Nucleon strangeness form factors and moments of PDF

Takumi Doi∗, Mridupawan Deka†, Shao-Jing Dong**, Terrence Draper**,
Keh-Fei Liu**, Devdatta Mankame**, Nilmani Mathur‡ and Thomas Streuer§

∗Graduate School of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan
email: doi@ribf.riken.jp
†Institute of Mathematical Sciences, Chennai- 600013, India
∗∗Department of Physics and Astronomy, University of Kentucky, Lexington KY 40506, USA
‡Department of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 40005, India
§Institute for Theoretical Physics, University of Regensburg, 93040 Regensburg, Germany

Abstract.

The calculation of the nucleon strangeness form factors from \(N_f = 2 + 1\) clover fermion lattice QCD is presented. Disconnected insertions are evaluated using the \(Z(4)\) stochastic method, along with unbiased subtractions from the hopping parameter expansion. We find that increasing the number of nucleon sources for each configuration improves the signal significantly. We obtain \(G_s(0) = -0.017(25)(07)\), which is consistent with experimental values, and has an order of magnitude smaller error. Preliminary results for the strangeness contribution to the second moment of the parton distribution function are also presented.

PACS: 13.40.-f, 12.38.Gc, 14.20.Dh

INTRODUCTION

Understanding the structure of the nucleon from QCD has been one of the central issues in hadron physics. In particular, the strangeness content of the nucleon attracts a great deal of interest lately. It is also an ideal probe for the virtual sea quarks in the nucleon. Extensive experimental/theoretical studies indicate that the strangeness content varies depending on the quantum number carried by the \(s\overline{s}\) pair: the scalar density is about 0–20% of that of up, down quarks, the quark spin is about −10 to 0% of the nucleon, and the momentum fraction is only a few percent of the nucleon. In general, the uncertainties in the strangeness matrix elements are quite large in both experiments and theories. Under these circumstances, it is desirable to provide the definitive quantitative results using lattice QCD.

The challenge in the lattice QCD calculation of strangeness matrix elements resides in the evaluation of the so-called disconnected insertion (DI). In fact, it requires the calculation of all-to-all propagators, which is prohibitively expensive compared to the connected insertion (CI). Consequently, there are only a few DI calculations [1, 2, 3], where the all-to-all propagators are stochastically estimated [4]. In this proceeding, we report the improvement of the calculation of all-to-all propagators using the stochastic method along with unbiased subtractions from the hopping parameter expansion [5], and the increment of the number of nucleon sources [6, 7]. We present the results for the strangeness contribution to the electromagnetic form factors [7] and the second moment of the nucleon. The preliminary result for the first moment of the nucleon is presented in Ref. [8].

FORMALISM AND SIMULATION PARAMETERS

We employ \(N_f = 2 + 1\) dynamical configurations with nonperturbatively \(\Theta(a)\) improved clover fermion and RG-improved gauge action generated by CP-PACS/JLQCD Collaborations [9]. We use \(\beta = 1.83\) and \(c_{\text{sea}} = 1.7610\) configurations with the lattice size of \(L^3 \times T = 16^3 \times 32\), which corresponds to \((2\text{fm})^3\) box in physical spacial size with the lattice spacing of \(a^{-1} = 1.625\text{ GeV}\) [9]. For the hopping parameters of \(u,d\) quarks (\(\kappa_{\text{ud}}\)) and \(s\) quark (\(\kappa_s\)), we use \(\kappa_{\text{ud}} = 0.13825, 0.13800, \text{and} 0.13760\), which correspond to \(m_u = 0.60\), \(0.70\), and \(0.84\text{ GeV}\), respectively, and \(\kappa_s = 0.13760\) is fixed. We perform the calculation only at the dynamical quark mass points, where 800 configurations are used for \(\kappa_{\text{ud}} = 0.13760, \text{and} 810\) configurations for \(\kappa_{\text{ud}} = 0.13800, 0.13825\).
The nucleon matrix elements can be obtained through the calculation of 3pt function $\Pi_3^{\text{pol}}$ (as well as 2pt function $\Pi_2^{\text{pol}}$), defined by

$$\Pi_3^{\text{pol}}(\vec{p},t_2; \vec{q},t_1; \vec{p}' = \vec{p} - \vec{q},t_0) = \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p} \cdot (\vec{x}_2 - \vec{x}_0)} \cdot e^{+i\vec{q} \cdot (\vec{x}_1 - \vec{x}_0)} \langle 0 | T [ \chi_N(\vec{x}_2,t_2) J(\vec{x}_1,t_1) \bar{\chi}_N(\vec{x}_0,t_0) ] | 0 \rangle,$$

where $\chi_N$ is the nucleon interpolating field and $J$ is the insertion operator. Since there is no strange quark as a valence quark in the nucleon, the 3pt is a DI which entails a multiplication of the nucleon 2pt correlator with the current quark. For the evaluation of the quark loop, we use the stochastic method [4], with $Z(4)$ noises in color, spin and space-time indices. We generate independent noises for different configurations, in order to avoid possible auto-correlation.

In the stochastic method, it is quite expensive to achieve a good signal to noise ratio (S/N) just by increasing $N_{\text{noise}}$. In view of this, we use many nucleon point sources $N_{\text{src}}$. Since there is no strange quark as a valence quark in the nucleon, the 3pt is a DI which entails a multiplication of the nucleon 2pt correlator with the current quark.

**STRANGENESS ELECTROMAGNETIC FORM FACTORS**

The formulas for Sachs electric (magnetic) form factors $G_E^\mu$ ($G_M^\mu$) are given by

$$R_\mu^\pm(\Gamma_{\text{pol}}) \equiv \frac{\text{Tr} \left[ \Gamma_{\text{pol}}^\pm \Pi_{2\text{pt}}(\vec{0},t_2; \pm \vec{q},t_1; -\vec{q},t_0) \right]}{\text{Tr} \left[ \Gamma_{\text{pol}}^\pm \Pi_{2\text{pt}}(\pm \vec{q},t_1; t_0) \right]} \cdot \frac{\text{Tr} \left[ \Gamma_{\text{pol}}^\pm \Pi_{2\text{pt}}(\vec{0},t_1; t_0) \right]}{\text{Tr} \left[ \Gamma_{\text{pol}}^\pm \Pi_{2\text{pt}}(\vec{0},t_2; t_0) \right]}$$

and

$$G_E^\mu(\vec{Q}^2) = \pm R_\mu^\pm(\Gamma_{\text{pol}} = \Gamma_{\text{pol}}^\pm), \quad G_M^\mu(\vec{Q}^2) = \pm \frac{E_N^\mu + m_N}{E_N^\mu - i e_i q_j} R_{\mu\nu}^\pm(\Gamma_{\text{pol}} = \Gamma_{\text{pol}}^\pm),$$

where $J_{\mu}(x + \mu/2) = \frac{1}{2} \left[ \bar{q}(x)(1 - \gamma_\mu) U_\mu(x) q(x + \mu) - \bar{q}(x + \mu)(1 + \gamma_\mu) U_\mu(x) q(x) \right]$ is the point-split conserved vector operator, $\{i,j,k\} \neq 4, \Gamma_{\text{pol}}^\pm \equiv (1 \mp \gamma_\mu)/2, \Gamma_{\text{pol}}^\pm \equiv (\pm i)/2 \times (1 \mp \gamma_\mu) \gamma_\nu \gamma_\kappa$ and $E_N^\mu \equiv \sqrt{m_N^2 + \vec{q}^2}$. The upper sign corresponds to the forward propagation ($t_2 \gg t_1 \gg t_0$), and the lower sign corresponds to the backward propagation ($t_2 \ll t_1 \ll t_0$).

**FIGURE 1.** $R_E^\pm$ (left) and $R_M^\pm$ (right) with $\kappa_{ud} = 0.13760, N_{\text{src}} = 64$ (circles) and $N_{\text{src}} = 4$ (triangles), plotted against the nucleon sink time $t_2$. The dashed line is the linear fit where the slope corresponds to the form factor.
we consider only the leading dependence on \( Q^2 \) and the pole mass \( Q_{gs} \) at \( Q^2 = 0 \).

\[ \langle \chi^2 \rangle_{s-\bar{s}} = \int_0^1 dx x^2 (s(x) - \bar{s}(x)) \]

This integral can be obtained by

\[ \frac{\text{Tr} \left[ \Gamma_{\pm}^{3q} \Pi_{F_{g,0}} (\pm \bar{\mu}, t_2; \pm \bar{\mu}_0) \right]}{\text{Tr} \left[ \Gamma_{\pm}^{3q} \Pi_{F_{g,0}} (\pm \bar{\mu}, t_2; \mu_0) \right]} = \pm p_z^2 \langle \chi^2 \rangle_{s-\bar{s}}, \]

which eliminates the \( Q^2 \) dependence of the form factors. For the magnetic form factor, we employ the dipole form, \( G_M(Q^2) = G_M(0)/(1 + Q^2/\Lambda^2)^2 \), where reasonable agreement with lattice data is observed. For the electric form factor, we employ the vector current conservation, \( E = -\mu K_1 Q_3 \overline{E} \), which is obtained by heavy baryon chiral perturbation theory (HB \( \chi \)PT) [10]. The chiral extrapolated results are \( G_M(0) = -0.017(25), \Lambda = 0.58(16) \), \( \langle r_1^2 \rangle_M = -6 \frac{\partial G_M}{\partial Q^2} |_{Q^2=0} = -7.4(71) \times 10^{-3} \text{fm}^2 \) and \( g_E^2 = 0.027(16) \) (or \( \langle r_1^2 \rangle_E = -6 \frac{\partial G_E}{\partial Q^2} |_{Q^2=0} = -2.4(15) \times 10^{-3} \text{fm}^2 \)).

We examine the systematic uncertainties in the result of form factors. For the ambiguity of \( Q^2 \) dependence, we reanalyze the data using the monopole form, and obtain the results which are consistent with those from the dipole form. For the uncertainties in chiral extrapolation, we test two alternative extrapolations [7], and find that all results are consistent with each other. For the contamination from excited states, we employ the new projection operator [7] which eliminates the \( S_11 \) state, and conclude that such contaminations are negligible.

Our final result for the magnetic moment is \( G_M(0) = -0.017(25)(07) \), where the first error is statistical and the second is systematic from uncertainties of the \( Q^2 \) extrapolation and chiral extrapolation. We also obtain \( \Lambda = 0.58(16)(19) \) for dipole mass or \( \Lambda = 0.34(17)(11) \) for monopole mass, and \( g_E^2 = 0.027(16)(08) \). These lead to \( G_M(Q^2) = -0.015(23), G_E(Q^2) = 0.0022(19) \) at \( Q^2 = 0.1 \text{GeV}^2 \), where error is obtained by quadrature from statistical and systematic errors. In Fig. 2, we plot \( G_M(Q^2), G_E(Q^2) \), where the shaded regions correspond to the square-summed error. Compared to the global analysis of the experimental data, e.g., \( G_M(Q^2) = 0.29(21) \) and \( G_E(Q^2) = -0.008(16) \) at \( Q^2 = 0.1 \text{GeV}^2 \) [11], our results are consistent with them, with an order of magnitude smaller error [7].

**SECOND MOMENT OF THE NUCLEON**

The (asymmetry of) strangeness second moment of the nucleon \( \langle x^2 \rangle_{s-\bar{s}} = \int_0^1 dxx^2 (s(x) - \bar{s}(x)) \) can be obtained by

\[ \text{Tr} \left[ \Gamma_{\pm}^{3q} \Pi_{F_{g,0}} (\pm \bar{\mu}, t_2; \pm \bar{\mu}_0) \right] / \text{Tr} \left[ \Gamma_{\pm}^{3q} \Pi_{F_{g,0}} (\pm \bar{\mu}, t_2; \mu_0) \right] = \pm p_z^2 \langle \chi^2 \rangle_{s-\bar{s}} \]
with the three-index operator defined as

$$T_{3ii} \equiv -\frac{1}{3}\left[\bar{q}\gamma_4 D_i \bar{D} q + \bar{q}\gamma_i D_4 \bar{D} q + \bar{q}\gamma_i D_i \bar{D} q\right], \quad (5)$$

where $i \neq 4$, and the upper (lower) sign corresponds to the forward (backward) propagation as before.

In Fig. 3 (left), we plot the ratio of 3pt to 2pt for $\langle x^2 \rangle_{s-\bar{s}}$ in terms of $t_2$ for $\kappa_{ud} = 0.13760$, $\vec{p}^2 = (2\pi/La)^2$, where the summation of operator insertion time $t_1$ is taken as was done for the form factor analysis. Note that the linear slope corresponds to the signal for $\langle x^2 \rangle_{s-\bar{s}}$. One can clearly see that increasing $N_{src}$ reduces the error bar significantly again (about a factor of $\sqrt{N_{src}}$, i.e., almost ideally). In Fig. 3 (right), we plot the bare value of the $\langle x^2 \rangle_{s-\bar{s}}$ in terms of $(m_Ka)^2$, and perform the chiral extrapolation. We find that the result at each $\kappa_{ud}$ and the chiral extrapolated result are basically consistent with zero within the error-bar. For the final quantitative result, it is necessary to take the renormalization factor into account. Systematic uncertainties have to be examined as well. The study along this line is in progress.

ACKNOWLEDGMENTS

We thank the CP-PACS/JLQCD Collaborations for their configurations. This work was supported in part by U.S. DOE grant DE-FG05-84ER40154. TD is supported in part by Grant-in-Aid for JSPS Fellows 21-5985. Research of NM is supported by Ramanujan Fellowship. The calculation was performed at Jefferson Lab, Fermilab and the University of Kentucky, partly using the Chroma Library [12].

REFERENCES