Regensburger
DISKUSSIONSBEITRÄGE
zur Wirtschaftswissenschaft

University of Regensburg Working Papers in Business,
Economics and Management Information Systems

Identifying the Shocks behind Business Cycle Asynchrony in Euroland

Carsten Trenkler*, Enzo Weber**

June 13, 2012

Nr. 466

JEL Classification: C32, E32

Keywords: Common cycles, euro area, impulse responses, structural VAR, Wald test

* University of Mannheim, Center for Econometrics and Empirical Economics, L7, 3-5, D-68131 Mannheim, Germany, trenkler@uni-mannheim.de, phone: +49 (0)621 181-1852, fax: +49 (0)621 181-1931.
** Department of Economics and Econometrics, University of Regensburg, 93040 Regensburg, Germany. enzo.weber@wiwi.uni-regensburg.de, phone: +49 941 943 1952, and Institute for Employment Research (IAB), Institut für Ost- und Südosteuropaforschung.

Identifying the Shocks behind Business Cycle Asynchrony in Euroland*

Carsten Trenkler†, Enzo Weber‡

Abstract

This paper investigates which shocks drive asynchrony of business cycles in the euro area. Thereby, it unites two strands of literature, those on common features and on structural VAR analysis. In particular, we show that the presence of a common cycle implies collinearity of structural impulse responses. Several Wald tests are applied to the latter hypothesis. Results reveal that differences in the GDP dynamics in several peripheral countries compared to a euro zone core are triggered by idiosyncratic, and to a lesser extent also world, shocks. Additionally, real shocks prove relevant rather than nominal ones.

Keywords: Common cycles, euro area, impulse responses, structural VAR, Wald tes
JEL classification: C32, E32

---

*We are grateful to the participants of the economics research seminar at the Westfälische Wilhelms-Universität Münster. The research was supported by the Deutsche Forschungsgemeinschaft (DFG) through the SFB 884 'Political Economy of Reforms'. Part of the research was done while the the first author was visiting the Humboldt Universität zu Berlin. He thanks the Humboldt Universität for its hospitality.

†University of Mannheim, L7, 3-5, D-68131 Mannheim, Germany, trenkler@uni-mannheim.de, phone: +49 (0)621 181-1852, fax: +49 (0)621 181-1931

‡University of Regensburg, Department of Economics and Econometrics, D-93040 Regensburg, Germany, enzo.weber@wiwi.uni-regensburg.de, phone: +49 (0)941 943-1952, fax: +49 (0)941 943-2735, and Institute of Employment Research

1 Introduction

The European debt crisis brought the topic of an optimal size of currency unions back to the agenda. While commentators disagree on the question whether single countries should opt-out after the crisis has aggravated, there is broad consensus that the union was overstretched initially, and that subsequent enlargements were premature. Even though economic structures in some countries were rather distinct from those of the EU core, entry was not postponed in favour of reaching a higher degree of real convergence. Thus, national monetary policy and exchange rate flexibility might have been abandoned too early. For instance, arguments go that common monetary policy provided access to cheap capital in some "peripheral" countries and brought interest rates far below levels that would have been appropriate in view of the stance of these economies, see e.g. Christodoulakis (2009), amongst others. Then, domestic absorption and risk taking rose excessively, external deficits lead to debt accumulation. Doubts about the sustainability of this development have been playing a key role in the crisis.

Logically, beyond actual crisis management it must be questioned whether all member countries fulfil the prerequisites for the success of a common monetary policy within a homogeneous union. Such optimal currency area (OCA) criteria are well known in the literature; see Mundell (1961), McKinnon (1963) or, more recently, Dellas & Tavlas (2009). Among them, economic integration in the sense of synchronization of cyclical fluctuations in GDPs is of paramount importance. Recently, this was underlined by wide-spread concerns on unsound overheating in single countries like Spain or Ireland. Regarding their stance and timing, common policies run into obvious conflicts and structural problems within an environment marked by such disparities.

Common sense tells that a group of core countries in Europe are likely to share a similar business cycle, but that several further union members might fail to do so; within the academic literature see e.g. Bayoumi & Eichengreen (1997) or the survey of Artis (2003). While we will confirm this view, our study goes an essential step beyond this stage of merely analyzing whether business cycles are synchronous: As our main research question, we ask which structural driving forces underlie the lack of synchronization in economic activity. Answering this question shows promise to give important insight into the nature of macroeconomic heterogeneity within the EMU, in support of policy measures creating a stronger and more sustainable foundation of the union.

Empirical macroeconomics thinks of such driving forces of economic activity as stochastic shocks. To identify potential sources of asymmetry, we construct small macroeconometric models that measure structural innovations to GDPs both of the core union and one of the peripheral countries. By statistically comparing dynamic reactions on both sides we are able to separate out those shocks that systematically produce asymmetric cyclical variation. In this context, we deem a distinction by origin of disturbances especially important. In particular, we discriminate common (world) shocks, core EMU shocks and idiosyncratic shocks to the peripheral country. For that purpose we employ a structural vector autoregression (SVAR) with recursive contemporaneous restrictions; compare also Kim & Chow (2003). We further dig into the structure within the peripheral economies by additionally distinguishing shocks to aggregate demand and supply, compare Bayoumi & Eichengreen (1994). Here, we apply long-run restrictions following Blanchard & Quah (1989).

While much attention has been paid in the literature to the correlation of shocks among countries, we identify those shocks that trigger asynchronous dynamic reactions and are thus responsible for business cycle disparities. We define cyclical synchrony as the presence of common dynamics in national outputs, i.e. shared patterns of recessions and expansions. In particular, following the literature of serial correlation common features, see Engle & Kozicki (1993), we speak of a common cycle if there exists a linear combination of GDP growth rates which eliminates their complete autocorrelation structure. We show that this criterion implies collinearity of impulse responses in both VARs and structural systems. By implication, in case of asynchronous cycles the responses to at least one of the innovations cannot be collinear (or stands in a different ratio than the remaining responses). Proper identification of the model thus allows singling out economically interpretable shocks driving cyclical divergence.

We apply Wald tests to the hypotheses of identical (or collinear) impulse responses in SVARs, see e.g. Jordà (2009). In doing so, bootstrap versions of this test is proposed. Further on, we visualize our statistical results by use of conventional confidence intervals and conditional t-ratios. The economic significance of the shocks for GDP growth is assessed using variance decompositions.

The paper is organized as follows. The subsequent section introduces our data. Section 3 elaborates on the analytical framework and discusses empirical results. The last section concludes. Methodological details are presented in the appendix.

2 Data

We obtained real seasonally adjusted GDPs as well as implicit price deflators for the Eurozone members (except Luxembourg) from Eurostat. GDP growth rates and inflation rates are calculated as first differences of logs. The sample is 1991:1-2011:3, with the exceptions of Ireland and Spain, where quarterly data is available only from 1997:1 or 1995:1, respectively, as well as Greece, which we address below. The sample choice balances the needs of stable parameters on the one hand (thus, the German reunification is excluded) and sufficient power of statistical tests on the other hand. We ensure this relatively compact sample period by chain-linking available data for Belgium, Greece and Portugal in 1995:1, 2000:1 and 1995:1, respectively; data from the early 1990s data for Portugal was obtained from the National Institute of Statistics. US data was provided by the Bureau of Economic Analysis.

Greece is the country which has been hit hardest in the course of the European debt crisis. In fact, until today (2012) no recovery can be noticed. As is evident from the continuing public debate, the slump in growth rates is extremely persistent. In contrast, the long period before the crisis was characterized by negative serial correlation. E.g., the first-order autocorrelation coefficient in the sample 1991:1 until 2007:4 results as -0.32. When this sample is combined with the persistent crisis period, measured serial correlation drops to values near zero. This would create the false impression that transitory dynamics are practically absent from Greek GDP growth. Obviously, for our research question the full sample is inadequate. Consequently, we cut the sample in 2007:4 when analyzing Greek data.

3 Empirical Methods and Results

In a first step we analyze business cycle (a)synchrony by exclusively considering GDP growth rates for the countries or regions of interest. This setup allows to identify the geographic origin of the economic shocks that have caused the potential business cycle asynchrony.

In the following subsection we briefly describe the vector autoregressive (VAR) model setup and link the concept of common cycles to this framework. We will use a set of inference procedures based on the VAR setup. Short explanations of these procedures will be given successively in the subsequent subsections. More details on the model framework and inference procedures as well as further references are deferred to an econometric appendix. There, we also discuss drawbacks of some of our inference procedures and present additional results as part of a robustness analysis. These additional results confirm our empirical findings presented in the main body of the paper.

3.1 VAR Model Framework and Common Cycles

Let y_t be the $K \times 1$ time series vector that contains the variables of interest. In our case y_t will comprise GDP growth rates of different countries or regions. The vector y_t is assumed to follow a vector autoregressive process of order p, VAR(p), i.e.

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \quad t = 1, 2, \ldots, T, \tag{3.1}$$

where ν is a $K \times 1$ parameter vector, A_i, $i = 1, \ldots, p$, are $K \times K$ parameter matrices, and the pre-sample values $y_0, y_{-1}, \ldots, y_{-p+1}$ are assumed to be fixed. Moreover, all roots of $\det(A(z)) = 0$, where $A(z) = I_K - A_1 z - \cdots - A_p z^p$, are outside the unit circle to assure that y_t is integrated of order zero, $I(0)$. We further assume $u_t \sim iid(0, \Sigma_u)$, where Σ_u is a positive definite covariance matrix, and $E(u_t^4) < \infty$. These assumptions are made in order to asymptotically justify the application of the bootstrap method.

We define cyclical synchrony as the presence of common dynamics in national outputs, i.e. shared patterns of recessions and expansions. To be precise, following the literature on serial correlation common features, see Engle & Kozicki (1993), we speak of a common cycle if there exists a linear combination of GDP growth rates which eliminates their complete autocorrelation structure. Let $y_t = \Delta x_t$, such that x_t contains the logarithms of the GDP series of interest which are $I(1)$. Under appropriate assumptions and ignoring deterministic components, see e.g. Vahid & Engle (1993), x_t can be decomposed into $x_t = r_t + c_t$, where r_t is a random walk part, which is sometimes called the 'trend', and c_t represents the stationary cyclical part of x_t. Vahid & Engle (1993) show that if there exists a $K \times 1$ vector δ with $\delta' y_t = \delta' u_t$, then $\delta' c_t = 0$ and vice versa. In other words, the linear combination $\delta' y_t$ does not contain any serial correlation because δ eliminates the cyclical part of x_t such that the variables in x_t are said to share a common cycle. Obviously, the conditions $\delta' A_i = 0$, $i = 1, \ldots, p$, have to hold in the VAR (3.1) in case of a common cycle, i.e. business cycle synchrony. These restrictions can be checked by using a likelihood ratio (LR) test as is described in the next subsection.

Note that we focus on GDP growth rates without considering so-called cointegration relations in the GDP levels. In our empirical framework we deem that sensible for two reasons: First, our sample periods are relatively short due to data availability and the need to avoid severe structural breaks. Estimating long-run (cointegration) relations in such limited samples has proven to be notoriously difficult. Second, while cointegration between GDPs can appear with regard to real income convergence, see Bernard & Durlauf (1995), this concept refers to *per capita* income. However, since our interest lies in business cycles rather than long-run growth, we do not divide GDP by population.

3.2 Testing for Common Cycles: Core and Periphery

Our concept implies testing for cyclical synchrony of peripheral countries with a Eurozone core. From the history of the European unification process and taking into account the traditional strong linkages between their economies, we consider Germany, France, Italy, the Netherlands, Belgium and Austria as members of the core. It may be disputable to include Italy given the current situation of the public finances and their perception in the financial markets. However, the composition of the core, including Italy, proved sensible in tests for common cycles with Germany as described in the following.

We apply a likelihood ratio (LR) test in the VAR model framework introduced above, where y_t contains GDP growth rates of Germany and one further (potential) core country. To this end, we determine the restricted log-likelihood of the VAR (3.1) by imposing the restriction $\delta' A_i = 0$, $i = 1, \dots, p$, via numerical optimization. The unrestricted log-likelihood is obtained via maximum likelihood (ML) estimation of the unrestricted VAR in the usual way. This test is equivalently applied by Schleicher (2007) and Paruolo (2003) to the cointegrated case. The concept of common serial correlation testing with regard to business cycles goes back to the work of Engle & Kozicki (1993) and Vahid & Engle (1993).

The lag order p is pre-specified using Akaike's Information criterion (AIC) with a maximum lag order of $p_{max} = 4$, see e.g. Lütkepohl (2005, Chapter 4). We find $p = 1$ for all cases. The restriction of a common cycle with Germany is rejected for none of the core countries. Even when $\delta' = (1, -1)$ is imposed in the tests, i.e. the common cycles are assumed to be of common magnitude, all p-values are clearly larger than 0.10. Therefore, we define real output of the core union by adding up the GDPs of these six countries. Growth rates $y_{core,t}$ are calculated from the obtained sum.

Next, we test for common cycles between the core and each of the peripheral countries Greece, Ireland, Portugal, and Spain. AIC chooses lag lengths of 1, 1, 2, and 4, respectively. The residuals of the resulting models are not significantly serially correlated. Adjusted Portmanteau tests, see Lütkepohl (2005, Chapter 4), for no serial correlation up to 16 lags cannot reject the null hypothesis with p-values of 0.88, 0.30, 0.90, and 0.22, respectively. Thus, the GDP growth dynamics should be adequately captured by our models.

The LR tests reject the null hypothesis of a common cycle with the EMU core for all four peripheral countries. The p-values are 0.01, 0.01, 0.02, and 0.00 for Greece, Ireland, Portugal, and Spain, respectively. This confirms the conjecture that business cycles in the more peripheral countries are not synchronized with the core. Given this empirical finding, we proceed to our main research question: Which economic shocks are driving this disparity?

3.3 Identifying the Shocks Causing Business Cycle Asynchrony

To address this question we consider a three-dimensional system consisting of the GDP growth rates of the U.S., $y_{us,t}$, the Euro core countries, $y_{core,t}$, and one of the four peripheral countries, $y_{pc,t}$. This results in $y_t = (y_{us,t}, y_{core,t}, y_{pc,t})'$. As described below, additionally considering the US growth rate allows to distinguish between world, EMU core, and peripheral country-specific growth shocks. The latter will be labeled as idiosyncratic shock in the following.

According to the foregoing discussion, the business cycles of the EMU core and the peripheral country under consideration are synchronous in our extended VAR system if $\delta' y_t = \delta' u_t$ with $\delta = (0 \ 1 \ \delta_3)'$ and $\delta_3 \neq 0$. This implies that the so-called forecast error impulse responses of $y_{core,t}$ and $y_{pc,t}$ to shocks in all three error term components in u_t are perfectly collinear starting with the period after the shock. In other words, $y_{core,t}$ and $y_{pc,t}$ respond in the same ratio, given by δ_3, to all three shocks across all response horizons after impact. To see this, note first that the forecast error impulse responses of the variables in y_t at horizon h are given by the moving average (MA) parameter matrix $\Phi_h = \sum_{j=1}^{h} \Phi_{h-j} A_j$, $h = 1, 2, \ldots$, with $\Phi_0 = I_K$, see e.g. Lütkepohl (2005, Chapter 2). This follows intuitively from the fact that y_t can be given the MA(∞) representation $y_t = \mu + \sum_{i=0}^{\infty} \Phi_i u_{t-i}$ with $\mu = A(1)^{-1}\nu$. Hence, the (i, j)-element of Φ_h, $\phi_{ij,h}$, describes how the i-th growth rate responds to a unit shock in the j-th growth rate that has occurred h periods ago. Then, because of $\delta' A_i = \mathbf{0}$, $i = 1, \ldots, p$, we have $\delta' \Phi_h = \mathbf{0}$, $h = 1, 2, \ldots$, which implies that the forecast error impulse responses of $y_{core,t}$ and $y_{pc,t}$ are perfectly collinear.

Hence, a failure in cyclical synchrony of output is due to non-collinear responses of the GDP growth rates $y_{core,t}$ and $y_{pc,t}$ to at least one error term shock, i.e. the response ratio is not constant across horizons. Alternatively, the responses to one shock may stand in a different ratio than the responses to the other shocks. However, the forecast error impulse response setup is not suitable to single out economically interpretable shocks that drive cyclical divergence since it is based on contemporaneously correlated residuals.

In order to identify economically interpretable shocks that cause asynchrony we consider the so-called orthogonalized impulse responses. For our GDP growth rate systems, the

Cholesky decomposition can be used to achieve identification of the structural, i.e. economically interpretable, error terms. We apply $\Sigma_u = PP'$, where P is a lower triangular matrix, to define the structural error term vector $\varepsilon_t = (\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t})' = P^{-1}u_t$ that has an identity covariance matrix. Then, the orthogonalized impulse responses to the shocks in ε_t are collected in the matrices $\Theta_h = \Phi_h P$, $h = 0, 1, \ldots$, see e.g. Lütkepohl (2005, Chapter 2). Given this definition we have $\delta'\Theta_h = \mathbf{0}$ if $\delta'\Phi_h = \mathbf{0}$, $h = 1, 2, \ldots$. Hence, collinearity with respect to the forecast error impulse responses implies collinearity with respect to the orthogonalized responses from horizon $h = 1$ onwards. This applies to other structural impulse responses as well as long as they are obtained by $\Phi_h B$, $h = 0, 1, \ldots$, for some matrix B.

If $\theta_{2i,h}$ and $\theta_{3i,h}$ denote the orthogonalized impulse responses of the EMU core countries and the peripheral country under consideration after h periods to a unit shock in ε_{it}, $i = 1, 2, 3$, then the common cycle condition can be expressed in more detail by $\delta'\Theta_h = (\theta_{21,h} + \delta_3\theta_{31,h}, \theta_{22,h} + \delta_3\theta_{32,h}, \theta_{23,h} + \delta_3\theta_{33,h}) = \mathbf{0}$, $h = 1, 2, \ldots$.

Since $\Theta_0 = P$, the i-th growth rate in y_t can immediately respond to a shock in the structural error ε_{jt} with $j \leq i$ but not to a shock in ε_{jt} with $j > i$. Hence, the ordering of the variables matters for identification and should be chosen based on careful economic reasoning. Our ordering of the growth rates has been derived accordingly. By placing the U.S. growth rate first, a shock in the first component of ε_t, ε_{1t} can have an instantaneous effect on the EMU core and peripheral country growth rates. Therefore, we interpret ε_{1t} as a common (world) growth shock from the EMU countries' perspective. Since the peripheral country's growth rate is placed last in the system we rule out immediate responses of the EMU core and U.S. growth rates to a shock in ε_{3t} such that we can identify this shock as idiosyncratic to the peripheral country. Finally, the second component in ε_t, ε_{2t}, represents then a EMU core growth shock.

In the light of missing business cycle synchrony between the EMU core and the four peripheral countries we now seek to single out the shocks that drive the cyclical divergence. Ideally, we would like to apply LR tests to test against multiplicity of the impulse responses of $y_{core,t}$ and $y_{pc,t}$ to each of the shocks, respectively. To be precise, we are interested in the null hypotheses $H_0 : \theta_{2i,h} = -\delta_{3i}\theta_{3i,h}$ for all $h = 1, 2, \ldots$, for $i = 1, 2, 3$, respectively. Note that the multiplicity factors δ_{3i} can be different with respect to the three shocks at this testing stage. In contrast to the common cycle framework, however, one cannot uniquely impose multiplicity of impulse responses to a single shock on the VAR model (3.1). This follows from arguments similar to those in Trenkler & Weber (2012, 2013). Hence, it is not possible to obtain restricted parameter estimators under H_0, including the one for δ_{3i}, $i = 1, 2, 3$.

Thus, LR tests cannot be applied.[1]

Therefore we consider Wald tests which only require to estimate the VAR model under the alternative hypothesis, i.e. we only need unrestricted estimators. One could then test the null hypothesis of constant impulse response ratios of $y_{core,t}$ and $y_{pc,t}$ over a finite number of horizons with respect to a single shock. Besides being silent about the values of the response ratios, i.e. about δ_{3i}, $i = 1, 2, 3$, the main drawback of this approach is that the null hypothesis involves non-linear parameter restrictions. The Wald test is not invariant to re-parameterizations of non-linear restrictions in finite samples, see e.g. Greene (2008, Chapter 11). In fact, we found that our test results strongly depend on the specific parameterization. Therefore, we do not regard this approach as reliable.

Given the aforementioned problems, we decided to apply Wald tests for the equivalence of the structural impulse responses of $y_{core,t}$ and $y_{pc,t}$, i.e. we assume $\delta_{3i} = -1$,$i = 1, 2, 3$. This results in linear restrictions in terms of the involved MA parameters. Hence, the null hypothesis for analyzing whether the i-th structural shock contributes to business cycle asynchrony is

$$H_0 : \theta_{2i,h} = \theta_{3i,h} \text{ for all } h = 1, \dots, H. \tag{3.2}$$

By focussing on the equivalence of the responses we test a stronger requirement than is needed for the existence of a common cycle. However, a test for equivalence contains the additional, economically relevant, information that the growth rate responses are of identical magnitude. This can be regarded as an important support for a common currency area. In contrast, different response amplitudes may impair the conduct of a consistent monetary policy for the whole union. Furthermore, the figures below indicate that the empirical impulse response shapes are quite different for the EMU core and the peripheral countries in the cases of rejections of equivalence. Hence, an economically plausible alternative to the choice $\delta_{3i} = -1$ is not evident. Finally, equivalence of the impulse responses of the EMU core and peripheral country growth rates is rejected for at least one structural shock for each of the four systems considered. Therefore, at least one shock can be identified to be the cause of business cycle asynchrony. Thus, it is not necessary to analyze whether asynchrony is due to the fact that the impulse responses of $y_{core,t}$ and $y_{pc,t}$ stand in a different ratio with respect to three structural shocks.

We set $H = 4$ for the test since we want to focus on medium-run responses due to our interest in business cycle (a)synchrony. Moreover, choosing a large value for H can dilute

[1] We provide more detailed information on this issue and the WALD tests explained in the following in the econometric appendix, see section A.2.

significant response differences given that the individual responses are quite close to zero after response horizon 4 in our setups. The short samples on which we have to rely would aggravate this problem.

We use three different versions of a Wald-test to check for the equivalence of the relevant orthogonalized impulse responses. The first test is the usual asymptotic Wald-test. For convenience, we abbreviate both the test and the corresponding Wald statistic by $\widehat{W}$.[2] $\widehat{W}$ requires to apply the asymptotic covariance matrix of the estimated impulse responses across the H response horizons. Since the impulse response coefficients are smooth functions of the VAR parameters, the asymptotic covariance matrix can be obtained via the Delta method, compare e.g. Lütkepohl (2005, Chapter 4). The Wald statistic is asymptotically χ^2-distributed with H degrees of freedom. It is known that inference based on the Delta method can be misleading in the context of impulse response analysis when only small samples are available, see e.g. Kilian (1998b, 1999), Lütkepohl (1996). Therefore, we have also applied two bootstrap-based test versions.

Following the proposal of Jordà (2009), we use a bootstrap covariance matrix estimator in the Wald statistic. The covariance matrix is estimated from 2500 bootstrap realizations of the orthogonal impulse responses. This approach is, in fact, analogous to the ones used in the literature for obtaining bootstrap confidence intervals for impulse responses or bootstrap t-ratios for parameter estimators in (structural) VARs. We implement the test, labeled as $\widehat{W}^B$, by recursively generating the bootstrap data via the VAR model structure. We use parameter estimators that are bias-corrected according to Pope (1990), as has been recommended by Kilian (1998a, 1999) for computing bootstrap confidence intervals for impulse responses. We have adopted this recommendation since the determination of the covariance matrix represents a strongly related setup. Accordingly, the bootstrap impulse response coefficients are obtained from the biased-corrected bootstrap VAR parameter estimates.

Finally, we consider a standard bootstrap Wald-test, say $\widehat{W}_b^*$, for which the p-value is determined based on 2500 bootstrap realizations of the Wald statistic $\widehat{W}$. The bootstrap data are generated like in the previous test setup, i.e. using biased-corrected VAR parameter estimators.

The lag order p for the VAR models has again been pre-specified using AIC with $p_{max} = 4$. The obtained lag orders as well as the sample sizes and results of the adjusted Portmanteau test are summarized in Table 1. The latter clearly indicate that residuals of the resulting models contain no significant serial correlation structure.

[2] The same notational approach will be applied to the other test procedures introduced next.

Table 1. Summary of Samples, VAR Lag Orders, and Results of Adjusted Portmanteau-Tests for GDP Growth Rate Systems

Peripheral Country	Sample	VAR Lag Order	Q(16) p-values
Greece	1991:2-2007:4	$p = 2$	0.879
Ireland	1997:2-2011:3	$p = 1$	0.297
Portugal	1991:2-2011:3	$p = 2$	0.897
Spain	1995:2-2011:3	$p = 3$	0.218

Note: Q(16) refers to the adjusted Portmanteau test on serial correlation in the residuals up to 16 lags. The test is described e.g. in Lütkepohl (2005, Chapter 4).

Table 2 summarizes the results of the Wald tests on significant differences in the responses of the core growth rate and the corresponding growth rate of the peripheral country. Clearly, there is strong evidence that the responses to the idiosyncratic shocks are significantly different in case of all four systems. Only for the Irish system one of the tests indicates solely borderline significance. While there is only weak evidence for the Spanish system that also the EMU core shock may cause business cycle asynchrony, the world shock seems to be more important in causing cycle divergence. We observe a clear rejection of response equivalence for the Spanish system and p-values close to the 10% significance level for the Greek system.

Note that the results are quite robust across the three Wald tests considered. Therefore, we focus on on the test $\widehat{W}^B$ in the following for two reasons. First, as already mentioned above, it represents the analogue to the usual bootstrap approaches of obtaining confidence intervals for impulse responses. Second, $\widehat{W}^B$ has the additional advantage of being also applicable in more general VAR model setups as described in the econometric appendix.

We complement the findings obtained by the Wald tests by presenting two further sets of empirical measures. First, we give a graphical representation of the impulse response results that allow for a more detailed analysis. Second, we assess the importance of the shocks causing the business cycle asynchrony by looking at so-called forecast error variance decompositions.

Figures 1 to 4 show for each system the following information. First, the responses of the EMU core growth rate (solid line) and of the corresponding peripheral country's growth rate (solid line with squares) to the respective shocks are given in the upper panels. In the

Table 2. Results of Wald-Tests (p-values) on Differences in the Impulse Responses of EMU Core and Peripheral Countries up to Response Horizon $H = 4$

Peripheral Country/ Sample (VAR order)	Asymptotic Wald-test $\widehat{W}$	Wald-test with Bootstrap Covariance Matrix Estimator $\widehat{W}^B$	Bootstrap Wald-test $\widehat{W}_b^*$
Greece: 1991:2-2007:4 ($p = 2$)			
World Shock	0.1193	0.1049	0.1328
EMU Core Shock	0.4535	0.5055	0.4248
Idiosyncratic Shock	0.0000	0.0000	0.0048
Ireland: 1997:2-2011:3 ($p = 1$)			
World Shock	0.2773	0.2104	0.3684
EMU Core Shock	0.7753	0.7914	0.7720
Idiosyncratic Shock	0.0000	0.0002	0.1492
Portugal: 1991:2-2011:3 ($p = 2$)			
World Shock	0.8199	0.8512	0.8236
EMU Core Shock	0.6798	0.7597	0.6648
Idiosyncratic Shock	0.0000	0.0001	0.0180
Spain: 1995:2-2011:3 ($p = 3$)			
World Shock	0.0188	0.0096	0.0388
EMU Core Shock	0.1148	0.1414	0.1188
Idiosyncratic Shock	0.0019	0.0012	0.0228

lower panels, the solid line displays the differences of the responses. The responses and their differences are depicted up to horizon 12, i.e. we go beyond what is considered in the Wald tests. Then, we present bootstrap confidence intervals for the response differences (dashed lines). These are equal-tailed Effron intervals based on biased-corrected parameters that are computed according to Kilian (1998a). These intervals allow to assess the significance of a single impulse response difference at a specific horizon h. Note, however, that the intervals do not take into account the correlation of the impulse response difference estimates across the horizons, i.e. they should just be interpreted pointwise. Therefore, we also present so-called conditional t-ratios, say $t_{h|h-1,\ldots,1}$, which have been suggested by Jordà (2009).

For each horizon h, $t_{h|h-1,\dots,1}$ is equal to the ratio of the estimated conditional impulse response difference and its standard error. The conditional impulse response difference at horizon h represents the response difference at horizon h conditional on the response differences at the previous horizons $i = 1, \dots, h-1$. Hence, the conditional t-ratios take into account the correlation of the impulse response differences across the response horizons. The respective Wald statistic testing the equivalence of the two impulse response sequences can be written as the sum of the squared conditional t-ratios, i.e. it is equal to $\sum_{h=1}^{H} t^2_{h|h-1,\dots,1}$. Hence, the conditional t-ratios provide a measure of the conditional contribution of the impulse response difference at horizon h to the overall difference measure. In other words, they describe the importance of horizon h conditional on the contribution that has already built up until horizon $h-1$. The conditional t-ratios can give a quite different impression of significance than the conventional (unconditional) confidence intervals in our setup. Moreover, even the sign of the unconditional and conditional response difference may be different. This is not only due to the different nature of conditional and unconditional inference but also due to the fact that the impulse response differences are often negatively correlated across the response horizons. This is in contrast to individual impulse response estimates which are typically positively correlated.

The conditional t-ratios are represented by the circles and their values can be read from the right-hand side vertical axis in the lower panels of the figures 1 to 4. A black circle indicates that the t-ratio is larger than 1.96 in absolute value. The conditional t-ratios are computed in relation to the Wald test $\widehat{W}^B$, i.e. we have $\widehat{W}^B = \sum_{h=1}^{H} t^2_{h|h-1,\dots,1}$.

Figures 1 to 4 contain a couple of interesting findings. First, the impulse responses typically go to zero quite quickly. This was one motivation to constrain the Wald test to the first four responses. In terms of usual bootstrap confidence intervals, not shown here, significance beyond $H = 4$ is only found for the Spanish growth rate responses to the idiosyncratic shock and for the responses of both the EMU core and Irland to the world shock. Second, the bootstrap confidence intervals for the impulse response difference are generally quite large indicating a high degree of overall estimation uncertainty. Third, there is no clear pattern across the systems regarding the horizons at which the impulse responses are significantly different. This applies no matter whether the conventional bootstrap confidence intervals or the conditional t-ratios are considered. Since the conditional t-ratios take account of the correlation of the impulse response difference estimators we focus on them for a more detailed interpretation.

The idiosyncratic shocks create (conditionally) significant response difference in the quarter immediately following the shock in case of Greece and Ireland but with a delay of two or three quarters in the cases of Portugal and Spain. Interestingly, for the Irish and Portuguese systems we observe both significant (conditional) positive and negative response differences. These findings result from the zigzag response patterns of the Irish and Portuguese growth rates. Furthermore, the usual confidence intervals indicate significant differences for the Spanish and Portuguese systems beyond horizon 4. However, the conditional t-ratios do not regard the respective horizons as relevant if usual 5% or 10% critical values are considered. Hence, once the information up to horizon 4 is taken into account the more remote horizons would not substantially contribute to the overall measure of response differences.

As regards the world shock, we observe some individual significant response differences for the Spanish, Irish, and Greek systems. For Ireland, the only significant difference is not strong enough to generate a rejection of the null hypothesis by the Wald test. Similarly, the significant difference at horizon 2 for the Greek system only translates into borderline significance of the Wald test statistic. The same comment applies to the effect of the EMU-shock for the Spanish system. To summarize, although there is no clear pattern across the systems, a significant difference in the responses is typically only observed for single horizons but not persistently for all considered response horizons.

We now turn to the forecast error variance decomposition. This tool allows to analyze the importance of the three identified shocks for the h-step forecast error variance of each variable of the system. For a description on how to obtain the decomposition and on the relevant inference methods we refer to Lütkepohl (2005, Chapters 2 and 3). Figure 5 summarizes the proportion of the h-step forecast error variances, $h = 1, \ldots, 12$, of the growth rate of the peripheral country accounted for by the world, EMU-core, and idiosyncratic shocks, respectively.

The idiosyncratic shock that causes business cycle divergence plays a crucial role for the growth rate variations in all four peripheral countries. The proportion of the forecast error variances accounted for by this shock lies between approximately 50% and 95% depending on the specific country and horizon. Hence, the business cycle asynchrony detected is clearly of relevance. Not surprisingly, the idiosyncratic shocks in Greece, Ireland, and Portugal are of no importance for the EMU core growth rate. By contrast, the Spanish shock matters to some extent. We do not show these results here to conserve space.

We further note that the world shock is relevant for Spain. Hence, this shock also substantially contributes to the asynchrony we have found for the Spanish case. Interestingly, the EMU core shock is of much lower importance for Spain than the world shock. For the Greek system the response differences to the world shock were close to the 10% significance level.

However, the world shock does not play a crucial role for the Greek growth rate. Hence, one would not attribute much importance of this shock when explaining the sources of the business cycle divergence between Greece and the EMU core countries.

3.4 Real and Nominal Idiosyncratic Shocks

While the previous analysis has focused on the regional origin of the impulses, we now dig deeper into the main driver of cyclical disparity: the idiosyncratic shocks. In detail, we seek to decompose these shocks into their real and nominal parts. This should give further insight into the nature and causes of non-synchronous development.

We distinguish between real and nominal shocks based on their long-run impacts. Specifically, we define that a nominal (or demand) shock has no long-run impact on real GDP. This standard assumption can be seen in the context of stylized AD-AS models, where changes in supply affect both production and the price level in the long run, but the income effects of demand shocks can only be transitory.[3] This framework was empirically implemented by Bayoumi & Eichengreen (1994), amongst others, based on the methodology of Blanchard & Quah (1989).

We extend the system with three GDP growth rates by the inflation rate of the peripheral country $\pi_{pc,t}$ in order to capture nominal effects: $y_t = (y_{us,t}, y_{core,t}, y_{pc,t}, \pi_{pc,t})'$. We maintain the short-run assumption from the previous section, i.e. the periphery shocks (both real and nominal) have no contemporaneous impact on EMU core and US economic growth. To capture these restrictions, we define the matrix P_Ξ which is lower triangular apart from the 3,4-element that is not restricted to zero. The latter means that $y_{pc,t}$ can immediately respond to a shock in the fourth error term component.

We employ the long-run restriction introduced in the previous paragraph within the block of the periphery variables GDP growth and inflation. Formally, the matrix of long-run effects is given by $\Xi = \sum_{i=0}^{\infty} \Theta_i = (I_K - A_1 - \cdots - A_p)^{-1} P_\Xi$. We assume $\Xi_{34} = 0$ to assure that the nominal (the fourth) shock has no long-run impact on GDP growth in the peripheral country. Thereby, we can distinguish the nominal from the real (the third) shock to the periphery.

In the three-dimensional systems we chose the lag length by AIC in order to ensure that the model captures the complex dynamics. Now we add further complexity by introducing

[3] Of course, this theoretical framework is not meant to provide a realistic description of the economy, but rather to fix ideas on the broad interpretation of our empirical results. By the same token, the terms ”demand” and ”nominal” as well as ”supply” and ”real” are used synonymously, even if they are not necessarily identical with regard to more detailed models.

Table 3. Results of Adjusted Portmanteau-Tests on Residual Autocorrelation and Wald-Test (p-values) on Differences in the Impulse Responses of EMU Core and Peripheral Countries up to Response Horizon $H = 4$

Peripheral Country/ Sample (VAR order)	Q(16)	Wald-test with Bootstrap Covariance Matrix Estimator $\widehat{W}^B$
Greece: 1991:2-1997:4 ($p = 1$)	0.4592	
Idiosyncratic Real Shock		0.0103
Idiosyncratic Nominal Shock		0.8823
Ireland: 1997:2-2011:3 ($p = 1$)	0.1121	
Idiosyncratic Real Shock		0.0131
Idiosyncratic Nominal Shock		0.9702
Portugal: 1991:2-2011:3 ($p = 1$)	0.9564	
Idiosyncratic Real Shock		0.6369
Idiosyncratic Nominal Shock		0.3915
Spain: 1995:2-2011:3 ($p = 3$)	0.2397	
Idiosyncratic Real Shock		0.0172
Idiosyncratic Nominal Shock		0.6792

Note: Q(16) refers to the adjusted Portmanteau test on serial correlation in the residuals up to 16 lags. The test is described e.g. in Lütkepohl (2005, Chapter 4).

a fourth variable, the inflation rate of the peripheral country. In view of the limited number of observations we must ensure that model complexity does not overstrain the potentialities of the empirical basis. Indeed, using AIC leads to lag specifications allowing hardly any conclusions based on statistical significance. However, clearly separating the effects of the two peripheral shocks is key in this part of the analysis. Therefore, we decided to follow the more parsimonious Bayesian information criterion (BIC), see e.g. Lütkepohl (2005, Chapter 4). Results concerning serial correlation of the errors were still satisfactory, see below. Only for Spain we had to increase the chosen lag order from $p = 1$ to $p = 3$ to avoid highly significant residual autocorrelation. Information on the final models considered and the results of the adjusted Portmanteau test are given in the first two columns of Table 3.

We test for equivalence of the responses of the core and peripheral GDP growth rates using the Wald test $\widehat{W}^B$ introduced in the previous subsection. The results of $\widehat{W}^B$ are provided in Table 3. We focus on the real and nominal idiosyncratic shocks given our interest in decomposing the effects found for the idiosyncratic shocks in the growth rate systems.

The findings of the previous subsection suggest that idiosyncratic shocks are the most important drivers of the lack of synchronization of business cycle dynamics. This is also the case for the four-dimensional systems including the inflation rate. Moreover, we see in this respect from Table 3 that supply shocks tend to be relevant rather than demand shocks with long-run effects restricted to zero. To be precise, with the exception of Portugal, the GDP growth rates of the EMU core and the respective peripheral country significantly respond differently to the idiosyncratic real shock. In contrast, the idiosyncratic nominal shock does not cause significantly different responses for any of the systems.

4 Conclusion

The underlying paper explored the origins of non-synchronized business cycles in the euro area. In a monetary union, such asymmetries can be seen as a lack of economic integration. In the light of the OCA theory, difficulties in the course of the conduct of a common monetary policy come as a logical consequence. Therefore, shedding light on the drivers of diverging behavior is of high interest for deciding about the enlargement of existing currency areas, for managing the European debt crisis and for dealing with the structural impediments that became manifest in the euro zone.

We developed an approach suitable for identifying the deep economic shocks that trigger asymmetry in GDP dynamics. We state that the presence of a common cycle implies collinear impulse responses in dynamic systems and show that the same holds for structural models. By implication, the absence of common serial correlation must result from the GDP reactions to at least one structural innovation. We detect the concerned shocks by identifying structural VARs and applying different Wald tests to the hypotheses of collinear impulse responses.

In line with previous studies we find a core of European countries sharing a common business cycle. However, the more peripheral countries Greece, Ireland, Portugal, and Spain reveal systematic deviations from the core cycle. Tellingly, this group coincides with those countries which are most severely affected by the European debt crisis.

The analysis of impulse responses shows that idiosyncratic shocks to the periphery countries are the most important drivers of the lack of synchronization of business cycles. Evidently, dissipating these disturbances is still a sluggish process, so that persistent deviations from growth dynamics of the EMU core remain. Beyond, in some cases we find asymmetric reactions to world shocks. In contrast, impulses originating in the EMU core trigger comparable dynamic reactions through the whole union. Regarding this direction, economic integration already seems to be well advanced. However, in the opposite direction strong idiosyncrasies are still in place. Among the latter, local real shocks tend to be relevant rather than local nominal shocks with long-run effects restricted to zero. This implies that aggregate demand, even if in some cases considerably different from the euro zone average, is unlikely to be behind the deviating cyclical behavior in Europe. Instead, structural reasons come to the fore. In the light of the experience from the European debt crisis this does not seem implausible: fundamental problems in structurally critical fields such as the labor market, the real estate market and the state sector have been identified as key issues in the severe decline of the peripheral countries.

On the one hand, this conclusion seems to draw a pessimistic picture: idiosyncratic structural impediments affect the performance of the periphery and hinder deeper integration in the monetary union. However, on the other hand it offers a clear prospect for further progress: structural reforms, some of which are already underway, are likely to help seize control of the problem of excessive debt – but they also bear the potential to remove obstacles to convergence of business cycles. For the functioning of the monetary union, both aspects are of paramount importance.

The concept of common cycles is well established in the literature. The current paper demonstrated how to connect this framework to structural analysis in order to gain insights into the fundamental driving forces of cyclical asynchrony. Thereby, it is obvious that our model choice only sets the stage for further advancement in this field. Applying additional identification schemes and considering different country constellations provides promising lines of future research.

A Econometric Appendix

In this appendix we present details on the inference procedures applied in the paper. Moreover, we provide some additional empirical results as part of a robustness analysis using alternative test versions.

A.1 Impulse Responses

We are interested in testing hypotheses on the orthogonalized impulse response coefficients defined in section 3.3, i.e. on the coefficients in Θ_h, $h = 1, 2, \ldots, H$, using the Wald test framework. To this end, it is helpful to define[4]

$$\mathbf{\Theta}_H = \text{vec}(\Theta(1, H)) = \text{vec}(\Theta_1, \Theta_2, \ldots, \Theta_H).$$

For setting up the Wald statistic we need an appropriate estimator of $\mathbf{\Theta}_H$ and its (asymptotic) covariance matrix. First let $\boldsymbol{\alpha} = \text{vec}(A_1, \ldots, A_p)$. Then, $\hat{\boldsymbol{\alpha}} = \text{vec}(YM_1Z'(ZM_1Z')^{-1})$ is the OLS estimator of $\boldsymbol{\alpha}$, where $Y = (y_1, \ldots, y_T)$, $Z = (Z_0, \ldots, Z_{T-1})$ with $Z_t = (1, y_t', \ldots, y_{t-p+1}')'$, and $M_1 = I_T - \mathbf{1}_T(\mathbf{1}_T'\mathbf{1}_T)^{-1}\mathbf{1}_T'$ with $\mathbf{1}_T$ being a $T \times 1$ vector of ones. From Lütkepohl (2005, Proposition 3.1), it follows that

$$\sqrt{T}(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}) \xrightarrow{d} N(0, \Sigma_{\boldsymbol{\alpha}}), \tag{A.1}$$

where $\Sigma_{\boldsymbol{\alpha}} = (\text{plim}(ZM_1Z')/T)^{-1} \otimes \Sigma_u$. We obtain $\hat{\Sigma}_{\boldsymbol{\alpha}} = T(ZM_1Z')^{-1} \otimes \hat{\Sigma}_u$ with $\hat{\Sigma}_u = (T-p)^{-1} \sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$ as the corresponding residual covariance matrix estimator.

Next, we have $\hat{\Phi}_h = \sum_{j=1}^{h} \hat{\Phi}_{h-j} \hat{A}_j$ with $\hat{\Phi}_0 = I_K$. Define $\mathbf{\Phi}_H = \text{vec}(\Phi_1, \Phi_2, \ldots, \Phi_H)$ and $\hat{\mathbf{\Phi}}_H = \text{vec}(\hat{\Phi}_1, \hat{\Phi}_2, \ldots, \hat{\Phi}_H)$. Then, from Lütkepohl (2005, Proposition 3.6, Remark 7) we obtain the following limit result

$$\sqrt{T}(\hat{\mathbf{\Phi}}_H - \mathbf{\Phi}_H) \xrightarrow{d} N\left(0, \Omega_{\hat{\mathbf{\Phi}}_H}\right), \tag{A.2}$$

where $\Omega_{\hat{\mathbf{\Phi}}_H} = \boldsymbol{G}\Sigma_{\boldsymbol{\alpha}}\boldsymbol{G}'$ with $\boldsymbol{G} = (G_1', \ldots, G_H')'$ and

$$G_h = \sum_{i=0}^{h-1} J\left(\boldsymbol{A}'\right)^{h-1-i} \otimes \Phi_i, \quad h = 1, \ldots, H,$$

where $J = [I_K : 0 : \cdots : 0]$ is a $(K \times Kp)$ matrix and

$$\boldsymbol{A} = \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I_K & 0 & \cdots & 0 & 0 \\ 0 & I_K & \cdots & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_K & 0 \end{bmatrix}.$$

[4]The following framework is adopted from Jordà (2009) to our specific testing setup. Note, that some of the involved quantities differ from Jordà (2009) due to the use of a finite-order VAR model and a different ordering of the matrices in $\Theta(1, H)$.

We need the estimator $\hat{\Omega}_{\hat{\mathbf{\Phi}}_H} = \hat{\boldsymbol{G}}\hat{\Sigma}_{\boldsymbol{\alpha}}\hat{\boldsymbol{G}}'$, where $\hat{\boldsymbol{G}}$ is obtained by replacing the unknown quantities with the obvious estimators defined beforehand.

From $\hat{\Sigma}_u$ we obtain $\hat{P}$ via a Cholesky decomposition. This results in the estimator $\hat{\Theta}_h = \hat{\Phi}_h\hat{P}$, $h = 1, 2 \ldots H$, summarized in $\hat{\mathbf{\Theta}}_H = \text{vec}(\hat{\Theta}(1, H)) = \text{vec}(\hat{\Theta}_1, \hat{\Theta}_2, \ldots, \hat{\Theta}_H)$. We then have from Lütkepohl (2005, Proposition 3.6) analogously to Jordà (2009, Section IV)

$$\sqrt{T}(\hat{\mathbf{\Theta}}_H - \mathbf{\Theta}_H) \xrightarrow{d} N\left(0, \Omega_{\hat{\mathbf{\Theta}}_H}\right), \tag{A.3}$$

where

$$\Omega_{\hat{\mathbf{\Theta}}_H} = (I_H \otimes (P' \otimes I_K))\Omega_{\hat{\mathbf{\Phi}}_H}(I_H \otimes (P \otimes I_K)) + 2\bar{\mathbf{\Phi}}_H C D_K^+(\Sigma_u \otimes \Sigma_u)D_K^{+\prime}C'\bar{\mathbf{\Phi}}'_H \tag{A.4}$$

with $\bar{\mathbf{\Phi}}_H = [(I_K \otimes \Phi'_1), (I_K \otimes \Phi'_2), \ldots, (I_K \otimes \Phi'_H)]'$, $C = L'_k\{L_K(I_{K^2}+K_{KK})(P\otimes I_K)L'_K\}^{-1}$, $D_K^+ = (D'_K D_K)^{-1}D'_K$, where D_K is the duplication matrix such that for any square $K \times K$ matrix B, $\text{vec}(B') = D_K\text{vech}(B)$, L_K is the elimination matrix such that $\text{vech}(A) = L_K\text{vec}(A)$, and K_{KK} is the commutation matrix such that $\text{vech}(B') = K_{KK}\text{vec}(B)$. Plugging $\hat{\Phi}_1, \ldots, \hat{\Phi}_H$,$\hat{\Omega}_{\hat{\mathbf{\Phi}}_H}$, $\hat{P}$, and $\hat{\Sigma}_u$ into the expression for $\Omega_{\hat{\mathbf{\Theta}}_H}$ yields the estimator $\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}$.

A.2 Wald Statistics

As pointed out in section 3.3 it is not possible to uniquely impose the null hypotheses H_0 : $\theta_{2i,h} = -\delta_{3i}\theta_{3i,h}$ for all $h = 1, 2, \ldots$, for $i = 1, 2, 3$, respectively, on the VAR model (3.1). This is the case whether or not δ_{3i} is pre-specified. Our setup is related to the framework of codependence considered in Trenkler & Weber (2012, 2013). From their results one can see that if $\delta_i = (0\ 1\ \delta_{3i})$, $i = 1, 2, 3$, imposes additional restrictions on the MA parameters to the ones tested under H_0, then one cannot uniquely impose the restrictions of interest. Hence, restricted ML estimation and, therefore, LR testing is not possible.

A potential alternative would be to consider WALD tests on the null hypothesis

$$H_0 : \theta_{2i,1}/\theta_{3i,1} = \theta_{2i,h}/\theta_{3i,h} \text{ for all } h = 2, \ldots, H, \tag{A.5}$$

for some pre-specified upper bound H. One has to use such a finite upper bound for the response horizons when applying the WALD test setup, because the corresponding test statistic will be directly expressed in terms of the relevant MA parameters in Θ_i, $i = 1, 2, \ldots$. The null hypothesis (A.5) involves non-linear parameter restrictions such that the Wald test is not invariant to re-parameterizations of the considered restrictions in finite samples. In fact, we can re-express (A.5) in many different ways. Unfortunately, the test results strongly depend on the specific parameterization such that rejections and non-rejections may occur for the same set of restrictions tested. One likely reason is that some of the impulse response coefficients are relatively close to zero, while others are clearly different from zero. In connection with our small sample sizes, this may be critical since ratios (or products) are involved in the test statistics in our setup. Moreover, the approach of testing (A.5) does not deliver information about the multiplicity factors δ_{3i}, $i = 1, 2, 3$. Therefore, we rely on WALD tests with linear restrictions in terms of the relevant MA parameters. This is achieved by pre-specifiying δ_{3i}. In our case we set $\delta_{3i} = -1$, $i = 1, 2, 3$.

Define the selector matrix $S_{ij} = I_H \otimes e_j' \otimes e_i$, $i, j = 1, \ldots, K$, where e_m is the m-th column of I_K for $m = i, j$. Thus, $S_{ij}\mathbf{\Theta}_H$ contains the responses of the i-th variable in y_t, y_{it}, to a shock in the j-th component of the structural error term vector ε_t, ε_{jt}, over the response horizons $h = 1, \ldots, H$. Setting $S = (I_H : -I_H)(S_{ij}' : S_{mj}')'$, the equivalence of the orthogonal impulse responses of y_{it} and y_{mt} to a shock in ε_{jt} over the horizons $h = 1, \ldots, H$ is represented by the null hypothesis

$$H_0 : S\mathbf{\Theta}_H = 0. \tag{A.6}$$

Note that (A.6) just re-states the hypothesis (3.2) using general matrix algebra. We consider the usual Wald statistic

$$\widehat{W} = T\left(S\hat{\mathbf{\Theta}}_H\right)'\left(S\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}S'\right)^{-1}\left(S\hat{\mathbf{\Theta}}_H\right). \tag{A.7}$$

Given our assumptions, $\widehat{W}$ is asymptotically χ^2 distributed with H degrees of freedom if the null hypothesis (A.6) is true.

Jordà (2009) suggested to replace $T^{-1}\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}$ in (A.7) by a bootstrap-based covariance matrix estimator, say $\hat{\Omega}^B_{\hat{\mathbf{\Theta}}_H}$. We follow his proposal by adopting the bootstrap framework of Kilian (1998a). We generate $B = 2500$ sets of bootstrap data of size T, the sample size available for the respective country. The bootstrap data are recursively obtained via the VAR model structure exactly as in Kilian (1998a), i.e. we use bias-corrected estimators of the VAR parameters $A_1, \ldots, A_p$, which are computed according to Pope (1990). For each bootstrap data set the orthogonalized impulse response coefficients estimates $\hat{\mathbf{\Theta}}^b_H$, $b = 1, 2, \ldots, B$, are obtained as for the observed data but using again bias-corrected estimates of the VAR parameter estimators. Thereby, the related framework of Kilian (1998a) for computing bootstrap confidence intervals is matched. Accordingly, we have

$$\hat{\Omega}^B_{\hat{\mathbf{\Theta}}_H} = \frac{1}{B-1}\sum_{b=1}^{B}(\hat{\mathbf{\Theta}}^b_H - \bar{\hat{\mathbf{\Theta}}}^b_H)(\hat{\mathbf{\Theta}}^b_H - \bar{\hat{\mathbf{\Theta}}}^b_H)', \tag{A.8}$$

where $\bar{\hat{\mathbf{\Theta}}}^b_H = B^{-1}\sum_{b=1}^{B}\hat{\mathbf{\Theta}}^b_H$. Inserting (A.8) instead of $T^{-1}\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}$ into (A.7) gives the Wald statistic $\widehat{W}^B$. As mentioned in subsection 3.3, $\widehat{W}^B$ represents the analogue to the usual bootstrap approaches of obtaining confidence intervals for impulse responses or bootstrap t-ratios for parameter estimators in (structural) VARs.

Finally, we consider a standard bootstrap Wald test. To this end, bootstrap data are generated as described before. Then, a bootstrap Wald test statistic, say $\widehat{W}_b^*$, $b = 1, \ldots, B$, is computed for each bootstrap data set. However, the null hypothesis for the bootstrap Wald statistics is $H_0 : S\mathbf{\Theta}_H = S\hat{\mathbf{\Theta}}_H$ such that the bootstrap statistic is defined by

$$\widehat{W}_b^* = T\left[S\left(\hat{\mathbf{\Theta}}_{H,b}^* - \hat{\mathbf{\Theta}}_H\right)\right]'\left[S\hat{\Omega}_{\hat{\mathbf{\Theta}}_{H,b}}^* S'\right]^{-1}\left[S(\hat{\mathbf{\Theta}}_{H,b}^* - \hat{\mathbf{\Theta}}_H)\right],$$

where $\hat{\mathbf{\Theta}}_{H,b}^*$ and $\hat{\Omega}_{\hat{\mathbf{\Theta}}_{H,b}}^*$ represent the counterparts of $\hat{\mathbf{\Theta}}_H$ and $\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}$ estimated from the b-th bootstrap data set, respectively. These adjustments are necessary since the null hypothesis (A.6) cannot be imposed when generating the bootstrap data in our VAR setup, compare e.g. Horowitz (2001). The p-value for the bootstrap Wald test is obtained in the usual way by

$$p^*(\widehat{W}) = \frac{1}{B}\sum_{b=1}^{B} I(\widehat{W}_b^* > \widehat{W}),$$

where $I(\cdot)$ is the indicator function.

A.3 Bootstrap Confidence Intervals and Conditional t-ratios

The bootstrap confidence intervals presented in Figures 1 to 4 for the impulse response differences are computed according to Kilian (1998a). To be precise, the percentile intervals are derived from the empirical bootstrap distribution of the differences of the two respective impulse responses at a specific horizon h.

To derive the conditional t-ratios used in the empirical analysis we adapt the framework of Jordà (2009, Section II.B) to our specific situation of impulse response differences. First, we need the triangular factorization of $\hat{\Omega}_H^D$, the (estimated) covariance matrix of the impulse response differences contained in $S\hat{\mathbf{\Theta}}_H$. Hence we consider $\hat{\Omega}_H^D = S\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}^B S' = \hat{L}\hat{D}\hat{L}'$, where $\hat{L}$ is a lower triangular matrix having a unit principal diagonal and $\hat{D}$ is a diagonal matrix. Then, the conditional orthogonal impulse response differences of y_{it} and y_{mt} to a shock in ε_{jt} over the horizons $h = 1, \ldots H$ are collected in $\hat{\Psi}_H = \hat{L}^{-1}S\hat{\mathbf{\Theta}}_H$. Hence, we directly determine the conditional impulse response differences. That is, we do not consider the differences of individual conditional impulse responses. This is important to note since the latter approach would not appropriately take into account the correlation structure of the impulse response differences across the H response horizons. The correlation structure, however, is appropriately captured by directly applying the triangular factorization to $\hat{\Omega}_H^D$.

Accordingly, the h-th element of the vector $\hat{\Psi}_H$, say $\hat{\psi}_h$, is the impulse response difference conditional on the impulse response differences $\hat{\psi}_{h-1}, \dots, \psi_1$.

Denoting the h-th diagonal element of $\hat{D}$ by $\hat{d}_h$, we obtain the conditional t-ratio at horizon h by $t_{h|h-1,\dots,1} = \hat{\psi}_h / \hat{d}_h$, $h = 1, \dots, H$. It is straightforward to show that $\widehat{W}^B = \sum_{h=1}^{H} t^2_{h|h-1,\dots,1}$.

A.4 VAR(∞) Representation: Robustness Analysis

As has been mentioned in the literature, the asymptotic covariance matrix of the orthogonale impulse responses $\Omega_{\hat{\mathbf{\Theta}}_H}$ can be singular for a subset of the VAR parameter space, see e.g. Lütkepohl (2005, Chapter 4). If that is the case for the true VAR model (3.1), then the Wald statistic (A.7) is not asymptotically $\chi^2(H)$ distributed.

A way to circumvent this problem is to allow the VAR process to be of infinite order and increase the fitted order with growing sample size, see e.g. Lütkepohl (2005, Chapters 4 and 15), Lütkepohl (1996), and Lütkepohl & Poskitt (1996). To be precise, the model (3.1) is generalized to

$$y_t = \nu + \sum_{i=1}^{\infty} A_i y_{t-i} + u_t.$$

The MA coefficient matrices Φ_h, $h = 0, 1, \dots$, are still obtained via the recursion introduced in section 3.3 such that we further have $\Theta_h = \Phi_h P$. It is assumed that a finite-order VAR process of order p_T is fitted to y_t with $p_T = o(T^{1/3})$. The latter condition is satisfied if the AIC criterion is used in connection with with $p_{max,T} = o(T^{1/3})$, see e.g. Poskitt (2003). Based on the estimated VAR(p_T), the estimators $\hat{\mathbf{\Phi}}_H$ and $\hat{\mathbf{\Theta}}_H$ are obtained as described in subsection A.1. Then, under some further regularity conditions, it follows from Lütkepohl (2005, Proposition 15.4)

$$\begin{aligned} \sqrt{T}(\hat{\mathbf{\Phi}}_H - \mathbf{\Phi}_H) \ &\xrightarrow{d} N\left(0, \Omega^{\infty}_{\hat{\mathbf{\Phi}}_H}\right), \\ \sqrt{T}(\hat{\mathbf{\Theta}}_H - \mathbf{\Theta}_H) \ &\xrightarrow{d} N\left(0, \Omega^{\infty}_{\hat{\mathbf{\Theta}}_H}\right), \end{aligned}$$

where, using the representation of Lütkepohl (1996),

$$\Omega^{\infty}_{\hat{\mathbf{\Phi}}_H} = \left[\Sigma_u^{-1} \otimes \sum_{j=0}^{k-l} \Phi_j \Sigma_u \Phi'_{j+l-k} \right]_{k,l=1,\dots,H}$$

and $\Omega^{\infty}_{\hat{\mathbf{\Theta}}_H}$ is obtained by replacing $\Omega_{\hat{\mathbf{\Phi}}_H}$ in (A.4) with $\Omega^{\infty}_{\hat{\mathbf{\Phi}}_H}$. The estimator $\hat{\Omega}^{\infty}_{\hat{\mathbf{\Theta}}_H}$ is obtained by replacing the unknown quantities with the respective estimators that are obtained in the

Table 4. Results of Wald-Tests (p-values) on Differences in the Impulse Responses of EMU Core and Peripheral Countries up to Response Horizon $H = 4$

Peripheral Country/ Sample (VAR order)	Asymptotic Wald-test $\widehat{W}^{\infty}$	Bootstrap Wald-test $\widehat{W}_b^{*,\infty}$	Wald-test with Bootstrap Covariance Matrix Estimator $\widehat{W}^{B}$
Greece: 1991:2-2007:4 ($p = 2$)			
World Shock	0.1183	0.0428	0.1049
EMU Core Shock	0.5700	0.3236	0.5055
Idiosyncratic Shock	0.0003	0.0000	0.0000
Ireland: 1997:2-2011:3 ($p = 1$)			
World Shock	0.3014	0.0368	0.2104
EMU Core Shock	0.9814	0.5832	0.7914
Idiosyncratic Shock	0.0950	0.0116	0.0002
Portugal: 1991:2-2011:3 ($p = 2$)			
World Shock	0.9819	0.9056	0.8512
EMU Core Shock	0.7367	0.4364	0.6648
Idiosyncratic Shock	0.0693	0.0228	0.0001
Spain: 1995:2-2011:3 ($p = 3$)			
World Shock	0.0621	0.0340	0.0096
EMU Core Shock	0.5413	0.4180	0.1414
Idiosyncratic Shock	0.1144	0.0608	0.0228

usual way. We can then define the Wald statistic for the null hypothesis (A.6) as

$$\widehat{W}^{\infty} = T\left(S\hat{\boldsymbol{\Theta}}_H\right)'\left(S\hat{\Omega}^{\infty}_{\hat{\boldsymbol{\Theta}}_H}S'\right)^{-1}\left(S\hat{\boldsymbol{\Theta}}_H\right),$$

which is asymptotically χ^2 distributed with H degrees.

Some comments are in order. First, from an applied point of view, the assumption of an infinite-order VAR does not affect model selection and estimation. One only has to use a different covariance matrix estimator for the impulse response coefficient estimators. Second, we present the results for $\widehat{W}^{\infty}$ for the four GDP growth rate systems in the second column of Table 4. We keep on using AIC in connection with $p_{max,T} = 4$ to determine the lag order.

The maximum lag order was chosen following Lütkepohl (1996) such that $p_{max,T}$ is approximately equal to $T^{1/3}$ for our samples sizes. From Table 4, we see that the p-values of the Wald statistics $\widehat{W}^{\infty}$ fall compared to the ones associated with $\widehat{W}$, compare Table 2. This is not surprising since the use of $\hat{\Omega}^{\infty}_{\hat{\mathbf{\Theta}}_H}$ instead of $\hat{\Omega}_{\hat{\mathbf{\Theta}}_H}$ leads to *larger* variance expressions in the Wald statistic. In other words, the use of $\hat{\Omega}^{\infty}_{\hat{\mathbf{\Theta}}_H}$ robustifies inference at the expense of efficiency. This is also obvious from the simulation results of Lütkepohl (1996).

Third, the simulation results of Lütkepohl (1996) show that the asymptotic Wald test based on the infinite-order assumption can perform quite poorly in finite samples. Therefore, we have also considered a bootstrap Wald test version for the current framework. The corresponding bootstrap test statistics $\widehat{W}^{*,\infty}_b$ are obtained analogously to $\widehat{W}^{*}_b$. Fourth, the Wald statistic $\widehat{W}^{B}$ obtained using the bootstrap covariance matrix estimator $\hat{\Omega}^{B}_{\hat{\mathbf{\Theta}}_H}$ is asymptotically valid independent of whether a finite- or infinite-order VAR framework is assumed. This follows from Inoue & Kilian (2002) since the bootstrap scheme does not need to be adjusted due to the VAR(∞) assumption as pointed out by Kilian & Kim (2011). The results of $\widehat{W}^{B}$ are reproduced in 4 to simplify the comparison.

Obviously, our main conclusions continue to hold with respect to the asymptotic and the bootstrap tests. Hence, generalizing the VAR setup to an infinite-order framework is not crucial for our empirical analysis. Interestingly, the p-values of the bootstrap test $\widehat{W}^{*,\infty}_b$ are smaller than those associated with $\widehat{W}^{\infty}$. Thus, the empirical distribution of the bootstrap test statistics $W^{*,\infty}_b$, $b = 1, \dots, 2500$, is shifted to the left compared to the χ^2 distribution with four degrees of freedom.

References

Artis, M. J. (2003). Reflections on the optimal currency area (OCA) criteria in the light of EMU, *International Journal of Finance & Economics* **8**: 297–307.

Bayoumi, T. & Eichengreen, B. (1994). One money or many? Analyzing the prospects for monetary unification in various parts of the world, *Princeton Studies in International Finance* **74**: 1–44.

Bayoumi, T. & Eichengreen, B. (1997). Ever closer to heaven? An optimum-currency area index for European countries, *European Economic Review* **41**: 761–770.

Bernard, A. B. & Durlauf, S. N. (1995). Convergence in international output, *Journal of Applied Econometrics* **10**(2): 97–108.

Blanchard, O. & Quah, D. (1989). The dynamic effects of aggregate demand and supply disturbances, *American Economic Review* **79**: 655–673.

Christodoulakis, N. (2009). Ten years of EMU: convergence, divergence and new policy priorities, *National Institute Economic Review* **108**: 86–100.

Dellas, H. & Tavlas, G. (2009). An optimum-currency-area odyssey, *Journal of International Money and Finance* **28**: 1117–1137.

Engle, R. F. & Kozicki, S. (1993). Testing for common features, *Journal of Business and Economic Statistics* **11**: 369–380.

Greene, W. H. (2008). *Econometric Analysis*, 6th edn, Prentice Hall.

Horowitz, J. L. (2001). The bootstrap, *in* J. J. Heckmann & E. E. Leamer (eds), *Handbook of Econometrics, Vol. 5*, North-Holland, Amsterdam.

Inoue, A. & Kilian, L. (2002). Bootstrapping smooth functions of slope parameters and innovation variances in VAR (∞) models, *International Economic Review* **43**(2): 309–332.

Jordà, Ò. (2009). Simultaneous confidence regions for impulse responses, *Review of Economics and Statistics* **91**: 629–647.

Kilian, L. (1998a). Confidence intervals for impulse responses under departures from normality, *Econometric Reviews* **17**: 1–29.

Kilian, L. (1998b). Small-sample confidence intervals for impulse response functions, *Review of Economics and Statistics* **80**: 218–230.

Kilian, L. (1999). Finite-sample properties of percentile and percentile-t bootstrap confidence intervals for impulse responses, *Review of Economics and Statistics* **81**: 652–660.

Kilian, L. & Kim, Y. J. (2011). How reliable are local projection estimators of impulse responses?, *Review of Economics and Statistics* **93**: 1460–1466.

Kim, Y. & Chow, H. K. (2003). Optimum currency area in Europe: an alternative assessment, *Economics Letters* **81**: 297–304.

Lütkepohl, H. (1996). Testing for zero impulse responses in vector autoregressive processes, *Journal of Statistical Planning and Inference* **50**: 1–20.

Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Berlin: Springer-Verlag.

Lütkepohl, H. & Poskitt, D. S. (1996). Testing for causation using infinite-order vector autoregressive processes, *Econometric Theory* **12**: 61–87.

McKinnon, R. I. (1963). Optimum currency areas, *American Economic Review* **53**: 717–725.

Mundell, R. A. (1961). A theory of optimum currency areas, *American Economic Review* **51**: 657–665.

Paruolo, P. (2003). Common dynamics in I(1) VAR systems, *Discussion Paper 2003/35*, Department of Economics, University of Insubria.

Pope, A. (1990). Biases of estimation of multivariate non-gaussian autoregressions, *Journal of Time Series Analysis* **11**: 249–258.

Poskitt, D. S. (2003). On the specification of cointegrated autoregressive moving-average forecasting systems, *International Journal of Forecasting* **19**(3): 503–519.

Schleicher, C. (2007). Codependence in cointegrated autoregressive models, *Journal of Applied Econometrics* **22**: 137–159.

Trenkler, C. & Weber, E. (2012). Codependent VAR models and the pseudo-structural form, *Working Paper ECON 12-10*, University of Mannheim.

Trenkler, C. & Weber, E. (2013). Testing for codependence of non-stationary variables, *Applied Economics* **45**: 1953–1964.

Vahid, F. & Engle, R. F. (1993). Common trends and common cycles, *Journal of Applied Econometrics* **8**: 341–360.

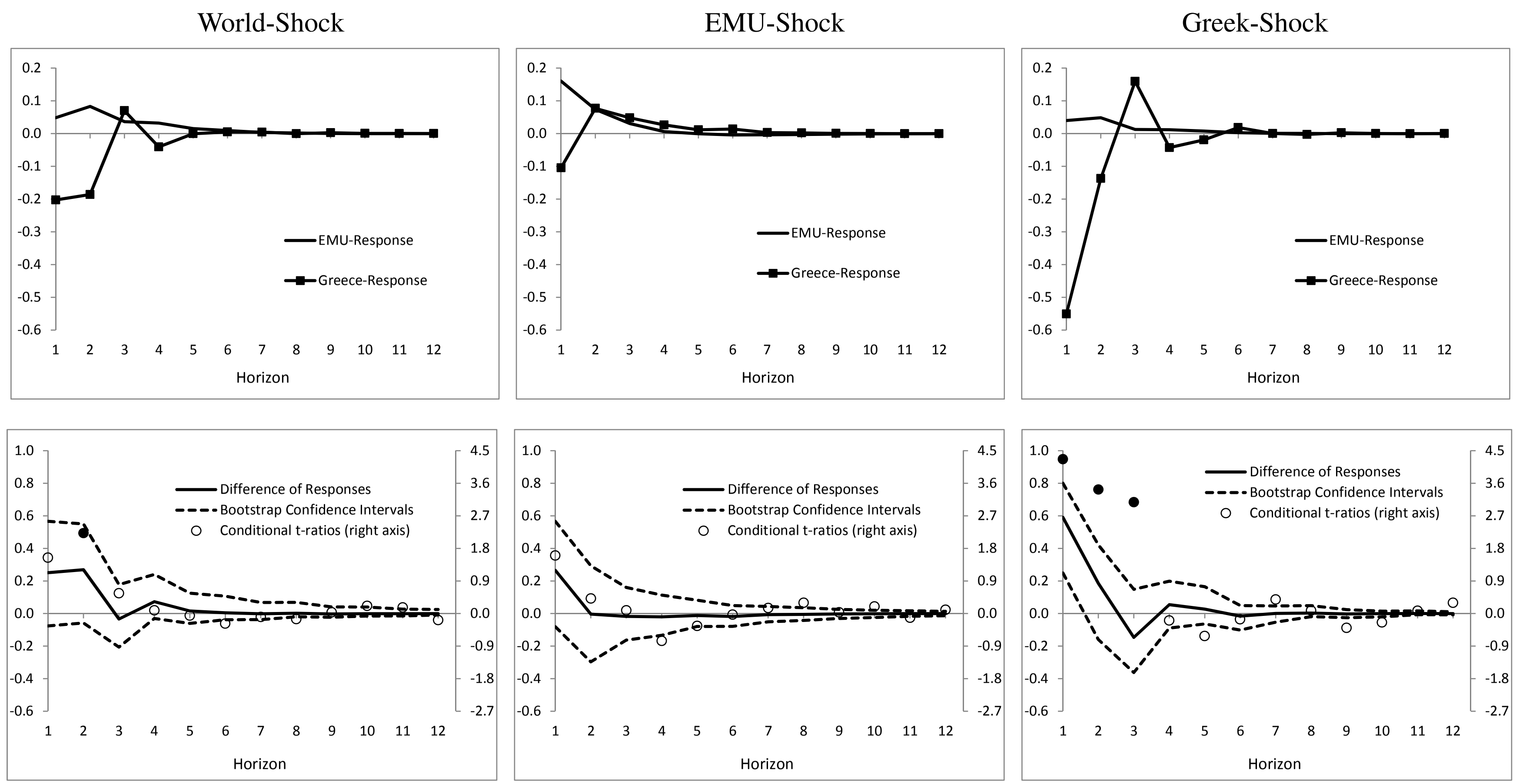

Figure 1. Results of impulse response analysis for Greek system, Sample: 1991:2-1997:4, VAR order $p = 2$. Conditional t-ratios larger than 1.96 in absolute value (see right axis) are indicated by a black circle.

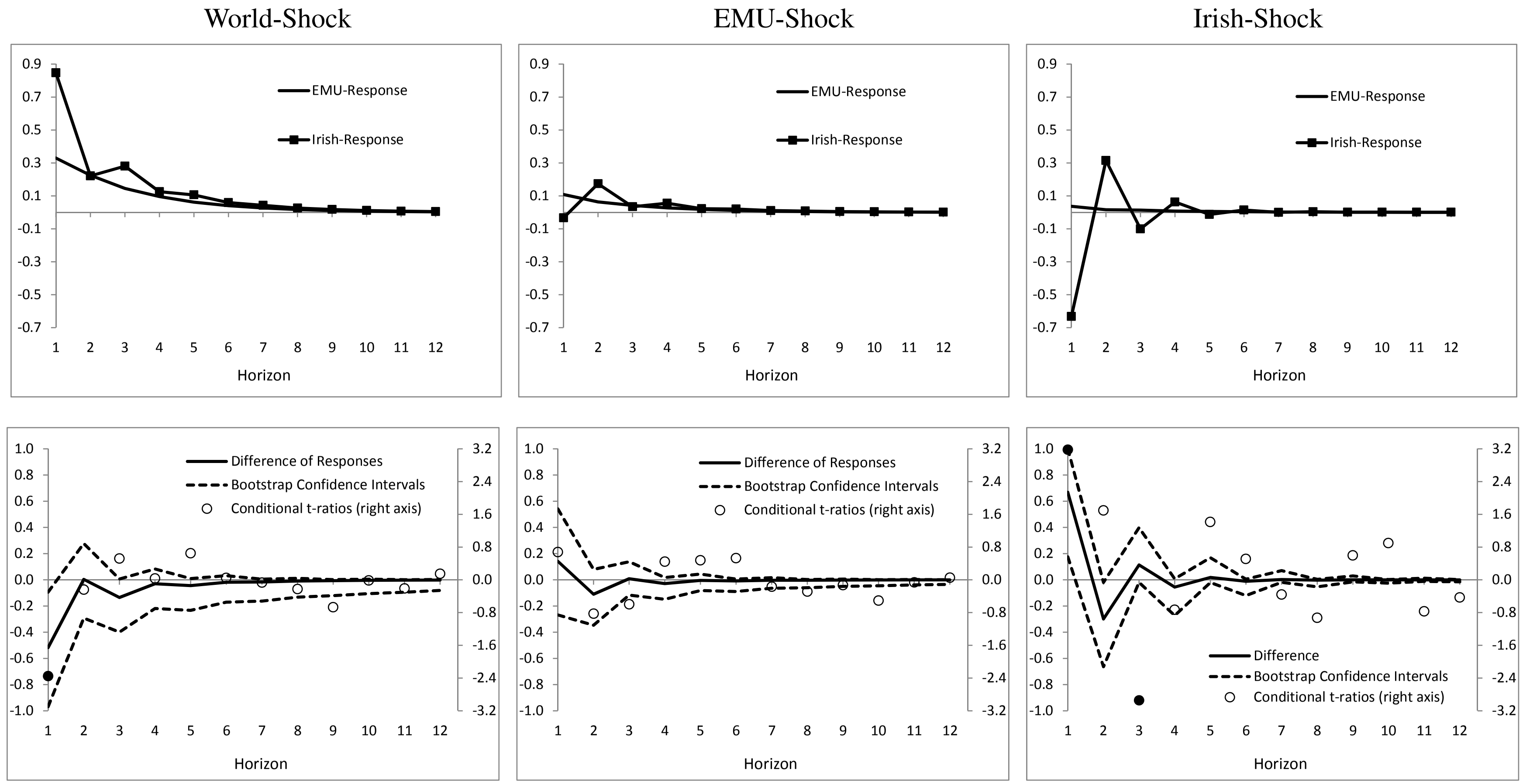

Figure 2. Results of impulse response analysis for Irish system, Sample: 1997:2-2011:3, VAR order $p = 1$. Conditional t-ratios larger than 1.96 in absolute value (see right axis) are indicated by a black circle.

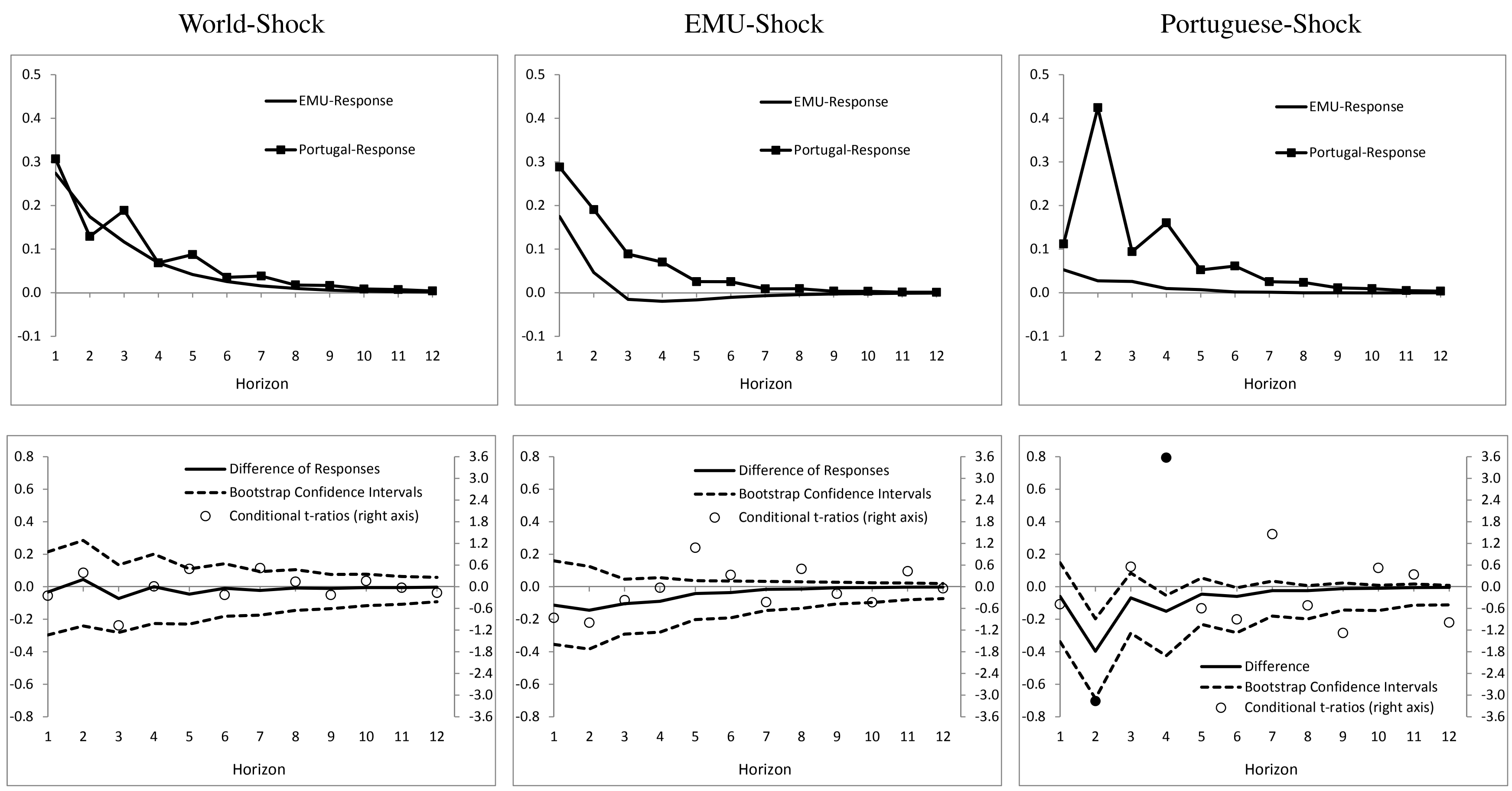

Figure 3. Results of impulse response analysis for Portuguese system, Sample: 1991:2-2011:3, VAR order $p = 2$. Conditional t-ratios larger than 1.96 in absolute value (see right axis) are indicated by a black circle.

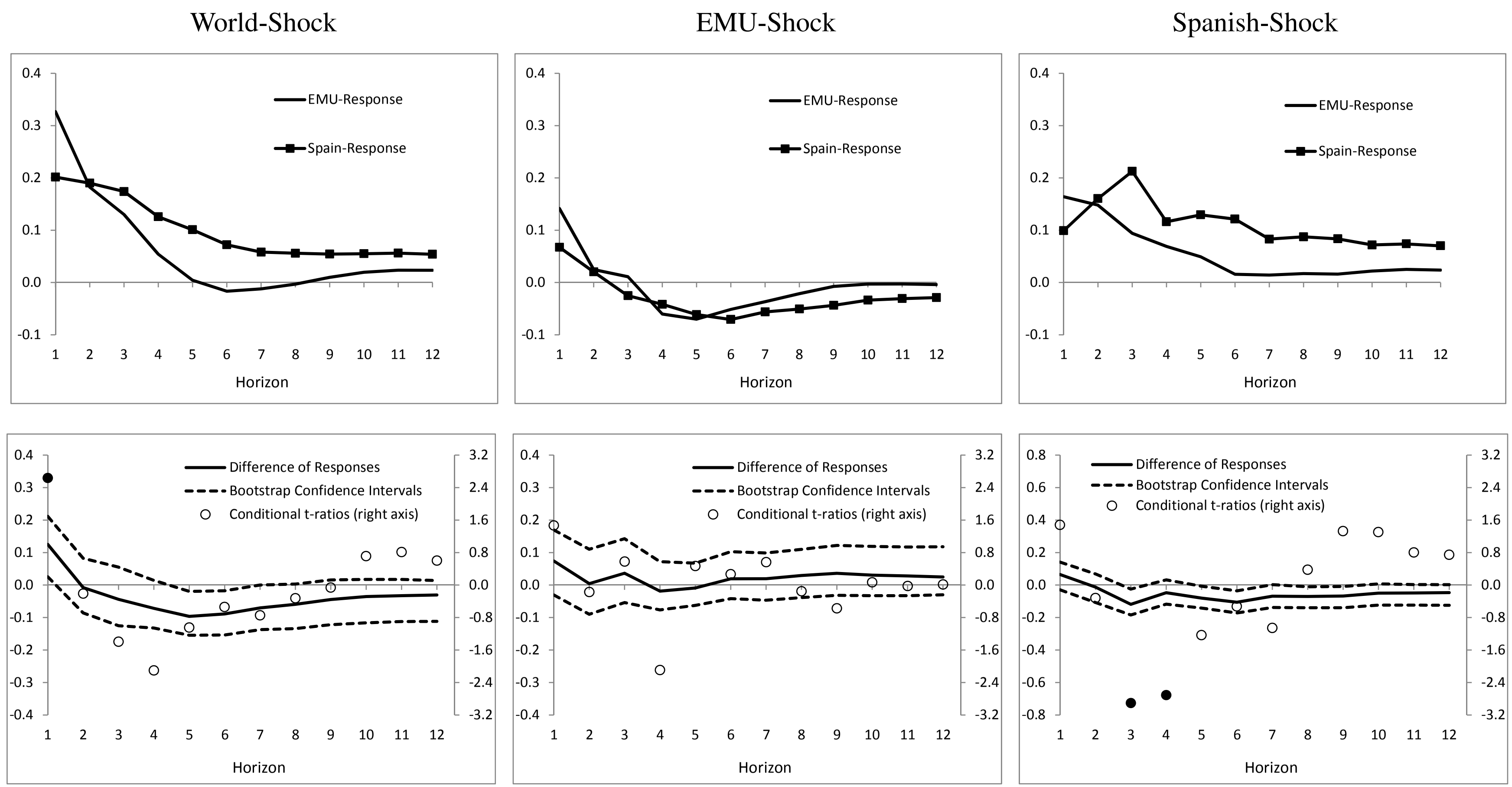

Figure 4. Results of impulse response analysis for Spanish system, Sample: 1995:2-2011:3, VAR order $p = 3$. Conditional t-ratios larger than 1.96 in absolute value (see right axis) are indicated by a black circle.

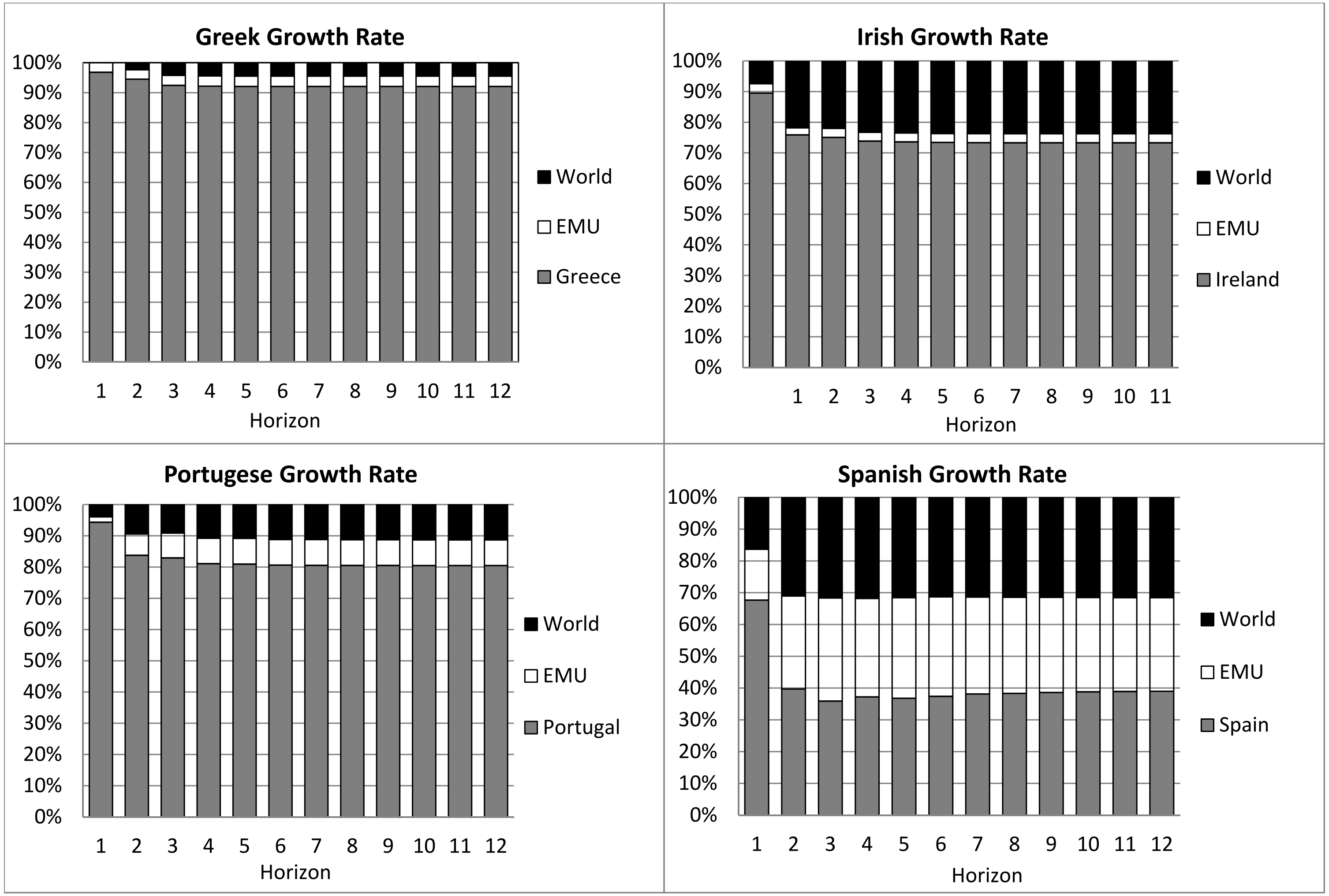

Figure 5. Forecast error variance decompositions of Greece growth rate (Sample: 1991:2-1997:4, VAR order $p = 2$), of Irish growth rate (Sample: 1997:2-2011:3, VAR order $p = 1$), of Portuguese growth rate (Sample: 1991:2-2011:3, VAR order $p = 2$), Spanish growth rate (Sample: 1995:2-2011:3, VAR order $p = 3$)