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Baryon Wave Functions from Lattice QCD

V. M. Braun*, M. Göckeler*, R. Horsley†, Y. Nakamura**, D. Pleiter*, P. E. L. Rakow‡, A. Schäfer*, R. Schiel*, G. Schierholz§, H. Stüben¶, P. Wein* and J. M. Zanotti
[QCDSF Collaboration]†

*Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany
†School of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK
**RIKEN Advanced Institute for Computational Science, Kobe, Hyogo 650-0047, Japan
‡Theoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
§Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany
¶Konrad-Zuse-Zentrum für Informationstechnik Berlin, 14195 Berlin, Germany

Abstract. We describe an ongoing project by the QCDSF collaboration to calculate baryon wave functions at small inter-quark separation using lattice QCD. Preliminary results are presented for the wave functions at the origin and momentum fractions carried by the valence quarks in the nucleon and its negative parity partner, $N^*(1535)$. The calculations are done using two flavors of dynamical (clover) fermions on lattices of different volume and pion masses down to $m_\pi \simeq 180$ MeV.

Keywords: Lattice QCD, Nucleon Wave Function, Nucleon Resonances
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Understanding the nucleon structure in terms of quarks and gluons presents an ultimate goal of the theory of strong interactions. The full nucleon wave function is very complicated and remains elusive so that progress in the theory has mainly been in establishing the areas where QCD predictions only require limited nonperturbative input. In particular, hard exclusive reactions involving large momentum transfer from a point-like source to the final state baryon are mostly sensitive to the wave functions at small transverse separations between the constituents, usually called distribution amplitudes.

To be specific, consider the valence three-quark Fock state of the nucleon corresponding to zero angular momentum $L=0$ [1, 2, 3]:

$$\begin{aligned} |P\uparrow\rangle^{\ell_z=0} &= 2\frac{\varepsilon^{abc}}{\sqrt{6}}\int\frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\left[\psi_1^{L=0}(x_i,\vec{k}_i)+(k_1^+k_2^- - k_1^-k_2^+)\,\psi_2^{L=0}(x_i,\vec{k}_i)\right] \\ &\quad\times\left\{\left|u_a^\uparrow(x_1,\vec{k}_1)u_b^\downarrow(x_2,\vec{k}_2)d_c^\uparrow(x_3,\vec{k}_3)\right\rangle - \left|u_a^\uparrow(x_1,\vec{k}_1)d_b^\downarrow(x_2,\vec{k}_2)u_c^\uparrow(x_3,\vec{k}_3)\right\rangle\right\} \end{aligned}$$

Here $k^\pm = k_x \pm ik_y$ and $\psi_{1,2}^{L=0}(x_i,\vec{k}_i)$ are the two contributing light-cone wave functions written in terms of momentum fractions x_i and transverse momenta $\vec{k}_i$ of the quarks. The integration measure is defined as $\int[dx] = \int_0^1 dx_1dx_2dx_3\delta(\sum x_i - 1)$. In hard processes the contribution of $\psi_1^{L=0}$ is dominant whereas $\psi_2^{L=0}$ gives rise to a power-suppressed correction, i.e. a correction of higher twist. The leading-twist distribution amplitude $\varphi_N(x_i)$ can be thought of as the light-cone wave function integrated over the transverse

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momenta

$$\varphi_N(x_1,x_2,x_3;\mu) \;=\; \int_{|k|<\mu}[d^2\vec{k}]\;\psi_1^{L=0}(x_1,x_2,x_3;\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

so it is a function of longitudinal momentum fractions only.

Alternatively, $\varphi_N(x_i)$ can be defined from a matrix element of a nonlocal light-ray operator that involves quark fields of given helicity $q^{\uparrow(\downarrow)} = (1/2)(1\pm\gamma_5)q$ [4]:

$$\begin{aligned}&\langle 0|\varepsilon^{ijk}\Big(u_i^{\uparrow}(a_1 n)C\not{n}u_j^{\downarrow}(a_2 n)\Big)\,\not{n}d_k^{\uparrow}(a_3 n)|N(P)\rangle\\ &\qquad\qquad = -\frac{1}{2}f_N\,P\cdot n\,\not{n}u_N^{\uparrow}(P)\int[dx]\,e^{-iP\cdot n\sum x_i a_i}\,\varphi_N(x_i).\end{aligned} \tag{1}$$

Here P_μ, $P^2 = m_N^2$, is the proton momentum, $u_N(P)$ the usual Dirac spinor in relativistic normalization, n_μ an arbitrary light-like vector $n^2 = 0$ and C the charge-conjugation matrix. The Wilson lines that ensure gauge invariance are not shown for brevity. The normalization constant f_N is defined by

$$\langle 0|\varepsilon^{ijk}\big(u_i C\not{n}u_j\big)(0)\gamma_5\not{n}d_k(0)|N(P)\rangle = f_N P\cdot n\not{n}\,u_N(P)\,. \tag{2}$$

On the lattice one can calculate moments of the DA

$$\varphi^{lmn} = \int[dx]\,x_1^l x_2^m x_3^n\,\varphi(x_i)\,,$$

which are related to matrix elements of local three-quark operators with covariant derivatives, see [5] for details. The normalization is such that $\varphi^{000} = 1$ and the first moments correspond to the average momentum fractions carried by the three valence quarks $\langle x_1\rangle = \varphi^{100}$, etc.

The subleading twist-4 nucleon distribution amplitudes [4, 6] correspond to the contributions of the light-cone wave functions with $L = 1$ [7] or, equivalently, "minus" quark field components in the light-cone quantization. They involve two new normalization constants λ_1^N, λ_2^N:

$$\begin{aligned}&\langle 0|\varepsilon^{ijk}\big(u_i C\gamma_\mu u_j\big)(0)\gamma_5\gamma^\mu d_k(0)|N(P)\rangle = \lambda_1^N m_N\,u_N(P),\\ &\langle 0|\varepsilon^{ijk}\big(u_i C\sigma_{\mu\nu}u_j\big)(0)\gamma_5\sigma^{\mu\nu}d_k(0)|N(P)\rangle = \lambda_2^N m_N\,u_N(P).\end{aligned} \tag{3}$$

The operators on the l.h.s. of Eqs. (1)–(3) do not have definite parity. Thus the same operators couple also to $N^*(1535)$ and one can define the corresponding leading-twist distribution amplitude by

$$\begin{aligned}&\langle 0|\varepsilon^{ijk}\Big(u_i^{\uparrow}(a_1 n)C\not{n}u_j^{\downarrow}(a_2 n)\Big)\,\not{n}d_k^{\uparrow}(a_3 n)|N^*(P)\rangle\\ &\qquad\qquad = \frac{1}{2}f_{N^*}\,P\cdot n\,\not{n}u_{N^*}^{\uparrow}(P)\int[dx]\,e^{-iP\cdot n\sum x_i a_i}\,\varphi_{N^*}(x_i)\,,\end{aligned}$$

where, of course, $P^2 = m_{N^*}^2$. The normalization constants f_{N^*}, $\lambda_1^{N^*}$, $\lambda_2^{N^*}$ are obtained by similar substitutions, see Ref. [8].

TABLE 1. The set of lattices used in our calculations

κ	m_π/ MeV	Size	# Configurations
$\beta = 5.29,\ a = 0.0716$ fm			
0.13590	660	$24^3 \times 48$	901
0.13620	428	$24^3 \times 48$	850
0.13632	306	$24^3 \times 48$	540
0.13632	295	$32^3 \times 64$	950
0.13632	288	$40^3 \times 64$	858
0.13640	182	$48^3 \times 64$	798 + more soon
$\beta = 5.40,\ a = 0.0604$ fm			
0.13610	722	$24^3 \times 48$	687
0.13625	622	$24^3 \times 48$	1180
0.13640	503	$24^3 \times 48$	1037

The calculation of the moments of distribution amplitudes in lattice QCD requires the following steps:

- Find lattice (discretized) operators that transform according to irreducible representations of spinorial group $\overline{H(4)}$
- Calculate non-perturbative renormalization constants for these operators
- Compute matrix elements of these operators on the lattice from suitable correlation functions
- Extrapolate $m_\pi \to m_\pi^{\text{phys}}$, lattice volume $V \to \infty$ and lattice spacing $a \to 0$

Irreducibly transforming $\overline{H(4)}$ multiplets for three-quark operators have been constructed in Ref. [9]. Non-perturbative renormalization and one-loop scheme conversion factors RI-MOM$\to$ $\overline{\text{MS}}$ have been calculated in Ref. [10]. A consistent perturbative renormalization scheme for the three-quarks operators in dimensional regularization has been found [11] and the calculation of two-loop conversion factors using this scheme is in progress.

The matrix elements of interest are calculated from correlation functions of the form $\langle \mathscr{O}_{\alpha\beta\gamma}(x)\overline{\mathscr{N}}(y)_\tau\rangle$, where $\mathscr{N}$ is a smeared nucleon interpolator and $\mathscr{O}$ is a local three-quark operator with up to two derivatives, and applying the parity "projection" operator $(1/2)(1 \pm m\gamma_4/E)$ [12]. In this way we get access to the normalization constants, the first and the second moments of the distribution amplitudes. Calculation of yet higher moments is considerably more difficult because one cannot avoid mixing with operators of lower dimension.

The correlation functions were evaluated using $N_f = 2$ dynamic Wilson (clover) fermions on the set of lattices specified in Table 1. The calculations on the largest $48^3 \times 64$ lattice with the smallest pion mass $m_\pi \sim 180$ MeV are still in progress. The results reported below are based on about one half of the available gauge configurations for this case.

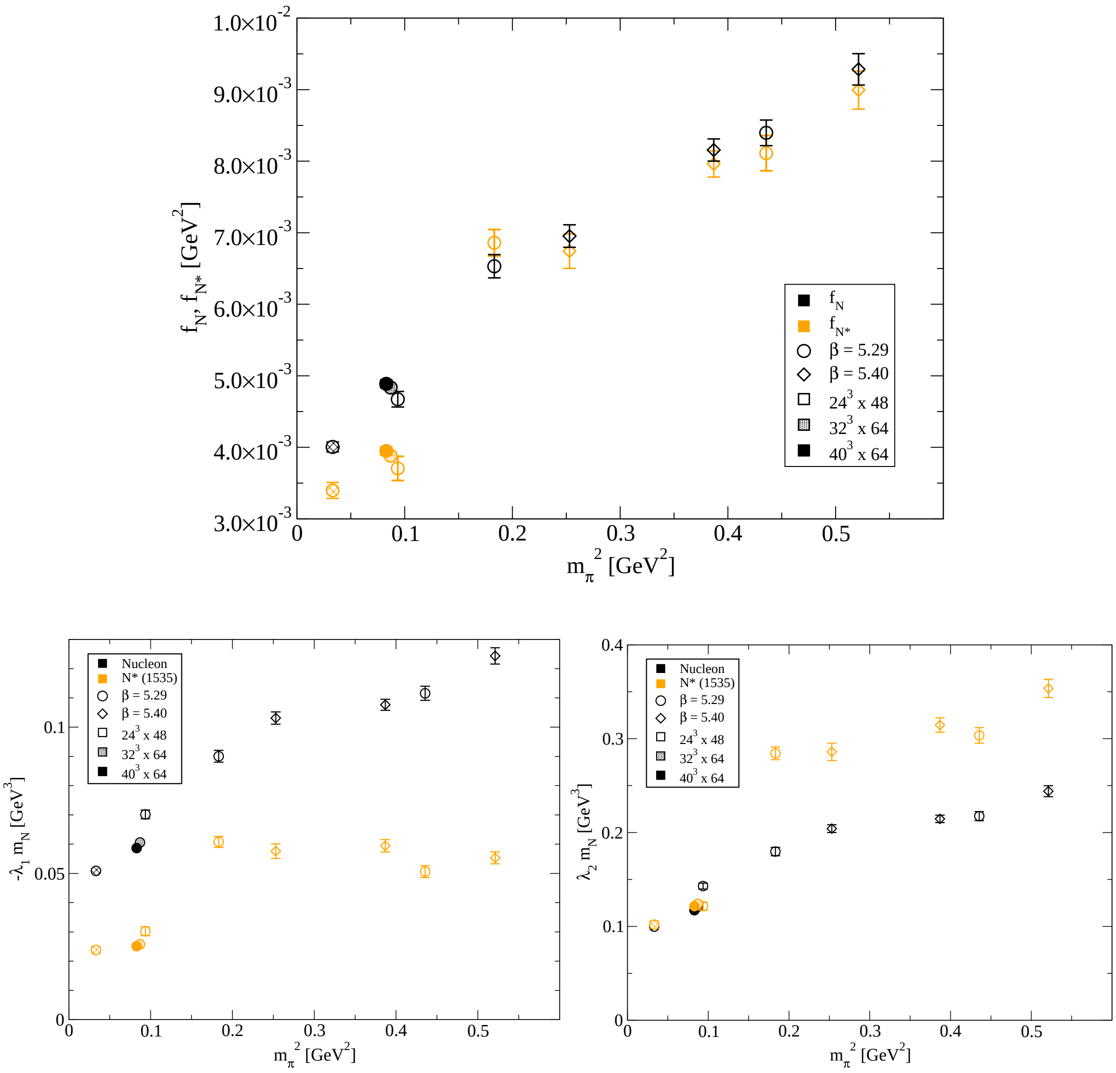

FIGURE 1. Decay constants for the nucleon and $N^*(1535)$ as a function of the pion mass. The errors shown are statistical only.

Our preliminary results for the normalization constants are summarized in Fig. 1. The extrapolation of the results for the nucleon to the physical pion mass and infinite volume as well as the analysis of the related systematic errors are in progress. This analysis will be done using one-loop chiral perturbation theory. The necessary expressions have been worked out in Ref. [13]. Whereas the pion mass dependence of nucleon couplings is generally in agreement with expectations, we observe a large difference (up to a factor of three) in $N^*(1535)$ couplings calculated with heavy and light pions: All three couplings drop significantly in the transition region where the decay $N^* \to N\pi$ opens up. This effect can be due to the change in the structure of the wave function, but also to contamination of our $N^*(1535)$ results by the contribution of the πN scattering state, or some other lattice artefact. We will try to clarify this issue.

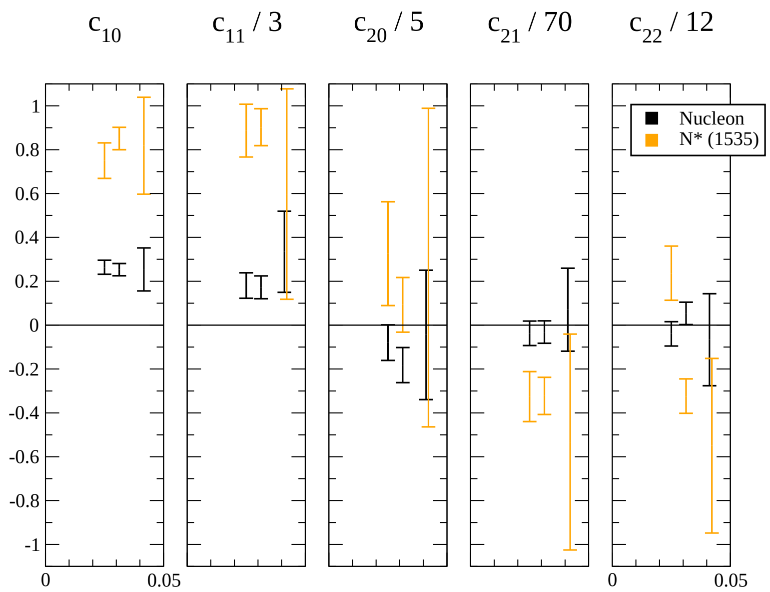

FIGURE 2. Shape parameters c_{ik} for the nucleon and $N^*(1535)$ distribution amplitudes as a function of the lattice size $1/L$ for $a = 0.0716$ fm and the pion mass $m_\pi \sim 300$ MeV. The errors are statistical only.

Moments of the distribution amplitude can be related by simple algebra to the expansion over multiplicatively renormalizable contributions [6]

$$\begin{aligned}\varphi_N(x_i;\mu) &= 120x_1x_2x_3\Big\{1+c_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}} \\ &\quad + c_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} + c_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}} \\ &\quad + c_{21}\,(1-4x_2)\,(x_1-x_3)\,L^{\frac{40}{9\beta_0}} + c_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \ldots\Big\}\end{aligned}$$

where $L = \alpha_s(\mu)/\alpha_s(\mu_0)$. We refer to the constants c_{ik} in this expansion as shape parameters. They are obtained from the constrained fit to φ^{mnl} with $m+n+l = 1,2$ such that the momentum conservation conditions $\varphi^{mnl} = \varphi^{(m+1)nl} + \varphi^{m(n+1)l} + \varphi^{mn(l+1)}$ are imposed. Our results for c_{ik} for three lattices with the same spacing $a = 0.0716$ fm and pion mass $m_\pi \sim 300$ MeV are plotted as a function of the lattice size $1/L$ in Fig. 2. We find clear signals for the first order coefficients c_{10} and c_{11} whereas the second order coefficients $c_{2k}, k = 0,1,2$ are in general consistent with zero to our accuracy, with the exception of c_{21} for $N^*(1535)$. Both parameters, c_{10} and c_{11}, for $N^*(1535)$ are much larger than for the nucleon, which means that the momentum sharing between the quarks in less symmetric. This is illustrated in Fig. 3 where the leading-twist distribution amplitudes of the nucleon (left) and $N^*(1535)$ (right) are shown in barycentric coordinates $x_1 + x_2 + x_3 = 1$.

To summarize, we have calculated the normalization constants f_N, λ_1, λ_2 and the first two moments of the quark distribution amplitude for the nucleon and $N^*(1535)$ resonance on a set of lattices using $N_f = 2$ dynamic Wilson clover fermions. In particular the results at three lattice volumes for $m_\pi \simeq 300$ MeV provide a control over the finite

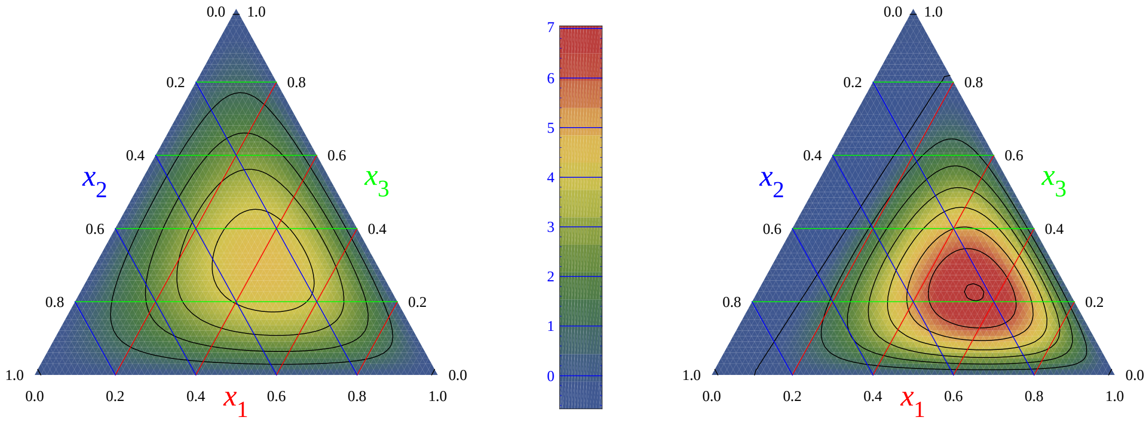

FIGURE 3. Leading-twist distribution amplitudes of the nucleon (left) and $N^*(1535)$ (right) in barycentric coordinates $x_1 + x_2 + x_3 = 1$.

volume dependence. Chiral perturbation theory predictions for all quantities are being worked out and the final analysis is in progress. Extension of this study to $N_f = 2+1$ is planned and will be the next big step.

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