Baryon Wave Functions from Lattice QCD

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Abstract. We describe an ongoing project by the QCDSF collaboration to calculate baryon wave functions at small inter-quark separation using lattice QCD. Preliminary results are presented for the wave functions at the origin and momentum fractions carried by the valence quarks in the nucleon

Understanding the nucleon structure in terms of quarks and gluons presents an ultimate goal of the theory of strong interactions. The full nucleon wave function is very complicated and remains elusive so that progress in the theory has mainly been in establishing the areas where QCD predictions only require limited nonperturbative input. In particular, hard exclusive reactions involving large momentum transfer from a point-like source to the final state baryon are mostly sensitive to the wave functions at small transverse separations between the constituents, usually called distribution amplitudes.

To be specific, consider the valence three-quark Fock state of the nucleon corresponding to zero angular momentum $L = 0$ [1, 2, 3]:

$$\langle P^\uparrow | \ell_z = 0 \rangle = \frac{2 e^{abc}}{\sqrt{6}} \int [dx] \int_{-1}^{1} \left\{ \psi_{1,2}^{\ell_z = 0}(x_i, \bar{k}_i) \psi_{1,2}^{\ell_z = 0}(x_i, \bar{k}_i) \right\}$$

Here $k^\pm = k \pm ik_y$ and $\psi_{1,2}^{\ell_z = 0}(x_i, \bar{k}_i)$ are the two contributing light-cone wave functions written in terms of momentum fractions $x_i$ and transverse momenta $\bar{k}_i$ of the quarks. The integration measure is defined as $\int [dx] = \int_0^1 dx_1 dx_2 dx_3 \delta(\sum_i x_i - 1)$. In hard processes the contribution of $\psi_1^{\ell_z = 0}$ is dominant whereas $\psi_2^{\ell_z = 0}$ gives rise to a power-suppressed correction, i.e. a correction of higher twist. The leading-twist distribution amplitude $\varphi_N(x_i)$ can be thought of as the light-cone wave function integrated over the transverse
derivatives, see [5] for details. The normalization is such that
\[ P \]
where, of course, \( P \) normalizes the matrix. The Wilson lines that ensure gauge invariance are not shown for brevity. The
\[ L \]
quarks moments correspond to the average momentum fractions carried by the three valence quarks \( \frac{1}{3} \). The
\[ \rho \]
operators couple also to
\[ N \]
operators. The operators on the l.h.s. of Eqs. (1)–(3) do not have definite parity. Thus the same
\[ \lambda \]
operators couple also to \( N^s \) (1535) and one can define the corresponding leading-twist distribution amplitude by
\[ \langle 0 | e^{ijk} (u_i (a_1 n) \gamma \mu u_j (a_2 n) ) \gamma_5 d_k (a_3 n) | N^s (P) \rangle \]
\[ = \frac{1}{2} f_{N^s} P \cdot n \gamma_5 u_{N^s} (P) \int [dx] e^{-i P \cdot n} \gamma_5 x | N^s (x) \rangle, \]
where, of course, \( P^2 = m_{N^s}^2 \). The normalization constants \( f_{N^s} \), \( \lambda_1^{N^s} \), \( \lambda_2^{N^s} \) are obtained by
similar substitutions, see Ref. [8].
The calculation of the moments of distribution amplitudes in lattice QCD requires the following steps:

- Find lattice (discretized) operators that transform according to irreducible representations of spinorial group $\tilde{H}(4)$
- Calculate non-perturbative renormalization constants for these operators
- Compute matrix elements of these operators on the lattice from suitable correlation functions
- Extrapolate $m_\pi \to m_\pi^{\text{phys}}$, lattice volume $V \to \infty$ and lattice spacing $a \to 0$

Irreducibly transforming $\tilde{H}(4)$ multiplets for three-quark operators have been constructed in Ref. [9]. Non-perturbative renormalization and one-loop scheme conversion factors $\text{RI-MOM} \to \overline{\text{MS}}$ have been calculated in Ref. [10]. A consistent perturbative renormalization scheme for the three-quarks operators in dimensional regularization has been found [11] and the calculation of two-loop conversion factors using this scheme is in progress.

The matrix elements of interest are calculated from correlation functions of the form $\langle \bar{\mathcal{N}}(x) \mathcal{N}(y) \rangle$, where $\mathcal{N}$ is a smeared nucleon interpolator and $\mathcal{O}$ is a local three-quark operator with up to two derivatives, and applying the parity “projection” operator $(1/2)(1 \pm m\gamma_5/E)$ [12]. In this way we get access to the normalization constants, the first and the second moments of the distribution amplitudes. Calculation of yet higher moments is considerably more difficult because one cannot avoid mixing with operators of lower dimension.

The correlation functions were evaluated using $N_f = 2$ dynamic Wilson (clover) fermions on the set of lattices specified in Table 1. The calculations on the largest $48^3 \times 64$ lattice with the smallest pion mass $m_\pi \sim 180$ MeV are still in progress. The results reported below are based on about one half of the available gauge configurations for this case.
Our preliminary results for the normalization constants are summarized in Fig. 1. The extrapolation of the results for the nucleon to the physical pion mass and infinite volume as well as the analysis of the related systematic errors are in progress. This analysis will be done using one-loop chiral perturbation theory. The necessary expressions have been worked out in Ref. [13]. Whereas the pion mass dependence of nucleon couplings is generally in agreement with expectations, we observe a large difference (up to a factor of three) in $N^*$ couplings calculated with heavy and light pions: All three couplings drop significantly in the transition region where the decay $N^* \to N\pi$ opens up. This effect can be due to the change in the structure of the wave function, but also to contamination of our $N^*$ (1535) results by the contribution of the $\pi N$ scattering state, or some other lattice artefact. We will try to clarify this issue.
Moments of the distribution amplitude can be related by simple algebra to the expansion over multiplicatively renormalizable contributions \[6\]

\[
\phi_N(x;i;\mu) = 120 x_1 x_2 x_3 \left\{ 1 + c_{10}(x_1 - 2x_2 + x_3)L^{8\pi^0} + c_{11}(x_1 - x_3)L^{14\pi^0} + c_{20} \left[ 1 + 7(x_2 - 2x_1 x_3 - 2x_2^2) \right] L^{14\pi^0} + c_{21}(1 - 4x_2)(x_1 - x_3)L^{32\pi^0} + c_{22} \left[ 3 - 9x_2 + 8x_2^2 - 12x_1 x_3 \right] L^{32\pi^0} + \cdots \right\}
\]

where \(L = \alpha_s(\mu)/\alpha_s(\mu_0)\). We refer to the constants \(c_{ik}\) in this expansion as shape parameters. They are obtained from the constrained fit to \(\phi^{nml}\) with \(m + n + l = 1,2\) such that the momentum conservation conditions \(\phi^{nml} = \phi^{(m+1)n} + \phi^{m(n+1)} + \phi^{m(l+1)}\) are imposed. Our results for \(c_{ik}\) for three lattices with the same spacing \(a = 0.0716\) fm and pion mass \(m_\pi \sim 300\) MeV are plotted as a function of the lattice size \(1/L\) in Fig. 2. We find clear signals for the first order coefficients \(c_{10}\) and \(c_{11}\) whereas the second order coefficients \(c_{2k}\), \(k = 0,1,2\) are in general consistent with zero to our accuracy, with the exception of \(c_{21}\) for \(N^*(1535)\). Both parameters, \(c_{10}\) and \(c_{11}\), for \(N^*(1535)\) are much larger than for the nucleon, which means that the momentum sharing between the quarks in less symmetric. This is illustrated in Fig. 3 where the leading-twist distribution amplitudes of the nucleon (left) and \(N^*(1535)\) (right) are shown in barycentric coordinates \(x_1 + x_2 + x_3 = 1\).

To summarize, we have calculated the normalization constants \(f_N\), \(\lambda_1\), \(\lambda_2\) and the first two moments of the quark distribution amplitude for the nucleon and \(N^*(1535)\) resonance on a set of lattices using \(N_f = 2\) dynamic Wilson clover fermions. In particular the results at three lattice volumes for \(m_\pi \sim 300\) MeV provide a control over the finite
FIGURE 3. Leading-twist distribution amplitudes of the nucleon (left) and $N^\pi(1535)$ (right) in barycentric coordinates $x_1 + x_2 + x_3 = 1$.

volume dependence. Chiral perturbation theory predictions for all quantities are being worked out and the final analysis is in progress. Extension of this study to $N_f = 2 + 1$ is planned and will be the next big step.

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REFERENCES


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