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Two-point functions of quenched lattice QCD in Numerical Stochastic Perturbation Theory

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Abstract. We summarize the higher-loop perturbative computation of the ghost and gluon propagators in $SU(3)$ Lattice Gauge Theory. Our final aim is to compare with results from lattice simulations in order to expose the genuinely non-perturbative content of the latter. By means of Numerical Stochastic Perturbation Theory we compute the ghost and gluon propagators in Landau gauge up to three and four loops. We present results in the infinite volume and $a \to 0$ limits, based on a general fitting strategy.

Keywords: Lattice gauge theory, stochastic perturbation theory, gluon propagator, ghost propagator, Landau gauge
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1. INTRODUCTION

This talk summarizes our work on the higher-loop perturbative gluon and ghost propagators in Landau gauge [1, 2]. The Monte Carlo study of both propagators, which are closely related to each other by Schwinger-Dyson equations (SDE), has attracted much attention outside the lattice community by phenomenologists working on infrared QCD in general and hadron physics (see our original papers for further references). Taken together, both propagators provide us with a definition and the momentum dependence of the running coupling $\alpha_s(q^2)$ directly based on the ghost-gluon vertex.

A simple connection between the two propagators exists in the extreme infrared, both beeing powerlike in a scaling or massive in a decoupling solution. This nonuniqueness reflects the Gribov problem. The effect of nontrivial vacuum structure (vortices, instantons) is manifest also in the gluon propagator, in the intermediate momentum range around $\mathcal{O}(1\text{ GeV})$ where the SDE approach suffers from truncation ambiguities and where nonperturbative lattice calculations are unrivalled. In order to follow the onset of nonperturbative effects, it is desirable to approach this momentum range from high momenta within higher-order perturbation theory. While ordinary diagrammatic lattice perturbation theory (LPT) soon gets too involved to be pursued, Numerical Stochastic Perturbation Theory (NSPT, for a recent review see Ref. [3] and references therein), provides a powerful tool to perform high-loop computations.

[1] Speaker

2. NSPT IN A NUTSHELL

NSPT has its roots in stochastic quantization and is based on a modified Langevin equation equipped with stochastic gauge fixing. We use here a version for quenched lattice QCD with Wilson gauge action. Actually, it is a hierarchy of first-order evolution equations associated with various parts of the gauge link fields U and gluon fields A exposed by an expansion in powers of the lattice coupling $g \propto 1/\sqrt{\beta}$:

$$U_{x,\mu} = \sum_{l\geq 0} \beta^{-l/2} U_{x,\mu}^{(l)}, \quad A_{x+\frac{\hat{\mu}}{2},\mu} = \sum_{l\geq 1} \beta^{-l/2} A_{x+\frac{\hat{\mu}}{2},\mu}^{(l)}. \quad (1)$$

These different orders are separately dealt within the code. The maximal addressable order of perturbation theory is thus limited by the available computing resources (cpu time and memory).

The Langevin simulation is implemented in an Euler scheme with a finite evolution time step. Before the estimator for the gauge dependent ghost and gluon propagators can be evaluated, we have to fix the gauge to the minimal Landau gauge. For this purpose, a sequence of configurations (separated by $\mathcal{O}(50)$ Langevin time steps) is subjected to a Fourier-accelerated gauge-fixing procedure, after which the individual gluon fields, $A_\mu^{(l)}$ (associated with particular perturbative order g^l) are transversal within machine precision.

The propagators are evaluated taking the long-time average of coefficients, order by order in a loop expansion in even powers of g. Contributions from odd powers vanish within the statistical errors. As for any Langevin simulation, one then has to take the limit to vanishing time step. In order to get results comparable with the prac-

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tice of LPT, the continuum limit and the limit of infinite volume must be performed. NSPT results for *finite* lattice volume and spacing can be confronted directly with standard MC results for a given β, provided the definitions of the studied observables is the same.

The gluon two-point function in n-loop order is defined as a convolution of the bilinears of gluon fields (in momentum space) in complementary orders ($A_\mu^{(l)} = T^a A^{a,(l)}$, $p_\mu(k_\mu) = 2\pi k_\mu/(aN)$):

$$\delta^{ab} D_{\mu\nu}^{(n)}(p(k)) = \left\langle \sum_{l=1}^{2n+1} \left[\widetilde{A}_\mu^{a,(l)}(k) \widetilde{A}_\nu^{b,(2n+2-l)}(-k) \right] \right\rangle_U . \tag{2}$$

In Landau gauge we consider $\sum_{\mu=1}^4 D_{\mu\mu}^{(n)} \equiv 3D^{(n)}$ and use the dressing functions ($\hat{p}_\mu(k_\mu) = (2/a)\sin(\pi k_\mu/N)$)

$$J^{G,(n)}(p) = p^2\, D^{(n)}(p(k))\,, \hat{J}^{G,(n)}(p) = \hat{p}^2\, D^{(n)}(p(k))\,. \tag{3}$$

The color diagonal ghost propagator in momentum space is the color trace in the adjoint representation

$$G(p(k)) = \frac{1}{8} \langle \mathrm{Tr}_{\mathrm{adj}}\, M^{-1}(k) \rangle_U\,. \tag{4}$$

In (4) $M^{-1}(k)$ is the Fourier transform of the inverse FP operator in lattice coordinate space. It is expanded in terms of products of various $A^{(l)}$, with the term $M^{(j)}$ collecting all terms of order g^j. This structure allows to express $M^{-1}(k)$ also as an expansion in orders of g in a recursive way. Again we use the ghost dressing functions

$$J^{(n)}(p) = p^2\, G^{(n)}(p(k))\,, \hat{J}^{(n)}(p) = \hat{p}^2\, D^{(n)}(p(k))\,. \tag{5}$$

As an example the cumulatively summed perturbative gluon dressing function for various volumes is shown in Fig. 1 using $g^2 = 6/\beta = 1$.

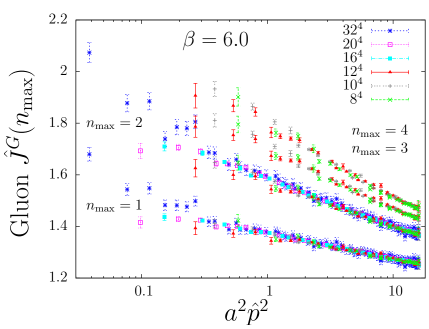

FIGURE 1. The cumulatively summed perturbative gluon dressing function for various volumes.

A reasonable "convergence" of the NSPT results up to few loops (three or four are available now) requires a small bare coupling g. However, g is known to be a poor expansion parameter [4]. One can speed up convergence by "boosting", i.e., trading the bare coupling constant by an effective "boosted" coupling $g_b^2 = g^2/P_{\mathrm{pert}}(g^2) > g^2$. Here P_{pert} is defined by the average perturbative plaquette determined also within our Langevin simulations. The effect of the boosted coupling being larger is overcompensated by the rapid decay of the expansion coefficients with increasing order n.

We illustrate the effect of "boosting" the perturbative expansion and confront the boosted dressing functions with corresponding new Monte Carlo (MC) data of the Berlin group [5] adopting the same definitions for the propagators and the gauge fixing as in NSPT. This is shown in Figs. 2,3 where also the bare and the boosted inverse couplings β and β_{boost} are given. As expected, boosting moves the NSPT data closer to the MC results, but they cannot be reached completely, certainly not at $\beta = 6.0$.

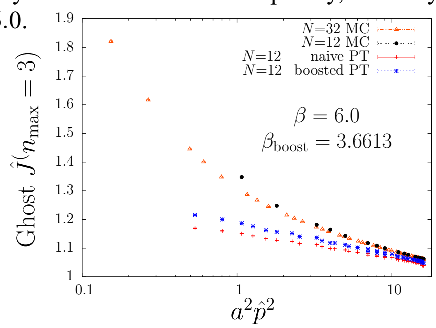

FIGURE 2. The ghost dressing function in three loops for naive and boosted NSPT comapred to MC data for a 12^4 lattice.

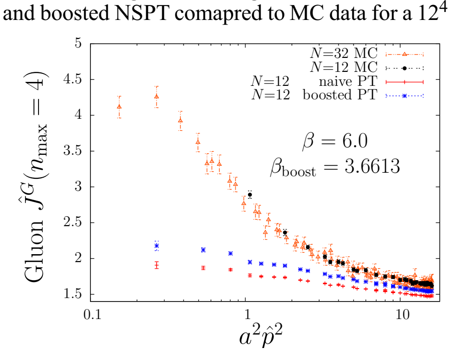

FIGURE 3. Same as in Fig. 2 for the gluon dressing function in four loops.

Here we define the renormalization-group invariant running coupling α_s by the ghost-gluon vertex in a particular (minimal) MOM scheme (see e.g. [6]). It is given in terms of the bare gluon and ghost dressing functions J^{G} and J as follows:

$$\alpha_s(p(k)) = \frac{6}{4\pi\beta} \hat{J}(p(k),\beta)^2\, \hat{J}^G(p(k),\beta)\,. \tag{6}$$

The α_s calculated from the NSPT dressing functions, both summed up to the orders available, is compared to the MC results at $\beta = 6.0$ and 9.0. The corresponding data is shown in Figs. 4,5 again for naive and boosted

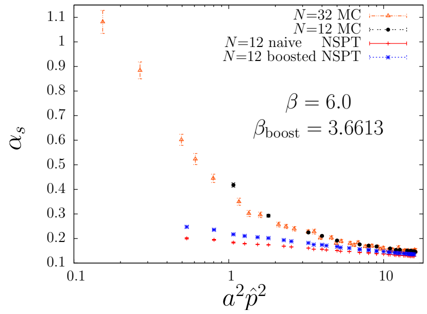

FIGURE 4. Comparing naive and boosted PT (based on NSPT data) for the running coupling constant α_s to corresponding MC data for a 12^4 lattice at $\beta = 6.0$.

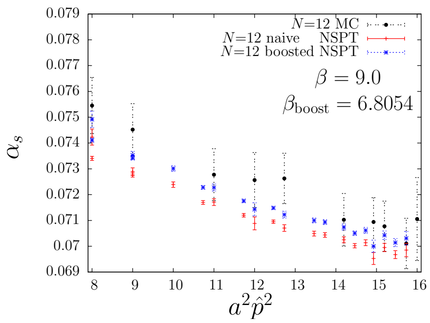

FIGURE 5. Same as in Fig. 4 at $\beta = 6.0$ zoomed into the large momentum region.

perturbation theory. We see that the running coupling from MC simulations is approached at large momenta from below up to 7% for $\beta = 6.0$ and practically approached within the present errors for $\beta = 9.0$, i.e. in an effectively deconfined phase.

3. DRESSING FUNCTIONS IN THE LIMITS $V \to \infty$ AND $ap \to 0$

In the RI'-MOM scheme the dressing functions are just the wave function renormalization constants for the ghost and gluon fields at the renormalization point $\mu^2 = p^2$. The NSPT data available at various volumes and lattice momentum realizations allow us to find the perturbative dressing functions to three-loop accuracy in the bare coupling including the non-logarithmic contributions. Via standard transformations the results can be transformed to the renormalized coupling in the preferred scheme. To find those constants, a fitting procedure has been proposed which takes into account both hypercubic and finite volume effects (for details see [1]). As result we get ($L \equiv \log(pa)^2$)

$$J(a,p,\beta) = 1 + \frac{1}{\beta}\Big[-0.0854897\,L + 0.525314\Big]$$
$$+\frac{1}{\beta^2}\Big[0.0215195\,L^2 - 0.358423\,L + 1.4872(57)\Big]$$
$$+\frac{1}{\beta^3}\Big[-0.0066027L^3 + 0.175434\,L^2 \qquad (7)$$
$$-1.6731(1)\,L + 4.94(27)\Big]$$

$$J^G(a,p,\beta) = 1 + \frac{1}{\beta}\Big[-0.24697\,L + 2.29368\Big]$$
$$+\frac{1}{\beta^2}\Big[0.08211L^2 - 1.48445\,L + 7.93(12)\Big]$$
$$+\frac{1}{\beta^3}\Big[-0.02964L^3 + 0.81689\,L^2 \qquad (8)$$
$$-8.13(3)\,L + 31.7(5)\Big]$$

The results for one-loop lattice perturbation theory are known for a long time, the higher-loop non-leading log's and constant contributions are our predictions for the Landau gauge.

4. CONCLUSION

We have calculated the gluon propagator in Landau gauge up to four and the ghost propagator up to three loops in NSPT. The dressing functions summed using boosted PT are compared to recent MC measurements of the Berlin Humboldt University group. At large lattice momenta the dressing functions with more than four loops will match the MC measurements, thus enabling a fair accounting of the perturbative tail taking care of discretization effects. This can be used as an alternative to fitting the high momentum tail of MC results by continuum-like formulae.

We worked out the relation to standard LPT in limits $V \to \infty$ and $pa \to 0$. For this aim we developed a fitting strategy for lattice artifacts and finite-size corrections. We find good agreement with known one-loop results of diagrammatic LPT and present original two- and three-loop results for the propagators.

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