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Citation: AIP Conf. Proc. **1350**, 321 (2011); doi: 10.1063/1.3601431
View online: http://dx.doi.org/10.1063/1.3601431
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1350&Issue=1
Published by the American Institute of Physics.

Transverse momentum distributions inside the nucleon from lattice QCD

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Abstract. We study transverse momentum dependent parton distribution functions (TMDs) with non-local operators in lattice QCD, using MILC/LHPC lattices. Results obtained with a simplified operator geometry show visible dipole deformations of spin-dependent quark momentum densities.

Keywords: transverse momentum; parton distribution functions; lattice; QCD
PACS: 12.38.Gc, 13.88.+e, 13.85.Ni

INTRODUCTION

Transverse momentum dependent parton distribution functions (TMDs) describe the longitudinal and transverse motion of quarks (or gluons) in a nucleon with large momentum $\mathbf{P} = (0, 0, \mathbf{P}_z)$. The quark momentum k in terms of light cone coordinates $k^\pm \equiv (k^0 \pm k^3)/\sqrt{2}$, $\mathbf{k}_\perp = (\mathbf{k}_x, \mathbf{k}_y)$ scales like $k^+ : \mathbf{k}_\perp : k^- \sim P^+ : 1 : (P^+)^{-1}$ with the large momentum component P^+ of the nucleon. TMDs resolve the dependence on $x \equiv k^+/P^+$ and transverse momentum $\mathbf{k}_\perp$, see illustration Fig. 1 a). In spin-polarized channels at leading twist, TMDs encode dipole- or quadrupole-shaped deformations of the nucleon in the $\mathbf{k}_\perp$-plane, see, e.g., Fig. 1 b) for a result from first explorative lattice QCD calculations [1, 2, 3]. Our studies have been motivated by a history of successful lattice computations of x-moments of generalized parton distributions (GPDs), providing images of the nucleon in the impact parameter, $\mathbf{b}_\perp$-, plane, see [4] for a review. A remaining theoretical problem concerns the precise form of the correlator defining TMDs in the continuum, see [5, 6] and references therein. In its basic form, it is given by [7]

$$\Phi_q^{[\Gamma]}(x, \mathbf{k}_\perp; P, S; \mathscr{C}) \equiv \int dk^- \int \frac{d^4 l}{(2\pi)^4}\, e^{-ik\cdot l}\, \underbrace{\frac{1}{2} \langle P, S|\ \bar{q}(l)\, \Gamma\, \mathscr{U}[\mathscr{C}_l]\, q(0)\ |P, S\rangle}_{\widetilde{\Phi}_q^{[\Gamma]}(l, P, S; \mathscr{C})} \Big|_{k^+ = xP^+}$$

$$= \frac{1}{P^+} \underbrace{\int \frac{d(l\cdot P)}{2\pi}\, e^{-i(l\cdot P)x}}_{\not{\int}_X} \underbrace{\int \frac{d^2 \mathbf{l}_\perp}{(2\pi)^2}\, e^{i\mathbf{l}_\perp\cdot \mathbf{k}_\perp}}_{\not{\int}_M} \widetilde{\Phi}_q^{[\Gamma]}(l, P, S; \mathscr{C}) \Big|_{l^+ = 0} \quad (1)$$

DIFFRACTION 2010
AIP Conf. Proc. 1350, 321-324 (2011); doi: 10.1063/1.3601431

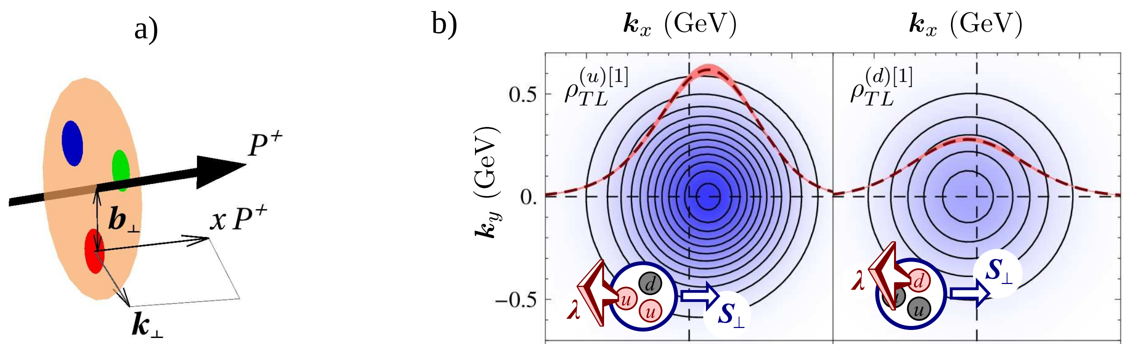

FIGURE 1. a) Illustration of quark degrees of freedom in the nucleon at large momentum. b) Dipole-deformed x-integrated densities obtained with straight gauge links at a pion mass $m_\pi \approx 500\,\mathrm{MeV}$. The insets display the spin polarization of the quarks (arrow pointing out of the paper plane) and of the nucleon (arrow pointing right).

where Γ is a Dirac matrix. The Wilson line $\mathscr{U}[\mathscr{C}_l]$ running along a continuous path $\mathscr{C}_l$ from l to 0 ensures gauge invariance of the expression. For the SIDIS and Drell-Yan scattering process, the Wilson line extends to infinity along a direction v that needs to be chosen (almost) lightlike, such that the cross section factorizes into hard, perturbative parts and soft contributions, see, e.g., Ref. [8]. Based on its symmetry transformation properties, the above correlator can be parametrized in terms of TMDs [9, 10, 11], e.g.,

$$2\,\rho_{TL}^{(q)} \equiv \Phi_q^{[\gamma^+ + \lambda\gamma^+\gamma^5]} = f_{1,q} + \lambda\,\frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{m_N} g_{1T,q} + \left[\frac{\mathbf{S}_j \varepsilon_{ji} \mathbf{k}_i}{m_N}\, f_{1T,q}^\perp\right]_{\text{odd}}, \tag{2}$$

Here λ is the longitudinal quark polarization, and Λ and $\mathbf{S}_\perp$ are longitudinal and transverse nucleon polarization, respecively. The leading-twist TMDs $f_{1,q}$, $g_{1T,q}$, $f_{1T,q}^\perp$ are real-valued functions of x and $\mathbf{k}_\perp^2$.

STRAIGHT LINK TMDS FROM THE LATTICE

In light of the uncertainties about the precise form of the continuum correlator, and to develop our methods, our first lattice studies employ a simple operator geometry that does not relate to a specific scattering process: We connect the quark fields with a direct, straight Wilson line. For the resulting "process-independent" TMDs, the "naively time-reversal odd" functions, such as the Sivers function $f_{1T,q}^\perp$ in Eq. (2), vanish exactly.

In our approach, we calculate matrix elements $\langle P,S|\,O\,|P,S\rangle$ from ratios of three- and two-point functions using the same techniques as GPD calculations by the LHP collaboration in Ref. [12]. We also use the same sequential propagators and quark propagators, calculated by LHPC with domain-wall valence fermions on top of asqtad-improved staggered MILC gauge configurations [13, 14, 15] with 2+1 quark flavors at a lattice spacing $a \approx 0.12\,\mathrm{fm}$. The difference with respect to GPD calculations is that we directly insert the non-local operator $O \equiv \bar{q}(l)\Gamma\mathscr{U}[\mathscr{C}_l]q(0)$ in our three-point function. The Wilson line $\mathscr{U}[\mathscr{C}_l]$ is approximated as a step-like product of HYP-smeared link-variables as illustrated in Fig. 2 a). To renormalize, we use the renormalization condition

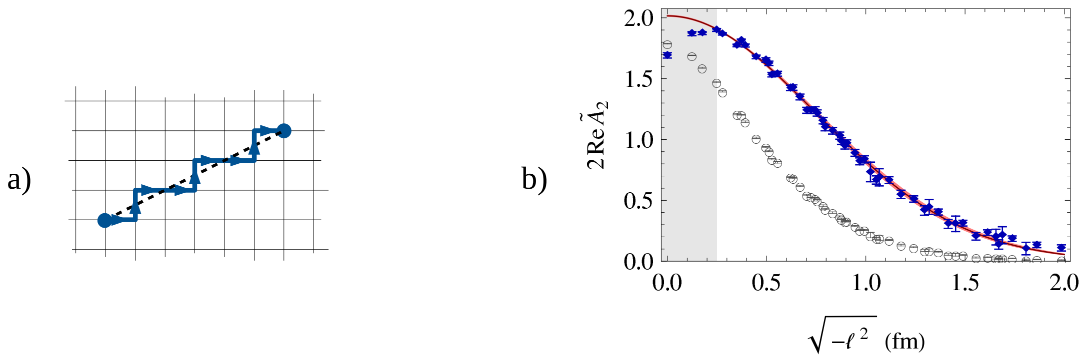

FIGURE 2. a) Representation of a straight Wilson line (dashed line) as a step-like product of link variables. b) Amplitude $\widetilde{A}_2(l^2,0)$ for up quarks at a pion mass $m_\pi \approx 500\,\mathrm{MeV}$, using straight gauge links.

employed in Refs. [16, 17], see Ref. [2] for details. At present, we lack an interpretation of this renormalization condition in terms of a physical scale in the context of TMD factorization.

The connection between the matrix elements $\tilde{\Phi}^{[\Gamma]}$ and TMDs is established through a parametrization in terms of Lorentz-invariant amplitudes $\widetilde{A}_i(l^2, l\cdot P)$. For straight Wilson lines, we obtain in analogy to the parametrization in terms of amplitudes $A_i(k^2, k\cdot P)$ [9]:

$$\begin{aligned}
\tilde{\Phi}^{[\gamma^\mu]} &= 2P^\mu \widetilde{A}_2 + 2i\,{m_N}^2\, l^\mu \widetilde{A}_3\,, \\
\tilde{\Phi}^{[\gamma^\mu\gamma^5]} &= -2m_N S^\mu \widetilde{A}_6 - 2i\,m_N P^\mu (l\cdot S)\widetilde{A}_7 + 2\,{m_N}^3\, l^\mu (l\cdot S)\widetilde{A}_8\,.
\end{aligned}$$

The TMDs are then obtained by

$$f_1(x,\mathbf{k}_\perp^2) = 2 \int\!\!\!\!\!\!X \int\!\!\!\!\!\!M\, \widetilde{A}_2(l^2, l\cdot P)\,, \qquad g_{1T}(x,\mathbf{k}_\perp^2) = 4m_N^2 \partial_{\mathbf{k}_\perp^2} \int\!\!\!\!\!\!X \int\!\!\!\!\!\!M\; \widetilde{A}_7(l^2, l\cdot P)\,.$$

Our Euclidean lattice approach is restricted to the determination of amplitudes $\widetilde{A}_i$ for $l^0 = -il_4 = 0$, i.e., to the region $l^2 < 0$, $|l\cdot P| \le \sqrt{-l^2}|\mathbf{P}|$, where $\mathbf{P}$ is the selected three-momentum of the nucleon on the lattice. Nevertheless, x-integrated TMDs and densities are directly accessible: Integrating Eq. (1) with respect to x removes $\int\!\!\!\!\!\!X$ and sets $l\cdot P$ to zero. Correspondingly, the x-integral of, e.g., f_1 becomes $\int_{-1}^{1} dx\, f_1(x,\mathbf{k}_\perp^2) \equiv f_1^{[1]}(\mathbf{k}_\perp^2) = 2\int\!\!\!\!\!\!M\, \widetilde{A}_2(l^2,0)$. In Fig. 2 b), we show unrenormalized lattice data for $\widetilde{A}_2(l^2,0)$ as open symbols and renormalized data as solid points. Studies of the Wilson line operator at different lattice spacings indicate that discretization errors become large for link lengths $l \lesssim 0.25\,\mathrm{fm}$, i.e., in the gray shaded region of Fig. 2 b). The curve and statistical error band correspond to a Gaussian fit to the renormalized data in the range $\sqrt{-l^2} \ge 0.25\,\mathrm{fm}$. Similar fits for $\widetilde{A}_7$ enable us to calculate the "worm-gear" function $g_{1T}^{[1]}$, and correspondingly, the dipole deformed x-integrated density $\rho_{TL}^{(q)[1]}$ defined in Eq. (2) and shown in Fig. 1 b). For the Gaussian distributions, the strength of the deformation in

Fig. 1 b) can be expressed as an average transverse quark momentum shift [1]

$$\langle \mathbf{k}_x \rangle_{TL} = m_N \frac{\int d^2\mathbf{k}_\perp \, \mathbf{k}_\perp^2/(2m_N^2) \, g_{1T}^{[1]}(\mathbf{k}_\perp)}{\int d^2\mathbf{k}_\perp \, f_1^{[1]}(\mathbf{k}_\perp)} = \begin{cases} 67(5)\,\mathrm{MeV} & \text{(up)} \\ -30(5)\,\mathrm{MeV} & \text{(down)} \end{cases},$$

which we find to be rather insensitive to the renormalization condition used. Reference [18] reveals a remarkable similarity of our results for $\langle \mathbf{k}_x \rangle_{TL}$ with a light-cone constituent quark model [19], despite the unphysically large quark masses employed in our lattice calculation.

An ongoing project with staple-shaped gauge links can potentially address TMDs specific to SIDIS or the Drell-Yan process, including T-odd functions responsible for single-spin asymmetries.

ACKNOWLEDGMENTS

We are grateful to the LHP and MILC collaborations, for providing us gauge configurations and propagators. We thank Vladimir Braun, Meinulf Göckeler, Gunnar Bali, Markus Diehl, Alexei Bazavov, and Dru Renner for helpful discussions. Our software uses the `Chroma`-library [20], and we use USQCD computing resources at Jefferson Lab. We acknowledge support by the Emmy-Noether program and the cluster of excellence “Origin and Structure of the Universe” of the DFG (Ph.H. and B.M.), SFB/TRR-55 (A.S.) and the US Department of Energy grant DE-FG02-94ER40818 (J.N.). Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.

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