AIP | Conference Proceedings

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Citation: AIP Conf. Proc. **1343**, 536 (2011); doi: 10.1063/1.3575086
View online: http://dx.doi.org/10.1063/1.3575086
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=1343&Issue=1
Published by the American Institute of Physics.

The running of the Schrödinger functional coupling from four-flavour lattice QCD with staggered quarks

Paula Perez Rubio* and Stefan Sint†

*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
†School of Mathematics, Trinity College Dublin, Dublin 2, Ireland

Abstract. We present preliminary results for the running coupling in the Schrödinger functional scheme in QCD with four flavours. A single-component staggered quark field is used on lattices of size $(L/a)^3 \times (L/a \pm 1)$. This provides us with 2 different regularisations of the same renormalized coupling, and thus some control over the size of lattice artefacts. These are found to be comparatively large, calling for a more refined analysis, which still remains to be done.

Keywords: strong coupling, lattice QCD, renormalization group

INTRODUCTION

QCD is a very economical theory: with just a few parameters, namely the quark masses and the coupling constant it is very predictive. If one fixes the parameters by matching a corresponding number of hadronic observables, one should be able to compute the renormalised parameters in dimensional regularisation at any high energy scale such as the one set by the Z-boson mass. A major problem consists in the wide scale differences which need to be bridged: at low energies one needs to avoid finite volume effects on the hadronic quantities, whereas the renormalisation scales needed to safely reach the perturbative domain must still be small in lattice units. Taken together these requirements constitute the problem of large scale differences for lattice QCD. A solution has been spelt out some time ago [1] and is based on a combination of finite size scaling techniques [2] and the use of intermediate renormalisation scheme where the scale is set by the linear extent of the space-time volume, i.e. $\mu = 1/L$. Practical schemes which are gauge-invariant, non-perturbatively defined and amenable to a perturbative treatment are not easy to find. The best solution to date is provided by the Schrödinger functional [3, 4] by which one means the Euclidean functional integral of QCD on a hyper-cylinder (cf. fig. 1),

$$\mathscr{Z}[C',C] = \int_{\text{fields}} \exp(-S) \tag{1}$$

with (inhomogeneous) Dirichlet boundary conditions for the spatial gluon components

$$A_k(x)|_{x_0=0} = C_k, \qquad A_k(x)|_{x_0=T} = C'_k, \tag{2}$$

and homogeneous Dirichlet conditions for half of the quark and anti-quark fields [4]. The Schrödinger functional allows to define a renormalized coupling by choosing a 1-parameter family of spatially constant Abelian

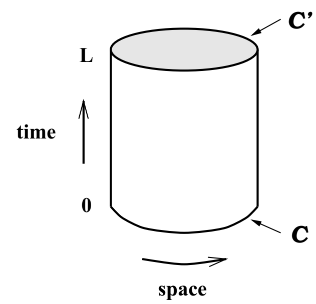

FIGURE 1. The Schrödinger functional is defined on a space-time manifold given by a hyper-cylinder with Dirichlet boundary conditions in time.

boundary fields $C_k(\eta)$ and $C'_k(\eta)$ such that the induced background field B_μ represents an absolute minium of the action [3], corresponding to a colour electric field. The coupling $\bar{g}(L)$ is then defined as the response coefficient to the change of the colour electric field. It is obtained by differentiating the effective action,

$$\Gamma[B] = -\ln \mathscr{Z}[C',C], \tag{3}$$

with respect to η,

$$\frac{1}{\bar{g}^2(L)} \propto \frac{\partial}{\partial \eta}\Gamma[B]\bigg|_{\eta=0}, \tag{4}$$

with $T = L$ and all quark masses set to zero [5]. The proportionality constant is defined such that $\bar{g} = g_0$ at the tree-level of perturbation theory. Given the definition of the coupling one proceeds to defined the so-called step-scaling function (SSF)

$$\sigma(u) = \bar{g}^2(2L)|_{u=\bar{g}^2(L)}, \quad \Rightarrow \ln 2 = \int_{\sqrt{\sigma(u)}}^{\sqrt{u}} \frac{\mathrm{d}g}{\beta(g)}, \tag{5}$$

The IX International Conference on Quark Confinement and the Hadron Spectrum - QCHS IX
AIP Conf. Proc. 1343, 536-538 (2011); doi: 10.1063/1.3575086

which is an integrated version of the β-function. For fixed u, the SSF can be computed from data for pairs of lattices with linear sizes L/a and $2L/a$,

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L). \qquad (6)$$

Repeating the computation for a range of u-values yields the function $\sigma(u)$ in some interval $[u_{\min}, u_{\max}]$ and it is then possible to step up the energy scale recursively:

$$u_0 = \bar{g}^2(L_{\min}), \quad u_k = \sigma(u_{k-1}) = \bar{g}^2(2^k L_{\min}), \qquad (7)$$

for $k = 1, 2, \ldots$. After 7-8 steps, scale differences of O(100) are bridged. At the lower end one needs to relate $L_{\max}$ to hadronic input by computing e.g. $F_\pi L_{\max}$, which is of O(1) and does not pose a scale problem. At high energies one may use perturbation theory to compute

$$g^2_{\overline{\rm MS}}(\mu = 1/L_{\min}) = \bar{g}^2(L_{\min}) + c_1 \bar{g}^4(L_{\min}) + {\rm O}(\bar{g}^6). \quad (8)$$

The scale problem for the coupling is thus solved, and one may then use a similar procedure for the running quark masses and renormalisation constants of composite operators. Existing studies of the running coupling in QCD for $N_{\rm f} = 0, 2, 3, 4$ quark flavours [6]-[9], are based on the lattice formulation with Wilson quarks. Here we use the Schrödinger functional with staggered quarks [10]-[14] to obtain results for $\sigma(u)$ in four-flavour QCD. As $\sigma(u)$ is a universal quantity the outcome will be directly comparable to the results from Wilson quarks [9].

THE SET-UP AND RESULTS

A peculiarity of the Schrödinger functional with staggered quarks is the requirement that the number of lattice points in the time direction must be odd while spatial lattice sizes are even [10, 11]. It is therefore not possible to set $T = L$ exactly. We define $T' = T + sa$ with $s = \pm 1$ and take the continuum limit at fixed $T' = L$ [12, 14]. This modified continuum approach required a major effort to re-compute the O(a) improvement coefficient $c_{\rm t}$ at the tree and one-loop level of perturbation theory [14]. This counterterm eliminates O(a) effects caused by the Dirichlet boundary conditions. A further fermionic counterterm coefficient is known at tree-level, which means that the cancellation of boundary O(a) effects is not yet on a par with Wilson fermions where these coefficients are known to 2-loop and 1-loop accuracy, respectively [15].

The parameter $s = \pm 1$ provides us with 2 regularisations of the coupling which we use to define the lattice SSF $\Sigma(u, a/L)$ in 4 different ways:

normal SSF: $\bar{g}^2(2L)|_{s=\pm 1}$ is considered as function of $\bar{g}^2(L)|_{s=\pm 1}$

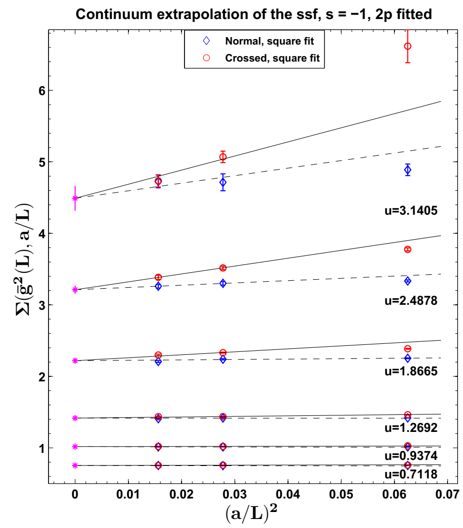

FIGURE 2. The normal (diamonds) and crossed (circles) definitions of the SSF where the argument u is defined by $\bar{g}^2(L)|_{s=-1}$ (numerical values are given in the plot). The rightmost points correspond to $L/a = 4$ data and did not enter the fits (solid and dashed lines).

crossed SSF: $\bar{g}^2(2L)|_{s=\pm 1}$ is considered as function of $\bar{g}^2(L)|_{s=\mp 1}$

Furthermore, the complete cutoff effects at one-loop order are known and have been removed from the data. Numerical simulations have been carried out using the Hybrid Monte Carlo algorithm [16] and a customized version of the MILC code [17]. Data were produced for a range of β-values and lattices with $L/a = 4, 6, 8, 12, 16$. The analysis of autocorrelations was done using the method of [18]. We performed combined continuum extrapolations for the normal and crossed definitions of the SSFs, assuming leading O(a^2) effects and leaving out the $L/a = 4$ data. The results for the case $s = -1$ are shown in figs. 2 and 3, and the continuum results are collected in fig. 4. While the data for the normal definition of the SSF shows relatively little a-dependence, cutoff effects seem to be quite a bit larger for the crossed definition.

CONCLUSIONS

We have computed the running SF coupling in four-flavour QCD with staggered fermions. Two variants of the regularisation are used which allows us to monitor the size of cutoff effects. The cutoff effects seem to be large compared to Wilson quarks, especially in the case of the crossed definition of the SSF. Assuming the dominance of O(a^2) effects is probably too optimistic.

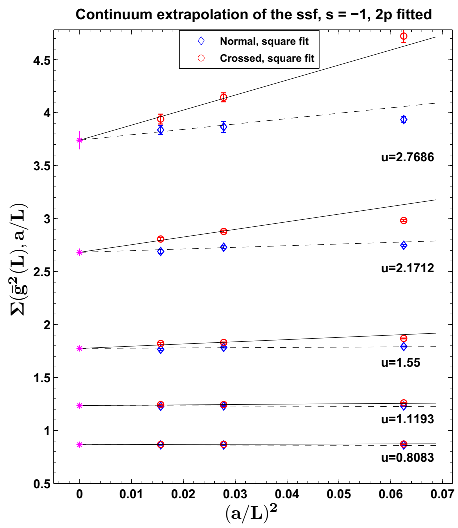

FIGURE 3. The same as fig. 3 but for different u-values.

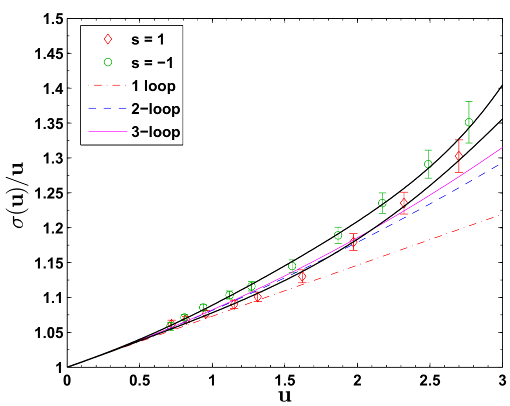

FIGURE 4. The SSF $\sigma(u)$. The dotted-dashed, dashed and solid lines represent the successive perturbative approximations up to 3-loop order. The diamonds represent $s = 1$ data and circles $s = -1$. The thick solid lines are the respective fit functions, with the difference indicating the size of current systematic errors.

A refined analysis, together with a detailed comparison with Wilson quarks and the determination of the Λ-parameter still remains to be done.

There are various ways to improve on the current results. For instance, one might use the data at smaller couplings to estimate the two-loop cutoff effects and subtract these from the data at the larger couplings [8]. Furthermore, with some improvements in the code, it could be envisaged to extend the study to lattices with $L/a = 24$, which would greatly enhance the leverage ot the continuum extrapolation.

ACKNOWLEDGMENTS

This research was supported by the Research Executive Agency (REA) of the European Union under Grant Agreement number PITN-GA-2009-238353 (ITN STRONGnet) and by the Spanish Government through an FPU grant. We are grateful for the support from the Trinity Centre for High-Performance Computing (TCHPC) and the Irish Centre for High-End Computing (ICHEC) where the simulations were carried out.

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