Extracting current-induced spins: spin boundary conditions at narrow Hall contacts

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Abstract. We consider the possibility to extract spins that are generated by an electric current in a two-dimensional electron gas with Rashba-Dresselhaus spin-orbit interaction (R2DEG) in the Hall geometry. To this end, we discuss boundary conditions for the spin accumulations between a spin-orbit coupled region and contact without spin-orbit coupling, i.e. a normal two-dimensional electron gas (2DEG). We demonstrate that in contrast to contacts that extend along the whole sample, a spin accumulation can diffuse into the normal region through finite contacts and detected by e.g. ferromagnets. For an impedance-matched narrow contact the spin accumulation in the 2DEG is equal to the current induced spin accumulation in the bulk of R2DEG up to a geometry-dependent numerical factor.

1. Introduction

In recent years, there has been an increasing impetus towards generating and detecting spin accumulations and spin currents in nonmagnetic systems. Conventional means of achieving this goal are to use ferromagnets and magnetic fields to inject and/or detect spins [1]. Recently, spin generation based on two related effects, current induced spin accumulation [2, 3, 4] and current induced transverse spin current [6] (known as the spin Hall effect), has attracted considerable attention. In Ref. [6], the spin Hall effect was caused by the spin-orbit (SO) interaction of impurities and the effect is then called "extrinsic". The "intrinsic" SHE caused by a band structure with SO-induced spin splittings was proposed by Sinova *et al.* [9] for the R2DEG and Murakami *et al.* [10] for the hole gas in bulk III-V semiconductors with significant SO interaction. After an initial controversy, it is now generally agreed that in the diffuse regime the SHE vanishes in the bulk of a 2DEG with k-linear (Rashba and/or Dresselhaus) SO coupling [11, 12, 13], but remains finite for extrinsic SO coupling, intrinsic SO coupling in two-dimensional hole systems, and near the edges of a finite diffusive R2DEG [12, 14]. The spin Hall effect has been observed in semiconductor electron [16] and hole [17] systems by the detection of edge spin accumulations with optical methods, and in metals by the electrical detection of spin currents via ferromagnetic leads [18]. Although initial theoretical investigations of the SHE and current-induced spin accumulation has been on bulk disordered conductors using Kubo, Keldysh or Boltzmann formalism [7, 9, 10, 11, 12, 13, 14, 19, 20], it is now understood that the bulk conductivity is not necessarily related to experimentally relevant quantities such as local spin accumulations probed by local optical or electrical probes. In this respect, a more local approach based on spin diffusion equations is advantageous [12, 13]. However, spin diffusion equations have to be supplemented by suitable boundary conditions that have observable consequences. There have been many proposals in that direction [14, 22, 21, 23, 24, 25, 26], but a consensus has not been reached so far.

Here, we focus on the boundary conditions between a (half infinite) 2DEG with finite Rashba type spin orbit coupling (R2DEG) and a (half infinite) 2DEG without spin-orbit coupling connected by a contact that is narrow on the scale of the system, but wider than the mean free path. Such a boundary has been considered by Refs. [23, 26], but for an infintely wide contact region, for which it could be shown that no spin accumulation could diffuse into the 2DEG [26]. We shall show below, however, that for a *narrow* (as opposite to wide) contact, the spin accumulation in the 2DEG is equal to the bulk value of the spin accumulation in R2DEG up to a numerical constant which depends on the geometry that is smaller but can be of the order of unity. These results prove that current induced spins *can* be extracted to a region with small spin-orbit coupling in which the spin lifetime is very long and used for spintronics applications, thus confirming our previous results [14].

This article is organized as follows: we define our model and derive spin diffusion equations in section 2. In section 3, we first recapitulate the symmetry relations for

conductances with respect to measuring the spin accumulation in a normal region with ferromagnetic leads. Next we apply these relations to demonstrate that the spin accumulation from the R2DEG can be extracted into a 2DEG region. In section 4, we focus on a model for a small contact between the R2DEG and the 2DEG and solve it to demonstrate the principle of spin extraction to a region with vanishing SO interaction. The numerical simulations for the diffuse R2DEG—2DEG heterostructure are reported in section 5.

2. Spin diffusion equations in a 2D electron gas with Rashba spin-orbit coupling

In this paper we focus on a disordered finite size 2DEG with Rashba type spin-orbit coupling, noting that the effects of a significant Dresselhaus term can be included straightforwardly. Throughout the paper we shall assume that all length scales of this finite region are much larger than the elastic mean free path such that spin transport is governed by diffusion equations [12, 13] is valid. In this section, we proceed to derive these spin diffusion equations for later convenience.

In 2×2 spin space, our system is defined by the Hamiltonian:

$$H = \frac{\mathbf{p}^2}{2m} + \alpha \mathbf{p} \cdot (\boldsymbol{\sigma} \times \boldsymbol{z}) + U(\mathbf{x}) + V(\mathbf{x}) \tag{1}$$

where $\mathbf{x}$ and $\mathbf{p}$ are the (two-dimensional) position and momentum operators, respectively, $\boldsymbol{\sigma}$ is the vector of Pauli spin matrices (the 2x2 unit vector is implied with scalars), $\boldsymbol{z}$ is the unit vector normal to the 2D plane, and α parameterizes the strength of the SO interaction that can be position dependent e.g. due to local external gates, and $V(\mathbf{x}) = \sum_{i=1}^{N} \phi(\mathbf{x} - \boldsymbol{X}_i)$ the impurity potential, modelled by N impurity centers located at points $\{\boldsymbol{X}_i\}$, which for the sake of simplicity we assume to be spherically symmetric, $U(\mathbf{x})$ is a smooth potential that confines the system to a finite region but allows a few openings to reservoirs.

2.1. Rashba Green function

Our starting point is the impurity averaged Green's function $G(k) = (\hbar^2 k^2/2m + \hbar\alpha\boldsymbol{\eta} \cdot \mathbf{k} - E - i\hbar/\tau)^{-1}$, where $\boldsymbol{\eta} = \boldsymbol{z} \times \boldsymbol{\sigma}$, and τ is the momentum lifetime. In terms of its components, $G(k)$ is given by

$$\frac{2m}{\hbar^2} G(\mathbf{k}) = \frac{1}{2} \left(\frac{1}{k^2 - k_+^2} + \frac{1}{k^2 - k_-^2} \right)$$
$$+ \frac{k_\alpha \mathbf{k} \cdot \boldsymbol{\eta} - k_\alpha^2/2}{k_+^2 - k_-^2} \left(\frac{1}{k^2 - k_+^2} - \frac{1}{k^2 - k_-^2} \right), \tag{2}$$

where $k_\pm^2 = k_F^2 + k_\alpha^2/2 \pm k_\alpha \sqrt{k_F^2 + k_\alpha^2/4} + 2mi/(\tau\hbar)$, $k_\alpha = 2m\alpha/\hbar$ and $k_F = \sqrt{2mE_F/\hbar^2}$. The real space Green function is then obtained by a Fourier transform:

$$G(\mathbf{x}, E_F) = \frac{im}{2\hbar^2} \left[-\frac{1}{2}(H_0(k_+x) + H_0(k_-x)) - \frac{k_\alpha^2/2}{k_+^2 + k_-^2}(H_0(k_+x) - H_0(k_-x)) \right.$$

$$+ \frac{i\boldsymbol{\eta} \cdot \hat{\mathbf{x}} k_\alpha}{k_+^2 - k_-^2} (k_+ H_0(k_+ x) - k_- H_0(k_- x)) \Bigg], \tag{3}$$

where $x = |\mathbf{x}|$. We note that we only need the large $k_F x$ asymptotics of $G(x)$, because we are interested in dilute disorder. The conventional approximation [27] is to expand $G(x)$ to leading order in $1/(k_F r)$ and k_α/k_F:

$$G(\mathbf{x}, E_F) \approx -\frac{im}{2\hbar^2} \sqrt{\frac{2}{k_F x}} e^{ik_F x - i\pi/4 - x/2l} e^{-ik_\alpha \mathbf{x} \cdot \boldsymbol{\eta}/2}, \tag{4}$$

where $l = \hbar k_F \tau / m$. This level of approximation is sufficient for most spin-orbit related applications such as the calculation of Dyakonov-Perel spin relaxation, spin precession, weak antilocalization etc. However in order to study current induced spin accumulation and SHE in diffusive systems, it is necessary to go to higher order in $m\alpha/\hbar k_F$ and $1/(k_F x)$. With these correction terms the asymptotic Green function becomes:

$$G(\mathbf{x}, E_F) \approx \frac{-im}{2\hbar^2} \sqrt{\frac{2}{k_F x}} e^{ik_F x - i\pi/4 - x/2l} \Bigg[e^{-ik_\alpha \mathbf{x} \cdot \boldsymbol{\eta}/2} \left(1 - \frac{k_\alpha}{4k_F} \hat{\mathbf{x}} \cdot \boldsymbol{\eta} \right)$$
$$- \frac{3i}{8k_F x} e^{ik_\alpha \mathbf{x} \cdot \boldsymbol{\eta}/2} + \frac{i}{8k_F x} \left(e^{ik_\alpha x/2} + e^{-ik_\alpha x/2} \right) \Bigg], \tag{5}$$

where $\hat{\mathbf{x}} = \mathbf{x}/x$ In the next subsection, we will use this expression to derive spin diffusion equations for a R2DEG.

2.2. Diffusion equation

We first focus on the equation of motion of the density matrix with coherent spin components. It can be shown that in the limit $E_F \tau/\hbar \gg 1$, the energy resolved density matrix satisfies the following equation [12, 13, 23, 24]:

$$\rho_a(\mathbf{x}, \omega) = \frac{1}{2\pi\nu\tau} \int d^2 x' \mathcal{K}_{ac}(\mathbf{x}, \mathbf{x}'; \omega) \rho_c(\mathbf{x}', \omega), \tag{6}$$

where $\rho_a = \mathrm{tr}(\rho\sigma_a)$, summation over repeated indices is implied, and ν is the density of states and

$$\mathcal{K}_{ab}(\mathbf{x}, \mathbf{x}'; \omega) = \frac{1}{2} \mathrm{Tr} \left(\sigma_a G^R(\mathbf{x}, \mathbf{x}'; E + \omega) \sigma_b G^A(\mathbf{x}', \mathbf{x}; E) \right). \tag{7}$$

Multiplying $\rho(E)$ with the density of states and integrating over energy, we obtain the densities and polarizations, whereas accumulations are obtained by directly integrating over energy. The diffusion equation is obtained by expanding Eq. 7 to second order in spatial gradients. In a homogeneously disordered system we have:

$$\rho_a(\mathbf{x}) = \frac{1}{2\pi\nu\tau} \int d^2 r \mathcal{K}_{ac}(\mathbf{r}) \rho_c(\mathbf{r} + \mathbf{x})$$
$$\approx \frac{1}{2\pi\nu\tau} \int d^2 r \mathcal{K}_{ac}(\mathbf{r}) (\rho_c(\mathbf{x}) + \mathbf{r} \cdot \nabla \rho_c(\mathbf{x}) + r_i r_j \partial_i \partial_j \rho_c(\mathbf{x})), \tag{8}$$

where $\rho_a(\mathbf{x}) = \rho_a(\mathbf{x}; 0)$. We now use the asymptotic expression Eq. (5) for the Green's function and insert the resulting expression in to Eq. (8). The spatial integrals are

elementary and lead to the following equations for the vector components of the density matrix, $s_i = \rho_i/2$ and $n = \rho_0$:

$$D\nabla^2 n - 4K_{s-c}(\boldsymbol{\nabla} \times \mathbf{s})_z = 0 \tag{9}$$

$$D\nabla^2 s_3 - 2K_p(\boldsymbol{\nabla} \cdot \mathbf{s}) = \frac{2s_3}{\tau_s} \tag{10}$$

$$D\nabla^2 \mathbf{s} + 2K_p\boldsymbol{\nabla}s_3 - K_{s-c}(\mathbf{z} \times \boldsymbol{\nabla})n = \frac{\mathbf{s}}{\tau_s} \tag{11}$$

Here $D = v_F^2\tau/2$, $\tau_s = \tau(1 + 4\xi^2)/2\xi^2$ (the Dyakonov-Perel spin relaxation time), $K_{s-c} = \alpha\xi^2/(1 + 4\xi^2)$, $K_p = \hbar k_F\xi/m(1 + 4\xi^2)^2$ and $\xi = \alpha p_F\tau/\hbar$. A similar expansion for the spin current, this time to first order in the spatial gradients, produces the analog of Fick's law for spin diffusion:

$$j_j^i = \frac{\nu v_F\xi}{1 + 4\xi^2}\left(\delta_{i3}\left(s_j - \epsilon_{jm3}\frac{\alpha\tau}{2}\nabla_m n\right) - \delta_{ij}s_3\right) - \nu D\nabla_j s_i. \tag{12}$$

When supplied with suitable boundary conditions the diffusion equations (9-11) and the spin current expression (12) can be solved to obtain all spin and charge conductances. Here, we are mainly interested in the boundary between a R2DEG and a 2DEG (for hard wall boundary conditions see Refs. [21, 23, 24]). In this case, the boundary conditions require the continuity of the spin current [14, 26]

$$\left.\frac{\nu v_F\xi}{1 + 4\xi^2}\left(\delta_{i3}\left(\mathbf{n} \cdot \mathbf{s}^R - \frac{\alpha\tau}{2}\mathbf{z} \cdot (\mathbf{n} \times \nabla)n\right) - n_i s_3^R\right)\right|_0 - \left.\nu D\mathbf{n} \cdot \nabla s_i^R\right|_0 = \left.\nu D\mathbf{n} \cdot \nabla s_i^N\right|_0, \tag{13}$$

where $\mathbf{s}^R$ and $\mathbf{s}^N$ are the spin accumulations in the R2DEG and the 2DEG respectively, and $\mathbf{n}$ is the unit normal vector at the interface. A common choice for the matching condition for the spin accumulation at the interface is to assume that the spin accumulations are continuous (see e.g. Ref. [1]):

$$\left.\mathbf{s}^R\right|_0 = \left.\mathbf{s}^N\right|_0. \tag{14}$$

This condition has been criticized recently in Ref. [26] in which it was demonstrated that for an infinite interface with a constant electric field parallel to it:

$$\left.\left(\mathbf{s}^R + \frac{\alpha\tau}{2}\mathbf{n}\left(\mathbf{n} \cdot (\mathbf{z} \times \boldsymbol{\nabla}n)\right)\right)\right|_0 = \left.\mathbf{s}^N\right|_0 \tag{15}$$

We first note that when the charge current is perpendicular to the interface, such as for a two-probe configuration [42], these two boundary conditions agree and no controversy exists. However, for an infinite interface where the charge current density is homogeneous, the difference between these two boundary conditions is drastic: if Eq. (14) is valid, a current induced spin accumulation diffuses into the 2DEG. On the other hand, if Eq. (15) is valid, the spin accumulation vanishes in the 2DEG. We solve this conundrum below by showing that for a contact smaller than the spin relaxation length (as assumed in Ref. [14]), the two boundary conditions lead to results that agree up to a numerical factor of the order of unity. We therefore conclude that it *is* possible to extract spin accumulation to the 2DEG and detect it with a ferromagnet.
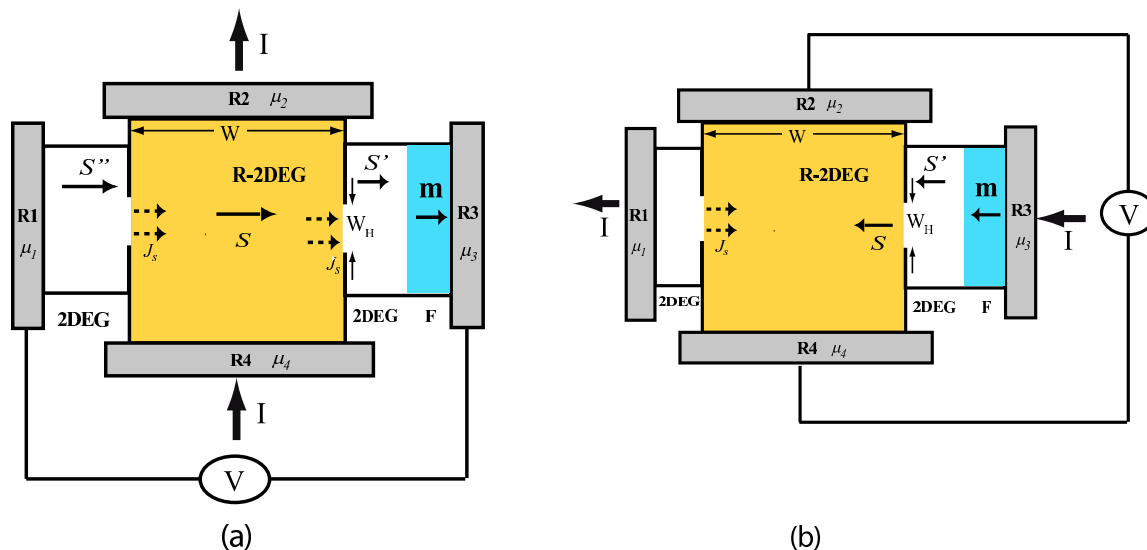
Figure 1. Setup for detection of current induced spins

3. Onsager's relations and the spin boundary conditions

In this section we provide a general symmetry argument based on Onsager's relations, that that proves viability of electric detection of the SHE and the current induced spin accumulation by finite size contacts. Let us start by addressing the symmetry properties of multiprobe conductances relevant for the combination of a spin-orbit coupled region with a ferromagnet via a normal region (Fig. 1), using Onsager's relations [28, 29, 30, 31, 32]. We are particularly interested in the setup shown in Fig. 1. The configuration in Fig. 1a is designed to measure the spin accumulation in the 2DEG injected from the neighbouring R2DEG. The voltage signal V directly observes boundary conditions between R2DEG and 2DEG when the charge current is *parallel* to the boundary. The setup in Fig. 1b, on the other hand, measures how much spin is injected into the R2DEG from the ferromagnet through the 2DEG. V measures directly the spin boundary conditions for a charge current *perpendicular* to the boundary. Onsager relations relate these two conductances, enabling us to relate the boundary conditions when the current is parallel or perpendicular to the boundary.

3.1. Onsager's relations

A generic SO-coupling operator consists of combinations of velocity and spin operators that are invariant under time reversal. When the spin-orbit coupled region is brought into contact with a Ferromagnetic region, the Hamiltonian of the combined system has the symmetry $TH(\mathbf{m})T^{-1} = H(-\mathbf{m})$, where $\mathbf{m}$ is a unit vector in the direction of the magnetization of the ferromagnet and T the time-reversal operator. We now focus on a the specific four-probe setups in Fig. 2 (for a more general discussion see Ref. [32]). The currents in the leads and the respective chemical potentials of the reservoirs are related in linear response as $I_i = \sum_j G_{ij}\mu_j$. We now use the Landauer-

Büttiker formalism to obtain G_{ij}. The scattering matrix for the spin orbit (SO) coupled region and the ferromagnetic region is given respectively by S_{SO} and $S_{\mathbf{m}}$. The symmetry properties of these matrices are self-duality (reflecting the presence of spin-orbit coupling) $S_{SO} = \Sigma_2 S_{SO}^T \Sigma_2$, and $S_{\mathbf{m}} = \Sigma_2 S_{-\mathbf{m}}^T \Sigma_2$, where Σ_2 is block diagonal in the Pauli matrix σ_y [33]. We are interested in the block structure of S_{SO} singling out lead 3 combining the SO and F regions:

$$S_{SO} = \begin{pmatrix} r_{SO} & t'_{SO} \\ t_{SO} & r'_{SO} \end{pmatrix} \tag{16}$$

where the matrix r_{SO} includes all reflections and transmissions that begin and end in the leads 1, 2 and 4. Using the rules for combining S-matrices, we obtain the joint S-matrix of the combined SO|F region:

$$t = t_{\mathbf{m}}[1 - r'_{SO}r_{\mathbf{m}}]^{-1}t_{SO} \tag{17}$$

$$t' = t'_{SO}[1 - r_{\mathbf{m}}r'_{SO}]^{-1}t'_{\mathbf{m}} \tag{18}$$

$$r = r_{SO} + t'_{SO}r_{\mathbf{m}}[1 - r'_{SO}r_{\mathbf{m}}]^{-1}t_{SO} \tag{19}$$

$$r' = r'_{\mathbf{m}} + t_{\mathbf{m}}[1 - r'_{SO}r_{\mathbf{m}}]^{-1}r'_{SO}t'_{\mathbf{m}} \tag{20}$$

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \tag{21}$$

Using these rules we obtain the symmetries of the combined S matrix: $\Sigma_2 t^T(\mathbf{m})\Sigma_2 = t'(-\mathbf{m})$ and $\Sigma_2 r^T(\mathbf{m})\Sigma_2 = r(-\mathbf{m})$ which in turn leads to the Onsager relations. For the two probe configuration, $G(\mathbf{m}) = G(-\mathbf{m})$. For the four probe configuration the transmission probabilities satisfy $T_{ij}(\mathbf{m}) = \mathrm{tr}(t_{ij}t_{ij}^{\dagger}) = T_{ji}(-\mathbf{m})$. Focusing on the current/voltage configuration: $I_1 = -I_3$, $I_2 = -I_4$, $eV_1 = \mu_3 - \mu_1$ and $eV_2 = \mu_4 - \mu_2$ [29] the relation between currents and voltages can be expressed as [30]:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} \alpha_{11}(\mathbf{m}) & -\alpha_{12}(\mathbf{m}) \\ -\alpha_{21}(\mathbf{m}) & \alpha_{22}(\mathbf{m}) \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \tag{22}$$

where the coefficients $\alpha_i j$ can be found in Eqs. (4.a-4d) of Ref. [30]. The Onsager relations can then be expressed as:

$$\alpha_{ij}(\mathbf{m}) = \alpha_{ji}(-\mathbf{m}). \tag{23}$$

If we choose (say) I_1 equal to zero, the relation between the applied current and the spin-Hall voltage is: $I_2 = V_1(\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})/\alpha_{12}$. For phase incoherent conductors, we can ignore the interference terms that arise while obtaining the transmission probabilities, but the Onsager relations Eq. (23) are unaffected. For a general analysis based on Kubo formula see Ref. [32]

This analysis implies the equivalence of two Hall measurements: (i) setting I_1 equal to zero and detecting V_1 generated by an applied I_2 (Fig. 1a) and (ii) switching magnetization, setting I_2 equal to zero and detecting V_2 (Fig. 1b). In other words, driving a current I_2 through the system and detecting the spin Hall voltage with a ferromagnetic contact is equivalent to driving a spin accumulation into the SO region
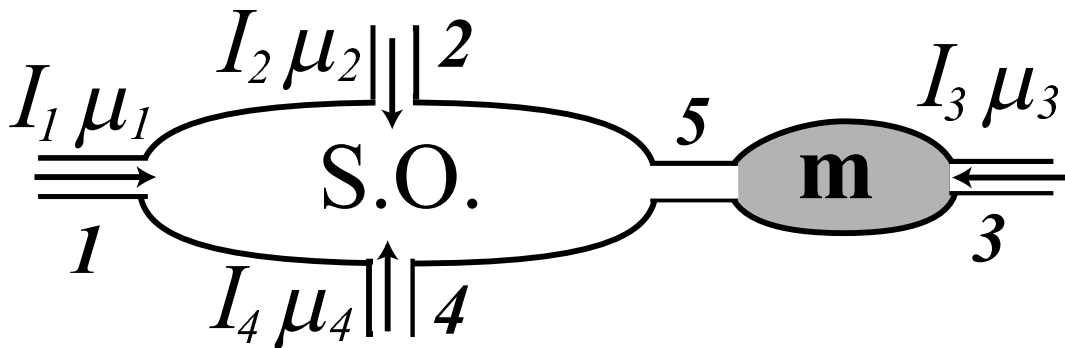
Figure 2. a generic four-probe setup for the detection of current-induced spins

via a ferromagnetic contact that leads to a real Hall voltage detected by normal contacts. In the next subsection, we shall exploit this symmetry to gain insight to the boundary conditions for a R2DEG|2DEG interface.

3.2. four-probe setup and boundary conditions

We now use the Onsager relations from the previous subsection to better understand the spin boundary value problem. Consider the four-probe setup in Fig. 1. When the ferromagnetic lead is a Hall contact, the vanishing spin transfer derived by Ref. [26] for a (infinitely) wide contact seems to imply that there is neither spin accumulation nor spin current near the ferromagnetic reservoir and therefore no Hall voltage. On the other hand, in the Onsager equivalent measurement, spins are injected from the ferromagnet into the normal region. Since in this case the current is perpendicular to the boundary, the spin accumulations can be matched [26] and a spin accumulation in the SO region exists. However, the diffusion equation (9) implies that a spin accumulation gives rise to a voltage drop in the spin orbit region [34, 18]. Onsager's relations discussed in the previous section imply that these two voltages must be the same provided the injected currents are the same. Thus the result for an infinite contact that a current-induced spin accumulation can not enter Hall contacts [26] appears to be misleading. In the following we shall demonstrate that the spin accumulations around the Hall contact must be close up to a numerical factor around the Hall contact.

We now focus on the current-voltage setup in Fig. 1b. In this case the current is perpendicular to the boundary, so the spin accumulations are continuous across an ideal R2DEG|2DEG interface. Assuming a diffusive ferromagnet magnetized parallel to the current direction and ignoring the resistivity of the normal region, we obtain the spin current polarized in the magnetization direction entering the R2DEG:

$$I_s^m \propto \frac{I}{L_s} \frac{\delta D}{\Lambda}, \tag{24}$$

where $L_s = \sqrt{D\tau_s}$ is the (Dyakonov-Perel) spin relaxation length in the R2DEG and

$$\Lambda\left(\mathbf{m}\right) = L_s^{-1} D\nu_R \mathbf{m} \cdot \boldsymbol{\mu} + L_{sF}^{-1} D_F \nu_F (1 - \delta D^2/4). \tag{25}$$

Here, L_{sF}, D_F, ν_F are the spin relaxation length, diffusion constant and average density of states in the ferromagnet, respectively, $\delta D = (\nu_+ D_+ - \nu_- D_-)/(\nu_F D_F)$, $\nu_\pm$ and $D_\pm$ are the density of states and diffusion constants of the majority and minority spin electrons, $\boldsymbol{\mu}$ is a linear function of $\mathbf{m}$ of order unity that depends on the details of the geometry of the contact. The spin accumulation in the SO region localized within a depth of L_s at the contact aperture. acts as a dipole source for the diffusion equation:

$$\nabla^2 n = \boldsymbol{\nabla} \cdot \mathbf{P}, \tag{26}$$

with dipole density $\mathbf{P} = -4K_{s-c}(\mathbf{z} \times \mathbf{s})/D$. We then estimate the potential drop in the Hall direction to be:

$$\phi = \frac{K_{s-c}}{D} \frac{1}{W} \int d\mathbf{r} s(\mathbf{r}), \tag{27}$$

which is proportional to the integrated spin accumulation

$$\int d\mathbf{r} s(\mathbf{r}) \approx L_s I \frac{\delta D}{\Lambda}. \tag{28}$$

The potential drop is therefore:

$$\phi_b = \frac{\alpha \tau}{L_s} \frac{I}{W} \frac{\delta D}{\Lambda} = \frac{\alpha}{v_F} \frac{\xi}{\sqrt{1 + \xi^2}} \frac{I}{W} \frac{\delta D}{\Lambda}, \tag{29}$$

up to a numerical constant.

   We now focus on the potential drop in the Onsager-equivalent setting in Fig. 1a. According to the boundary condition Eq. (15), the current induced spin accumulation does not enter the normal region. Then the potential drop at the ferromagnet|2DEG interface would be zero in contradiction to Onsager's relations. Let us assume that the spin accumulations at the R2DEG and 2DEG near the contact are equal to each other up to a numerical constant Z, i.e. $s_{2DEG} = Z s_{R2DEG}$. Then the calculation of the potential drop proceeds similar to Ref. [32]. Again ignoring the resistance of the 2DEG region, we obtain a potential drop as:

$$\phi_a = Z \frac{\alpha}{v_F} \frac{\xi}{\sqrt{1 + \xi^2}} \frac{I}{W} \frac{\delta D}{\Lambda}, \tag{30}$$

up to a numerical factor. Comparing with Eq. (29) and noting that we have ignored all numerical factors in the calculations above, we conclude that Z must be a numerical factor of the order unity in order to satisfy Onsager's relations. In the next section we shall consider a model for a narrow contact and show that this is indeed the case.

4. Model for spin accumulation near a contact

In this section we focus on the current density and spin accumulation near a finite contact between a half-infinite R2DEG and a half-infinite 2DEG (Fig. 3a). The model we adopt is sketched in Fig. 3b. Asymptotically, we have a constant current density in the left region (R2DEG) in the y direction whereas in the right region (2DEG) the charge current density vanishes. The two regions are divided by an infinitely thin and
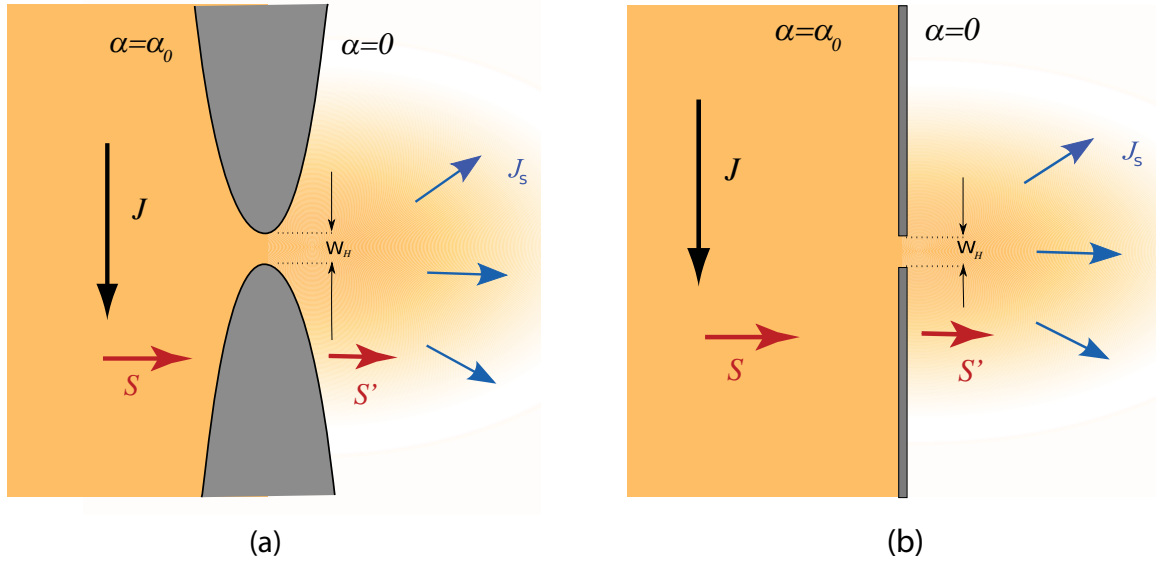
Figure 3. Geometry of the contact: (a) 2D electron gas with a constriction in the middle. On the left side there is an applied homogeneous current density which is modified near the opening. On the right side, the current density far away from the contact as well as the net charge current flowing from the left region to the right region is zero. However, there are a finite spin current and a finite spin accumulation in the right region. The respective mobilities of the left and right regions are assumed to be the same but the Rashba coefficients are different. (b) an idealized version of (a) used in the calculations of this section. The origin is chosen at the center of the opening with width W_H.

high potential barrier, except for an opening (the contact) of size W_H centered at $(0, 0)$.

We note that the solution to this problem closely follows that of an analogous one in magnetostatics [35]. We proceed by expressing the chemical potential n in terms of the (yet undetermined) solution ϕ of the Laplace equation:

$$
\begin{aligned}
n &= \frac{J_0 y}{\nu D} + \phi(x, y) \quad \text{if } x < 0 \\
n &= -\phi(x, y) \qquad\quad \text{if } x > 0,
\end{aligned}
\tag{31}
$$

where J_0 is the bulk current density in the R2DEG. The asymmetric behaviour of ϕ in left and right regions is dictated by the current continuity at $x = 0$. The boundary conditions are:

$$
\begin{aligned}
\phi(0, y) &= -\frac{J_0 y}{2\nu D} \quad \text{if } |y| < W_H/2 \\
\frac{\partial \phi(0, y)}{\partial x} &= 0 \qquad\quad \text{if } |y| > W_H/2.
\end{aligned}
\tag{32}
$$

Next we expand ϕ in terms of the modes of the Laplace equation:

$$
\phi(x, y) = \int_0^\infty dk \, A(k) e^{-k|x|} \sin(ky).
\tag{33}
$$

The solution to the diffusion equation with the above boundary conditions then reduces to that of a dual integral equation:

$$\int_0^\infty dk \, A(k) \sin(ky) = -\frac{J_0 y}{2\nu D} \quad \text{if } |y| < W_H/2$$

$$\int_0^\infty dk \, k A(k) \sin(ky) = 0 \qquad \text{if } |y| > W_H/2. \tag{34}$$

Such integral equations arise commonly in potential theory for mixed boundary conditions (see Ref. [35] for the solution in 3D). In our case the solution is

$$A(k) = -\frac{j_0 W_H}{4\nu D} \frac{J_1(kW_H/2)}{k}. \tag{35}$$

We may now express the spin accumulations in terms of $A(k)$. For the sake of simplicity, we at first disregard the precession term, proportional to K_p, in the spin diffusion equations Eqs. (9-11). We shall be particularly interested in the question whether current-induced spin accumulation in the spin-orbit coupled region can leak out of the contact, into the normal (i.e. no spin-orbit interaction) region. In the bulk of the R2DEG, the current is in the y direction, so the current-induced spin accumulation is polarized in the x direction. Then the general solution to the spin diffusion equations in the R2DEG region is given by:

$$s_x(x, y)^- = \frac{\alpha\tau}{2} \left(\frac{J_0}{\nu D} + \frac{\partial \phi(x, y)}{\partial y} \right) + \delta s_x(x, y), \tag{36}$$

where δs_x satisfies the source-free (i.e. zero charge current) diffusion equation that can be expanded as:

$$\delta s_x(x, y) = \int_0^\infty dk \, B(k) e^{-\kappa|x|} \cos(ky), \tag{37}$$

where $\kappa = \sqrt{k^2 + L_s^{-2}}$. For the 2DEG side $(x > 0)$, a similar expansion gives:

$$s_x^+(x, y) = \int_0^\infty dk \, D(k) e^{-k|x|} \cos(ky). \tag{38}$$

Using the boundary conditions that the spin current is continuous and s_x is discontinuous by an amount equal to $(\alpha\tau/2)dn/dy$ [26], we find that the accumulation in the 2DEG satisfies:

$$D(k) = -\frac{\alpha\tau}{2} k A(k) - (\kappa/k) B(k), \tag{39}$$

and $D(k)$ is determined from $A(k)$, through the following dual integral equations:

$$\int_0^\infty dq \, D(q) \left(1 + \frac{q}{\sqrt{q^2 + \lambda^2}} \right) \cos(q\bar{y}) = -\int_0^\infty dq \, A(q) \frac{W\alpha\tau q^2}{\sqrt{q^2 + \lambda^2}} \cos(q\bar{y}) \tag{40}$$

if $|\bar{y}| < 1$, and

$$\int_0^\infty dq \, q D(q) \cos(q\bar{y}) = 0 \tag{41}$$

if $|\bar{y}| > 1$. Here we have introduced dimensionless variables $q = kW_H/2$, $\bar{y} = 2y/W_H$ and $\lambda = W_H/2L_s$. In the limit $\lambda \gg 1$ (wide contact), expanding Eq. (40) to leading

order in λ^{-1} we obtain that $D(k)$ vanishes like λ^{-1}, in agreement with Ref. [26]. In the opposite limit $\lambda \ll 1$ (narrow contact), we again expand Eq. (40), this time to leading order in λ. We then identify the resulting integral equation with the y derivative of Eq. (34) times $\alpha\tau/2$. Thus we show that $D(k) = -\frac{\alpha\tau}{2}kA(k)/2$ solves Eq. (40) up to order λ^2 corrections. Then the spin accumulation in the 2DEG near a narrow contact is given by:

$$s_x^+(0, y) \approx \frac{\alpha\tau}{4}\frac{dn(0, y)}{dy} = \frac{\alpha\tau J_0}{8\nu D}. \tag{42}$$

We see that the spin accumulation in the 2DEG does not vanish even when the mobilities of both sides are equal. For comparison, we also calculate the spin accumulation under the assumption that there is no jump in the accumulations. We obtain that in this case the spin accumulation is twice as large as $s_x^+(0, y)$. The presence of the term proportional to K_p generates z-polarized spin currents going into the 2DEG, owing to the precession of y polarized spin accumulation as it diffuses out of the R2DEG, but does not change the general picture presented above. We conclude that the choice of the boundary condition for spin accumulation near a narrow contact is not important qualitatively, because either boundary condition produces identical result up to a numerical factor, in agreement with the Onsager's relations.

5. Numerical results

In this section, we shall provide a numerical demonstration of the results of the previous section, *i.e.* the possibility of extracting spin accumulations to a normal region with small contacts. We focus on the discretized version of the hamiltonian (1). Discretization with lattice spacing a yields the following tight-binding representation of $\mathcal{H}_0$ [36]:

$$\mathcal{H}_0 = \frac{\hbar^2}{2ma^2}\left\{\sum_{n,m}(4 + \bar{U})c_{n,m}^\dagger c_{n,m} + \sum_{n,m}\left(\left[- c_{n,m}^\dagger c_{n+1,m}\right.\right.\right. \tag{43}$$

$$\left.\left.\left. - c_{n,m}^\dagger c_{n,m+1} + i\bar{\alpha}c_{n,m}^\dagger\sigma_y c_{n+1,m} - i\bar{\alpha}c_{n,m}^\dagger\sigma_x c_{n,m+1}\right] + \text{H.c.}\right)\right\}$$

where $n(m)$ is the $x(y)$-coordinate of the site (n, m), $\bar{\alpha} = (ma/\hbar)\alpha$. The abbreviation $c_{n,m}^\dagger = (c_{n,m,+}^\dagger, c_{n,m,-}^\dagger)$ was used, where $c_{n,m,\sigma}^\dagger$ $(c_{n,m,\sigma})$ creates(annihilates) an electron at site (n, m) with spin orientation σ with respect to the $\hat{z}$-direction. We also define the spin precession length $L_{\text{SO}} = \pi a/\bar{\alpha}$, which is related to L_s by $L_{\text{SO}} = 2\pi L_s$ in the dirty limit, but remains well-defined for ballistic systems where there is no spin relaxation. In this model, instead of dilute localized scatterers, we shall assume Anderson disorder: the dimensionless onsite potential $\bar{U}$ is set to a different random value $\bar{U} \in [-U_0/2; U_0/2]$ at each lattice site (n, m) of the disordered region, where U_0 accounts for the strength of the disorder [43]. The parameter U_0 is related to the momentum relaxation rate τ and the electron mean free path $l = v_{\text{F}}\tau$ by:

$$\tau = 48a^2\frac{m}{\hbar U_0^2}, \qquad l = 48a\frac{\sqrt{\epsilon_{\text{F}}}}{U_0^2}, \tag{44}$$
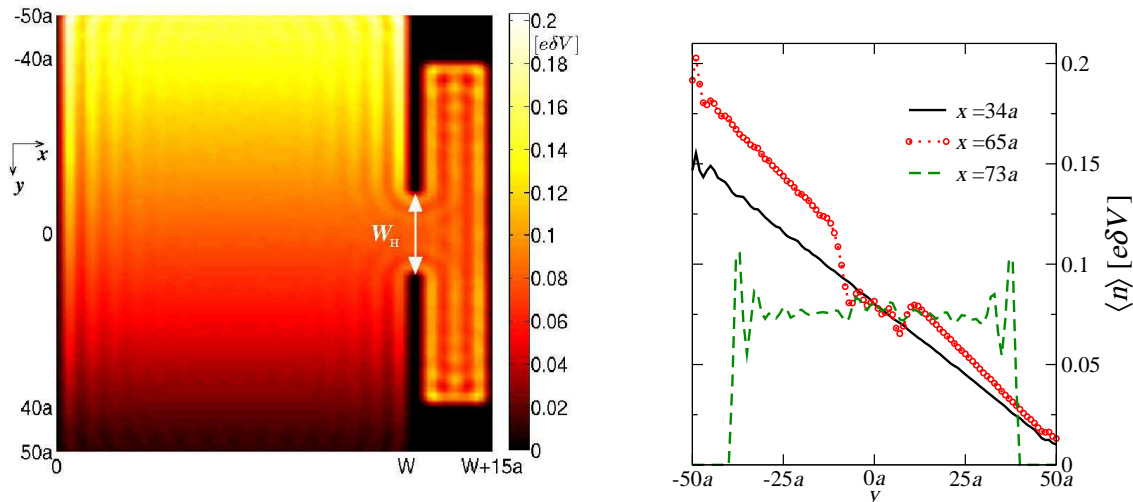
Figure 4. Left panel: Geometry used for numerical calculations: A disordered wire with spin orbit coupling and width W connected to two clean leads with spin-orbit coupling and to a disordered side-pocket of size $14a \times 80a$ without spin-orbit coupling. The colourplot shows the nonequilibrium density $\langle n \rangle$ averaged over 60000 disorder configurations for a system with $L_{SO} = 35a$, $W = 68a$, $W_H = 20a$. The rapid oscillations are due to the finite number of channels in the wire. Nevertheless, the slow varying part satisfies the diffusion equation. Right panel: Electron density $\langle n \rangle$ of the system shown in the left Panel as a function of vertical coordinate y for fixed horizantal coordinate $x = 34a$ (black solid line), $x = 65a$ (red circles) and $x = 73a$ (green dashed line).

where $\epsilon_F = (\hbar^2/2m^*a^2)^{-1}E_F$ and E_F is the Fermi energy. In the rest of this section, we choose $U_0 = 2$ and $\epsilon_F = 0.38$ in order to ensure that the transport through the system is diffusive. With this choice of parameters the mean free path $l \approx 7.4a$ is smaller than any length scale characterizing the system.

In order to study the spin accumulation extracted to a normal region we focus on the setup shown in Fig. 4, where a normal region (*i.e.* $\bar{\alpha} = 0$) with a size of $80a \times 14a$ is attached to a Rashba spin-orbit coupled wire of infinite length, width W and constant finite spin orbit coupling $\bar{\alpha} > 0$ via a contact of size W_H. Disorder of strength U_0 is present inside the normal region and in the spin-orbit region for $-50a < y < 50a$. We shall use the nonequilibrium Green function method [44] to calculate the lesser Green function $G^<(\vec{r}; \vec{r}')$ which is related to spin accumulation according to

$$s_x(\vec{r}) = -\frac{1}{2}i \operatorname{Tr}[\sigma_x G^<(\vec{r}; \vec{r})] \tag{45}$$

and to the electron density through

$$n(\vec{r}) = -i \operatorname{Tr}[G^<(\vec{r}; \vec{r})]. \tag{46}$$

Here, we focus on the ensemble averaged accumulations $\langle s_x \rangle$ and $\langle n \rangle$. The variances are also of interest [40, 41], but we shall not consider them here.
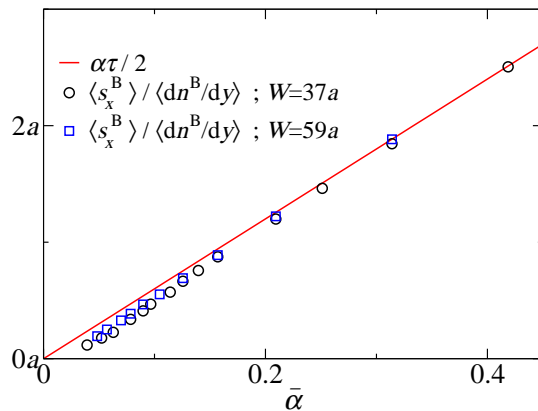
Figure 5. The ratio $\langle s_x^{\mathrm{B}} \rangle / \langle \mathrm{d}n^{\mathrm{B}}/\mathrm{d}x \rangle$ as a function of $\bar{\alpha}$ calculated numerically for two different geometries with $W_{\mathrm{H}} = 20a$ and $W = 37a$ (black dots), $W = 59a$ (blue squares) and estimated as in Eq. (11) (red line). $\langle s_x^{\mathrm{B}} \rangle$ and $\langle \mathrm{d}n^{\mathrm{B}}/\mathrm{d}x \rangle$ have been evaluated by averaging over 20000 disorder configurations as well as over the area indicated by the blue square shown in the bottom panel of Fig. 6.

We apply a small bias δV between the chemical potentials of the top and the bottom lead and generate a current in y direction. The left panel of Fig. 4 shows the electron density $\langle n \rangle$ inside the system when a current is passed from the top to the bottom. Due to the disorder in the central region ($-50a < y < 50a$) the electron density decreases from top to bottom. In the right panel of Fig. 4 we show the dependence of $\langle n \rangle$ on y for three different values of x. We observe that $\langle n \rangle$ decreases linearly in the bulk of the spin-orbit region (solid line), showing that the system is diffusive. For $x = 65a$ (circles) the side contact at $x = 68a$ disturbs the homogeneous current flow. Inside the normal region, $x = 73a$, $\langle n \rangle$ is approximately constant (dashed line).

The current driven by δV, generates a spin accumulation in the bulk of the R2DEG. According to Eq. (10), $\langle s_x^{\mathrm{B}} \rangle = (\alpha\tau/2)(\langle \mathrm{d}n^{\mathrm{B}}/\mathrm{d}y \rangle)$ in the bulk. Our simulations agree well with the diffusive result as shown in Fig. 5 for large enough $\bar{\alpha}$. For smaller values of $\bar{\alpha}$, L_{SO} becomes comparable to the overall length of the disorder region $L = 100a$. In this regime ballistic processes can no longer be neglected, causing slight deviations from the diffusive theory.

Having demonstrated that our numerical system is diffusive, we now focus on the spin accumulation in the normal region. In Fig. 6, we show the spin density $\langle s_x \rangle$ averaged over 50000 impurity configurations inside three distinct systems with $L_{\mathrm{SO}} = 25a$. We note that in agreement with Ref. [26], when the interface between R2DEG and 2DEG is infinite (top left panel), the spin accumulation in the 2DEG is much smaller than the bulk spin accumulation. Nevertheless, when the size of the contact is made smaller (top right panel), we observe that the spin accumulation inside the normal region increases, reaching a comparable value to the spin accumulation in the bulk when the size of the opening is comparable to L_{SO} (bottom Panel). In order to demonstrate this further, we evaluate $\langle s_x^{\mathrm{B}} \rangle$ by averaging the spin accumulation in the bulk over
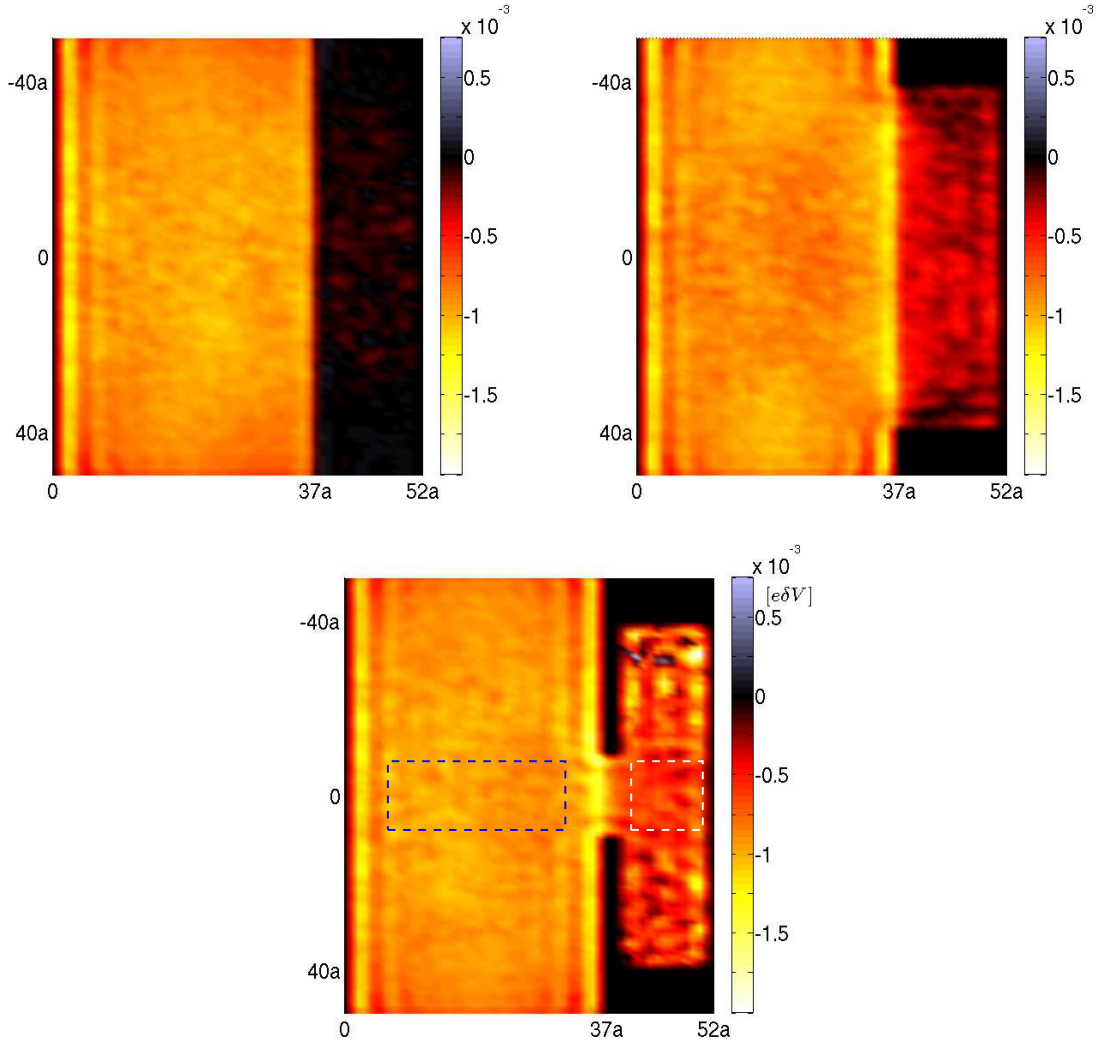
Figure 6. Top left panel: Spin accumulation $\langle s_x \rangle$ in a quantum wire of width $W = 52a$ with an abrupt drop of the SO coupling strength at $x = 37a$ from the constant $\bar{\alpha} = \pi/25$ ($L_{SO} = 25a$) for $x < 37a$ to zero on the other side. Top right panel: Spin accumulation $\langle s_x \rangle$ for a system as shown in Fig. 4 with $W = 37a$, $W_H = 80a$ and $L_{SO} = 25a$. Bottom panel: Same as top right panel with $W_H = 20a$. In all three panels, $\langle s_x \rangle$ is obtained by averaging over 50000 disorder configurations.

the blue square shown in Fig. 6 and $\langle s_x^P \rangle$ by averaging the spin accumulation in the normal conducting side-pocket over the white square shown in Fig. 6. In Fig. 7 we plot the ratio $\langle s_x^P \rangle / \langle s_x^B \rangle$ as a function of L_{SO}/W_H, for various values of system and contact sizes. We observe that starting from small L_{SO}/W_H, the spin accumulation increases with L_{SO}/W_H, approaching to $\approx 0.5 - 0.7$. This value is in between the estimates 0.5 and 1.0 based on diffusion equations using the boundary conditions Eq. (15) and Eq. (14) respectively. For small values of L_{SO}/W_H (Fig. 7, left panel), $\langle s_x^P \rangle / \langle s_x^B \rangle$ is of order (L_{SO}/W_H) in agreement with the analytical calculation above. We note, however, that in this limit the system we considered is close to the clean limit $L_{SO} \sim l$, where
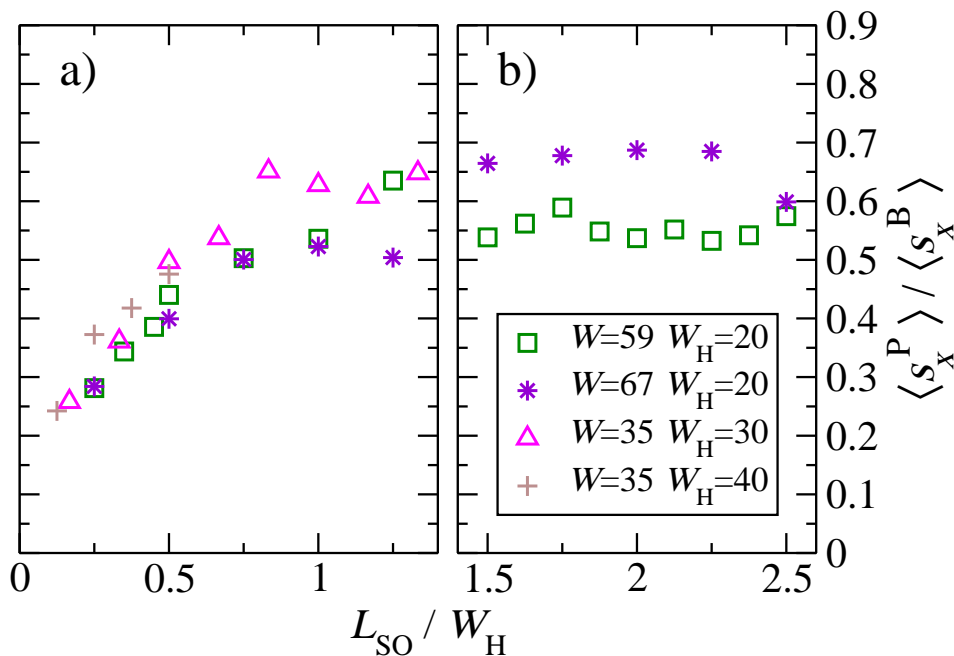
Figure 7. Left Panel: Average spin accumulation inside the normal region $\langle s_x^P \rangle$ relative to the accumulation in the bulk of the spin-orbit region $\langle s_x^B \rangle$ for various geometries averaged over 20000 disorder configurations as a function of L_{SO}/W_H. Right Panel: $\langle s_x^P \rangle / \langle s_x^B \rangle$ for two different geometries averaged over 60000 disorder configurations.

deviations from the diffusion equations might be expected. Currently, we are working on larger systems in order to explore small L_{SO}/W_H in the dirty limit [45].

6. Conclusions

In this work, we considered the problem of extracting current-induced spins generated in a region with spin-orbit coupling into a region with vanishing (or small) spin-orbit coupling, where the spin relaxation time is long. To this end we focused on the spin boundary conditions between a spin-orbit coupled region and a normal region. Although for an infinite interface the spins are confined to the spin-orbit region via the boundary spin Hall effect, we have shown by solving a model problem as well as doing numerical simulations that for a finite interface the spin accumulations generated in the spin-orbit region can be extracted to a normal region. The amount of extracted spin accumulation is equal to that of the spin-orbit region up to a geometrical factor of order unity.

7. Acknowledges

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