Chiral Dynamics of the S11(1535) and S11(1650) Resonances Revisited

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Abstract. We analyze s-wave pion-nucleon scattering in a unitarized chiral effective Lagrangian including all dimension two contact terms. We find that both the S11(1535) and the S11(1650) are dynamically generated, but the S31(1620) is not. We further discuss the structure of these dynamically generated resonances.

Keywords: Pion–baryon interactions Chiral Lagrangians Baryon resonances

INTRODUCTION

Pion-nucleon scattering has traditionally been the premier reaction to study the resonance excitations of the nucleon. In particular, in the S11 partial wave, one finds two close-by resonances at 1535 and 1650 MeV, which overlap within their widths of about 100 MeV. It was pointed out early in the framework of unitarized coupled-channel chiral perturbation theory [2] that this resonance might not be a three-quark (pre-existing) resonance but rather is generated by strong channel couplings, with a dominant KΣ−KΛ component in its wave function. This analysis was extended in Ref. [3], where within certain approximations the effects of 3-body ππN channels were also included. Further progress was made in Ref. [4], where the S11 phase shift was fitted from threshold to about \( \sqrt{s} \approx 2 \) GeV together with cross section data for \( \pi^- p \to \eta n \) and \( \pi^- p \to K^0 \Lambda \) in the respective threshold regions. This led to a satisfactory description of the S11 phase and a reasonable description of the inelasticity up to the \( \eta N \) threshold. More recently, it was pointed out in a state-of-the-art unitary meson-exchange model that there is indeed strong resonance interference between the two S11 resonances, as each of these resonances provides an energy-dependent background in the region of the other [5].

In view of these developments and our attempts to construct a unitary and gauge-invariant model for Goldstone-boson photoproduction off nucleons based on coupled-channel unitarized chiral perturbation theory [6], we consider in this letter the two s-waves S11 and S31 in pion-nucleon scattering. We work in the framework of a coupled-channel Bethe-Salpeter equation (BSE) including in the driving potential all local terms of second order in the chiral counting, thus going beyond the often used approximation of simply including the leading order Weinberg-Tomozawa interaction. Further, we do not perform the often used on-shell approximation. Our investigation is restricted to center-of-mass energies below 1.8 GeV, as required for the future meson photoproduction studies. As we will show, both resonances in the S11 partial wave are dynamically generated, even if the scattering data are fitted only up to \( \sqrt{s} = 1.56 \) GeV. Quite in contrast, the S31(1620) resonance is not generated by the coupled-channel dynamics. We also analyze the structure of the dynamically generated resonances as revealed through their coupling to the various meson-baryon channels.

FORMALISM

We consider the process of meson–baryon scattering at low energies. The s-wave interaction near the thresholds is dominated by the Weinberg-Tomozawa contact term, derived from the effective chiral Lagrangian

\[
L_{\phi}^{(1)} = \langle \bar{B}(i\gamma_\mu D^\mu - m_0)B \rangle + \frac{D/F}{2i} \langle \bar{B} \gamma_\mu \gamma_5 [u^\mu, B]_\pm \rangle, 
\] (1)

where \( D/F \) is the Goldstone-boson propagator in the chiral limit. The Dirac matrices \( \gamma_\mu \) and \( \gamma_5 \) are multiplied together in the usual way. The notation \( \langle \cdots \rangle \) denotes the expectation value in the vacuum state, and \( \gamma_5 \) is the fifth component of the gamma matrices.

The Bethe-Salpeter equation (BSE) is a powerful tool for including long-range forces in the scattering amplitude. It is derived from the Lagrangian \( L_{\phi}^{(1)} \) by requiring the invariance of the amplitude under Lorentz transformations. The BSE is a non-linear integral equation that describes the scattering of two particles in terms of their relative motion.

The solution of the BSE provides the scattering amplitude, which can be used to calculate various observables such as the cross section and the differential and total cross sections. The BSE is solved numerically using various techniques, such as the Dyson equation method or the Bethe-Salpeter equation method.

The BSE is particularly useful for studying the properties of resonances, such as their mass, width, and coupling constants. It is also useful for studying the dynamics of the scattering process, such as the exchange of mesons and baryons.

In conclusion, the BSE is a powerful tool for studying the properties of resonances and the dynamics of the scattering process. It provides a framework for understanding the interactions between mesons and baryons at low energies, and it is an essential tool for studying the strong interactions of the nucleon.
At first chiral order there are also the Born graphs, describing the $s$-channel and $u$-channel exchanges of an intermediate nucleon. The full inclusion of these graphs in the driving term of the Bethe-Salpeter equation leads to conceptional and practical difficulties, which have not yet been solved to the best of our knowledge, see [1] for further details. Therefore, we will approximate our interaction kernel by a sum of contact terms. To go beyond the simple Weinberg-Tomozawa potential, we shall include the full set of meson-baryon vertices from the second order chiral Lagrangian. These terms may lead to sizeable corrections to the leading-order results, see e.g. the calculation of NNLO corrections on meson-baryon scattering lengths within SU(3) ChPT [7].

We denote the overall four-momentum, in- and outgoing meson momenta by $p$, $q_1$ and $q_2$, respectively. For the meson-baryon scattering amplitude $T(q_2, q_1; p)$ and chiral potential $V(q_2, q_1; p)$ the integral equation to solve reads

\[ T(q_2, q_1; p) = V(q_2, q_1; p) + \int \frac{d^4l}{(2\pi)^4} V(q_2, l; p) S(l; -l) \Delta(l) T(l, q_1; p), \]

(2)

where $S$ and $\Delta$ represent the baryon (of mass $m$) and the meson (of mass $M$) propagator, respectively, and are given by $iS(p) = i/(p - m + i\epsilon)$ and $i\Delta(k) = i/(k^2 - M^2 + i\epsilon)$. Since we are dealing with coupled channels, $T$, $V$, $S$ and $\Delta$ are matrices in channel space. In view of a later application to photoproduction off protons, we restrict ourselves to meson-baryon channels with strangeness $S = 0$ and electric charge $Q = \pm 1$. This leaves us with the following channels: \{$p\pi^0$, $n\pi^+$, $p\eta$, $AK^+$, $\Sigma^0K^+$, $\Sigma^+K^0$\}.

The loop diagrams appearing in the BSE Eq. (2) are in general divergent and require renormalization. Without going into details here, we preserve the analytic structure of the loop integrals by utilizing dimensional regularization and just replacing the divergent part by a subtraction constant. The purely baryonic integrals are set to zero from the beginning, which is in effect, similar to the EOMS regularization scheme advocated in [8]. As it was argued in [6] it is not possible to express the terms necessary to absorb the divergencies in the BSE as counterterms derived from a local chiral potential itself, see [9]. In this spirit we apply the usual $\overline{\text{MS}}$ subtraction scheme, keeping in mind that the modified loop integrals are still scale-dependent. This regularization scale ($\mu$) is used as a fitting parameter, reflecting the influence of higher order terms not included in our potential.

\section*{RESULTS AND DISCUSSION}

Throughout the present work we use the following numerical values (in GeV) for the masses and the meson decay constants: $F_\pi = F_\eta/1.3 = 0.924$, $F_K = 0.113$, $M_{\pi^0} = 0.135$, $M_{\pi^+} = 0.1396$, $M_\eta = 0.5478$, $M_{K^+} = 0.4937$, $M_{K^0} = 0.4977$, $m_p = 0.9383$, $m_n = 0.9396$, $m_{\Lambda} = 1.1557$, $m_{\Sigma^0} = 1.1926$ and $m_{\Sigma^+} = 1.1894$.

There are 17 free parameters in the present experimental approach, given by the 14 LECs of the NLO Lagrangian, as well as three subtraction constants, corresponding to the logarithms of the undetermined regularization scales (in GeV), i.e. $\log(\mu_\pi)$, $\log(\mu_K)$ and $\log(\mu_\eta)$. We consider experimental data for $s$-wave $\pi N$ scattering up to $W = 1.56$ GeV, i.e. partial wave amplitudes $S_{11}$ and $S_{31}$ provided by the SAID-program at GWU, see [10]. Comparing an earlier analysis by the Karlsruhe group [11] to the current one, we assign for the energies below $W = 1.28$ GeV an absolute systematic error of 0.005 and for higher energies an error of 0.030 to the partial wave amplitudes.

For the best fit, found using the MINUIT library, with a $\chi^2_{\text{dof}} = 1.23$ we obtain the following parameter set (all $b_i$ in GeV$^{-1}$)

\[
\log(\mu_\pi, \eta, K) = \{-0.924, -2.18, +0.581\}, \quad b_{11,11} = \{-0.082, -0.118, -1.890, -0.215, -0.963, +0.218, -1.266, +0.609, -0.633, +1.920, -0.919\}, \quad b_{D,F} = \{-0.768, +0.641, -0.098\}.
\]

All parameters are of natural size and LECs agree with the estimates from the SU(3) to SU(2) matching relations provided in [7]. However we are only able to estimate the computational errors on the above parameters within the MIGRAD (MINUIT) minimization procedure, which appear to be negligible.

In Fig. 1 we present the result of our approach for the $S_{11}$ and $S_{31}$ partial waves. The low-energy region is reproduced for both isospin $3/2$ and $1/2$ reasonably well. For the two $s$-wave scattering lengths, we obtain $a_{1/2} = 145.8 \times 10^{-3}/M_{\pi^+}$ and $a_{3/2} = -91.6 \times 10^{-3}/M_{\pi^+}$, to be compared with the direct extraction of these scattering lengths from the GWU solution, $a_{1/2} = 174.7 \pm 2.2 \times 10^{-3}/M_{\pi^+}$ and $a_{3/2} = (-89.4 \pm 1.7) \times 10^{-3}/M_{\pi^+}$.\footnote{Note that no special weight was put on the threshold region in our fits.}

Within the fit region we reproduce the $S_{11}(1535)$. At the same time the $S_{31}(1620)$ resonance is not reproduced by our approach, which is in agreement with the current state of knowledge that the first $S_{31}$ resonance does not have a

\footnote{Note that no special weight was put on the threshold region in our fits.}
prominent dynamically generated component. Moreover, after fixing the $S_{11}$ partial wave in the energy region up to $\sqrt{s} = 1.560$ GeV every curve with minimized $\chi^2_{\text{dof}}$ possesses a second structure between the $K\Lambda$ and $K\Sigma$ threshold. Obviously this corresponds to the well-known $S_{11}(1650)$ resonance and is predicted here only by demanding a good description in the low-energy and the first resonance region. We conclude that the $S_{11}(1650)$ can also be described as a dynamically generated resonance, just like the $S_{11}(1535)$.

For the mass and width of these states we perform the analytic continuation of the $T^{11}_{NN}$ into the complex $s$-plane to the appropriate Riemann sheets, see [1] for details. We end up with $W_{1535} = (1.506 - 0.140 i)$ GeV and $W_{1650} = (1.682 - 0.042 i)$ GeV, which is similar to values of similar approaches [5] and [13], as well to those of various phenomenological models listed in [12]. To analyze the structure of these states we consider the on-shell scattering matrix in the vicinity of the two poles, where it takes the form $T^{i\ell}_{ij}(s) \approx \frac{g_i g_j}{s - s_{11}}$, with $g_i, g_j$ the complex coupling constant for the initial (final) transition of the meson-baryon system. For the $S_{11}(1535)$ we find that the largest component is the $K\Lambda$ one and that the coupling to $\eta N$ is significantly bigger than the $\pi N$ ones, in agreement with the empirical fact that the $S_{11}(1535)$ couples dominantly to $\eta N$. For the $S_{11}(1650)$ the $K\Sigma$ component is dominant and the $K\Lambda$ one is completely negligible. As for the lower-lying resonance, the coupling to $\eta N$ is bigger than the one to $\pi N$.

ACKNOWLEDGMENTS

This work was supported in part by DFG (TR 16). The speaker thanks the organisers for the opportunity to participate on the conference, and also the Bonn Cologne Graduate School for the partial financial support.

REFERENCES


FIGURE 1. Real and imaginary part of the $S_{11}$ (left) and $S_{11}$ (right) partial wave amplitude compared with the SAID-data (WI08-analysis). Full curves correspond to the best fit, the dashed ones to fits with slightly worse $\chi^2_{\text{dof}}$. The bold vertical line limits the region of the fit, where in the non-fit region single energy values are taken from the SAID-data.