Theory of the ac Spin-Valve Effect

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The spin-valve complex magnetoimpedance of symmetric ferromagnet–normal-metal–ferromagnet junctions is investigated within the drift-diffusion (standard) model of spin injection. The ac magnetoresistance—the real part difference of the impedances of the parallel and antiparallel magnetization configurations—exhibits an overall damped oscillatory behavior, as an interplay of the diffusion and spin relaxation times. In wide junctions the ac magnetoresistance oscillates between positive and *negative* values, reflecting resonant amplification and depletion of the spin accumulation, while the line shape for thin tunnel junctions is predicted to be purely Lorentzian. The ac spin-valve effect could be a technique to extract spin transport and spin relaxation parameters in the absence of a magnetic field and for a fixed sample size.

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Electrical spin injection from a ferromagnetic (F) to a nonmagnetic (N) conductor is essential for spintronics [1,2]. Predicted by Aronov [3] and first realized by Johnson and Silsbee [4–6], it is now a well-established concept. A biased ferromagnetic-nonmagnetic junction generates a nonequilibrium spin accumulation within the spin diffusion length at the interface, building a nonequilibrium resistance [5,7,8]. In an FNF junction this nonequilibrium resistance gives rise to the difference in the junction resistances for parallel (P) and antiparallel (AP) magnetization orientations of the F regions—the giant magnetoresistance (GMR) [9,10]. Drift-diffusion theory along with the spin accumulation concept successfully describes magnetoresistance effects in charge neutral [11,12] as well as in space-charge systems [13,14], enabling one to obtain relevant spin-related materials parameters [15], such as the spin relaxation times.

Recently, Rashba has generalized the spin-polarized drift-diffusion theory to the alternating current (ac) regime [8,16]. We apply this theory and investigate the complex impedance $\mathcal{Z}(\omega)$ of symmetric FNF junctions. We show that the real part of the spin-valve magnetoimpedance (we call it here ac magnetoresistance) $\Delta\mathcal{Z} = \mathcal{Z}_{\mathrm{AP}} - \mathcal{Z}_{\mathrm{P}}$ of the junctions exhibits damped oscillations as a function of frequency. The oscillation period is given by the diffusion time through the normal layer. In mesoscopic junctions (of sizes up to the spin relaxation length L_s), the ac magnetoresistance can be *negative* at experimentally accessible frequencies, meaning that the antiparallel configuration has a lower ac resistance than the parallel one. The negative ac magnetoresistance is a consequence of a resonant spin accumulation effect, namely, a resonant spin amplification in the P configuration and a resonant spin depletion in the AP one. In nanoscale junctions (with sizes much less than L_s), with tunnel contacts, the oscillation period is large, leaving a nice Lorentzian profile with the width of the spin relaxation rate. A one-parameter fit to the line shape (either damped oscillator or Lorentzian) determines the spin relaxation time τ_s.

We present the ac spin-valve effect as an alternative to other methods that measure τ_s of nonmagnetic conductors, such as the conduction electron spin resonance, spin pumping, or the Hanle effect, which require magnetic fields, or to the dc spin injection method (in vertical or lateral geometries), which requires studying various sample sizes (distances to electrodes) to extract the spin diffusion length [17]. In a sense the ac spin-valve effect is similar to the Hanle effect, which is widely used to find spin relaxation times in metals and semiconductors [4,18], but the role of the magnetic field is taken by the frequency; in the Hanle effect too the signal in general oscillates as a function of magnetic field, with a modified Lorentzian shape in the diffusive regime [2].

A microwave measurement of τ_s in the absence of a magnetic field may be important as in many conductors the spin relaxation time depends strongly on it; a striking case is aluminum in which τ_s decreases by an order of magnitude as the magnetic field increases from 0.05 to 1.3 T [19]. Still, τ_s obtained by spin resonance tend to be, for a given temperature, much greater than that obtained from transport techniques, as catalogued for Al and Cu in Ref. [20]. The case of Au is even more striking, as spin resonance shows that at low temperatures the ratio of τ_s to the momentum relaxation time is about one, while transport techniques predict the ratio to be about 100 [21,22]; at room temperature, at which phonons are relevant, the ratio is about 10, as measured by spin pumping [23] which requires both magnetic field and nanoscale transparent junctions. For extracting bulk spin relaxation times it may be preferable to work with tunnel contacts and mesoscopic samples, so that spin relaxation is not strongly influenced by the interface and surface effects. (Various techniques for measuring τ_s as well as useful data are given in the review Ref. [15].) The ac spin-valve method could potentially explore nano and mesoscopic spin valves, in both vertical and lateral geometries, at no magnetic field applied to the normal conductor, and provide the spin relaxation times at a fixed sample size [24].

We consider a symmetric FNF junction as comprising two FN junctions in series, see Fig. 1. Each FN junction has a contact (c) region with a spin-dependent conductance; otherwise spin is assumed to be preserved at the contact. The spin-valve dc magnetoresistance $\Delta\mathcal{R} = \Delta\mathcal{Z}(\omega = 0)$ of a symmetric FNF junction, whose N region has width d and the F regions have widths much greater than the spin diffusion lengths, can be expressed analytically within the drift-diffusive regime [1,8,16,25]. This dc formula has a straightforward extension to the harmonic ac regime, and we write the complex magnetoimpedance as

$$\Delta\mathcal{Z}(\omega, d) = \frac{8r_N(\omega)[r_F(\omega)P_{\sigma_F} + r_c P_{\Sigma_c}]^2 e^{d/L_{sN}(\omega)}}{[r_F(\omega) + r_c + r_N(\omega)]^2 e^{2d/L_{sN}(\omega)} - [r_F(\omega) + r_c - r_N(\omega)]^2}, \tag{1}$$

by indicating the complex frequency-dependent quantities (labeled by the region N and F),

$$\tau_s(\omega) = \tau_s/(1 - i\omega\tau_s), \tag{2}$$

$$L_s(\omega) = L_s/\sqrt{1 - i\omega\tau_s}, \tag{3}$$

$$r(\omega) = r/\sqrt{1 - i\omega\tau_s}. \tag{4}$$

Here τ_s is the spin relaxation time, $L_s = \sqrt{D\tau_s}$ is the spin diffusion length, D is the diffusivity, and $r = L_s/\sigma$ is the effective resistance, with σ denoting the conductivity. The effective contact resistance is $r_c = (\Sigma_\uparrow + \Sigma_\downarrow)/4\Sigma_\uparrow\Sigma_\downarrow$, with Σ_λ the contact conductance of spin λ. Finally, P_{σ_F} and P_{Σ_c} denote the spin polarization of the conductivity and conductance of the F and contact region, respectively. Driving ac is assumed to be harmonic with the angular frequency $\omega = 2\pi f$, i.e., $j(t) \propto e^{-i\omega t}$.

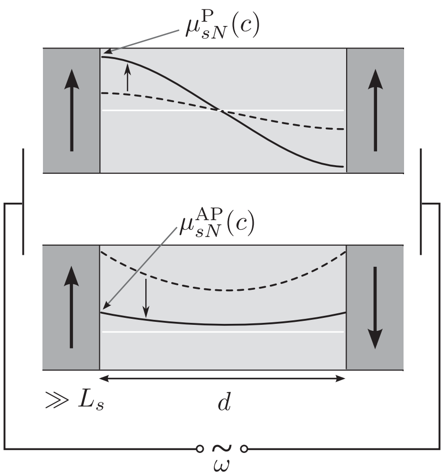

FIG. 1. Scheme of an FNF spin valve. The spacer N region has width d and the sizes of the ferromagnetic electrodes are assumed greater than the corresponding spin diffusion length L_s. In the dc regime the parallel configurations result in smaller spin accumulation (dashed line) than in the antiparallel one, demonstrated by the positive dc spin-valve magnetoresistance. In the ac regime, this can be reversed (solid): at certain frequency ranges there can be a resonant spin amplification in the parallel and spin depletion in the antiparallel configuration, resulting in a negative ac magnetoresistance.

We analyze the spin-valve impedance, based on Eq. (1), for a realistic model Py/Cu/Py junction, with the following experimentally obtained data [17,20] at the temperature $T = 4.2$ K: $L_{sN} = 1$ μm, $\tau_{sN} = 42$ ps, $D_N = 238$ cm^2 s^{-1}, $r_N = 14$ fΩ m^2, $L_{sF} = 5.5$ nm, $\tau_{sF} = 0.6$ ps, $D_F = 0.5$ cm^2 s^{-1}, $r_F = 0.42$ fΩ m^2, $P_{\sigma_F} = 0.22$. For the contact characteristics we employ [26]: $r_c(\simeq r_F) = 0.5$ fΩ m^2 and $P_{\Sigma_c} = 0.4$, so the contact interface is generic, neither tunnel nor transparent. The specific spin resistivities r_F, r_N, and r_c and hence the spin valve $\Delta\mathcal{Z}(\omega, d)$ are evaluated for a unit cross section. In the experiment one divides these resistivities by the actual conductor cross sections, which could be $10^{-3} - 1$ μm^2.

Figure 2 presents the calculated magnetoresistance. In Fig. 2(b) we show the dc magnetoresistance as a function of the d. With increasing d the magnetoresistance exponentially decreases, as the injected spin accumulation is damped. The plot in Fig. 2(c) shows the ratio of the ac to the dc magnetoresistance, $\mathrm{Re}[\Delta\mathcal{Z}(f, d)]/\Delta\mathcal{R}(d)$, as a function of d and frequency $f = \omega/2\pi$. For a given d, the ac magnetoresistance oscillates as a function of f, between positive and negative values. The negative peaks are considerable fractions (tens of percents) of the dc values. On the d-f plot the oscillations show hyperbolic stripes. In Fig. 2(a) the oscillations are shown for $d = 4$ μm. For thin samples, the dependence on f is rather weak for this generic junction. We will see below that for tunnel junctions the dependence becomes Lorentzian.

To be specific, consider $d = L_{sN} = 1$ μm. The ac magnetoresistance remains positive for $f < 34.8$ GHz. Further increase in the driving frequency leads to a negative ac spin-valve magnetoresistance: $\mathrm{Re}(\Delta\mathcal{Z}) < 0$. For $d = 3L_{sN} = 3$ μm the spin-valve magnetoresistance remains positive up to the frequency $f \approx 6$ GHz; then it becomes negative for 6 GHz $\lesssim f \lesssim 26.1$ GHz. There should be more oscillations observable at larger values of d, but at the cost of exponentially reducing the magnitude, see Fig. 2(a). We will see below that the relevant time scale parameter for the oscillations is the diffusion time through

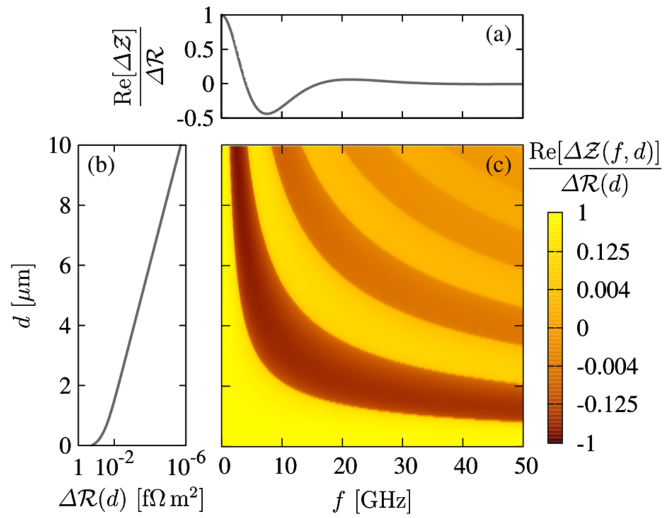

FIG. 2 (color online). ac spin-valve effect in a model Py/Cu/Py junction. (a) Calculated ac/dc ratio of the spin-valve magnetoresistance as a function of $f = \omega/2\pi$ for $d = 4\ \mu$m. (b) Calculated dc spin-valve magnetoresistance as a function of d. (c) Calculated ac/dc ratio of the spin-valve magnetoresistance as a function of d and driving frequency f. The visible light and dark bands of equal signs are separated by the node lines, $\mathrm{Re}[\Delta Z(f, d)] = 0$.

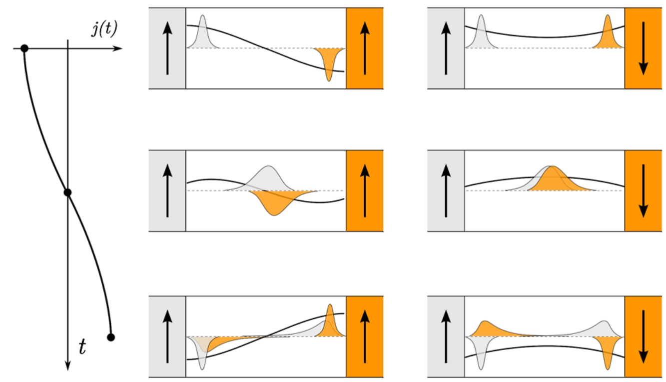

FIG. 3 (color online). Mechanism for the resonant amplification and depletion of the spin accumulation in an ac-driven FNF junction for parallel (P) and antiparallel (AP) configurations. The solid lines within the horizontal of the N spacer represent actual profiles of the nonequilibrium spin accumulation $\mu_{sN}^{\mathrm{P/AP}}(x, t)$, which correspond to the harmonic ac signal $j(t)$ shown on the left. The injected and extracted spin packets and their diffused and spread positions are shown at three distinct times $t = 0$, $t = T_N/4$, and $t = T_N/2$, where T_N is the characteristic diffusion time across the N conductor. Frequency of the driving ac is close to $1/T_N$ and tones of the packets correspond to F conductors which initially emitted them.

the spacer layer. For our model junction, a reasonable parameter range for measuring the ac oscillations would be the sample sizes $L_{sN} \lesssim d \lesssim 4L_{sN}$. The involved frequency, $f = \omega/2\pi$, ranges are 1 GHz–50 GHz, experimentally well accessible.

Mathematically, the spin-valve oscillations appear naturally. The real dc transport parameters become in the ac case complex, see Eqs. (2)–(4). The imaginary part of $d/L_{sN}(\omega)$ gives rise to the ac exponential $e^{d/L_{sN}(\omega)}$ in Eq. (1) with the trigonometric character and hence a certain oscillatory behavior of the complex spin-valve impedance $\Delta Z(\omega)$. For the frequencies $\omega \ll \tau_{sN}^{-1} (\ll \tau_{sF}^{-1})$ the imaginary part of $d/L_{sN}(\omega)$ plays no role, see Eq. (3). The ac magnetoresistance exhibits changes on the scales of the relaxation rate $1/\tau_s$ or the diffusion rate through the spacer. These provide the practical limit for the use of microwaves in the experiment.

We now give a qualitative picture of the predicted oscillatory behavior, including the negative ac spin-valve magnetoresistance. First, we show that the spin-valve impedance $\Delta Z(\omega)$ is related to the contact values of the spin accumulations in N, for P and AP configurations. From the standard spin injection model for a symmetric FNF junction we derive the following formula [27]:

$$\frac{\mu_{sN}^{\mathrm{P}}(c, t) - \mu_{sN}^{\mathrm{AP}}(c, t)}{j(t)} = \frac{r_F(\omega) + r_c}{r_F(\omega)P_{\sigma_F} + r_c P_{\Sigma_c}} \Delta Z(\omega). \quad (5)$$

Here $\mu_{sN}^{\mathrm{P/AP}}(c, t)$ represent the actual nonequilibrium spin accumulation in the N spacer for P and AP configurations, respectively, at the left FN contact interface c (see Fig. 1) and $j(t)$ is the driving harmonic ac. To understand the ac magnetoresistance oscillations, one needs to look at the contact spin accumulation only.

The qualitative picture is in Fig. 3, which shows P and AP configurations at 3 times $t = 0$, $t = T_N/4$, and $t = T_N/2$, where $T_N = \tau_{sN} d^2/L_{sN}^2 = d^2/D_N$ is the diffusion time through N. The resonant spin amplification and depletion effect happens if the period of the driving current $j(t)$ is close to the N spacer diffusion time T_N, this case is shown in Fig. 3.

At time $t = 0$ the current j is negative and electrons are injected from the left and extracted to the right electrodes, leaving behind positive and negative spin accumulations, indicated in Fig. 3 by diffusive packets (the sample is locally charge neutral, only spin is redistributed nonuniformly). The dynamics of these spin packets is governed by diffusion and relaxation, but not by bias voltage. This is because in the N spacer, there is no spin-charge coupling and spin and charge transports are decoupled, see [2]. At time $t = T_N/4$ the current vanishes, $j = 0$, as well as the spin injection and extraction. In the meantime the spin packets diffusively spread and reach the center of the N spacer. At $t = T_N/2$ the spin packets reach the other contacts. Now the current is fully reversed: in the P configuration the new spin packet is injected at the right electrode, *amplifying* the initial injected spin packet that has traveled from the left. In the AP configuration, the new spin packet of the opposite sign is injected at the right, *depleting* the initial injected spin. Similarly at the left electrode. The left contact difference $\mu_{sN}^{P}(c) - \mu_{sN}^{AP}(c)$ at $t = T_N/2$ becomes negative, the actual current $j > 0$ and, according to Eq. (5), we get negative ac magnetoresistance, $\mathrm{Re}(\Delta Z) < 0$.

The qualitative resonance condition, $\omega T_N \approx \pi$, is equivalent to $L_{sN}/d = \sqrt{\omega\tau_{sN}/\pi}$. In practice, to see the negative ac magnetoresistance one prefers $d \approx L_{sN}$, so that $\omega\tau_{sN} \approx \pi$, which is the microwave regime. If $d \gtrsim L_{sN}$, as in our model shown in Fig. 2, then the oscillations can be observed at lower frequencies, but at the cost of decreasing the magnitude of the ac magnetoresistance due to spin relaxation. This need not be an issue with tunnel contacts, as the precision of measuring higher resistances is higher. On the other hand, no oscillations (within the GHz regime) should be seen for nanoscale junctions, for $d \ll L_{sN}$. We will show below that this important regime gives a Lorentzian profile.

We now turn to the case of a junction with tunnel contacts, $r_F, r_N \ll r_c$. (In general, using tunnel barriers allows us to adopt the junction resistance and size independently, providing maximal flexibility in the device design.) In this case Eq. (1) reduces to

$$\Delta Z \approx \frac{4r_N}{\sqrt{1 - i\omega\tau_{sN}}} \frac{P_{\Sigma_c}^2}{\sinh[\frac{d}{L_{sN}}\sqrt{1 - i\omega\tau_{sN}}]}, \quad (6)$$

where $L_{sN} = \sqrt{D_N \tau_{sN}}$. A single-parameter ($\tau_{sN}$, knowing d and D_N) fit of a measurement of the ω dependence of the tunnel spin-valve impedance (relative to the dc value) to Eq. (6) can determine the spin relaxation time of the normal region. The shape is illustrated in Fig. 4. Since τ_{sF} is typically an order or two magnitudes smaller than τ_{sN}, the ac effects do not play a significant role in the F electrodes.

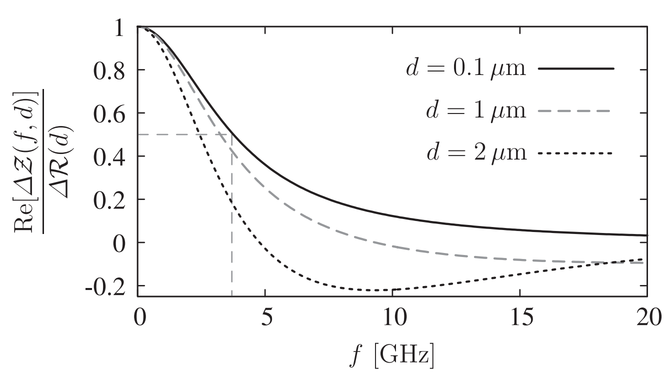

FIG. 4. Calculated ac/dc ratio of the FNF magnetoresistances for three different widths d of the N spacer with tunnel contacts to F. The solid line, $d = 0.1\ \mu$m, represents a Lorentzian line shape with its half-width determining the spin relaxation time τ_{sN}. The dotted line, $d = 2\ \mu$m, shows the first oscillation, with negative ac magnetoresistance. The dashed line, $d = 1\ \mu$m, is the intermediate case. The values for the parameters are as in Fig. 2, but with a greater tunnel resistance, $r_c = 5\ \mathrm{n}\Omega\,\mathrm{m}^2$.

For $d \gtrsim L_{sN}$ we can approximate ΔZ as follows:

$$\Delta Z(\omega, d, \tau_{sN}) \approx 8\frac{r_N P_{\Sigma_c}^2}{\sqrt{1 - i\omega\tau_{sN}}} e^{-d\sqrt{(1-i\omega\tau_{sN})/D_N\tau_{sN}}}. \quad (7)$$

Suppose we know the experimental value of the frequency ω_0 at which $\mathrm{Re}[\Delta Z(\omega_0, d)]$ vanishes. As an alternative to the fitting, the spin relaxation time can be given by the equation

$$\frac{1 + \sqrt{1 + \omega_0^2\tau_{sN}^2}}{\omega_0\tau_{sN}} = \tan\frac{d\omega_0}{\sqrt{2D_N}}\sqrt{\frac{\tau_{sN}}{1 + \sqrt{1 + \omega_0^2\tau_{sN}^2}}}, \quad (8)$$

which can be solved for τ_{sN} with simple numerics.

In the opposite important case of $d \ll L_{sN}$, Eq. (6) becomes a Lorentzian:

$$\Delta Z(\omega, d, \tau_{sN}) \approx 4\frac{r_N P_{\Sigma_c}^2}{d/\sqrt{D_N\tau_{sN}}} \frac{1 + i\omega\tau_{sN}}{1 + \omega^2\tau_{sN}^2}. \quad (9)$$

The half-width frequency $\omega_{1/2}$ at which $\mathrm{Re}[\Delta Z(\omega_{1/2}, d)] = \frac{1}{2}\Delta\mathcal{R}(d)$ determines the spin relaxation time according to $\tau_{sN} = 1/\omega_{1/2}$. This Lorentzian shape is rather robust for tunnel junctions, illustrated in Fig. 4, which also shows an intermediate case of $d \approx L_{sN}$.

For transparent contacts ($r_c \ll r_F, r_N$) and nanoscale junctions, $d \ll L_{sN}$, the magnetoimpedance is $\Delta Z(\omega, d) \approx 2r_F(\omega)P_{\sigma_F}^2$, the square root of the Lorentzian, but with the width of $1/\tau_{sF}$. The shape is therefore featureless in the microwave regime, unless the ferromagnetic contacts have a relatively large spin relaxation time, in which case τ_{sF} could be determined from the measured shape. In mesoscopic junctions oscillations should be visible.

In summary, we have presented a simple but robust theory of the ac spin-valve effect in symmetric FNF junctions and predict negative ac magnetoresistance due to resonant amplification and depletion of the spin accumulation in the normal metal region. The oscillating line shape allows a single-parameter (spin relaxation time) fitting for mesoscopic and nanoscale spin valves; for the latter a Lorentzian shape should be seen with tunnel contacts.

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