in eq. (39), the parameter γ then satisfies the relation

$$\gamma = \frac{g_U}{g_K} \left[1 + \frac{\phi(\varpi, \sigma_0)}{f'(\varpi, \sigma_0)} \right] \approx 0.832$$

where, g_U is the growth rate of marginal utility and g_K is the growth rate of consumption (identical to the growth rate of capital on a balanced growth path). The autoregressive parameter for the risk shocks, χ , is set to 0.90 so that the persistence is roughly the same as that of the productivity shocks.

The final two parameters are the adjustment cost parameters (κ_k, κ_h) . In their analysis of quarterly U.S. business cycle data, Christiano, Eichenbaum and Evans (2005) provide estimates of κ_k for different variants of their model which range over the interval (0.91, 3.24) (their model did not include housing and so there was no estimate for κ_h). Since our empirical analysis involves annual data, we choose a lower value for the adjustment cost parameter and, moreover, we impose the restriction that $\kappa_k = \kappa_h$. We assume that $\kappa_k = \kappa_h = 0.2$ implying that the (short-run) elasticity of investment and housing with respect to a change in the respective shadow prices is 5 (i.e. the inverse of the adjustment cost parameter). Given the estimates in Christiano, Eichenbaum, and Evans (2005), we think that these values are certainly not extreme. We also solve the model with no adjustment costs. As discussed below, the presence of adjustment costs improves the behavior of the model in several dimensions.

¹⁸ It is worth noting that, using financial data, Gilchrist et al. (2008) estimate σ_0 to be equal to 0.36 so our value is broadly in line with theirs.

Table 4: Business Cycle Properties (1975:1 - 2007:2)

Data: all series are Hodrick-Prescott filtered with the smoothing parameter set to 1600			
% S.D.			
GDP			1.2
Consumption			0.69
House Price Index (HPI)			1.9
Non - Residential Fixed Investment (NRFI)			4.5
Residential Fixed Investment (RFI)			8.7
Correlations			
GDP, Consumption			0.83
GDP, HPI			0.31
GDP, HPI (for pre 1990)			0.21
GDP, HPI (for post 1990)			0.51
NRFI, RFI			0.29
GDP, NRFI			0.81
GDP, RFI			0.30
GDP, Real Estate Loans (from 1985:1)			0.15
Real Estate Loans, HPI			0.47
Lead - Lag correlations	$i = -3$	$i = 0$	$i = 3$
$GDP_t, NRFI_{t-i}$	0.47	0.78	0.31
GDP_t, RFI_{t-i}	-0.27	0.20	0.32
$NRFI_{t-i}, RFI_t$	0.63	0.26	-0.27

Figure 1 and Table 4 show some of the cyclical and statistical features of the U.S. economy for the period from 1975 through the second quarter of 2007 using quarterly data.¹⁹ As mentioned in the Introduction, the stylized facts for housing are readily seen. i): Housing prices are much more volatile than output; ii) Residential investment is almost twice as volatile as non-residential investment; iii) GDP, consumption, the price of housing, non-residential - and residential investment all co-move positively; iv) and lastly, residential investment leads output by three quarters.

¹⁹ Note that quarterly data is used here only to present some of the broad cyclical features of the data. As mentioned in the text, we follow Davis and Heathcote (2005) and use annual data when calibrating the model. In comparing the model output to the data, we employ annual data for this exercise.

Table 5: Steady - State Values (Relative to GDP)

Variables	Lending channel model	Davis and Heathcote (D & H)	Data (1948 - 2007)
Capital Stock (K)	1.96	1.52	1.43
Residential structures stock ($P_d \times S$)	3.20	1.00	1.00
Private consumption ($PCE = c + r_h h$)	0.77	0.64	0.65
Nonresidential investment (i_c)	0.18	0.14	0.13
Residential investment (i_d)	0.05	0.04	0.05
Construction ($b = p_b x_b$)	0.05	0.05	0.05
Manufacturing ($m = p_m x_m$)	0.24	0.25	0.31
Services ($p_s x_s + r_h h$)	0.71	0.71	0.63

5 Results

5.1 Steady State Values, Second Moments and Lead - Lag Patterns

Table 5 shows some of the selected steady-state values (relative to steady-state GDP) from our model that includes the lending channel. These steady state values differ somewhat from those in Davis and Heathcote (2005) but the calibrated parameter values produce steady-state values that are broadly in line with the data.²⁰

Our main interest is in the business cycle, i.e. second moment, properties of the model, and the roles that risk shocks and adjustment costs play in affecting these properties. To that end, we first examine the importance of risk shocks in a model with no adjustment costs, then add adjustment costs to both stocks (capital and housing) individually and jointly, and conclude with some impulse response functions that help to illuminate the internal workings of the model

5.1.1 The Role of Risk Shocks

In order to examine the roles of risk shocks and the credit channel mechanism for house production, we compare simulated data generated under two scenarios. In the first case, only technology shocks to the intermediate sectors are present. This is identical to Davis and Heathcote's (2005) analysis

²⁰ As in Davis and Heathcote (2005), GDP is calculated as $gdp_t = y_{ct} + p_{dt}y_{dt} + r_{ht}h_t$ where r_{ht} is the rental rate of housing and is determined by the marginal rate of substitution between housing and consumption. Also note that private consumption (PCE) and services includes the rental value of housing.

Table 6: Standard Deviations relative to S.D.(GDP) - No adjustment costs

Variables (in relation to GDP)	Lending channel model		(D & H)	Data (\pm 2s.d.) (1948 - 2007)
Volatility of σ_ω	0.0	0.21		
Private consumption (PCE)	0.51	0.55	0.48	0.78 (\pm 0.15)
Labor (N)	0.39	0.51	0.41	1.01 (\pm 0.20)
Nonresidential investment (i_c)	3.05	5.19	3.21	2.51 (\pm 0.45)
Residential investment (i_d)	3.56	22.94	6.12	5.04 (\pm 0.98)
House price (p_h)	0.37	1.33	0.4	1.36 (\pm 0.31)
Construction output (x_b)	3.22	10.64	4.02	2.74 (\pm 0.53)
Manufacturing output (x_m)	1.51	1.47	1.58	1.85 (\pm 0.36)
Service output (x_s)	0.98	1.05	0.99	0.85 (\pm 0.16)
Construction labor (n_b)	1.55	10.27	2.15	2.37 (\pm 0.45)
Manufacturing (n_m)	0.38	0.39	0.39	1.53 (\pm 0.30)
Service (n_s)	0.37	0.61	0.37	0.66 (\pm 0.13)
Construction Investment (i_b)	1.93	10.36	25.9	9.69 (\pm 1.88)
Manufacturing Investment (i_m)	1.05	1.02	3.23	3.53 (\pm 0.69)
Service Investment (i_s)	1.03	1.10	3.43	2.91 (\pm 0.46)
Markup (\bar{s})	0.33	3.65		0.96 (\pm 0.22)
Risk premium (RP)	0.12	1.48		20.6 (\pm 4.92)

and so the role of the lending mechanism is highlighted. We then add risk shocks and set the volatility of the risk shocks so that the model matches the volatility of housing prices. The results from this exercise are presented in Tables (6) and (7); also included in the Tables are the corresponding values from Davis and Heathcote (2005) and the data. As a crude estimate of a 95% confidence interval, a two-standard deviation value is given and model values that fall within this range are reported in bold type.

As seen in Table (6), in the absence of risk shocks, the model output is quite similar to that in Davis and Heathcote (2005). The main difference is that the lending channel mutes the volatility of residential investment (i_d) and the volatility of the related intermediate sectors. In particular, construction investment (i_b) and manufacturing investment (i_m) are too low in the housing channel model. A critical deficiency is that the model can not replicate the volatility of housing prices (also true for the Davis and Heathcote (2005) framework). It is also worth noting that the two variables introduced by the housing channel, namely the markup (\bar{s}) and the risk premium (RP)

Table 7: Correlations

Variables	Lending channel model		D & H	Data (\pm 2s.d.)
σ_ω	0	0.21		
(GDP, PCE)	0.96	0.82	0.95	0.79 (+0.08,-0.12)
(GDP, p_h)	0.71	0.09	0.65	0.73 (+0.13,-0.22)
(i_c, PCE)	0.89	0.64	0.91	0.61 (+0.15,-0.20)
(i_d, PCE)	0.42	-0.19	0.26	0.66 (+0.13,-0.18)
(i_c, i_d)	0.26	-0.79	0.15	0.25 (+0.23,-0.27)
(i_d, p_h)	-0.68	-0.69	-0.2	0.34 (+0.22,-0.26)
(\bar{s}, p_h)	-0.21	0.83		0.55 (+0.21,-0.30)
(\bar{s}, GDP)	-0.20	-0.20		0.11 (+0.33,-0.36)
(RP, GDP)	0.20	-0.19		-0.65 (+0.29,-0.18)

exhibit much less volatility than seen in the data. When risk shocks are added, the volatility of housing prices is increased and matches (by construction) that seen in the data. But this increased volatility in housing prices produces counterfactual volatility in residential investment (i_d) and construction labor (n_b). On a more positive note, the volatility of construction investment (i_b) is now in line with the data. The risk shocks, as expected, result in a dramatic increase in the volatility of the markup and risk premium but the former variable is now too volatile while the latter remains too smooth relative to the data.

Turning to the contemporaneous correlations of some key variables as reported in Table (7), we again see that the housing model per se does not change too many of the features seen in the original Davis and Heathcote (2005) model. Note a key discrepancy between both models and the data is the correlation between residential investment and housing prices: in the data these variables co-move positively while, in the models, they are negatively correlated. Adding risk shocks does not improve matters along this dimension but also produces counterfactual negative correlations between residential investment and consumption, $Corr(i_d, PCE)$, and residential investment and housing prices, $Corr(i_d, p_h)$. These results are not surprising in that all of the shocks (technology shocks and risk shocks) are primarily supply shocks so that a (relatively) stable demand curve for housing is observed. Also, risk shocks are akin to a investment specific technology shock which typically moves investment and consumption in opposite directions (e.g.

Greenwood, Hercowitz, and Krusell (2000)). More favorably, the correlation between the markup variable and housing prices becomes positive in the presence of risk shocks and the risk premium is negatively correlated with GDP; both of these features are seen in the data. The markup and GDP are negatively correlated in the model and fall within the 95% confidence interval estimated from the data.

The conclusion from this exercise is that the inclusion of risk shocks in the credit channel/housing model provides an improvement over the basic Davis and Heathcote (2005) framework with respect to house price volatility but does so at the cost of greater volatility of several key real variables. These results suggest that adjustment costs might improve the model's characteristics.

5.1.2 Adding Adjustment Costs

Adjustment costs are added using the functional form given in eq. (17) and the model is solved under four different permutations of the adjustment costs parameter: $(\kappa_k, \kappa_h) = (0.0, 0.2)$. The second moments from the simulated data are presented in Tables (8) and (9). In all simulations, we adjust the volatility of risk shocks (σ_ω) so that the model matches the volatility of house prices as seen in the data.

Turning first to volatility (Table (8)), it is seen that when adjustment costs are added only to housing production, the model improves on several dimensions. First, the variance of the risk shocks is dramatically reduced; in terms of the coefficient of variation, this is reduced from roughly 100% to 30%. Gilchrist, et al. (2008) provide estimates of time-varying uncertainty using financial data and they report an average volatility of $\sigma_0 = 0.36$ and $\sigma_\omega = 0.14$ or a coefficient of variation of 38% so the reduction in risk shocks in the model is clearly not unreasonable.²¹ With adjustment costs in housing production, the volatility of real variables associated with the housing sector are reduced and now are in line with the data. Note that the volatilities of the markup

²¹ Gilchrist et al. use a two-state Markov process for $\sigma_{\omega t}$. Their estimates for this process are a low value of 0.25 and high value of 0.52 with a symmetric transition probability matrix with diagonal elements of 0.69. This corresponds roughly to the coefficient of variation reported in the text.

Table 8: Standard Deviations relative to GDP - The role of adjustment costs

Variables (in relation to GDP)	Lending channel model				(D & H)	Data (\pm 2s.d.) (1948 - 2007)
Adjustment cost κ_k	0.0	0.0	0.2	0.2		
Adjustment cost κ_h	0.0	0.2	0.0	0.2		
Volatility of σ_ω	0.21	0.07	0.16	0.06		
Private consumption (PCE)	0.55	0.52	0.68	0.64	0.48	0.78 (\pm 0.15)
Labor (N)	0.51	0.39	0.66	0.33	0.41	1.01 (\pm 0.20)
Nonresidential investment (i_c)	5.28	3.16	2.63	2.52	3.21	2.51 (\pm 0.45)
Residential investment (i_d)	23.5	4.82	16.2	4.67	6.12	5.04 (\pm 0.98)
House price (p_h)	1.36	1.36	1.36	1.36	0.4	1.36 (\pm 0.31)
Construction output (x_b)	10.9	3.27	7.97	3.41	4.02	2.74 (\pm 0.53)
Manufacturing output (x_m)	1.47	1.52	1.47	1.55	1.58	1.85 (\pm 0.36)
Service output (x_s)	1.05	0.99	0.90	0.97	0.99	0.85 (\pm 0.16)
Construction labor (n_b)	10.5	2.15	7.49	2.09	2.15	2.37 (\pm 0.45)
Manufacturing (n_m)	0.39	0.38	0.53	0.31	0.39	1.53 (\pm 0.30)
Service (n_s)	0.61	0.38	0.30	0.28	0.37	0.66 (\pm 0.13)
Construction Investment (i_b)	10.6	2.39	7.51	2.37	25.9	9.69 (\pm 1.88)
Manufacturing Investment (i_m)	1.02	1.05	1.02	1.04	3.23	3.53 (\pm 0.69)
Service Investment (i_s)	1.1	1.05	0.94	1.02	3.43	2.91 (\pm 0.46)
Markup (\bar{s})	3.73	1.76	3.1	1.7		0.96 (\pm 0.22)
Risk premium (RP)	1.51	0.69	1.25	0.67		20.6 (\pm 4.92)

parameter and risk premium are also reduced but neither matches what is seen in the data. When adjustment costs are only in capital accumulation (column 3), the model improves in related sectors (consumption and non-residential investment), but the housing sector and associated inputs are too volatile. Also the volatility of risk shocks necessary to produce the observed house price volatility is twice as great relative to the previous case. The model performs quite well when both capital and housing adjustment costs are present.

The contemporaneous correlations for several variables are reported in Table (9). Capital adjustment costs only (column 3) do not improve the model's behavior (and produce a counterfactually negative correlation of housing prices and GDP) while housing adjustment costs produce a positive correlation between residential investment and private consumption and non-residential and residential investment; albeit both are lower than observed in the data. When both adjustment costs are present, the model produces qualitatively many of the features seen in the data but

Table 9: Correlations

Variables	Lending channel model				D & H	Data (\pm 2s.d.)
	κ_k	κ_h				
Adjustment cost κ_k	0.0	0.0	0.2	0.2		
Adjustment cost κ_h	0.0	0.2	0.0	0.2		
σ_ω	0.21	0.07	0.16	0.06		
(GDP, PCE)	0.81	0.96	0.55	0.95	0.95	0.79 (+0.08,-0.12)
(GDP, p_h)	0.08	0.10	-0.03	0.16	0.65	0.73 (+0.13,-0.22)
(i_c, PCE)	0.64	0.89	0.78	0.91	0.91	0.61 (+0.15,-0.20)
(i_d, PCE)	-0.19	0.19	-0.43	0.20	0.26	0.66 (+0.13,-0.18)
(i_c, i_d)	-0.80	0.0007	-0.25	0.19	0.15	0.25 (+0.23,-0.27)
(i_d, p_h)	-0.68	-0.69	-0.86	-0.63	-0.2	0.34 (+0.22,-0.26)
(\bar{s}, p_h)	0.83	0.90	0.93	0.88		0.55 (+0.21,-0.30)
(\bar{s}, GDP)	-0.20	-0.03	-0.3	-0.02		0.11 (+0.33,-0.36)
(RP, GDP)	-0.20	-0.03	-0.30	-0.02		-0.65 (+0.29,-0.18)

several important inconsistencies are present. The model continues to produce a negative correlation between housing prices and residential investment (this feature is present for all variations of the model). Again, given that explicit demand shocks are not present in the model, this result is not unexpected. The cyclical properties of the two lending channel variables are mixed: while both are weakly countercyclical this is broadly in line with the observed behavior of the markup but does not match the strong countercyclical behavior of the risk premium.

The last set of housing stylized facts that is in question is the lead - lag patterns of residential and non-residential investments. Table 10 shows the results. As in Davis and Heathcote (2005), we also fail to reproduce this feature of the data. Consequently, the propagation mechanism of agency costs model does amplify prices and other real variables, but does not contribute in explaining the lead-lag features.

5.2 Dynamics: Impulse Response Functions

While the results discussed above provide some support for the housing cum credit channel model, the role of the lending channel is not easily seen because of the presence of the other impulse shocks (i.e., the sectoral productivity shocks). To analyze how the lending channel influences the effects of a risk shock, we initially analyze the model's impulse response functions to risk shocks under

Table 10: Lead - Lag Patterns: Different Levels of Adjustment Costs

Variables	Lending channel model				D & H	Data (± 2 s.d.)
Adjustment cost κ_k	0.0	0.0	0.2	0.2		
Adjustment cost κ_h	0.0	0.2	0.0	0.2		
σ_ω	0.21	0.07	0.16	0.06		
$(i_c [-1], GDP [0])$	0.37	0.54	0.49	0.54	0.45	0.25 (+0.24,-0.27)
$(i_c [0], GDP [0])$	0.29	0.93	0.68	0.95	0.94	0.75 (+0.10,-0.15)
$(i_c [1], GDP [0])$	0.14	0.32	0.35	0.60	0.33	0.47 (+0.18,-0.24)
$(i_d [-1], GDP [0])$	-0.07	0.10	-0.02	0.17	0.19	0.52 (+0.17,-0.23)
$(i_d [0], GDP [0])$	0.32	0.32	0.48	0.42	0.44	0.47 (+0.19,-0.24)
$(i_d [1], GDP [0])$	0.10	0.31	0.14	0.29	0.14	-0.22 (+0.27,-0.24)
$(i_c [-1], i_d [0])$	0.07	0.18	0.07	0.19	0.07	-0.37 (+0.26,-0.21)
$(i_c [0], i_d [0])$	-0.80	0.0007	-0.25	0.19	0.15	0.25 (+0.24,-0.27)
$(i_c [1], i_d [0])$	0.06	-0.09	-0.37	0.008	0.08	0.53 (+0.17,-0.23)

two different scenarios. In the first scenario, we set the monitoring cost parameter to zero ($\mu = 0$) while in the second scenario we use the value employed in the stochastic simulations ($\mu = 0.25$). With no monitoring costs, risk shocks should not influence the behavior of housing prices and residential investment (see eqs. (39), (56), and (57)). We also examine the economy's response to an innovation to productivity in the construction sector (this being the most important input into the residential investment good). The impulse response functions (to a 1% innovation in both shocks) for a selected set of key variables are presented in Figures 7-9. For all cases, both adjustment cost parameters are set equal to 0.2.

We first turn to the behavior of three key macroeconomic variables, namely GDP, household consumption (denoted PCE), and residential investment (denoted RESI) seen in Figure 7. The response to a technology shock to the construction sector has the predicted effect that GDP increases. Consumption also increases, but only slightly, while investment responds dramatically (recall, that this is in the presence of adjustment costs). This consumption/savings decision reflects agents response to the expected high productivity (due to the persistence of the shock) in the construction sector. Also note that monitoring costs (i.e. whether $\mu = 0$ or $\mu = 0.25$) play a rather insignificant role in the dynamic effects of a technology shock. And, as the model implies, when $\mu = 0$, risk shocks have no effect on the economy. (For this reason, in Figures 8 and 9,

we report only the responses for $\mu = 0.25$.) When $\mu = 0.25$, a risk shock which affects housing production results in a modest fall in GDP but a relatively dramatic fall in residential investment. Recall, as discussed in the partial equilibrium analysis of the credit channel model, an increase in productivity risk results in a leftward shift in the supply of housing; since residential investment is the primary input into housing, it too falls in response to the increased risk. Consumption responds positively which is consistent with models that have an investment specific technology shock (e.g. Greenwood, Hercowitz, and Krusell (2000)).

Figure 8 reports the impulse response functions of the housing markup, the risk premium on loans to the housing producers and the bankruptcy rate. A positive technology shock to the construction sector increases the demand for housing and, *ceteris paribus*, will result in an increase in the price of housing. This will result in greater lending to the housing producers which will result in a greater bankruptcy rate and risk premium; both of these effects imply that the housing markup will increase. Note the counterfactual implication that both the bankruptcy rate and the risk premium on loans will be procyclical; this was also the case in the original Carlstrom and Fuerst (1997) model and for exactly the same reason. In contrast, a risk shock produces countercyclical behavior in these three variables. Hence, this argues for inclusion of risk shocks as an important impulse mechanism in the economy.

Finally, we report in Figure 9, the impulse response functions of the prices of land and housing to the two shocks. A technology shock to the construction sector results in lower cost of housing inputs due to the increased output in residential investment so that the price of housing falls. However, the price of land, i.e. the fixed factor, increases. For an uncertainty shock, the resulting fall in the supply of housing causes the demand for the fixed factor (land) to fall and the price of the final good (housing) to increase.

In ending this section, a word of caution is needed in interpreting the quantitative magnitudes seen in the impulse response functions. In particular, note that the response of housing prices to a productivity increase in the construction sector is greater than the response due to a risk shock.

One might deduce that the housing sector and risks shocks play a minor role in the movement of housing prices. As the results from the full model (i.e. when the all technology and risk shocks are present) imply, such a conclusion would be incorrect (see Table 8).

5.3 Some Final Remarks

Our primary findings fall into two broad categories. First, risk shocks to the housing producing sector imply a quantitatively large role for uncertainty over the housing and business cycles. Second, our model can account for many of the salient features of housing stylized facts, in particular, housing prices are more volatile than output. Critically, however, adjustment costs, especially in housing accumulation, are necessary to mute the volatility in the real side of the economy produced by risk shocks. The lead - lag pattern of residential and non-residential, however, is still not reconciled within our framework.²²

For future research, modelling uncertainty due to time variation in the types of entrepreneurs would be fruitful. One possibility would be an economy with a low risk agent whose productivity shocks exhibit low variance and a high risk agent with a high variance of productivity shocks. Because of restrictions on the types of financial contracts that can be offered, the equilibrium is a pooling equilibrium so that the same type of financial contract is offered to both types of agents. Hence the aggregate distribution for technology shocks hitting the entrepreneurial sector is a mixture of the underlying distributions for each type of agent. Our conjecture is that this form of uncertainty has important quantitative predictions and, hence, could be an important impulse mechanism in the credit channel literature that, heretofore, has been overlooked. It also anecdotally corresponds with explanations for the cause of the current credit crisis: a substantial fraction of mortgage borrowers had higher risk characteristics than originally thought.

However, the most glaring omission in our analysis is that of demand disturbances. Incorporating shifts in housing demand that are fundamental based (i.e. without appealing to asset bubble

²² Recently, Jonas Fisher (2007) presents a model with household production which does produce the lead-lag pattern pattern observed in residential and non-residential investment.

explanations) which can produce large swings in housing prices remains a substantial modeling hurdle.

References

- Aoki, Kosuke, J. Proudman, and J. Vlieghe, (2004), "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach." *Journal of Financial Intermediation*, 13(4), pp. 414–35.
- Bloom, N., M. Floetotto and N. Jaimovich, (2008), "Really Uncertain Business Cycles," Stanford University Working Paper.
- Carlstrom, C.T. and T.S. Fuerst, (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 87, pp. 893 - 910.
- Carlstrom, C.T. and T.S. Fuerst, (1998), "Agency Costs and Business Cycles," *Economic Theory* (12), pp. 583-597.
- Chari V., Kehoe P. and Ellen R. McGrattan, (2007), "Business Cycles Accounting", *Econometrica* (75), pp. 781-836.
- Christiano, L., R. Motto, and M. Rostagno, (2003), "The Great Depression and the Friedman-Schwartz Hypothesis," *Journal of Money, Credit, and Banking*, 35, 1119-1198
- Christiano, L., R. Motto, and M. Rostagno, (2008), "Financial Factors in Business Cycles", working paper.
- Davis, Morris and J. Heathcote, (2005), "Housing and the Business Cycle", *International Economic Review*, Vol. 46, No. 3, pp. 751 - 784.
- Dorofeenko, V., G. Lee and K. Salyer, (2008), "Time Varying Uncertainty and the Credit Channel", *Bulletin of Economic Research*, 60:4, pp. 375-403
- Fernández-Villaverde, J., P. A. Guerrón-Quintana, J. Rubio-Ramirez, and M Uribe, (2009), "Risk Matters: The Real Effects of Volatility Shocks," NBER Working Paper 14875.
- Jonas D. M. Fisher, (2007), "Why Does Household Investment Lead Business Investment over the Business Cycle?," *Journal of Political Economy*, 115, pp. 141-168.
- Kahn, J.A., (2008), "What Drives Housing Prices," New York Federal Reserve Staff Report no. 345.
- Gilchrist, S., J. Sim, and E. Zakrajsek, (2008), "Uncertainty, Credit Spreads, and Investment Dynamics," FRB Working Paper.
- Glaeser, E. L. and J. Gyourko, (2006), "Housing Dynamics", NBER Working Paper No.12787.
- Greenwood, J., Hercowitz, Z. and Krusell, P. (2000). "The Role of Investment Specific Technological Change in the Business Cycle", *European Economic Review*, 44, pp. 91–115.
- Iacoviello, Matteo, "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle" (2005), *American Economic Review*, Vol. 95, No. 3 (June), pp. 739-764.
- Iacoviello, Matteo and Raoul Minetti, (2008), "The Credit Channel of Monetary Policy: Evidence from the Housing Market" *Journal of Macroeconomics*, 30, pp. 69-96.
- Iacoviello, Matteo and S.Neri, (2008), "Housing Market Spillovers: Evidence from an Estimated DSGE model", Boston College, Department of Economics, mimeo.

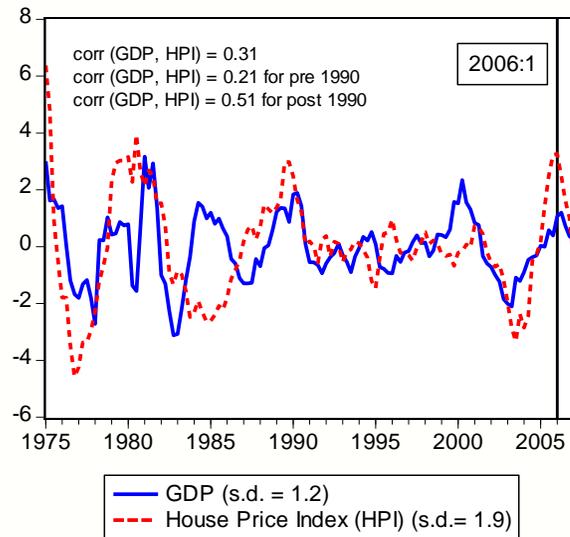
6 Data Appendix

- Loans: Federal Reserve Board, Statistics: Releases and Historical Data Assets and Liabilities of Commercial Banks in the U.S. - h.8. Seasonally adjusted, adjusted for mergers, billions of dollars. <http://www.federalreserve.gov/releases/h8/data.htm>
 - Total Loans: Total loans and leases at commercial banks.
 - Residential Real Estate Loans: Loans to residential sector excluding revolving home equity loans.
 - Commercial Real Estate Loans: Loans to commercial sector excluding revolving home equity loans.
 - Commercial and Industrial Loans (Business Loans): Commercial and industrial loans at all Commercial Banks.
 - Consumer Loans: Consumer (Individual) loans at all commercial banks.
- Gross Domestic Product (GDP), Personal Consumption Expenditures (PCE), Aggregate of gross private domestic investment (Non-RESI), Residential gross private domestic investment (RESI), and the Price Indexes for private residential Investment (PRESI) are all from the National Income and Product Accounts Tables (NIPA) at the Bureau of Economic Analysis.
 - To calculate the implied markup, \bar{s} , we used the house price index (HPI), residential investment (RESI) and the price for the RESI (PRESI).
 - We use the equation $p_{ht} = (1 - \zeta) y_{dt}^{-\zeta} p_{dt} \bar{s}_t$.
- House Price Index. (HPI): Constructed based on conventional conforming mortgage transactions obtained from the Federal Home Loan Mortgage Corporation (Freddie Mac) and the Federal National Mortgage Association (Fannie Mae). Source: The Office of Federal Housing Enterprise Oversight (OFHEO).

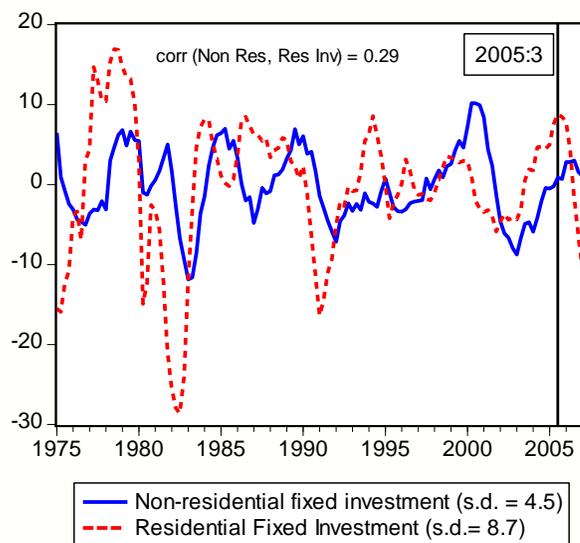
- Risk Premium: To calculate our risk premium, we used the spread between the prime rate and the three month commercial paper. These data can be obtained from Federal Reserve Bank of St. Louis, FRED Dataset under the category of Interest Rates.
 - <http://research.stlouisfed.org/fred2/categories/22>. These two series are monthly.
 - Commercial rate: CP3M, 3-Month Commercial Paper Rate :1971-04 till 1997-08.
 - Prime rate: MPRIME, Bank Prime Loan Rate: 1949-01 till 2009-08.

Figure 1: U.S. GDP, House Price, Non – and Residential Fixed Investments (1975:1 – 2007:2)

**Percent Deviation from Trend (using HP filter)
for U.S. Output and House Prices Quarterly (1975:1 - 2007:2)**



**Percent Deviation from Trend (using HP filter)
for U.S. Fixed Private Non- and Residential Investment Quarterly
(1975:1 - 2007:2)**



**Percent Deviation from Trend (using HP filter)
for U.S. GDP, Fixed Private Non- and Residential Investment Quarterly
(1975:1 - 2007:2)**

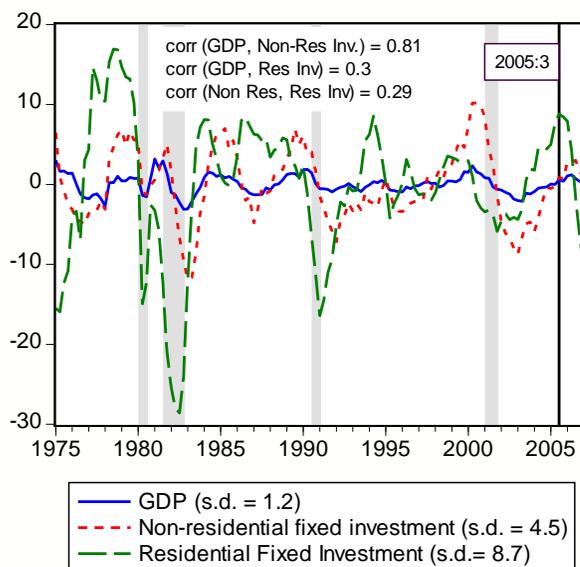


Figure 2: Different Loans at All U.S. Commercial Banks (1990:1 to 2007:7)

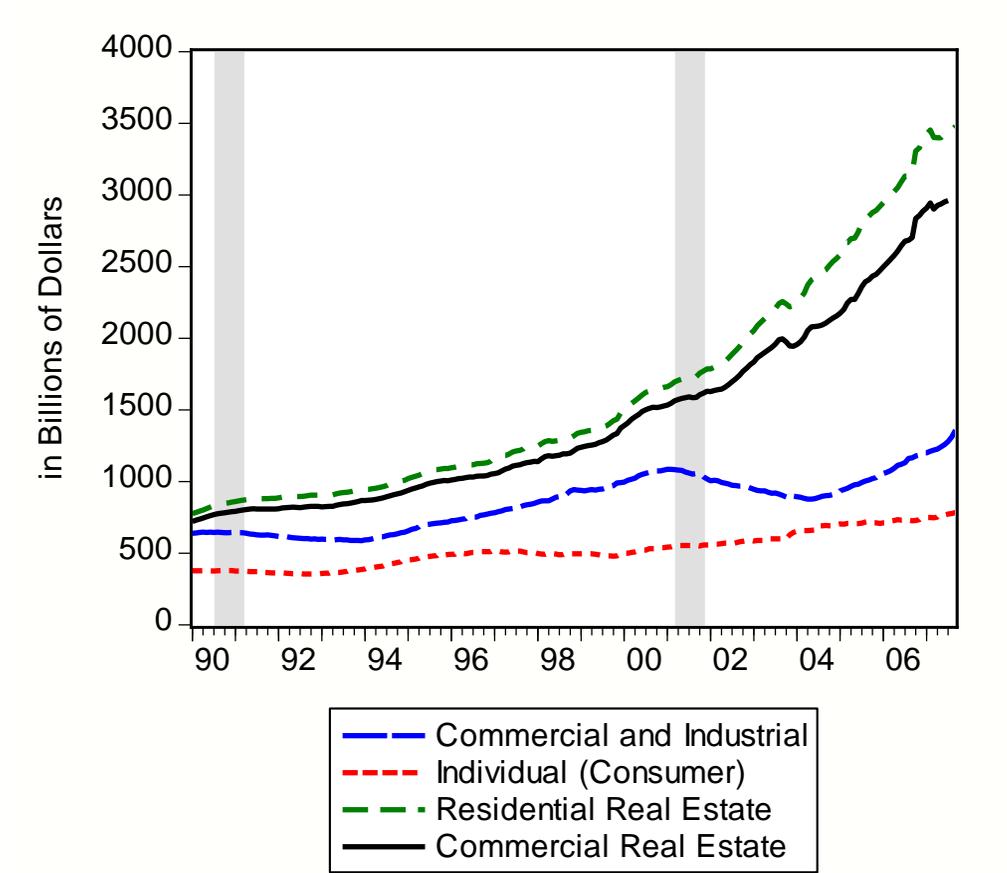
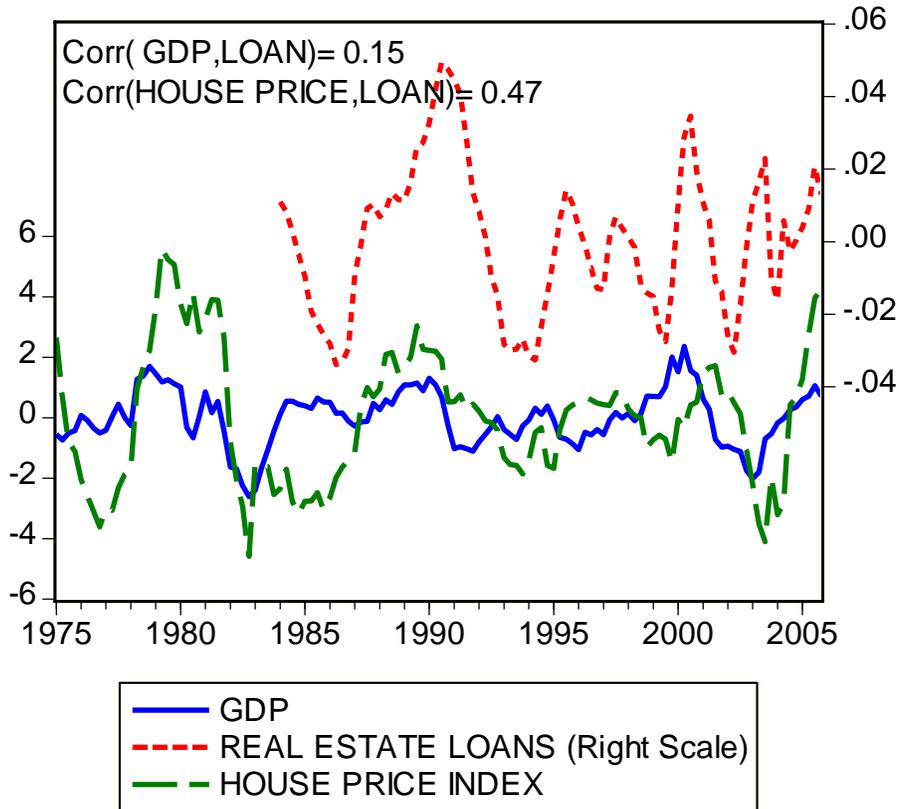


Figure 3: U.S. GDP, House Price and Residential Real Estate Loans

**Percent Deviation from trend (using HP filter):
U.S. Output Real Estate Loans and Price**



U.S. House Price and Real Estate Loans

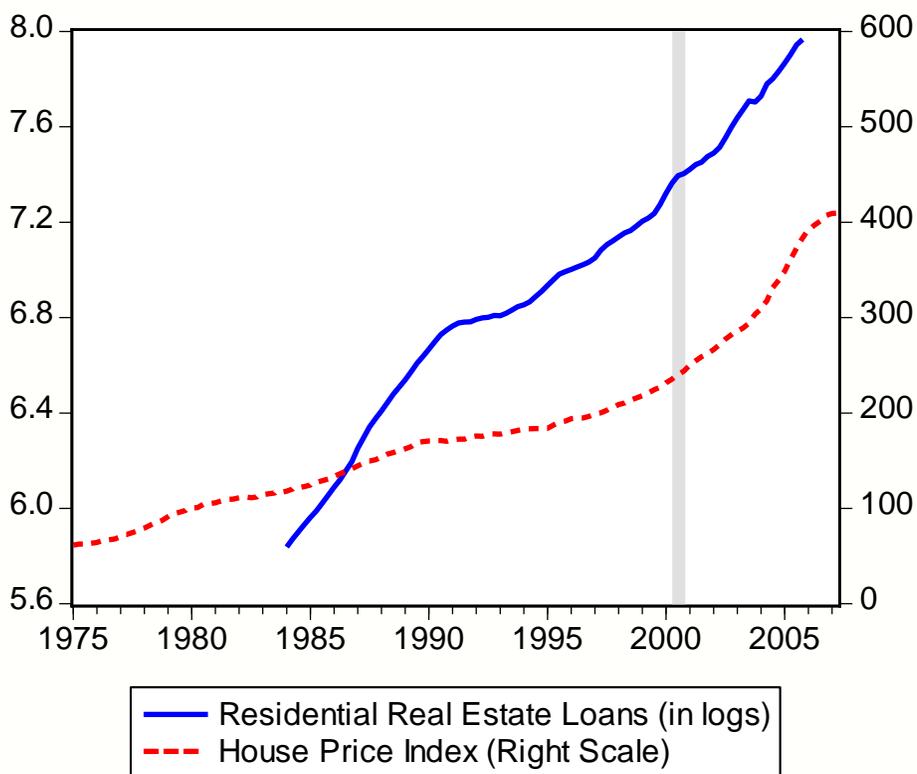
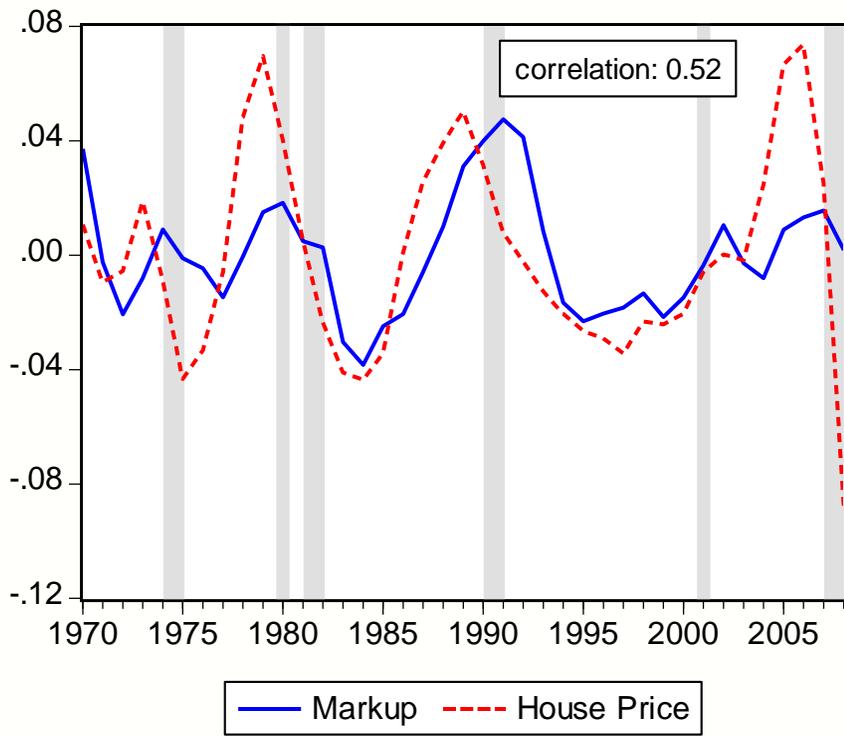


Figure 4: House Price, Housing Markups, and Risk Premium

House Price vs Markup: Annual 1970 - 2008 (HP filtered)



GDP vs Risk Premium: Annual 1970 - 2008 (HP filtered)

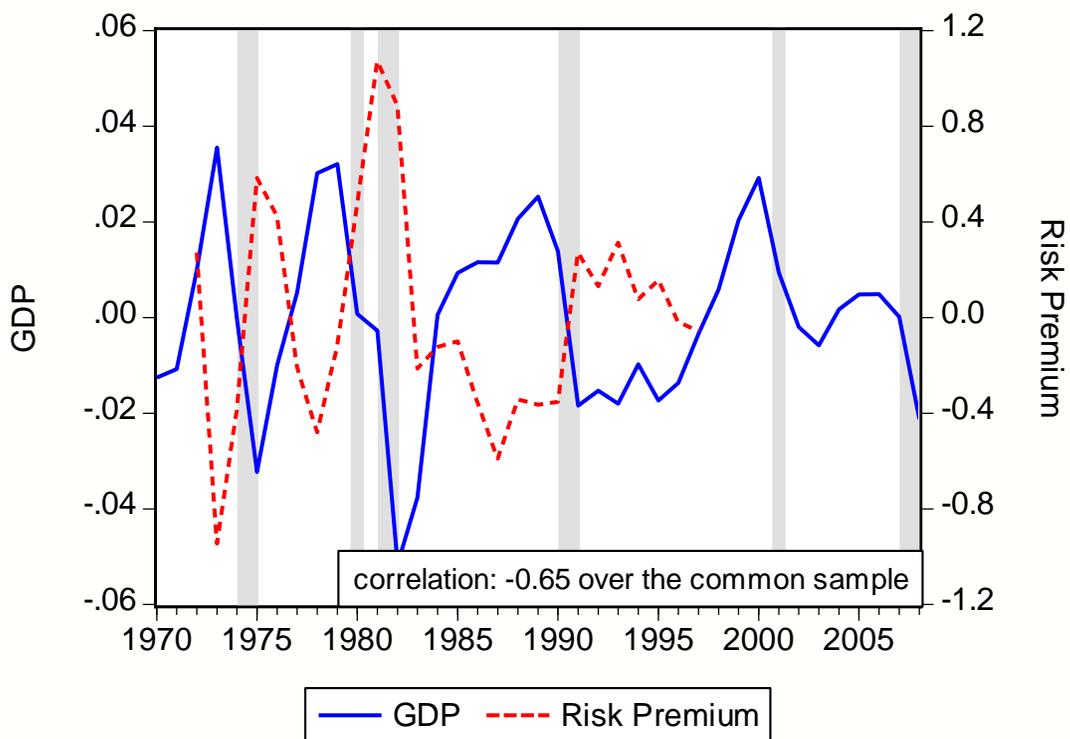


Figure 5: Flow of Funds in Credit Channel with Housing Model
 (Households land income, as well as Entrepreneur labor input and income are not shown)

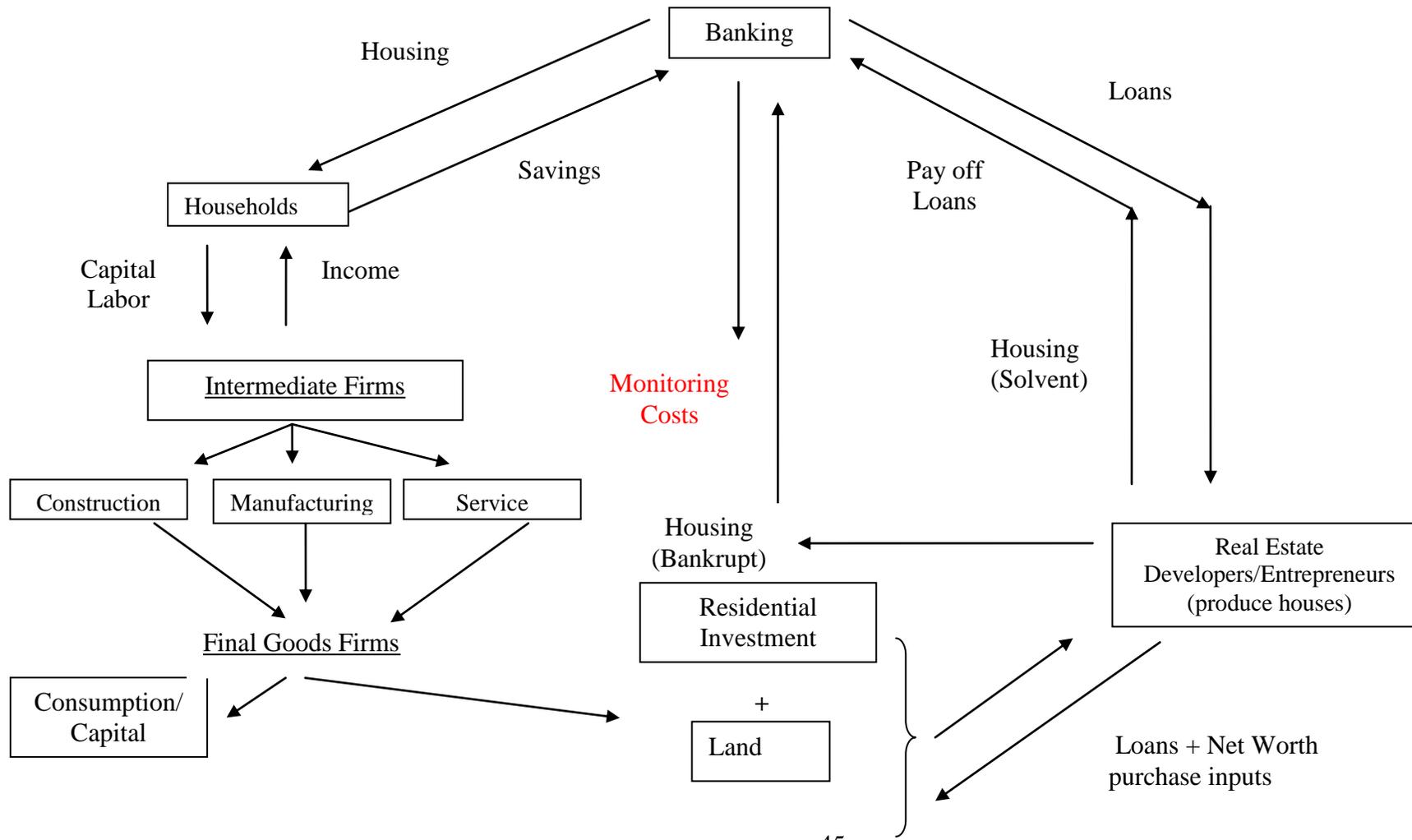


Figure 6: Technology and Uncertainty Shocks:
Effects on Housing Demand and Supply

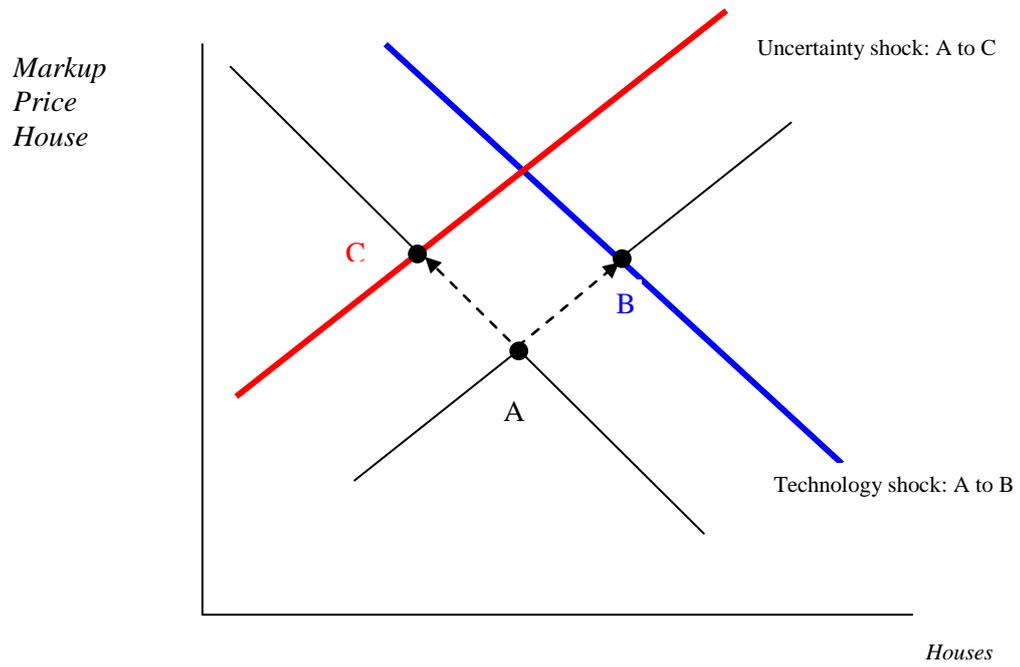


Figure 7: Response of Output, Private Consumption Expenditure, and Investment to 1% increase in Sector (Construction) Technology shocks and Uncertainty Shocks
(percentage deviations from steady-state values)

<u>Technology Shock to Construction sector</u>	<u>Uncertainty Shock to Housing Developer</u>
$\mu = 0.25$ (monitoring cost: effects of agency cost) $\mu = 0$ (no monitoring cost: multi-sector model with housing and no agency cost) $\kappa_k = \kappa_h = 0.2$ (adjustment cost)	

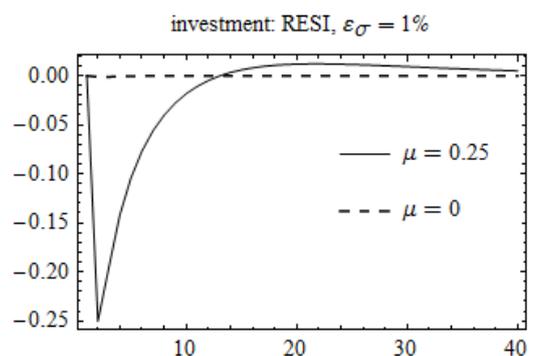
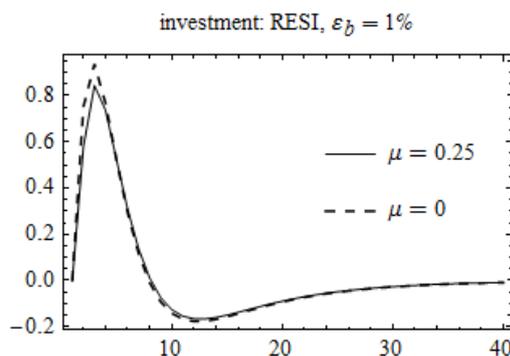
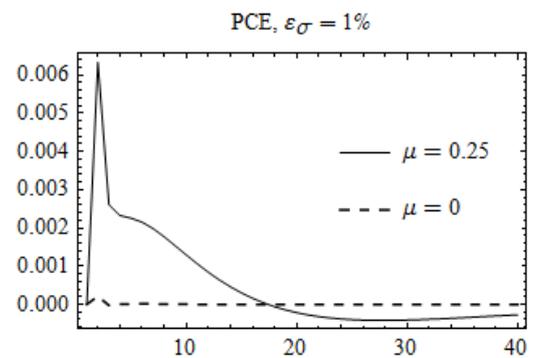
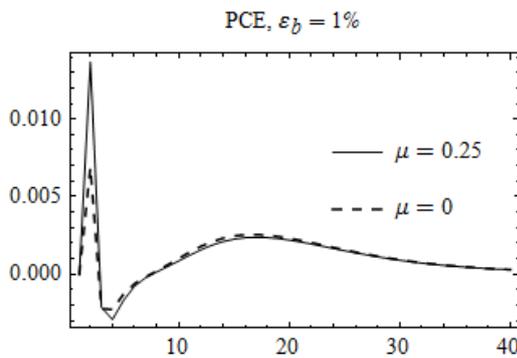
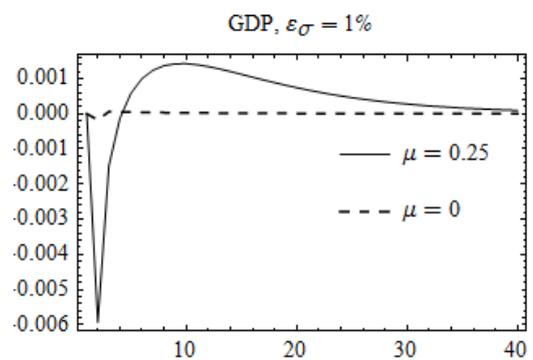
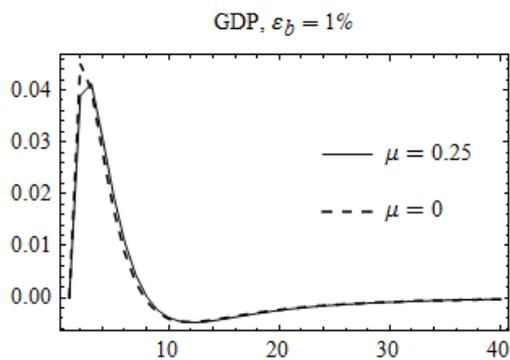


Figure 8: Response of Markup House Price, Risk Premia, and Bankruptcy Rate to a 1% increase in Sector (Construction) Technology shocks and Uncertainty Shocks (percentage deviations from steady-state values)

<u>Technology Shock to Construction sector</u>	<u>Uncertainty Shock to Housing Developer</u>
$\mu = 0.25$ (monitoring cost: effects of agency cost) $\kappa_k = \kappa_h = 0.2$ (adjustment cost)	

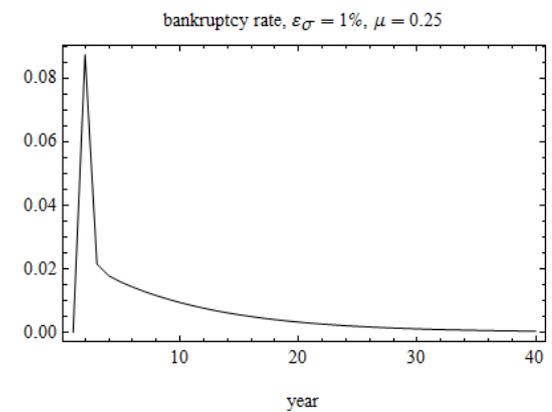
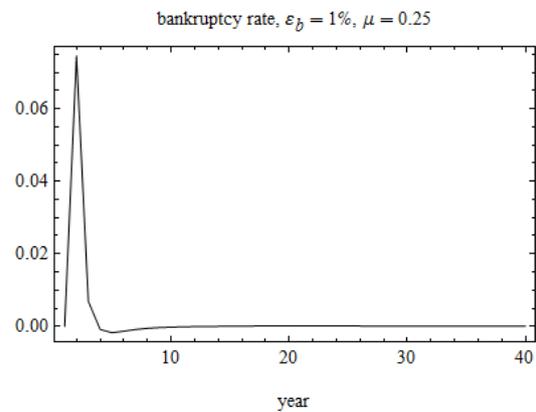
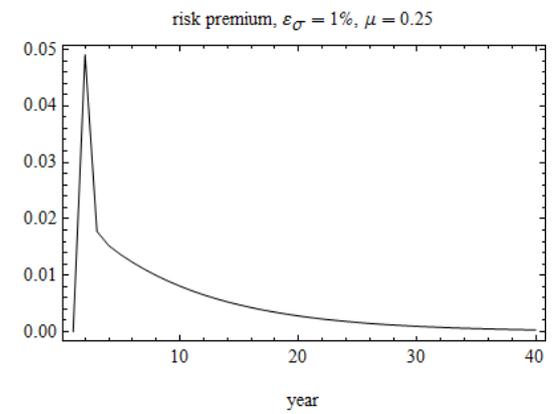
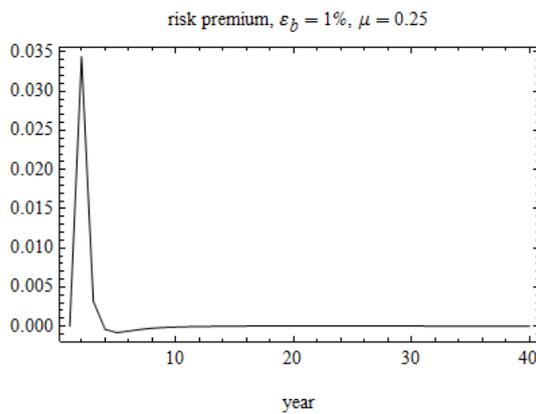
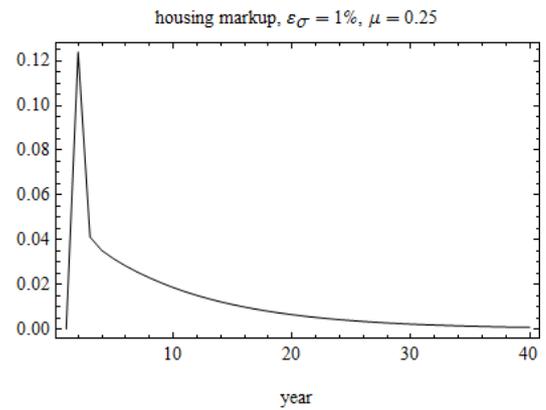
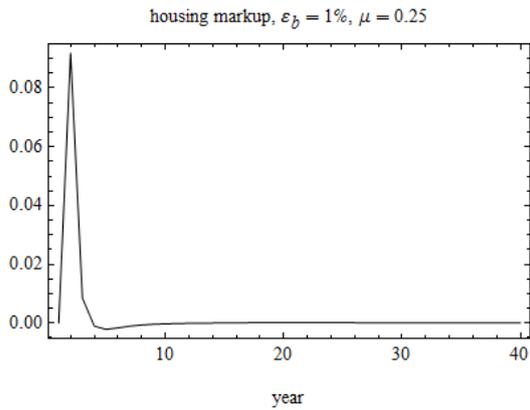


Figure 9: Response of Land Price and House Price to a 1% increase in Sector
 (Construction) Technology Shocks and Uncertainty Shocks
 (percentage deviations from steady-state values)

Technology Shock to Construction sector	Uncertainty Shock to Housing Developer
$\mu = 0.25$ (monitoring cost: effects of agency cost) $\kappa_k = \kappa_h = 0.2$ (adjustment cost)	

