Delegation, Risk, and Project Scope

by

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This paper studies a partial-contracting model where an agent may provide effort to increase a project’s scope before some later (operative) decisions have to be taken. Consistent with existing empirical evidence, we find a positive relationship between exogenous risk and delegation. That is, only if the exogenous risk is sufficiently large may the risk-neutral principal prefer to delegate authority over decisions to the risk-averse agent. Intuitively, for incentive reasons, the principal may optimally want to allow the agent to reduce his risk exposure. Nevertheless, even endogenous risk may be higher when the risk-averse agent has control.

1 Introduction

Motivation. Authority over decision-making is frequently delegated to lower levels of a hierarchy. For example, RAJAN AND WULF [2006] document that over the last decades entire layers of hierarchies in firms have been eliminated (i.e., more and more managers report directly to the CEO), and more managers are being appointed officers of the firm. This trend towards more delegation of authority raises the question under which circumstances it is optimal for a principal to grant lower tiers the right to take relevant decisions.1

Interestingly, there is evidence for a positive relationship between exogenous risk and delegation, i.e., there seems to be more delegation in settings that are more risky. At first sight, this observation might be surprising, because one could argue that risk-averse managers might try to reduce total firm risk at the expense of the expected return, which would not be in the best interest of (well-diversified)

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1 At the same time, the above evidence indicates that CEOs are getting directly connected deeper down in hierarchies.
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This line of reasoning would imply a negative relationship between risk and delegation, contrary to what has been found in the empirical literature. In particular, in a recent study on the retail banking sector NAGAR [2002] finds that high-growth, volatile, and innovative retail banks delegate more authority to branch managers. In the franchising context, LAFONTAINE [1992] considers the decision of potential franchisors either to operate a given store directly (i.e., to keep it company-owned) or to franchise it (where a franchisee has considerably greater autonomy in decision-making). Considering a variety of industries (such as fast-food restaurants, business aids and services, construction and maintenance, and nonfood retailing), LAFONTAINE [1992] finds that the higher is the exogenous risk (measured by the average proportion of discontinued outlets), the more likely is a given store to be franchised (for a survey of related results, see LAFONTAINE AND BHATTACHARYYA [1995]). Finally, ACEMOGLU et al. [2007] study three large data sets of French and British manufacturing firms and find that firms that are closer to the technological frontier, firms in more heterogeneous environments, and younger firms are more likely to decentralize decision-making.

In the present paper, we identify a novel channel through which higher exogenous risk might make delegation more desirable. As discussed above, delegation of authority may allow agents to reduce their risk exposure, and we show that this may in fact be beneficial from a principal’s perspective if large risk would stifle an agent’s initiative (i.e., delegation may be desirable for incentive reasons).

An Illustrative Example: The Hudson’s Bay Company Case. To illustrate our main idea, we briefly discuss the Hudson’s Bay Company case – a historic example that has frequently been employed to highlight the relevance of organizational design for firm performance (see, e.g., MILGROM AND ROBERTS [1992], ROBERTS [2004], and the references cited therein). In 1670, Hudson’s Bay Company (henceforth, HBC) was granted a royal monopoly by King Charles II of England for trade with all lands draining into the Hudson Bay. Having approximately fifteen times the size of the UK, at that time Hudson Bay was a trackless wilderness sparsely populated by some aboriginal people, but rich in animal fur, which was in high demand in Europe. Fur trade was HBC’s main business. HBC had set up half a dozen forts on the shores of Hudson Bay, waiting for customers seeking European-made goods in exchange for their furs. In the course, it amassed huge profits. Then, in 1779, the North West Company of Montreal (henceforth, NWC) entered the market. Initially, this did not seem to be a threat, because NWC faced a huge cost disadvantage: due to HBC’s royal monopoly, NWC could not ship goods through Hudson Bay. Instead, it was forced to first transport furs and other goods over land to Montreal, resulting in costs.

2 See AMIHUD AND LEV [1981] and MAY [1995] for discussions of this issue and for empirical evidence in the context of conglomerate mergers. In the context of firms’ financing decisions, LEWELLEN [2006] documents that managers’ behavior is influenced by their desire to reduce the riskiness of their personal income streams.

3 For a discussion of various empirical measures of risk, see LAFONTAINE [1992, pp. 271ff.].
twice as high as HBC’s. Nevertheless, by 1809 NWC had an 80% market share and was immensely profitable, while HBC was near bankruptcy.

How did this come about? In this connection, it is important to note that, while being in the same business, HBC and NWC had markedly different organizational designs. In the case of HBC, decision-making (for example, on prices and on how business was to be conducted) was centralized in headquarters in London. Moreover, given the geography and climate of Hudson Bay, there was only very limited possibility for communication between local employees and London: ships were able to bring in goods (and new instructions) from Europe only once a year. This lack of flexibility to conduct business as they saw fit stifled the local employees’ initiative to trade with people far from the bay: given the wilderness and the uncertainties of demand and supply, such trade involved huge risks. In contrast to HBC’s approach, NWC had erected dozens of trading posts inland right where the furs where collected. In addition, decision-making was delegated to local “Nor’ westers,” thereby giving them the opportunity to better adjust to the perceived risks and giving them an incentive to actually go to the remote areas. Initially, HBC was slow to react to NWC’s challenge, but eventually it simply copied NWC’s organizational design and – given its cost advantage – by 1820 had absorbed NWC through a merger.

As the Hudson’s Bay Company case illustrates, agents may be reluctant to increase the scope of operations if they anticipate that through later decisions a principal exposes them to a lot of risk. Frequently, it will be difficult for a principal to commit to a certain (less risky) course of action beforehand. Hence, delegating authority to the agent (thereby allowing him to proceed as he sees fit) may provide the principal with a credible way of reducing the agent’s risk exposure (and, as a consequence, may raise the agent’s initiative). Such considerations will be the more important, the higher is exogenous risk in the first place, and hence through this channel higher exogenous risk may make delegation more desirable from a principal’s perspective.

Model and Results. To formalize our idea, we consider a partial-contracting model where a risk-neutral principal (she) hires a risk-averse agent (him) to conduct a project. Only the agent is able to provide some noncontractible effort that expands the scope of the project. Subsequent decisions are taken under uncertainty and may be made by either the principal or the agent (in case the principal decides to delegate authority). In line with the emerging literature on partial contracting, we assume that only control over the decisions (not the decisions themselves) are contractible. Finally, uncertainty regarding the state of the world is resolved, and the payoffs of the parties are realized. In order to focus on the effect of risk on the desirability of delegation, we assume that the principal and the agent only differ in their risk attitudes, and we abstract from other potential conflicts of interest between the parties.

4 This form of governance was meant to counter the perceived danger of its (far away) employees frittering away or misappropriating profits.

5 Clearly, NWC’s organizational design was most likely also superior in making use of local knowledge.
We obtain the following results. First, in line with the empirical evidence, we find that, for sufficiently low levels of exogenous risk, the risk-neutral principal prefers to retain control over decisions. However, if there is a sufficiently pronounced risk-return trade-off, then, for sufficiently large exogenous risk, it is strictly optimal for the principal to delegate authority to the risk-averse agent. That is, in the model there is a positive relationship between exogenous risk and delegation. Second, this result provides an incentive-based reason why even a risk-neutral principal may find it optimal to allow an agent to reduce his risk exposure. Third, in the model the equilibrium project risk (measured by the variance of the project return) is endogenous and depends on the effort and decisions taken. Perhaps surprisingly, it is possible that even endogenous risk is higher when authority over decisions is delegated to the risk-averse agent (i.e., there might even be a positive relationship between endogenous risk and delegation).

The remainder of the paper is structured as follows. In section 2, we discuss the related literature. The model is introduced in section 3, and section 4 contains the results. Section 5 concludes. All proofs are relegated to the Appendix.

2 Related Literature

The present paper contributes to three strands of the literature. First, in its underlying idea, the paper is related to the literature that explains delegation through its function as a commitment device. That is, through delegation of authority to an agent (who behaves differently), a principal might be able to reduce time-inconsistency problems (where the principal prefers some behavior ex ante, to which, however, she cannot commit later on). On the one hand, such delegation might be advantageous in strategic interactions with third parties (see, e.g., ROGOFF [1985], VICKERS [1985], SAPPINGTON [1986], FERSHTMAN AND JUDD [1987], MELUMAD AND MOOKHERJEE [1989]). On the other hand, as in the present paper, delegation might be optimal because it convinces the agent that the principal will not interfere ex post, thereby raising the agent’s incentives ex ante (see, e.g., AGHION AND TIROLE [1997], BURKART, GROMB, AND PANUNZI [1997]). AGHION AND TIROLE [1997] consider a similar sequence of events, but they differ from the present paper in the interpretation of the effort of the agent. While in AGHION AND TIROLE [1997] the agent expends effort to acquire information about the prospects of various courses of action, in our model the agent’s effort increases the scope of the project, i.e., its size. Moreover, AGHION AND TIROLE [1997] do not study the relationship between risk and delegation.

Second, the paper is part of the literature that investigates how a positive relationship between exogenous risk and delegation might arise (see e.g., PRENDERGAST [2000], MOOKHERJEE [2006], and POITEVIN [2000]).
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[2002], ACEMOGLU et al. [2007]). PRENDERGAST [2002] mainly aims to explain a positive relationship between risk and pay-for-performance incentives that (in contrast to what is predicted by standard principal-agent theory) is frequently observed in empirical studies. In PRENDERGAST [2002] an agent has to specialize in one out of many tasks and subsequently chooses a variable effort. In contrast to the present paper, the agent is risk-neutral and has private information about the riskiness of output, and costly input and output monitoring by the principal are feasible. PRENDERGAST [2002] shows that the principal prefers to retain control over task choice if the risk is low (i.e., if the correct task is rather obvious). In this case, through input monitoring, the principal will ensure that the agent focuses on the desired task. On the other hand, if the risk is high (i.e., if it is unclear to the principal what the right task is), it is optimal for the principal to delegate task choice to the better-informed agent and to motivate him through pay for performance.9 In ACEMOGLU et al. [2007], the agent has superior information with respect to the correct course of action. The principal has access to publicly available information only and prefers to retain control if publicly available information is sufficiently precise. If it is too imprecise, the principal prefers to delegate authority to the agent. Hence, in both PRENDERGAST [2002] and ACEMOGLU et al. [2007] private information of the agent plays a crucial role.

Third, our paper adds to a literature that explores why risk reduction by an agent might be beneficial even from a risk-neutral principal’s perspective. At a basic level, this might be the case because risk-averse agents need to be compensated for the risk they bear (see, e.g., SMITH AND STULZ [1985]). In contrast, in our model risk reduction may increase the agent’s initiative. For an alternative explanation, see, e.g., DEMARZO AND DUFFIE [1995], who consider a model where an agent may engage in financial hedging, which results in profits that are more informative of project quality, thereby allowing better termination decisions.

3 The Model

A risk-neutral principal $P$ and a risk-averse agent $A$ conduct some project. Figure 1 illustrates the sequence of events. At date 0 the principal offers the agent a contract. Feasible contracts are discussed in more detail below.

8 For other recent explanations of a positive risk–incentive trade-off, see, e.g., CORE AND QIAN [2002], BAKER AND JORGENSEN [2003], RAITH [2003], and GUO AND OU-YANG [2006].

9 See also BESTER [2003], [2008], who focuses, not on the role of risk (which is taken to be exogenous), but, more generally, on the role of externalities caused by certain allocations of authority. In particular, BESTER [2003] assumes that larger projects impose higher (exogenously given) costs on the agent. In the present paper, such costs arise through a risk–return trade-off involved in decision-making. Under special circumstances a positive relationship between exogenous risk and delegation may emerge in BESTER [2003]. However, in contrast to the present paper, he assumes that decisions are made before effort is exerted.
At date 1, the agent may provide some unobservable effort $e \geq 0$, where for simplicity the disutility of effort is given by $1/2e^2$. The agent’s effort raises the scope of the project (i.e., the larger is $e$, the larger is the size of the project). At date 2, a decision $x \in [0, 1]$ has to be taken (by the principal or by the agent, whichever has the authority). We assume that $x$ directly determines the expected return per unit of the project; i.e., the expected return of the project is given by $e \cdot x$. For example, in the context of retail banking discussed in the Introduction, $e$ might represent the number of new credit customers a branch manager approaches. In this case, the expected return per customer might depend on whether an aggressive or a conservative credit policy $x$ is pursued. Alternatively, in the context of manufacturing, $e$ might represent some production quantity, and $x$ might be determined by the subsequent marketing strategy.\footnote{Important for our purposes is the fact that the agent’s marginal return to effort will depend on which decisions are taken later on. It will become clear below that, if this were not the case, in the present setting the principal would never find it optimal to delegate authority to the agent. See, e.g., GUO AND OU-YANG [2006], BAKER AND JORGENSEN [2003], and SUNG [1995] for principal–agent models employing a similar payoff structure.} Although $x$ determines the expected return, the true return $u$ per unit of the project is only realized at date 3. In the following, we want to allow for the possibility that there is a risk–return trade-off in decision-making; i.e., a more ambitious course of action (i.e., a higher $x$) might not only yield a higher expected return, but might also come with higher risk (i.e., it might lead to a higher variance of the project return). Frequently, it will be the case that a more ambitious goal involves greater risks, while the return to a more modest course of action will (almost) be predetermined. Consequently, we assume that $u$ is a normally distributed random variable with expected value $x$ and variance $r \cdot x^\gamma$, where $r > 0$ and $\gamma \geq 0$ are parameters. Our measure of exogenous risk is given by $r$, i.e., a larger $r$ will be interpreted as higher exogenous risk. For $\gamma = 0$, the variance of the project return is independent of $x$, and there is no risk–return trade-off. For $\gamma > 0$, there is a positive risk–return trade-off, i.e., in this case the variance of the project return is increasing in $x$. The literature on project selection typically assumes a strictly positive risk–return trade-off (see, e.g., HIRSHLEIFER AND SUH [1992], DEMSKI AND DYE [1999], CORE AND QIAN [2002]). Like the present paper, this literature considers settings where after effort provision some
decisions have to be taken. However, this literature does not consider delegation, but assumes that the agent has authority (for an exception, see, e.g., DUTTA AND REICHELSTEIN [2002], which, however, differs from our model in other important aspects).

As discussed above, the main concern of the paper is to investigate how the underlying riskiness of projects might influence the decision of the principal to delegate authority to the agent. In order to isolate this effect, we abstract from other potential conflicts of interest between the principal and the agent with respect to the decision, and assume that the parties only differ in their risk preferences. Formally, we assume that both the principal and the agent derive a (gross) payoff of $e \cdot u$ from conducting the project. These payoffs might, for example, represent private benefits that each of the parties derives from conducting the project (e.g., due to changes in reputation or career prospects). Alternatively, one could assume that the project generates a total gross return of two times $e \cdot u$, which the parties share equally.$^{11}$

The risk-neutral principal focuses on her expected payoff from the project, which is given by

\[ \pi(e, x) \equiv E[e \cdot u] = e \cdot x. \]  

(1)

The risk-averse agent evaluates his payoff with a concave utility function. For simplicity, we assume that the agent has exponential utility with constant absolute risk aversion $\rho > 0$. Hence, $A$’s expected utility can be represented by its certainty equivalent, which (net of effort costs) is given by$^{12}$

\[ a(e, x) \equiv e \cdot x - \frac{1}{2} \cdot \rho \cdot e^2 \cdot x^\gamma - \frac{1}{2} e^2. \]  

(2)

While the agent’s utility is increasing in the expected value of the project return, it is decreasing in its variance. In addition, the agent has to bear the effort costs.$^{13}$ The reservation utility of the agent is assumed to be zero.

In line with the emerging literature on partial contracting, we assume that only control over the decision, not the decision itself, is contractible.$^{14}$ Hence, at date 0 the principal (who is assumed to initially have control) may decide to delegate authority over the decision to the agent. That is, initial contracts take the form $[j; t]$, where $j \in \{A, P\}$ denotes which of the parties has authority and where $t$ denotes a transfer payment from $A$ to $P$.

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11 Note that under this interpretation the results do not depend on the assumption of equal sharing, but would continue to hold for any linear incentive contract.
12 For a proof of this claim, see, e.g., WOLFSTETTER [1999, p. 284 and Appendix D (R14)].
13 While constant absolute risk aversion allows one to derive explicit solutions, this assumption does not seem to be crucial, and at the cost of added complexity we expect our results to hold for more general utility functions.
14 See, e.g., AGHION, DEWATRIPONT, AND REY [2002], [2004], BESTER [2003], [2008], SCHMITZ [2005].
When deciding about whether to delegate authority to the agent, the principal aims to maximize her expected payoff subject to the agent’s participation constraint. Intuitively, as only control over the decision is contractible, the principal cannot commit not to behave opportunistically at the decision stage. That is, the principal cannot commit not to select the most profitable (but also most risky) course of action. Consequently, the principal faces the following trade-off. On the one hand, if the principal retains authority (P-control), she will select a large $x$ promising her a high expected return. However, in this case the agent anticipates that his payoff will be relatively risky, and hence the risk-averse agent’s incentive to increase the scope of the project will be relatively low. On the other hand, if the agent is granted authority (A-control), the agent may find it optimal to make a decision that, while promising only a moderate return, exposes him to less risk. As a consequence, while under A-control the agent may distort the decision (relative to the decision preferred by the principal), this will leave him with higher effort incentives than P-control. In particular, in the following we show that this trade-off may make it more attractive for the principal to delegate authority to the risk-averse agent at higher levels of exogenous risk $r$.

In a first step, we consider P-control and A-control in turn. Subsequently, we compare the principal’s payoffs under the two regimes in order to derive the principal’s optimal delegation decision. Given $j$-control, denote the equilibrium decisions by $x^j(e)$ and the equilibrium effort choices by $e^j$, where $j = P, A$. For the moment, assume that $P$ retains authority over the decision. In this case, it follows from (1) that, at date 2, the principal chooses $x^P(e) = 1$ for all $e$. Anticipating this, it follows from (2) that the agent selects his effort level such that

$$e^P = \arg \max_e \{a(e, 1)\} = \frac{1}{1 + \rho r},$$

where $e^P$ is decreasing in both exogenous risk $r$ and the risk-aversion parameter $\rho$.

To summarize:

**Lemma 1 (Equilibrium Outcome under P-control)** Suppose the principal has authority. In this case, she takes the decision that promises the highest expected return (i.e., $x^P(e^P) = 1$), and the agent selects an effort level given by $e^P = 1/(1 + \rho r)$.

Now suppose that the principal delegates authority to the agent. This implies that, at date 2, the agent takes some decision $x^A(e)$ maximizing his expected utility, i.e.,

$$x^A(e) \in \arg \max_{x} \{a(e, x)\},$$

and (2) implies$^{15}$

$$a_i(e, x) = e - \frac{1}{2} \cdot \rho \cdot e^2 \cdot \gamma \cdot x^{\gamma-1} \cdot r.$$

$^{15}$ Throughout, subscripts denote partial derivatives.
Hence, selecting a larger $x$ has two effects. First, it raises the expected return of the project. At the same time, it leads to a higher variance, which reduces the agent’s utility. This negative variance effect is the larger, the larger is the scope of the project (i.e., the larger is $e$). Intuitively, the larger the project is, the more the agent will be inclined to insure himself by choosing a course of action associated with less risk. Consequently, there exists a threshold level $\hat{e}$ for the scope of the project.

For effort levels below this threshold the implied risk is sufficiently small that the agent chooses $x = 1$ (just as the principal would do in the case of $P$-control). For levels of $e$ above $\hat{e}$, $A$ finds it optimal to distort $x$ downwards in order to reduce his risk exposure.

**Lemma 2 (Decision under $A$-Control)** Suppose the agent has authority. For sufficiently low effort levels, the agent selects the same decision as the principal. However, given sufficiently large effort levels, the agent prefers to distort the decision downward. Formally, for $e > \hat{e}$ we have $x^A(e) < 1$, and $x^A(e) = 1$ otherwise, where $\hat{e} \equiv \min\{2/\rho r, 2/\rho \gamma\}$.

Let us now turn to the agent’s effort choice at date 1, which solves

$$e^A \in \arg\max_e \{a(e, x^A(e))\} .$$

(6)

Lemma 2 raises the question under which circumstances the agent indeed finds it optimal to choose a project scope sufficiently large to imply a subsequent downward distortion of the decision. Only in this case will $A$-control and $P$-control lead to different equilibrium outcomes. First, if the risk–return trade-off is sufficiently weak (i.e., if $\gamma$ is sufficiently low), $A$-control will lead to the same equilibrium outcome as $P$-control. Second, suppose that the risk–return trade-off is sufficiently pronounced (i.e., assume that $\gamma$ is sufficiently large). If the level of exogenous risk $r$ is relatively low, the threshold value $\hat{e}$ is relatively large. That is, a relatively large project scope is required to make a downward distortion of the decision desirable for the agent (see Lemma 2), which, given convex effort costs, will not be optimal. As a consequence, $A$-control will again lead to the same equilibrium outcome as $P$-control. If, however, the exogenous risk $r$ is relatively large, then $\hat{e}$ is small, and the agent finds some $e$ above $\hat{e}$ profitable. In this case, $A$-control leads to a different outcome from $P$-control: relative to $P$-control, the agent chooses a larger project scope, but later on follows a course of action associated with less risk.

**Lemma 3 (Equilibrium Outcome under $A$-Control)** Suppose the agent has authority. If the risk–return trade-off is sufficiently pronounced and the exogenous risk is sufficiently large, then, compared to $P$-control, the equilibrium effort is larger, but the equilibrium decision is smaller. Formally, (i) $e^A > e^P$ and $x^A(e^A) < 1$ if $r > \tilde{r}$ and $\gamma > 2$, where $\tilde{r} \equiv 2/[\rho(\gamma - 2)]$, and (ii) $e^A = e^P$ and $x^A(e^A) = 1$ otherwise.

Hence, only in the case of Lemma 3(i) do the equilibrium decisions under $P$-control and $A$-control differ (i.e., only in such cases is there an ex post conflict of interest between the principal and the agent).
The principal delegates authority to the agent whenever she obtains a higher total payoff from doing so, i.e., if and only if
\[ \pi(e^A, x^A(e^A)) + t^A > \pi(e^P, x^P(e^P)) + t^P, \]
where \( t^j \) denotes the transfer payment from the agent to the principal under \( j \)-control for \( j = A, B \). The principal will set the transfer payment such that the agent’s participation constraint is binding, i.e., \( t^j = a(e^j, x^j(e^j)) \) for \( j = P, A \).

In the following, we show that whenever there is an ex post conflict of interest between the parties, the principal prefers to delegate authority to the agent. First, note that in this case we have \( t^A = a(e^A, x^A(e^A)) > a(e^P, x^P(e^P)) = t^P \); under \( A \)-control the agent chooses both \( e \) and \( x \), and hence his equilibrium utility (gross of the transfer payment) is higher than under \( P \)-control. Second, despite the fact that under \( A \)-control a smaller \( x \) is selected, the increase in effort is sufficiently large that the expected return of the project goes up too, i.e., \( \pi(e^A, x^A(e^A)) > \pi(e^P, x^P(e^P)) \) holds.

**Proposition 1 (Optimal Allocation of Control)** If the risk–return trade-off is sufficiently pronounced, then, for sufficiently large exogenous risk, the risk-neutral principal finds it strictly optimal to delegate authority to the risk-averse agent. Otherwise, the principal retains control. Formally, \( A \)-control is strictly optimal if both \( r > \hat{r} \) and \( \gamma > 2 \) hold.

Hence, Proposition 1 identifies a novel channel through which the empirically observed positive relationship between exogenous risk and delegation might emerge.

In equilibrium the agent responds to exogenous risk through a certain choice of effort level and decision. Consequently, the equilibrium project risk (i.e., the equilibrium variance of the project return) is endogenous. In the following, we study how this endogenous risk varies with exogenous risk (and hence, with the incidence of delegation). The equilibrium variance of the project return is given by
\[ V^* = \text{var}(e^j \cdot u), \]
where \( j = A \) if \( r > \hat{r} \) and \( \gamma > 2 \), and \( j = P \) otherwise, and where the variance of \( u \) is given by \( x^j \cdot r \). It can be shown that there is a hump-shaped relationship between exogenous risk \( r \) and \( V^* \) (see Figure 2). First, consider the case that \( P \)-control is optimal, where Lemma 1 and (8) imply that \( V^* = (e^P)^2 \cdot r \). Hence, on the one hand, an increase in \( r \) has a direct positive effect on \( V^* \). On the other hand, indirectly, larger values of \( r \) lead the agent to reduce his effort level (see Lemma 1), which reduces the equilibrium variance of the project return. As will be shown below, for low levels of \( r \) the former effect dominates, and endogenous risk is increasing in \( r \). However, for sufficiently large values of \( r \) the reduction in the effort level is sufficient to lead to a negative relationship between \( r \) and endogenous risk. Second, consider the case that \( A \)-control is optimal, implying that \( V^* = (e^A)^2 \cdot x^A(e^A) \cdot r \). In this case, the equilibrium variance is a decreasing function of the level of exogenous

\[16\] If the principal is indifferent, she is assumed to retain authority (for example, due to some (arbitrarily small) private benefits of control).
risk. Intuitively, note that both the agent's equilibrium decision and his effort are decreasing in the exogenous risk (see Lemmas 2 and 3), and these indirect negative effects jointly dominate the direct positive effect of $r$ on $V^*$. To summarize:

**Proposition 2 (Endogenous Risk)** The equilibrium variance of the project return is a hump-shaped function of exogenous risk. Formally, $V^*$ is a continuous function of $r$, and $V^*_r > (\leq) 0$ for all $r < (\geq) \tilde{r}$, where $\tilde{r} > 0$.

Interestingly, while Proposition 1 implies that in situations with larger exogenous risk it is more likely that authority is delegated to the risk-averse agent, Proposition 2 shows that an analogous positive relationship might exist with respect to endogenous risk and the incidence of delegation. It might very well be the case that endogenous risk is larger in cases where $A$ has authority than in cases where the risk-neutral principal retains control. Such a case is illustrated by points $y$ and $z$ in Figure 2.

### 5 Conclusion

Empirical studies provide evidence for a positive relationship between exogenous risk and delegation of authority to lower levels of a hierarchy. This paper makes three contributions. First, motivated by these empirical findings, we identify a novel channel through which more exogenous risk might lead to more delegation. We consider a partial-contracting model, where a risk-averse agent may exert effort to increase the scope of a project. Subsequent decisions may in principle be made
either by the risk-neutral principal or by the agent, and the decisions involve a risk–
return trade-off. That is, a course of action promising a higher expected return may
imply more risk. As control over the decisions, but not the decisions themselves, are contractible, the principal faces a commitment problem. On the one hand, if
the principal keeps authority, she will always choose the course of action yielding
the highest expected return. Thereby, however, the principal exposes the agent
to a lot of risk, and this may reduce the agent’s initiative. On the other hand, if
the principal delegates authority to the agent, the agent will proceed more cau-
tiously at the decision stage. Anticipating this, the agent has a higher incentive
to increase the project scope, making delegation the more attractive for the prin-
cipal, the larger is the exogenous risk. Hence, delegation might be optimal not
despite, but exactly because, the agent is risk-averse. Second, our findings pro-
vide an (incentive-based) rationale for why even a risk-neutral principal might
find it optimal to allow an agent to reduce the total firm risk. Finally, it turns
out that the relationship between endogenous risk and delegation is ambiguous.
Hence, perhaps surprisingly, it is possible that endogenous risk is larger when
the risk-averse agent has control. In general, however, the relationship between en-
dogenous risk and delegation depends on the underlying distribution of exogenous
risk.

Appendix

A.1 Proof of Lemma 2

Inspecting (5) reveals that \( a(e, x) \) is strictly concave in \( x \) if and only if \( \gamma > 1 \). First, suppose \( \gamma > 1 \). In this case, it immediately follows from (5) that

\[
x^A(e) = \sqrt{\frac{2}{\gamma pr}} \cdot \frac{1}{e} < 1
\]

if \( e > 2/\gamma pr \), and \( x^A(e) = 1 \) if \( e \leq 2/\gamma pr \). Second, suppose \( \gamma \leq 1 \). In this case, we have \( x^A(e) \in \{0, 1\} \). Given (2), it follows that \( x^A(e) = 1 \) if \( e - 1/2 \cdot \rho \cdot e^2 \cdot r \geq 0 \), that is, if \( e \leq 2/\rho r \). Taken together, this implies the result.

A.2 Proof of Lemma 3

In a first step, we prove that \( e^A = e^P \) and \( x^A(e^A) = x^P(e^P) = 1 \) if \( \gamma \leq 2 \). Define \( a^A(e) \equiv a(e, x^A(e)) \). First, suppose \( \gamma \leq 1 \). Lemma 2 and (2) imply

\[
a^A(e) = \begin{cases} e - \frac{1}{2} \cdot \rho \cdot e^2 \cdot r - \frac{\rho}{e} & \text{if } e \leq \frac{2}{\rho}, \\ -\frac{\rho}{e^2} & \text{if } e > \frac{2}{\rho}, \end{cases}
\]

and hence \( e > 2/\rho r \) cannot be optimal, because it is dominated by \( e = 0 \). As a con-
sequence, we have \( e^A = e^P \) if \( e^P = 1/(1 + \rho r) \leq 2/\rho r = \tilde{e} \) holds, which is indeed
the case. Second, suppose \( 1 < \gamma \leq 2 \). To prove the claim, we again show that in
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this parameter range some \( e > \hat{e} \) cannot be optimal. For all \( e > \hat{e} \), Lemma 2 and (2) imply

\[
a^A(e) = e \cdot x^A(e) - \frac{1}{2} \cdot r \cdot e^2 \cdot (x^A(e))' \cdot r - \frac{1}{2} e^2
\]

and hence

\[
a^A(e) = \left( \frac{2 - \gamma}{\gamma} \right) \cdot e \left( \frac{1}{1 - \gamma} \right) \cdot \hat{e} \cdot e - e.
\]

Consequently, if \( \gamma \leq 2 \), we have \( a^A(e) < 0 \) for all \( e > \hat{e} \), which implies \( e^A \leq \hat{e} \). Moreover, from Lemma 2 we know that \( x^A(e) = 1 \) for all \( e \leq \hat{e} \). Hence, the agent chooses the effort level that maximizes \( a(e, 1) \) subject to the constraint \( e \leq \hat{e} \), where \( a(e, 1) \) is strictly concave in \( e \). Thus, similarly to the above, we have \( e^A = e^P \) if

\[
e^P = \frac{1}{1 + \rho r} \leq \frac{2}{\gamma \rho r} = \hat{e}
\]

holds, which is indeed the case.

In a second step, we consider the parameter range \( \gamma > 2 \). To prove the result we show that it depends on the sign of \( a^A(e, 1) = a^A(\hat{e}) = 1 - \hat{e}(1 + \rho r) \) whether \( e^A \) lies above or below \( \hat{e} \). From the first part of the proof we know that \( a(e, 1) \) is strictly concave in \( e \). Moreover, (A4) implies that

\[
a^A(e) = - \left( \frac{2 - \gamma}{\gamma} \right) \left( \frac{1}{1 - \gamma} \right) \cdot e \left( \frac{1}{1 - \gamma} \right) \cdot \hat{e} \cdot e - 1 < 0,
\]

and hence \( e^A > \hat{e} \) if and only if

\[a^A(\hat{e}, 1) = 1 - \hat{e}(1 + \rho r) > 0 \iff 1 - \frac{2}{\gamma \rho r} : (1 + \rho r) > 0 \iff r > \frac{2}{\rho(\gamma - 2)} \equiv \tilde{r}.
\]

In this case, it follows from (A4) that

\[
e^A = \left( \frac{\gamma - 2}{\gamma} \right) \left( \frac{\hat{e} \gamma}{\gamma - 2} \right)^\frac{1}{2
\]

If, however, \( r < \frac{2}{\rho(\gamma - 2)} \), then by the same argument we have \( e^A = e^P \). It remains to show that \( e^A > e^P \) holds in the relevant parameter range (i.e., where \( \gamma > 2 \) and \( r > \tilde{r} \)). Note that \( a^A(\hat{e}, 1) > 0 \) implies \( e^A, e^P > \hat{e} \). Hence, as under both \( P \)-control and \( A \)-control the agent faces a concave problem, in order to prove the claim it
suffices to show that \( a^*_A(e) > a_e(e, 1) \) holds for all \( e > \hat{e} \). The envelope theorem and (2) imply
\[
\begin{align*}
a^*_A(e) > a_e(e, 1) & \iff x^A(e) - x^A(e)^\rho > 1 - \rho e \\
& \iff \frac{1}{2} \rho e \left[ 1 - x^A(e)^\rho \right] > \left[ 1 - \frac{1}{2} \rho e \right] - \left[ x^A(e) - x^A(e)^\frac{1}{2} \rho e \right],
\end{align*}
\]
which is satisfied for all \( e > \hat{e} \), because Lemma 2 implies that the left-hand side is strictly positive, while (4) and (2) imply that the right-hand side is strictly negative.

### A.3 Proof of Proposition 1

\( A \)-control can only be strictly optimal if it leads to a different equilibrium outcome from \( P \)-control, i.e., if both \( \gamma > 2 \) and \( r > \hat{r} \) (see Lemma 3) hold, which we assume in the following. First, note that \( t^A = a(e^A, x^A(e^A)) > a(e^P, x^P(e^P)) \geq a(e^P, 1) = t^P \). Second, we show that \( \pi(e^A, x^A(e^A)) > \pi(e^P, x^P(e^P)) \) holds as well, which in combination with (7) implies the result. Lemma 1 implies \( \pi(e^P, x^P(e^P)) = 1/(1 + z) \), where \( z \equiv \rho r \). Moreover, (A1) and (A6) imply
\[
\begin{align*}
x^A(e) = \frac{\gamma - 1}{\sqrt{e}} = \left( \frac{2}{e^2} \right)^{\frac{1}{\gamma - 1}} = \left( \frac{2}{\gamma} \right)^{\frac{1}{\gamma - 1}} \cdot e^{-\frac{1}{\gamma - 1}},
\end{align*}
\]
and
\[
\begin{align*}
x^A(e^A) \cdot e^A = \left( \frac{2}{e^2} \right)^{\frac{1}{\gamma - 1}} \cdot (e^A)^{\frac{1}{\gamma - 1}} = \left( \frac{1}{z} \right)^{\frac{1}{\gamma - 1}} \cdot \left( \frac{2}{\gamma} \right)^{\frac{1}{\gamma - 1}} \cdot \left( 1 - \frac{2}{\gamma} \right)^{\frac{1}{\gamma - 1}}.
\end{align*}
\]
Hence, we have
\[
\pi(e^A, x^A(e^A)) > \pi(e^P, x^P(e^P)) \iff x^A(e^A) \cdot e^A > x^A(e^P) \cdot e^P
\]
\[
\iff \left( \frac{1}{z} \right)^{\frac{1}{\gamma - 1}} \cdot \left( \frac{2}{\gamma} \right)^{\frac{1}{\gamma - 1}} \cdot \left( 1 - \frac{2}{\gamma} \right)^{\frac{1}{\gamma - 1}} > \frac{1}{1 + z}
\]
\[
\iff \left[ \frac{1}{z} \cdot \frac{2}{\gamma} \right] \cdot \left[ 1 - \frac{2}{\gamma} \right] > \frac{1}{1 + z}
\]
\[
\iff \left[ \frac{2}{\gamma} \right] \cdot \left( 1 - \frac{2}{\gamma} \right) > 1.
\]
Define \( f_z(z) = z^{-2/\gamma} + z^{(\gamma - 2)/\gamma} \). Note that at the boundary of the parameter range under consideration (i.e., at \( r = 2/|\rho(\gamma - 2)| \iff z = 2/(\gamma - 2) \)) the left-hand side of the above inequality is equal to 1. Hence, the above inequality is satisfied for all \( z > 2/(\gamma - 2) \) if \( f_z(z) > 0 \) for all \( z > 2/(\gamma - 2) \):
\[
\begin{align*}
f_z(z) > 0 & \iff -\frac{2}{\gamma} \cdot z^{-\frac{1}{\gamma} - 1} + \left( \frac{\gamma - 2}{\gamma} \right) \cdot z^{(\gamma - 2)/\gamma - 1} > 0 \\
& \iff -\frac{2}{\gamma} \cdot z^{-1} + \left( \frac{\gamma - 2}{\gamma} \right) > 0 \iff z > \frac{2}{\gamma - 2},
\end{align*}
\]
which concludes the proof.
A.4 Proof of Proposition 2

First, consider the case that $P$-control is optimal. Lemma 1 and (8) imply

$$V^* = \frac{1}{(1 + \rho r)^2} \cdot r$$

and

$$V^*_r = \frac{1}{(1 + \rho r)^2} - \frac{2\rho r}{(1 + \rho r)^3} = \left( \frac{1}{1 + \rho r} \right)^2 \cdot \left( 1 - \frac{2\rho r}{1 + \rho r} \right) = \left( \frac{1}{1 + \rho r} \right)^2 \cdot \left( \frac{1 - \rho r}{1 + \rho r} \right).$$

Hence, $V^*_r > 0 \iff r < 1/\rho$. Second, if $A$-control is optimal, it follows from (8), (A1), (A6), and Proposition 1 that

$$V^* = \left( \frac{2}{\gamma p} \right)^{\frac{\gamma - 1}{\gamma}} \cdot \left( \frac{\gamma - 2}{\gamma} \right)^{\frac{\gamma - 2}{\gamma}} \cdot r^{\frac{1}{\gamma}}.$$

and hence in this case we have $V^*_r < 0$ and $V^*_rr > 0$. Finally, note that in the case $\gamma > 2$ (where $A$-control is possibly optimal), we have $(e^p)^2 \cdot x^p(e^p)^\gamma \cdot r = (e^A)^2 \cdot x^A(e^A)^\gamma \cdot r \cdot \hat{r}$ if $r = \hat{r}$, and $1/\rho \leq \hat{r} \iff 1/\rho \leq 2/[\rho(\gamma - 2)] \iff \gamma \leq 4$, which in combination with Lemma 3 concludes the proof.

References


WOLFSTETTER, E. [1999], *Topics in Microeconomics*, Cambridge University Press: Cambridge, MA.

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